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# Precise Calculation of the Pionium Lifetime

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## 1 Introduction

The experiment DIRAC aims to measure the pionium ($\pi^+\pi^-$ atom or $A_{2\pi}$) lifetime with 10% accuracy. In order to be able to extract precise pion-pion scattering length data, the relation between lifetime and scattering lengths has to be known reliably and with high accuracy. Literature about this topic can be found in [1], [2], and for the most recent papers see Ref. [3],[10],[12].

The goal of this note is to summarize new developments in calculating the pionium decay width or lifetime from basic pion parameters. The progress in better understanding the theory of pionium physics goes parallel with the progress on the experimental side, i.e. tuning DIRAC for the detection of ionized pionium atoms or so-called "atomic pairs".

## 2 Decay of pionium

Pionium, the electromagnetic bound $\pi^+\pi^-$ system, decays predominantly into $\pi^0\pi^0$ ($\simeq$ 99.6%) via strong interaction. The formation as well as the subsequent decay of pionium into $2\pi^0$ are induced by isospin breaking mechanisms in the applied theory (QCD including photons). In 1954, Deser et al. [4] published an expression for the decay width of hadronic atoms (for $\pi^- p$ at leading order in isospin symmetry breaking). For pionium at leading order (LO) in isospin symmetry breaking the corresponding result can be found in [5]. Particularly, it was derived, that the width $\Gamma_{2\pi^0}^{LO}$ varies proportional to the square of the difference $a_0 - a_2$, $a_0$ and $a_2$ being the strong S-wave $\pi\pi$ scattering lengths with isospin $I$ =0 and 2, respectively:

$$\Gamma_{2\pi^0}^{LO} = \frac{2}{9}\alpha^3 p^* (a_0 - a_2)^2, \tag{1}$$

where $p^* = \left(M_{\pi^+}^2 - M_{\pi^0}^2 - \frac{1}{4}M_{\pi^+}^2\alpha^2\right)^{1/2}$ is the $\pi^0$ momentum in the pionium system. Schematically, formula (1) corresponds to

$$\Gamma_{2\pi^0}^{LO} \propto \{electromagnetic\ binding\} \cdot \{phase\ space\} \cdot \{strong\ decay\} \tag{2}$$

or

$$\Gamma_{2\pi^0}^{LO} \propto \{|\Psi(0)|^2\} \cdot \{p^*\} \cdot \{(a_0 - a_2)^2\}. \tag{3}$$

In the last five years the width of pionium (ground state) $\Gamma_{2\pi^0}$ has been calculated by considering in more details isospin breaking effects using the effective lagrangian framework, i.e. chiral perturbation theory (ChPT) [6],[9].

### 2.1 Pionium lifetime and $\pi\pi$ scattering lengths in standard ChPT

In the framework of "QCD including photons" based on standard ChPT (SChPT), Gasser et al. [3] take into account the electromagnetic interactions and the mass difference of the up and down quarks as isospin breaking effects. (In the following, $\alpha \simeq \frac{1}{137}$ and $(m_d - m_u)^2$ are counted as small parameters of order $\delta$.) In Ref. [6] a general expression for $\Gamma_{2\pi^0}$ at leading and *next-to-leading* order in isospin breaking was derived:

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^* \mathcal{A}^2(1+K), \tag{4}$$

where $\mathcal{A}$ and $K$ are expansions in powers of $\delta$ (isospin breaking parameter).

From the **experimental** viewpoint, the crucial question is the following one: Which quantities can be measured by the DIRAC experiment?
One sees

from relation (4), that the measurement of the width $\Gamma_{2\pi^0}$ or the pionium lifetime allows to precisely determine the $\pi^+\pi^- \to \pi^0\pi^0$ scattering amplitude $\mathcal{A}$ at threshold, if $K$ is known (see below). This amplitude $\mathcal{A}$ includes isospin breaking corrections, but is independent of any chiral expansion.

In order to be able to extract scattering lengths from the measured width, the quantity $\mathcal{A}$ is chirally expanded, being related to the difference of the S-wave scattering lengths $a_0 - a_2$, order by order (in chiral expansion). To get the dependence, $\mathcal{A}$ is expanded in powers of the isospin breaking parameter $\delta$ [3]:

$$\mathcal{A} = a_0 - a_2 + \mathcal{O}(\delta) = a_0 - a_2 + h_1(m_d - m_u)^2 + h_2\ \alpha + \mathcal{O}(\delta) = a_0 - a_2 + \epsilon, \tag{5}$$

where $a_0$ and $a_2$ are, *by definition*, the strong $\pi\pi$ scattering lengths in QCD at $e = 0$, $m_u = m_d$ and for $M_\pi = M_{\pi^+}$, and $\epsilon$ is the sum of the isospin breaking corrections at $\mathcal{O}(e^2p^2)$. The quantity $K$, also expanded in powers of $\delta$, looks as follows [3]:

$$K = f_1(m_d - m_u)^2 + f_2\ \alpha\ ln\alpha + f_3\ \alpha + \mathcal{O}(\delta).$$

The correction factor $(1 + K)$ turns out to be close to unity.

Evaluating $K$ (exact without chiral expansion) and $\mathcal{A}$ [12],[11],[3], the following values are derived:

$$\epsilon = (0.61 \pm 0.16)\cdot 10^{-2}, \quad K = 1.2\cdot 10^{-2}. \tag{6}$$

If formula (4) is rewritten in the *old-fashioned* form

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^*(a_0 - a_2)^2(1+\delta_\Gamma), \tag{7}$$

then one finds by inserting $a_0 - a_2 = 0.265$ according to Ref. [11]

$$\delta_\Gamma = 2\frac{\epsilon}{a_0 - a_2} + K = 5.8\cdot 10^{-2}. \tag{8}$$

Using the procedure described above, DIRAC will be in the position to provide from the experimental width $\Gamma_{2\pi^0}$ a value for $|a_0 - a_2|$.

From the **theoretical** viewpoint, Eq. (4) with (5), (6) and $a_0 - a_2 = 0.265 \pm 0.004$ [11] leads to the following prediction for the lifetime of pionium in the ground state [12]: $\tau_{2\pi^0} = (2.90 \pm 0.09)$ fs.

### 2.2 Pionium lifetime and $\pi\pi$ scattering lengths in generalized ChPT

In order to investigate the nature of spontaneous chiral symmetry breaking in QCD, the framework of generalized chiral perturbation theory (GChPT) [7],[8] allows to analyse the pionium lifetime as a function of $\pi\pi$ scattering lengths. These lengths depend on the value of the quark condensate, a crucial order parameter of QCD. In the scheme of GChPT the quark condensate (in the chiral limit) as a free parameter depends on the details of the chiral symmetry breaking mechanism, whereas the fundamental order parameter of spontaneous chiral symmetry breaking in QCD is $F_\pi$, the pion decay coupling constant. Therefore, the quark condensate parameter should be submitted to an experimental test.

The pionium lifetime $\tau_{2\pi^0}$, obtained in GChPT, depends on $(a_0 - a_2)$ as shown in Fig. 1 (see Ref. [10]). This means, that a measured lifetime can be interpreted in terms of the quark condensate: Values of $\tau_{2\pi^0}$ near 3 fs would confirm the large condensate as assumed in SChPT, whereas $\tau_{2\pi^0}$ values below 2.4 fs would favour the scheme of GChPT allowing also a small quark condensate.

The comparison of the predicted $\tau_{2\pi^0}$ for $a_0 - a_2 = 0.265$ in SChPT shows a complete agreement with the corresponding $\tau_{2\pi^0}$ in GChPT.

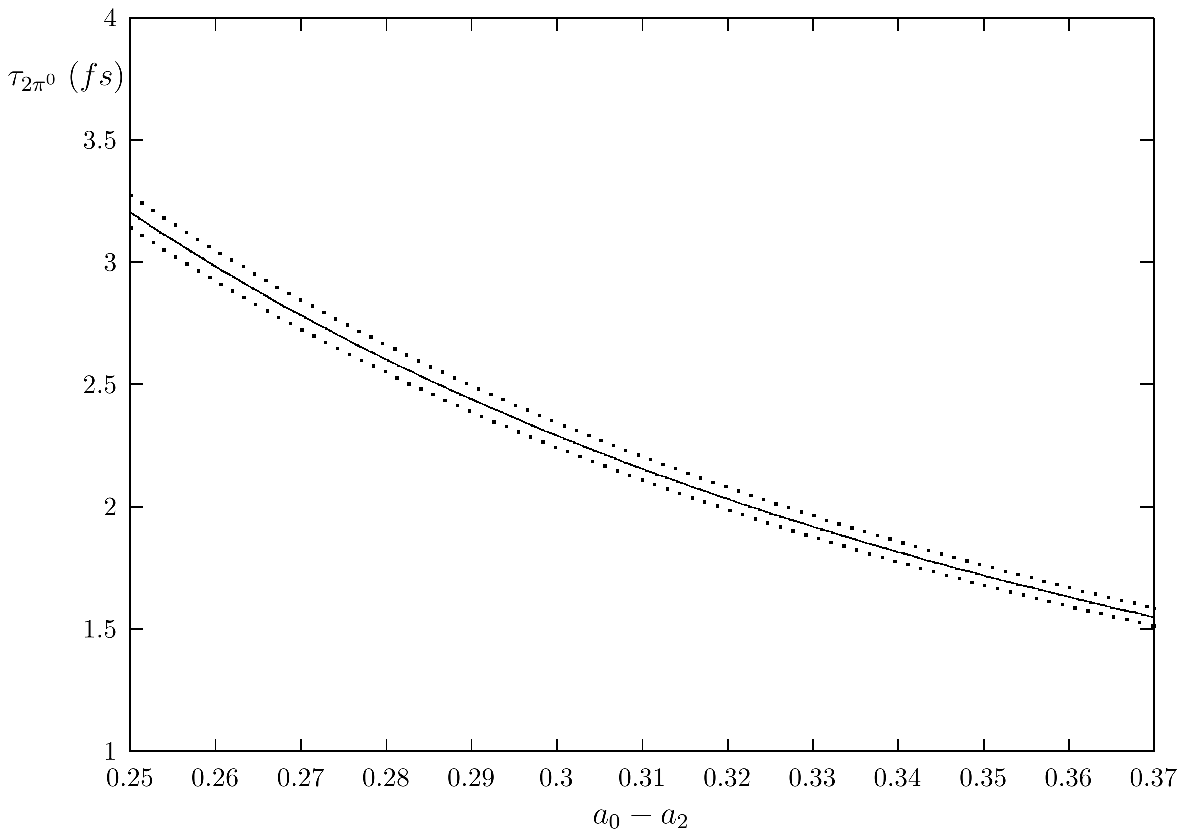


Figure 1: The pionium lifetime in dependence of the S-wave scattering length difference $(a_0 - a_2)$. The band drawn in dotted lines includes estimated uncertainties of about 2.5% (from H. Sazdjian, Ref. [10]).

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