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UNCERTAINTIES INDUCED BY MULTIPLE SCATTERING IN UPSTREAM DETECTORS OF THE DIRAC SETUP

(Part II: Relative Momentum Resolution)

M.Pentia, S.Constantinescu
National Institute for Physics and Nuclear Engineering, Bucharest, P.O.Box MG-6, RO-76900, ROMANIA.

Abstract

The *relative momentum* (Q) distributions for $\pi^+\pi^-$ atomic pairs are tracked along the upstream detector system of the DIRAC setup by Monte Carlo simulation. There are analyzed these distributions in the production point, at the target output and after track resonstruction. The Multiple Scattering (MS) contribution in the upstream detector elements is evaluated.

Contents

1 Introduction

The linear track reconstruction procedure furnishes the intercept position and slope parameters based on coordinate data from the tracking detectors of the DIRAC setup (see Fig.1). In the DIRAC Note 00-07 [1] we presented the first part of the paper, relative to Multiple Scattering (MS) induced errors on the reconstructed intercept position. Now we continue the study with the MS influence on the slope parameters and consequently on the *opening angle* Θ and on the *relative momentum* Q of the $\pi^+\pi^-$ atomic pairs from $A_{2\pi}$ breakup.

The *relative momentum* Q of the $\pi^+\pi^-$ atomic pairs is one of the main variable used in DIRAC experimental data analysis. We are studying the $A_{2\pi}$ production and its subsequent breakup into π^+ and π^- pairs, followed by particle transport within target and upstream detector tracking system of the DIRAC setup (Fig.1). The errors induced on the particle track due to MS in the target and upstream detector elements are careful studied, especially for the double track separation and for close opening angle and relative momentum measurements.

Using a specific $A_{2\pi}$ generator and the atomic breakup in the target, followed by a linear track reconstruction procedure with a nondiagonal error matrix, we expressed finally the track parameters and their errors. In such a way it was possible to study the changement in *opening angle* Θ and *relative momentum* Q of the $\pi^+\pi^-$ atomic pairs. The calculations have been done for the coordinate detector system configuration presented in Fig.1. The target (Ni) is placed in the origin of the coordinate system, oriented along the setup z-axis. The pointlike incident proton beam 24 GeV/c enter the target along the same z-axis.

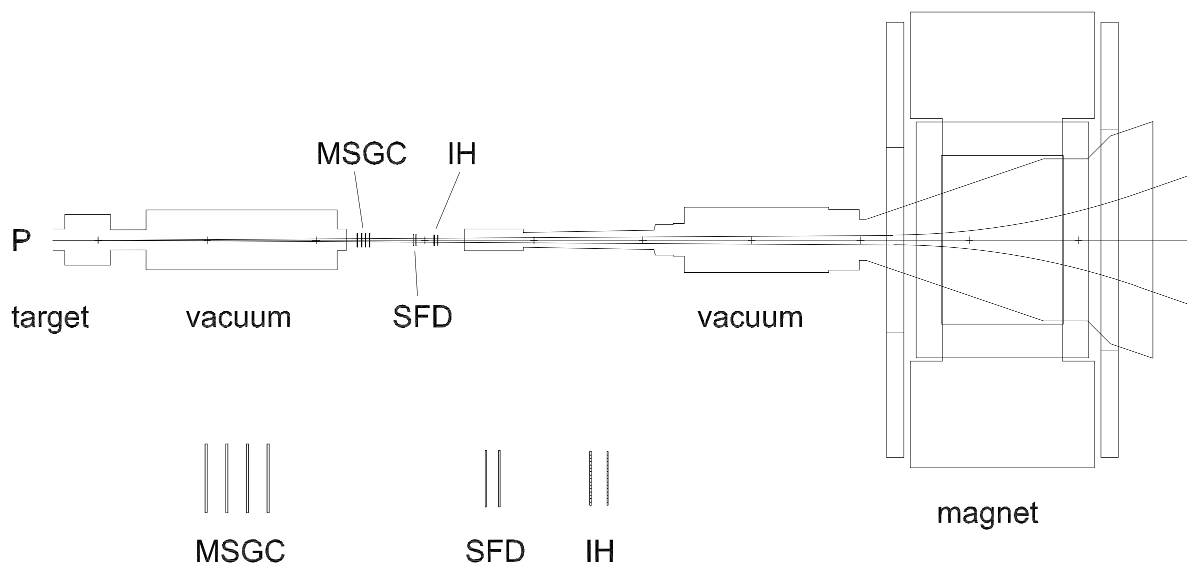

Figure 1: DIRAC upstream tracking detector system

2 The $\pi^+\pi^-$ atomic pair production

After production in hadron-nucleus interaction, relativistic $A_{2\pi}$ atoms ($1GeV/c < p_A < 8GeV/c$) are moving in the target. They can decay or, due to the electromagnetic interaction with the target material, can get excited or brokenup (ionized).

The target material and thickness are chosen in such a way that the $A_{2\pi}$ breakup competes with the decaying process. That is the atomic *interaction length* be similar to the *decay length* for a few GeV/c $A_{2\pi}$. Some of the targets used in our experiment and their characteristics are presented in Table 1. **Table 1.**

No.	Target material	Thickness (μm)	X_0 (cm)	$A_{2\pi}$ int.length (μm)
1	Be	2042.0	35.276	600.0
2	Ti	246.8	3.563	49.7
3	Ni	93.9	1.425	19.4
4	Pt	25.7	0.305	3.84

As a result of $A_{2\pi}$ breakup there are produced pion pairs. For angular and relativ momentum distribution of the resulted pion pairs in the CM system, we used the routine furnished by Oleg Gortchakov [2]. In Fig. 2 there are presented the path length distributions in Pt target, for incident protons (a), for $A_{2\pi}$ atoms (b), and for pions (c).

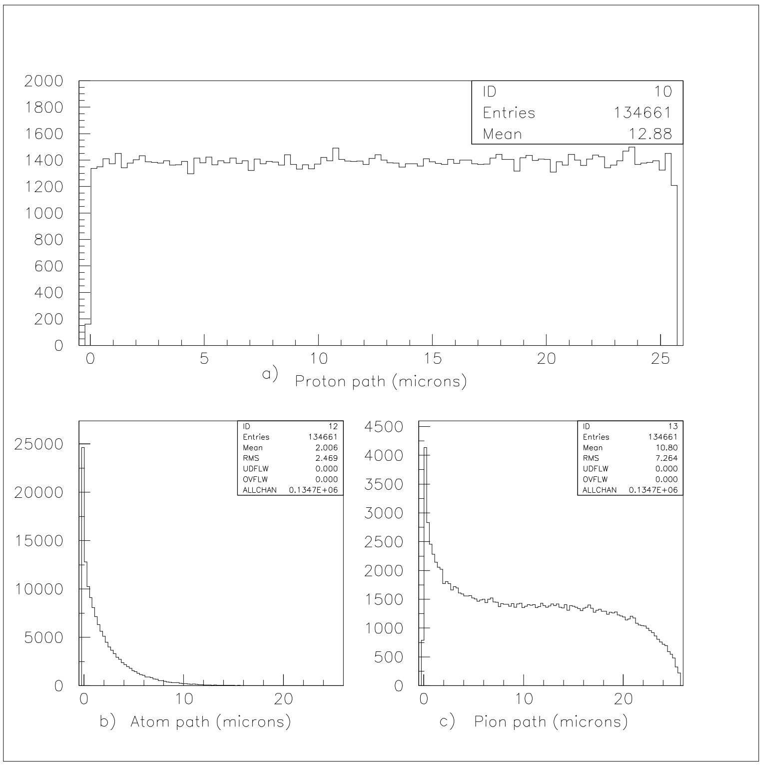

Figure 2: Path length distributions in Pt target, for **a.)** incident protons, **b.)** $A_{2\pi}$ atoms, **c.)** pions from atomic pairs

3 The particle track position errors due to multiple scattering

When a charged particle is crossing the elements of the experimental setup, it is subject to small deviations of the track due to MS. The effect is usually described by the theory of Molière (see for example Ref. [4]), which shows that, by crossing the material thickness s, the particle is subject to successive small-angle deflections, symmetrically distributed around the incident direction.

The Molière distribution of the scattering angle can be approximated by a Gaussian one [5]. The width of this distribution is the root mean square of the scattering angle [6]

$$\theta_0 = \frac{13.6MeV}{p\beta c} z_c \sqrt{\frac{s}{X_0}} \left[1 + 0.038 \ln\left(\frac{s}{X_0}\right)\right] \tag{1}$$

where p, βc and z_c are the momentum, velocity and charge number of the incident particle, and X_0 is the radiation length of the scattering medium.

For MS evaluation we had in view the target and all materials in the DIRAC configuration (see Fig.1), in the space between target and magnet. They are presented in Table 2.

A Monte-Carlo study of the particle transport within upstream part of the detector system has been done.

Table 2.

Ndet	Scattering material	z position (cm)	Thickness per X_0 ($\times 10^{-4}$)	θ_0 plane (mrad)	Detector resolution $\sigma_i^{det}(\mu m)$	MS error (π $2GeV/c$) $\sigma_{x_i}(\mu m)$
1	Mylar	229	8.7	0.14732	-	0.0
2	MSGC0	237.3	56.0	0.40974	40	12.27
3	MSGC1	239.4	56.0	0.40974	40	17.51
4	MSGC2	241.1	56.0	0.40974	40	24.59
5	MSGC3	243.2	56.0	0.40974	40	36.50
6	SciFi	287.78	117.9	0.42132	125	399.34
7	SciFi	290.28	117.9	0.42132	125	420.25

4 Track reconstruction parameters and their errors

The possibility to do an independent description of the MS data on x and y axis, allows a separate fit by a linear relation

$$\begin{aligned} x &= x_0 + \alpha_x z \\ y &= y_0 + \alpha_y z \end{aligned} \tag{2}$$

The least squares procedure uses the target point (0,0) and the coordinates (x_i, y_i) given by every coordinate detectors at z_i, together with the nondiagonal error matrix V_{ij} [3]

$$V_{ij} = \sum_{k=1}^{i-1} \theta_{0k}^2 (z_i - z_k)(z_j - z_k) + \delta_{ij} \cdot (\sigma_i^{det})^2 \tag{3}$$

For x-data set, the χ^2 in the matrix form is

$$\chi^2 = (X - HA_x)^T V^{-1} (X - HA_x) \tag{4}$$

where

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}; H = \begin{pmatrix} 1 & z_1 \\ 1 & z_2 \\ \vdots & \vdots \\ 1 & z_n \end{pmatrix}; A_x = \begin{pmatrix} x_0 \\ \\ \alpha_x \end{pmatrix} \tag{5}$$

Then the least squares criterion imposes

$$\frac{\partial \chi^2}{\partial A_x} = 0 \quad or \quad H^T V^{-1} (X - HA_x) = 0 \tag{6}$$

By solving the linear system (6) with respect to A_x we get the fit parameters

$$A_x = \begin{pmatrix} x_0 \\ \\ \alpha_x \end{pmatrix} = \left(H^T V^{-1} H\right)^{-1} \left(H^T V^{-1} X\right) \tag{7}$$

and the error of these parameters

$$V(A_x) = \begin{pmatrix} \sigma_{x_0}^2 & \rho\sigma_{x_0}\sigma_{\alpha_x} \\ \\ \rho\sigma_{\alpha_x}\sigma_{x_0} & \sigma_{\alpha_x}^2 \end{pmatrix} = \left(H^T V^{-1} H\right)^{-1} \tag{8}$$

where ρ is the correlation factor.

The error correlation matrix elements are given by

$$\rho_{ij} = \frac{< \delta x_i \delta x_j >}{\sqrt{< \delta x_i^2 >< \delta x_j^2 >}} \tag{9}$$

The uncorrelated position errors as for example the detector intrinsic resolution, have to be added in squares into the diagonal terms of the error matrix V.

As long as the V_{ij} matrix elements depend on both the kinematical characteristics of the detected particles and the tracking detector system configuration, the ρ_{ij} matrix elements are independent of particle characteristics, and is defined only by system configuration. For DIRAC upstream detector system the error correlation matrix is

$$\begin{pmatrix} 1. & .000 & .000 & .000 & .000 & .000 & .000 \\ .000 & 1. & .875 & .725 & .573 & .217 & .215 \\ .000 & .875 & 1. & .938 & .820 & .440 & .438 \\ .000 & .725 & .938 & 1. & .950 & .623 & .620 \\ .000 & .573 & .820 & .950 & 1. & .790 & .788 \\ .000 & .217 & .440 & .623 & .790 & 1. & .999 \\ .000 & .215 & .438 & .620 & .788 & .999 & 1. \end{pmatrix}$$

It must be pointed out that the track reconstructed parameter errors (8) do not depend on the particular track coordinate values (x_i, y_i), the errors depend only on the z_i layer position and on the θ_{0i} mean scattering angles, of the H and V matrices.

We applied the same procedure for y coordinate, in order to find the best fit parameters and their errors.

Finally the estimated linear track reconstructed parameters, based on the track data are intercept position $\hat{x}_0 \pm \sigma_x$, $\hat{y}_0 \pm \sigma_y$, and slope parameters $\hat{\alpha}_x \pm \sigma_\alpha$, $\hat{\alpha}_y \pm \sigma_\alpha$.

The target intercept estimations $\hat{x}_0 \pm \sigma_x$, $\hat{y}_0 \pm \sigma_y$, for both of the pion pair tracks, has been used for test the simulation procedure. This is done in Fig. 4, where it has represented the distribution for the variable $(x_{10} - x_{20})/\sqrt{(\sigma_{x1}^2 + \sigma_{x2}^2)}$. It is a Gaussian one with $\sigma = 1$.

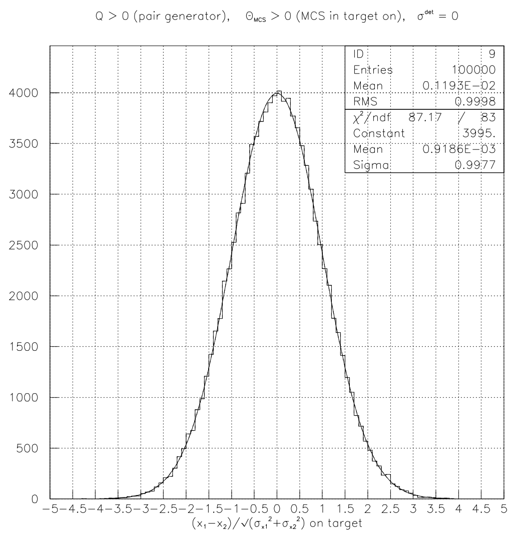

Figure 3: Distribution of the difference in pion pair target intercept points

5 Opening angle and relative momentum reconstruction

The slope parameter estimators $\hat{\alpha}_{ix}$ and $\hat{\alpha}_{iy}$, for the two atomic pions $i = 1, 2$, are used to define the two pion directions, and hence the opening angle Θ and with p_{ix}, p_{iy}, p_{iz} momentum components, we have finally the relative momentum Q.

$$\begin{aligned} \hat{\alpha}_{ix} &= tg\theta_{ix} = C_{ix}/C_{iz} \\ \hat{\alpha}_{iy} &= tg\theta_{iy} = C_{iy}/C_{iz} \end{aligned} \tag{10}$$

where C_{ix}, C_{iy}, C_{iz} are the directory cosines of the i-pion flight direction in the lab system. Using also the normalization relation, we get C_{ix}, C_{iy}, C_{iz} and finally the 4-momentum components of the atomic pions. They are used in angle expressions:

$$\begin{aligned} \cos\theta_1 &= (\vec{p}_1 \cdot \vec{p}_A)/(|\vec{p}_1||\vec{p}_A|) \\ \cos\theta_2 &= (\vec{p}_2 \cdot \vec{p}_A)/(|\vec{p}_2||\vec{p}_A|) \\ \cos\theta_{12} &= (\vec{p}_1 \cdot \vec{p}_2)/(|\vec{p}_1||\vec{p}_2|) \end{aligned} \tag{11}$$

The longitudinal components in the CM system are given by Lorentz transformation

$$\begin{aligned} p^*_{1l} &= \gamma(p_1 \cos\theta_1 - \beta E_1) \\ p^*_{2l} &= \gamma(p_2 \cos\theta_2 - \beta E_2) \end{aligned} \tag{12}$$

and transversal Q_t and longitudinal Q_l components of the relative momentum are:

$$\begin{aligned} Q_t &= p_1 \sin\theta_1 + p_2 \sin\theta_2 \\ Q_l &= p^*_{1l} - p^*_{2l} \end{aligned} \tag{13}$$

Finally the opening angle is:

$$\boxed{\Theta = arcos(\cos\theta_{12})}$$

and the relative momentum Q is:

$$\boxed{Q = \sqrt{Q_t^2 + Q_l^2}}$$

In Fig. 4 there are presented the Q_x and the Q distributions:
In the first line are the generated distributions.
The second line shows the distributions at the target output.
In the third line are shown the reconstructed distributions at the tracking system output.
In the forth line are the same reconstracted distributions but without use of the target point in the least sqares fit procedure. The Q_x resolution is $\sigma_{Q_x} = 1.01$ MeV/c, in accordance with the experimental measured value.

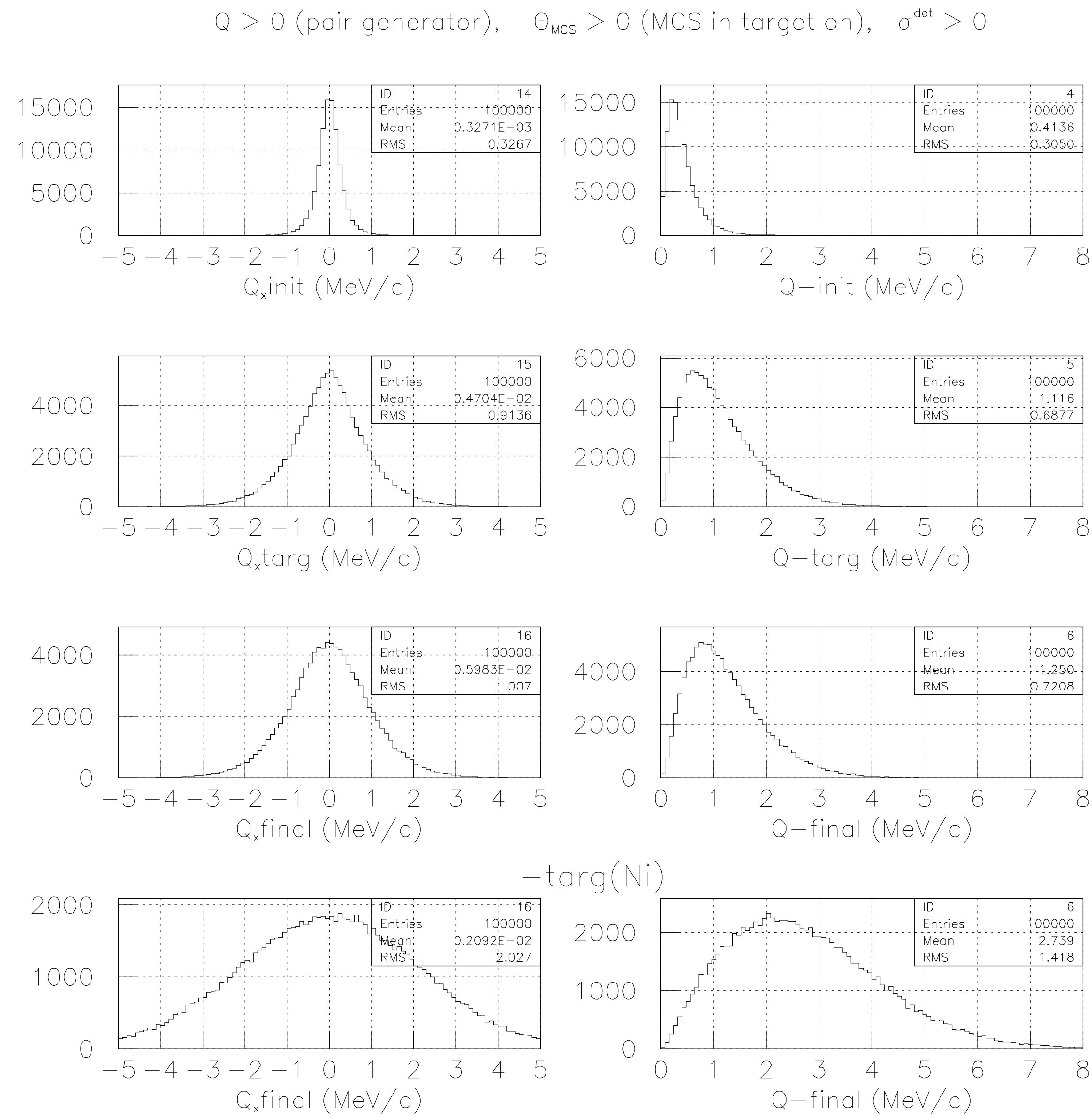

Figure 4: The Q_x and Q distributions of the $\pi^+\pi^-$ atomic pairs from the 4 GeV/c $A_{2\pi}$ breakup. **1-st line:** in the Ni target production point; **2-nd line:** at the target output; **3-rd line:** after the track reconstruction with the target point included in least squares procedure; **4-th line:** after the track reconstruction without use of the target point in the least squares procedure.

The relative momentum versus opening angle dependence is

$$Q^2 = (p_1 - p_2)^2 = p_1^2 + p_2^2 - 2p_1 p_2 \cos\Theta \tag{14}$$

The variance on Q, due to opening angle error is

$$\sigma_Q^2 = \frac{p_1^2 p_2^2}{Q^2} \cdot \sigma_{\cos\Theta}^2 \tag{15}$$

where $\sigma_{\cos\Theta} = \sin\Theta \cdot \sigma_\Theta$

$\sigma_\Theta = const.$, because it depends on α_x and α_y, which are specified exclusively by the tracking detector geometry.

If we turn "off" the target contribution (pion pairs generator and MS in the target), we have pure detector contribution to relative momentum resolution. The results are presented in Fig. 5.

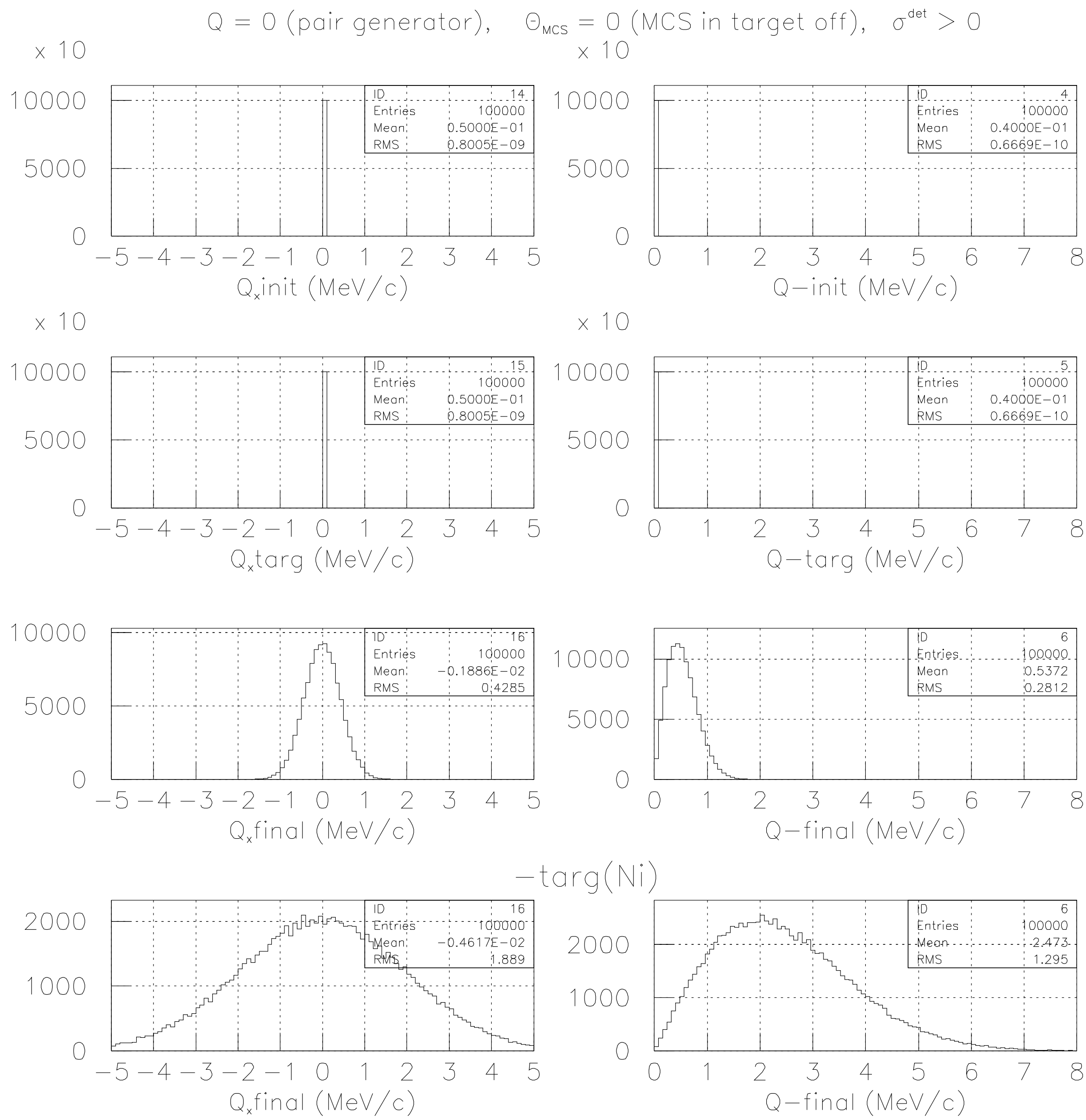

Figure 5: The Q_x and Q distributions of the $\pi^+\pi^-$ atomic pairs from the 4 GeV/c $A_{2\pi}$ breakup. **1-st line:** in the Ni target production point; **2-nd line:** at the target output; **3-rd line:** after the track reconstruction with the target point included in least squares procedure; **4-th line:** after the track reconstruction without using the target point in the least squares procedure.

The target contribution to the Q_x and Q resolution can also be seen in Fig. 6.

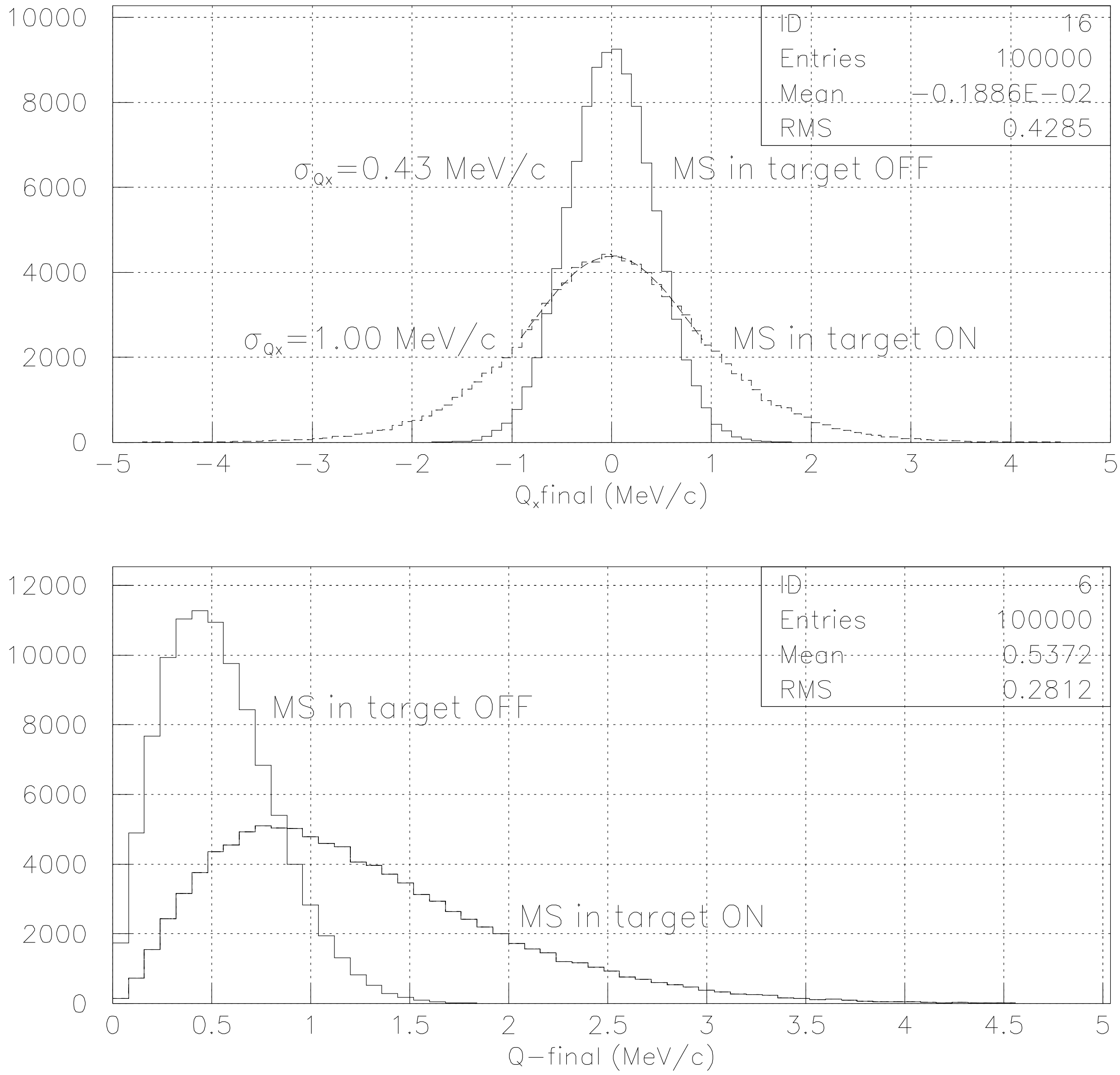

Figure 6: The Q_x and Q reconstructed distributions of the $\pi^+\pi^-$ atomic pairs from the 4 GeV/c $A_{2\pi}$ breakup in the Ni target

6 Conclusion

The relative momentum Q is the main variable for event selection in the DIRAC experiment. The processe of multiple scattering leading to the modification of the relative momentum has been carefully studied. In this paper we presented the results relative to multiple scattering influence during particle transport along with the Ni target and tracking detector system. The Table presents the upstream detector elements contribution to the relative momentum resolution.

Pair generator $Q > 0.\ MeV/c$	MS in target	MS in detectors	Intrinsic det. resolution	Q resolution (MeV/c)	Obs.
Yes	Yes	Yes	Yes	$\sigma_{Q_x} = 1.00$ $(RMS)_Q = 0.72$ $\sigma_{Q_l} = 0.17$	Setup resol.
No	No	Yes	Yes	$\sigma_{Q_x} = 0.43$ $(RMS)_Q = 0.28$ $\sigma_{Q_l} = 0.00$	No target contribution

References

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[6] D.E.Groom et al, Particle Data Group, Eur.Phys.J. C15, 1 (2000)