DIRAC NOTE 2002–08

Two target method for the lifetime measurements

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The method is based on the measurements the with two targets: the first one is the thick one-layer target and the second target consisting of a few layers with 1 mm gap between and having the same total thickness.

First let us consider the accuracy of our standard methord with fitting procedure. Number of the real coincidence events in the range of the small relative momentum for the standard, one-layer target is written:

$$N_R = N_{RA} - t \cdot N_{acc} = n_{A2\pi} + N_C + N_{nC} \ . \tag{1}$$

Here N_{RA} is the number of the real and accidental events in the central time peak; N_{acc} is the number accidental events in their time windows; t is the ratio of the time windows of the real to accidental; $n_{A2\pi}$ is the atomic pairs observed; N_C and N_{nC} are the numbers of the "Coulomb" and "non-Coulomb" pairs correspondingly.

The probability of $A_{2\pi}$ breakup, which is a ratio of the total number of the broken atoms $n^{\rm tot}_{A2\pi}$ to the total number of produced ones $N^{\rm tot}_{A2\pi}$, can be expressed via the number of atomic pairs in the selected range of the small relative momentum $n_{A2\pi}$ and the numbers of the "Coulomb" and "nonCoulomb" pairs in the same range N_C and N_{nC}:

$$P_{\rm br} = \frac{n^{\rm tot}_{A2\pi}}{N^{\rm tot}_{A2\pi}} = \frac{m \cdot n_{A2\pi}}{k' \cdot N_C} = \frac{n_{A2\pi}}{k \cdot N_C} = \frac{N_R - N_{Cf} - N_{nCf}}{k \cdot N_{Cf}} \ . \tag{2}$$

Here the f index stands for the numbers obtained as result of the standard fitting procedure. The coefficients m, k' and k are to be obtained from a Monte-Carlo simulation of processes of the atoms breakup and detection of pions pairs. In further calculations the value $k = 0.54$ for the range of the relative momentum $F < 2$ have been used.

The squared standard deviation of the observed atomic pairs number is written:

$$\begin{aligned} \sigma^2_{n_{A2\pi}} &= \sigma^2_{N_R} + \sigma^2_{N_{Cf}} + \sigma^2_{N_{nCf}} + 2 \cdot cov(N_{nCf}, N_{Cf}) + \\ & \quad 2 \cdot t \cdot cov(N_{Cf}, N_{acc}) + 2 \cdot t \cdot cov(N_{nCf}, N_{acc}) \ . \end{aligned} \tag{3}$$

Then squared standard deviation of the breakup probability is expressed as:

$$\sigma^2_{P_{\rm br}} = P^2_{\rm br} \cdot \left[\frac{\sigma^2_{n_{A2\pi}}}{n^2_{A2\pi}} + \frac{\sigma^2_{N_{Cf}}}{N^2_{Cf}} + 2 \cdot \frac{\sigma^2_{N_{Cf}} + cov(N_{nCf}, N_{Cf}) + t \cdot cov(N_{Cf}, N_{acc})}{n_{A2\pi} \cdot N_{Cf}} \right] \ . \tag{4}$$

For the second target consisting of a few layers with 1 mm gap between and having the same total thickness, the number of events in the same range of the small relative momentum (assuming the same number of primary interactions) is written as:

$$N2_R = N2_{RAc} - t \cdot N_{acc} = n2_{A2\pi} + N_C + N_{nC} \ . \tag{5}$$

The number of the atomic pairs $n2_{A2\pi}$ for this target is less and its dependence on the lifetime is much weaker in comparison with the standard target. Thus the following combination depends on the $A_{2\pi}$ lifetime in the unique way

$$V = \frac{N_R - N2_R}{N_{Cf}} = \frac{n_{A2\pi} - n2_{A2\pi}}{N_{A2\pi}/k} = \frac{N_{A2\pi}(P_{\rm br\,1} - P_{\rm br\,2}) \cdot k}{N_{A2\pi}} = (P_{\rm br\,1} - P_{\rm br\,2}) \cdot k \ . \tag{6}$$

Here $P_{\mathrm{br}\,1}$ and $P_{\mathrm{br}\,2}$ are the calculated probabilities of the atom breakup for each targets and k comes from the relation $N_{A2\pi} = kN_C$ (see Eq.2). Then squared standard deviation of V is written as:

$$\sigma_V^2 = V^2 \cdot \left[\frac{\sigma_{N_R}^2 + \sigma_{N2_R}^2}{(N_R - N2_R)^2} + \frac{\sigma_{N_{Cf}}^2}{N_{Cf}^2} + 2 \cdot t \cdot \frac{cov(N_{Cf}, N_{acc})}{(N_R - N2_R) \cdot N_{Cf}} \right] . \qquad (7)$$

Comparison of Eq.4 and Eq.7 shows that the contribution of N_{Cf}^2 and $\sigma_{N_{Cf}}^2$, which have the biggest systematic bias, are smaller for V relative to P_{br}. Thus the measurements of the values P_{br} and V allows us to obtain the lifetime in the two different approach and get an estimation of the probable systematic bias.

To get a numerical estimation of a required statistics the data collected in 2001 with the Nickel target have been used. In the first line of Table 1 the values N_R, N_{Cf} and $n_{A2\pi}$ are given for the present state of data processing for the range of the relative momentum $F < 2$. There are some known uncertainties which lead to decrease of the $n_{A2\pi}$ value, for this reason in the second line the value of $n_{A2\pi}$ is increased artificially to get the expected value of P_{br} for $\tau = 2.9 \cdot 10^{-15}$ s. Just the latter values have been used in all further calculations.

Table 1: Number of events selected from 2001 Nickel data.

N_R	N_{Cf}	$n_{A2\pi}$
17696 ± 135	14706 ± 250	2619 ± 261
17696 ± 135	13763 ± 250	3265 ± 259

The first column contains information for the standard fitting procedure, all other for for the two target method with different numbers of layers in the second target. The following line are shown: "Layers" is the number of layers in the target; "Thickness" is the thickness of one layer in μm; "$P_{\mathrm{br}\,i}$" is the calculated probabilities of the atom breakup, V is the variable defined in Eq.6; "Derivative" is $\partial P_{\mathrm{br}\,1}/\partial\tau$ for the first column and $\partial V/\partial\tau$ for all other; $\delta_{10\%}$ and $\sigma_{10\%}$ are the relative and absolute accuracy, respectively, of the measured value (P_{br} or V) which provides the 10% accuracy in the lifetime measurement; σ_{stat} is the accuracy in the measured values ($P_{\mathrm{br}\,1}$ or V) achieved (or could be achieved) with the Nickel 2001 date, for the case of two target method the data were divided into to equal parts; "Multiplication" is the factor by which the Nickel 2001 statistic should be increased to obtain 10% accuracy in the lifetime.

Thus the two target method with the number of layers in the second target between 10 and 20 looks like a very attractive addition to our standard fitting procedure.

Behaviours of P_{br} and V in dependence on the lifetime τ which have been employed for getting values for Table 2 are shown in Figs.1 and 2.

Table 2:

Layers	1	5	10	20
Thickness (μm)	100.	20.0	10.	5.
$P_{\mathrm{br}\,i}$	0.44704	0.33396	0.25373	0.17803
V		$6.106 \cdot 10^{-2}$	0.10439	0.14526
Derivative	$5.654 \cdot 10^{-2}$	$1.817 \cdot 10^{-2}$	$2.511 \cdot 10^{-2}$	$2.832 \cdot 10^{-2}$
$\delta_{10\%}$	$3.667 \cdot 10^{-2}$	$8.631 \cdot 10^{-2}$	$6.976 \cdot 10^{-2}$	$5.655 \cdot 10^{-2}$
$\sigma_{10\%}$	$1.640 \cdot 10^{-2}$	$5.270 \cdot 10^{-3}$	$7.282 \cdot 10^{-3}$	$8.214 \cdot 10^{-3}$
σ_{stat}	$4.167 \cdot 10^{-2}$	$1.942 \cdot 10^{-2}$	$1.927 \cdot 10^{-2}$	$1.911 \cdot 10^{-2}$
Multiplication	6.45	13.58	7.00	5.42

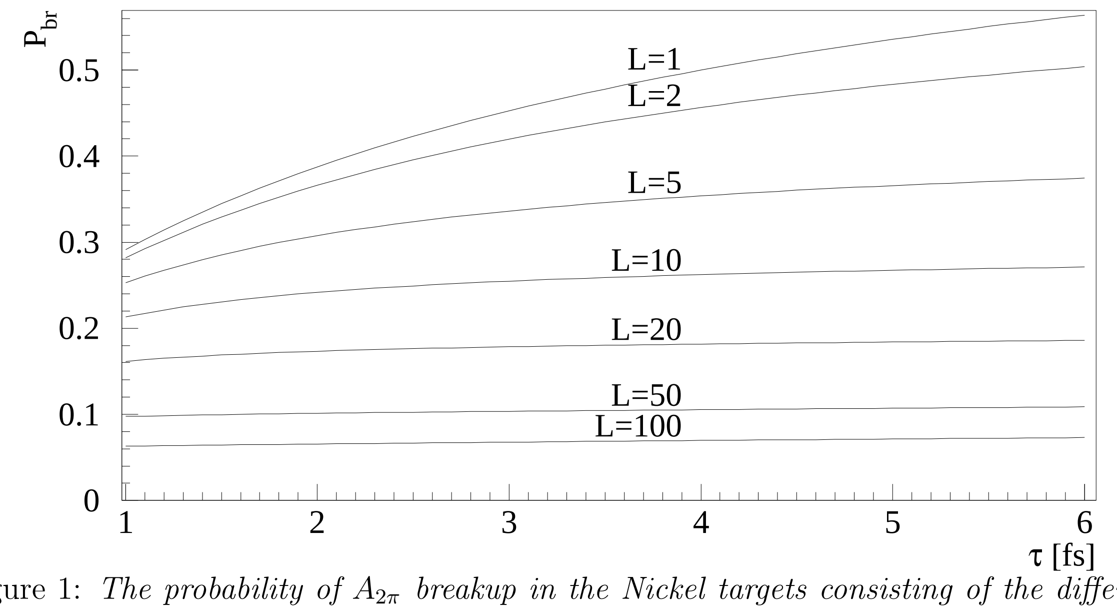

Figure 1: *The probability of $A_{2\pi}$ breakup in the Nickel targets consisting of the different number of layers (L) with 1 mm gap between and having the total thickness of 100 μm as a function of the lifetime τ.*

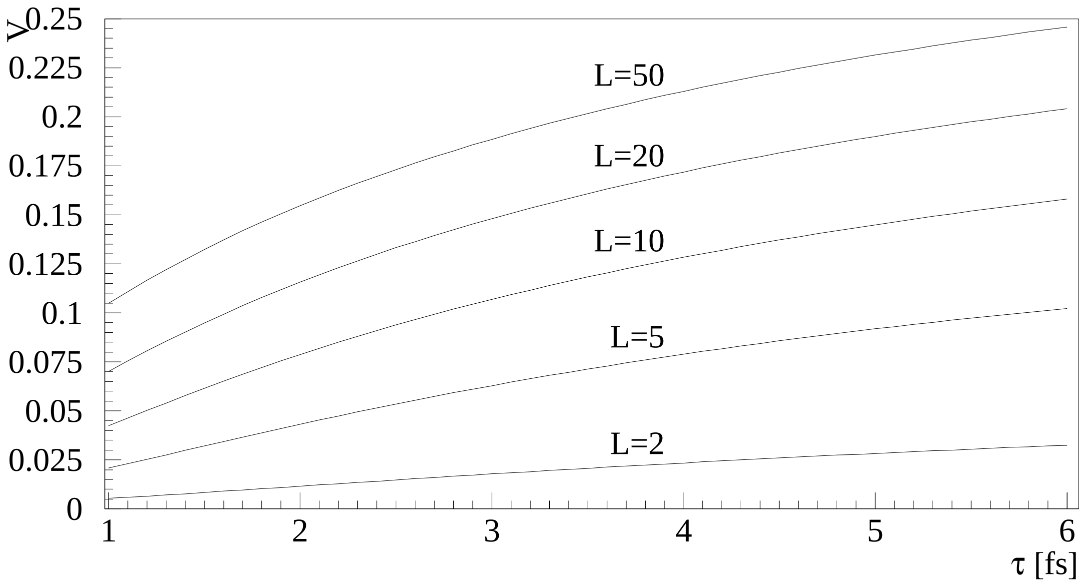

Figure 2: *Variable V defined in Eq.6 as a function of the $A_{2\pi}$ lifetime τ for the different number of layers (L).*