

## More on breakup probability.

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### Abstract

The result on the ponium breakup probability is compared for two different approaches to the  $A_{2\pi}$  - *atom* cross sections. Some direct results on the breakup probability are obtained for states with  $n \leq 10$ .

## 1 Introduction

The extrapolation method of  $A_{2\pi}$  lifetime measurement [1] uses the ponium breakup probability ( $P_{br}$ ) in a target of a definite width to obtain the measurement of lifetime. Hence, the precise knowledge of the dependence of  $P_{br}$  on lifetime is a master piece to obtain this result which will test the Chiral Perturbation Theory prediction.

In [2] the comparison of two different approaches to  $P_{br}$  calculation [2] [3] was made and shown to be in complete agreement. In both cases the matrix elements of the large differential equations system which must be solved in order to obtain the final result were calculated with a Fermi-Thomas-Moliere parametrization of the Coulomb potential of the target. Now, new and independent results have been obtained making use of a Dirac-Hartree-Fock description of this potential [4]. Furthermore, these results included the ionization cross-sections, which never before had been calculated for an atom in which both particles have the same mass (as ponium).

With all these new ingredients we have faced two tasks. First, a systematic comparison of the different cross-section and  $P_{br}$  results was made and second, the direct calculation of breakup probability as a function of the principal quantum number was performed. Unfortunately, it has been shown that the usage of ionization cross section does not leads to an independent result for the total breakup probability.

## 2 Breakup probability. A large differential equations system.

### 2.1 General remarks about the procedure to obtain the breakup probability of a population of $A_{2\pi}$ atoms in a target.

As it has been explained in [1], the probability for the different  $A_{2\pi}$  state populations in a material is given by the next system of differential equations:

$$\frac{dp_i(s)}{ds} = \sum_{j=1}^{\infty} a_{ij} p_j(s), \quad (1)$$

where the  $p_i$  are the different states population, and  $s$  is the distance in which these probabilities are evaluated. The  $a_{ij}$  are the different transition probabilities per unit of length that are directly related to the transition cross sections by:

$$a_{ij} = \frac{\sigma_i^j \rho N_0}{A}, \quad (2)$$

if  $i \neq j$  and by:

$$a_{ii} = -\frac{\sigma_i^{tot} \rho N_0}{A} - \begin{cases} 2m_\pi / p_A c \tau_n & \text{for nS states,} \\ 0 & \text{otherwise;} \end{cases} \quad (3)$$

if we talk about a diagonal element of the  $a_{ij}$  matrix. In these equations  $\rho$  is the density of the target material,  $A$  its atomic weight and  $N_0$  the Avogadro number. In equation (3)  $p_A$  is the  $A_{2\pi}$  momentum,  $\tau_n$  is the lifetime for the corresponding  $nS$  state,  $c$  is the speed of light and  $m_\pi$  the pion mass.

Hence, to know the breakup probability of a  $A_{2\pi}$  atom in a target of definite width, we need to solve this equations system extracting the information about the probability for the atoms to be annihilated, which will be denoted by  $P_{anh}$  throughout the text, the probability to remain in a discrete state with the principal quantum number beyond an upper limit  $n_{max}$  after the target, denoted by  $P_{dsc}$ , the probability to remain in a bound state with the principal quantum number  $n > n_{max}$ ,  $P_{tail}$  and the probability for the atoms to be broken up, denoted by  $P_{br}$ .

### 2.2 About cross sections.

As shown in [1] and [3] the total interaction cross sections in the Born approximation for  $A_{2\pi}$  - atom interaction are given by the expression:

$$\sigma_i^{tot} = \frac{1}{\pi\beta^2} \int_0^\infty |\tilde{U}(q)|^2 (1 - F_i^i(q)) q dq, \quad (4)$$

where  $\tilde{U}(q)$  is the Fourier transform of some convenient parameterization of the atomic potential and  $F_i^i(q)$  is the atomic form factor of the considered bound state<sup>1</sup>.

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<sup>1</sup>In this case it can be interpreted as the Fourier transformed of the charge density.

$n$	$l$	$m$	$\sigma_{nlm}^{(n+1)(l+1)(m+1)}$ [barn]	$\sigma_{nlm}^{(n+1)(l+1)m}$ [barn]	$\sigma_{nlm}^{(n+1)(l+1)m} / \sigma_{nlm}^{(n+1)(l+1)(m+1)}$
1	0	0	0.1354E+04	0.7456E+01	0.5509E-02
2	0	0	0.4544E+04	0.2085E+01	0.4589E-03
3	0	0	0.9262E+04	0.9387E+00	0.1013E-03
4	0	0	0.1478E+05	0.5248E+00	0.3550E-04
5	0	0	0.2041E+05	0.3302E+00	0.1618E-04
6	0	0	0.2546E+05	0.2244E+00	0.8814E-05
7	0	0	0.2963E+05	0.1621E+00	0.5471E-05
8	0	0	0.3332E+05	0.1216E+00	0.3650E-05
9	0	0	0.3713E+05	0.9332E-01	0.2514E-05

Table 1: Comparison of Z-parity conserved cross sections with Z-parity violated cross sections. All results from the [4] calculations.

Meanwhile, the transition cross sections are given by:

$$\sigma_i^f = \frac{1 - (-1)^{l-l'}}{2\pi\beta^2} \int_0^\infty |\tilde{U}(q)|^2 |F_i^f(q/2)|^2 q dq. \quad (5)$$

The results for discrete-discrete form factors are well known [3] and [4]. However, a difference in the calculation of the cross sections appears if different elections of the atomic potential parametrization are made. Hence, meanwhile in [3] a Thomas-Fermi-Molière parametrization was chosen, in [4] they used a Dirac-Hartree-Fock-Slater parametrization. However, as we shall see afterwards, the discrepancies between the difference in the atomic potential election are overcome because in [4] the incoherent cross-section ( $A_{2\pi}$ -Target interaction with electronic excitation) was also considered and in [3] neglected.

The cross-section calculations in [4] also included transitions which do not conserve  $Z$  - parity. These transitions were considered strictly forbidden in [3]. However in the calculation of [4] they are highly suppressed, as can be seen in Table 1 and the probability of the atoms to populate discrete states with odd  $Z$  - parity will be negligible<sup>2</sup>.

Finally, the results from [4] also included the discrete-continuum atomic form factors. And, as it will be shown, this allowed us to make direct calculations for some breakup probabilities.

## 2.3 A direct comparison of cross sections.

### 2.3.1 Total cross sections.

In Table 2 and Figure 1 a direct comparison between the cross-section results of [3] and [4] is performed. The results from [4] are the sum of incoherent and coherent cross-sections. For the Figure 1 we summed all the interaction cross sections over  $l$  and  $m$ , the angular quantum numbers, for either the [3] and [4] results. After that, the ratio between this sums for every  $n$  was found. We can observe that this ratio is almost 1 for low <sup>3</sup>  $n$  and

<sup>2</sup>We have made an estimation and  $P_{dsc} + P_{br} + P_{tail} + P_{anh} < 0.0007$  for odd Z-parity states.

<sup>3</sup>Anyway, [4] results are slightly larger when  $n = 1$ .

$n$	$l$	$m$	Results from [3]	Results from [4]	Results from [3]-Results from [4] [barn].	100 $\frac{\text{Results from [3]}-\text{Results from [4]}}{\text{Results from [3]}}$
1	0	0	0.5599E+04	0.5669E+04	-0.7074E+02	-1.26
2	0	0	0.5265E+05	0.5305E+05	-0.3962E+03	-0.75
3	0	0	0.1825E+06	0.1819E+06	0.5320E+03	0.29
4	0	0	0.4114E+06	0.4044E+06	0.6943E+04	1.69
5	0	0	0.7309E+06	0.7066E+06	0.2425E+05	3.32
6	0	0	0.1117E+07	0.1060E+07	0.5732E+05	5.13
7	0	0	0.1543E+07	0.1431E+07	0.1120E+06	7.26
8	0	0	0.1983E+07	0.1791E+07	0.1919E+06	9.68
9	0	0	0.2418E+07	0.2124E+07	0.2939E+06	12.16
10	0	0	0.2835E+07	0.2424E+07	0.4101E+06	14.47

Table 2: Comparison of [3] and [4] total interaction cross section results.

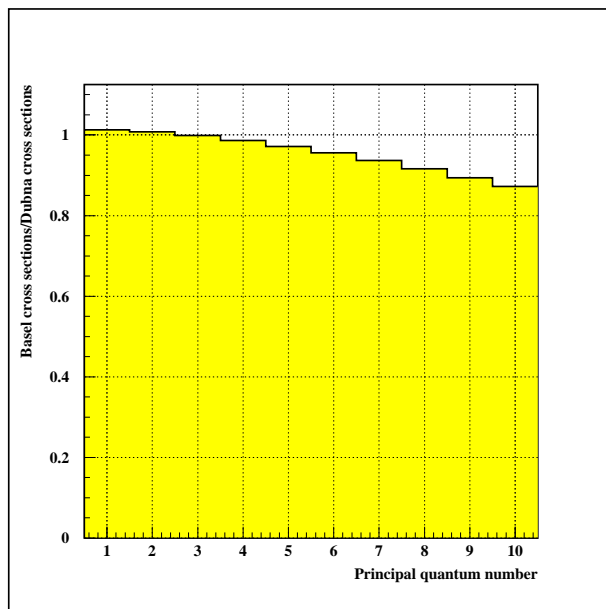


Figure 1: Rate between [4] cross sections and [3] cross sections. Summation over  $l$  and  $m$  was performed.

that systematically [3] results become larger as  $n$  increases. Nevertheless, the difference is under 5% up to  $n = 6$  and under 13% up to  $n = 10$ <sup>4</sup>.

### 2.3.2 Excitation cross sections.

A comparison of excitation (and de-excitation) cross section was also made. As the number of combinations in this case is too large, 6050 possibilities if Z-parity is conserved or 17317 if not, we only show those transitions with largest cross sections. This is verified for the  $n l m \rightarrow (n + 1) (l + 1) (m + 1)$  transition. In table 3 these results for the  $s$  states up to

<sup>4</sup>This could be due to the fact that the difference between the Hartree-Fock and the Thomas-Fermi-Molière descriptions of the atomic potential is higher at low  $q$ . Meanwhile, the atomic form factors differ from zero in a range of  $q \sim 1/(na_\pi)$  where  $a_\pi$  is the Bohr radius of pionium ( $a_\pi = 387fm$ ).

$n_i$	$l_i$	$m_i$	$n_f$	$l_f$	$m_f$	Results from [3]-Results from [4] barns.	100 $\frac{\text{Results from [3]}-\text{Results from [4]}}{\text{Results from [3]}}$
1	0	0	2	1	1	-.1095E+02	-0.82
2	0	0	3	1	1	0.7884E-01	0.00
3	0	0	4	1	1	0.1656E+03	1.76
4	0	0	5	1	1	0.6184E+03	4.01
5	0	0	6	1	1	0.1370E+04	6.29
6	0	0	7	1	1	0.2592E+04	9.24
7	0	0	8	1	1	0.4410E+04	12.96
8	0	0	9	1	1	0.6386E+04	16.08
9	0	0	10	1	1	0.7934E+04	17.61

Table 3: Comparison of some [3] and [4] excitation cross section results.

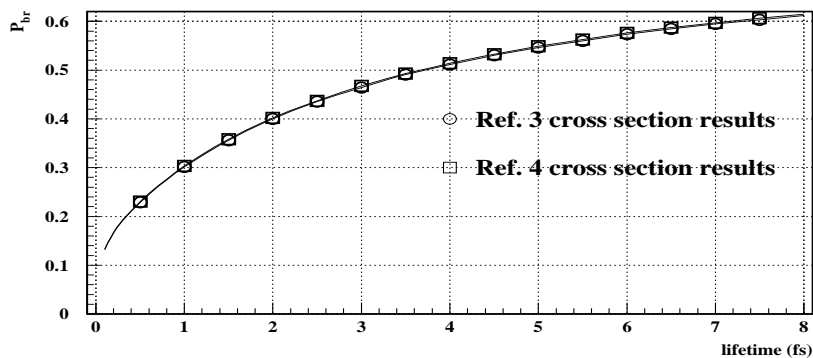


Figure 2: Breakup probability as a function of  $A_{2\pi}$  lifetime.

$n = 9$  can be seen.

The same conclusion as for total cross sections can be applied. Again small differences are observed and we see that they increase as  $n$  does. The results from [3] are systematically higher upon  $n = 3$ .

## 2.4 Comparison of breakup probability.

As a final comparison breakup probability has been calculated with both [4] and [3] cross sections as input with a Monte Carlo method described in [2]. The final result is shown in Figure 2. The largest absolute difference for the points of this plot is 0.6%. This small discrepancy can be understood because the main contribution to  $P_{dsc}$  and to  $P_{anh}$  is mainly due to populations and annihilations from low principal quantum number states. And for these states both results on cross-sections are almost equal.

## 3 Some studies about ionization cross sections.

One remarkable advantage of [4] results on  $A_{2\pi}$  cross sections is given by the fact that they have been able to calculate ionization cross sections for several bound states of the

atom (up to  $n = 10$ ).

This allowed us to obtain a new result concerning the probability of ionization of an atom from a definite bound state.

We have previously commented that the calculation of breakup probability can only involve a limited number of discrete states, we have set the limit in coincidence with the core number  $n_{max} = 7$ . So, in order to estimate any probability, we will distinguish between states with  $n \leq n_{max}$  and states with  $n > n_{max}$ . Hence we have already split the probability of those atoms to stay in a bound state after the target into two,  $P_{dsc}$  which gives into account of those atoms in states with  $n \leq n_{max}$  and  $P_{tail}$ , which refers to those states with  $n > n_{max}$ . The same procedure can be applied to the ionization probability. So, if we define  $P_{br}$  as the total breakup probability then:

$$P_{br} = P_{br}(n \leq n_{max}) + P_{br}(n > n_{max}). \quad (6)$$

This definition will be useful because with the new calculated ionization cross sections we are able to make a direct calculation of  $P_{br}(n \leq n_{max})$ . Figure 3 c) shows this result as a function of the principal quantum number  $n$ .

Since all these probabilities are related by

$$P_{br} = P_{br}(n \leq n_{max}) + P_{br}(n > n_{max}) = 1 - P_{dsc} - P_{tail} - P_{anh}, \quad (7)$$

the only unknowns can be grouped in:

$$P_{br}(n > n_{max}) + P_{tail} = 1 - P_{dsc} - P_{anh} - P_{br}(n \leq n_{max}). \quad (8)$$

Which, if  $P_{br}(n > n_{max}) + P_{tail}$ , was small, would allow us to enclose  $P_{br}$  in the inequality:

$$P_{br}(n \leq n_{max}) < P_{br} < 1 - P_{dsc} - P_{anh} \quad (9)$$

where, if both the upper and lower bound of inequality would be close one to the other we could obtain two independent estimations on  $P_{br}$ <sup>5</sup>. Unfortunately this is not true<sup>6</sup> and an additional hypothesis is needed in order to make a precise calculation of  $P_{br}$ . This hypothesis has been proposed in [1] and the main idea consists of making a fit of the discrete states probability to obtain  $P_{tail}$ <sup>7</sup>. With this we obtain results for the tail probabilities around 0.001. This also implies that there is a significant probability for the atoms to be broken from a state with  $n > n_{max}$ <sup>8</sup>.

<sup>5</sup>Actually  $P_{br} \simeq 1 - P_{dsc} - P_{anh}$ , meanwhile  $P_{br}(n \leq n_{max})$  is only a lower bound.

<sup>6</sup>For example, the results for a 95  $\mu m$  Nickel target are shown in Figure 3 were  $P_{br}(n > n_{max}) + P_{tail} \simeq 0.16$ .

<sup>7</sup>The proposed behavior of the discrete states tail would be of the kind:

$$P_{tail}(n) = \frac{a}{n^3} + \frac{b}{n^5}. \quad (10)$$

<sup>8</sup>In fact, the behavior of  $P_{br}(n \leq n_{max})$  with  $n$ , the principal quantum number of the state from which the atom ionizes, is compatible with a large tail that would enclose the main part of those atoms with  $n > n_{max}$ . This can be seen in plot c) of Figure 3.

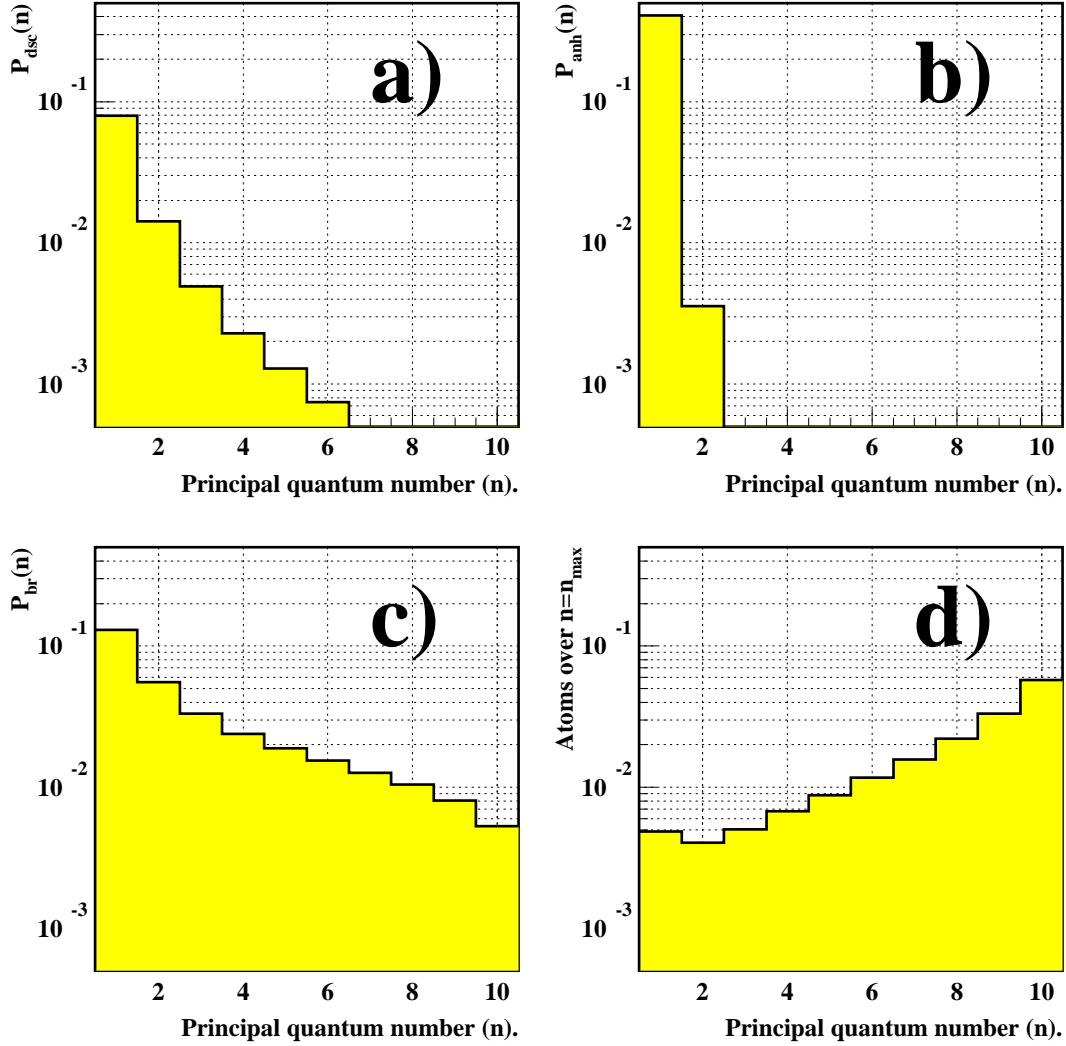


Figure 3: a) Probability for the atoms to leave the target in a bound state as a function of the principal quantum number,  $P_{dsc}(n)$ . b) Probability for the atoms to be annihilated from a state with the indicated principal quantum number,  $P_{anh}(n)$ . c) Probability for the atoms to be ionized from a state with the indicated principal quantum number  $P_{br}(n)$ . d) Probability for the atoms to suffer a transition from an initial state with the indicated principal quantum number into a final state with  $n > n_{max}$ . In all these four plots monochromatic atoms of  $4.7 \text{ GeV}/c$  momentum and  $3 \text{ fs}$  lifetime in a Nickel target of  $95. \mu\text{m}$  width were considered. Also  $n_{max} = 10$ .

### 3.1 One look inside the life of the $A_{2\pi}$ atoms.

We have just seen that there is a non negligible probability for the  $A_{2\pi}$  atoms to overcome the limit of bound states  $n_{max}$  which are involved in the  $P_{br}$  calculation. This is a fact

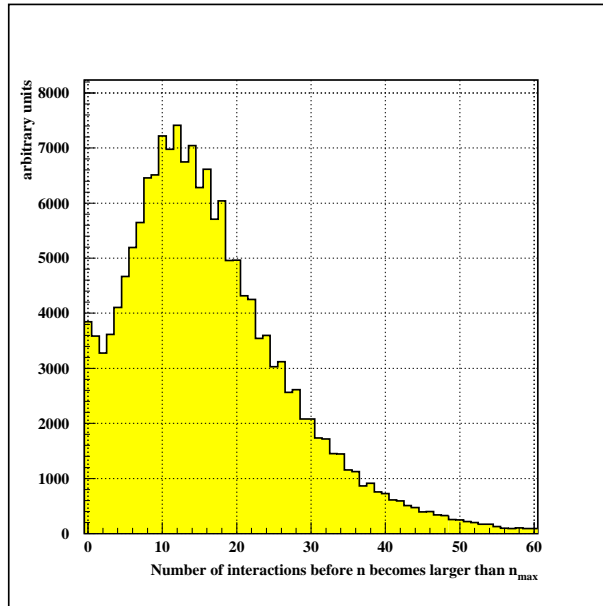


Figure 4: *Number of interactions for those atoms that have crossed to  $n > n_{max}$  states.*

that has a difficult solution. The first thing that can be done is to increase  $n_{max}$  but this process is limited by some physical reasons and also by calculation limits. However, we have increased the value of  $n_{max}$  from 7 to 10 and we had that  $P_{br}(n > n_{max})$  changed from 0.19 to 0.16.

This effect of large breakup probability tails could be explained as a consequence of the shortness of the mean free path lengths of those states with  $n > 1$  when compared to the target width. Hence, those  $A_{2\pi}$  atoms in the  $1s$  state can travel some tents of micron before they interact or annihilate, all other states with  $n \geq 2$  will suffer an interaction, at least every 1 micron. This implies that in a  $95 \mu m$  target they will interact several times. And in these interactions they will ionize or they will change to another bound state. As was mentioned, for any state, the most probable transition is  $n, l, m \rightarrow (n + 1), (l + 1), (m + 1)$  and therefore one natural possibility for some atoms will be a chain of transitions in the sequence:  $n = 1 \rightarrow n = 2 \rightarrow n = 3(\dots)n = n_{max} - 1 \rightarrow n = n_{max}$ . And once they are in a state with  $n = n_{max}$  they will have a non negligible chance to go to those states that have not been involved in the calculation. To illustrate this we have made the calculation of how many interactions suffer those atoms before they go to any state with  $n > n_{max}$  and we have obtained the distribution of Figure 4. This calculation was made with  $n_{max} = 10$  and it can be seen how the distribution peaks around this number.

## 4 Conclusions.

The main conclusion of this work is that from two independent calculations on  $A_{2\pi}$ -target interaction cross sections the same result is obtained on breakup probability within a 0.6% discrepancy. Also, the results on direct ionization cross sections allowed us to make direct calculations on the breakup probability for some core numbers. However, the slow decrease



of breakup probability with  $n$  made impossible to make an independent calculation on  $P_{br}$ .

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