DIRAC NOTE 06-04

Comments on DIRAC NOTE 06-03

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The following comments should help to clarify concepts, procedures and results of the *DIRAC note 06-03, "Measurement of Pionium Lifetime" by Adeva, Romero and Vasquez-Doce (Santiago)* in order to allow the collaboration to understand and eventually accept it.

General Remarks

1. References are made essentially to work of the Santiago group. This renders reading difficult as it remains unclear what is genuine, copy of standard procedures/formulas or modified standard. In particular a comparison of the results of the note with the DIRAC lifetime publication (Physics Letters B 619 (2005) 50) in May 2005 is missing. The results published there were thoroughly discussed, fully understood and accepted by the collaboration. Any deviation from these results, especially concerning errors, would have needed a critical and thorough analysis and discussion. This was not done.
2. “It should be understood that the Monte Carlo is entirely restricted to the description of experimental resolution..” (2nd paragraph of chapter 3) reveals a misunderstanding of the role of Monte Carlo simulation of the experiment, which is the quantitative understanding of the set-up, its functioning or disfunctioning, and the procedures. The note suffers somewhat from this misunderstanding (ad-hoc statements, lack of cross-checks etc.).

ARIANE version

ARIANE version 304-43 was used (page 2). This version does not exist in the public DIRAC. The last official version 304-40 was released on November 30, 2005. Version 304-43 is apparently an unchecked private version, unknown to the rest of the collaboration.

Formula 1 (see also appendix)

The definition of χ^2, using the notations of the note, is (see standard textbooks on statistics and error analysis, e.g. *WT Eadie, D. Drijard, FE James, M. Roos, and B. Sadoulet, Statistical Methods in Experimental Physics (North-Holland, Amsterdam and London, 1971)*)

$$\chi^2 = \sum_i \frac{\left\{N_p^i - \beta\alpha_{CC}\frac{N_{CC}^i}{N_{CC}} - \dots\right\}^2}{\sigma^2_{N_p^i - \beta\alpha_{CC}\frac{N_{CC}^i}{N_{CC}} - \dots}} = \sum_i \frac{\left\{N_p^i - \beta\alpha_{CC}\frac{N_{CC}^i}{N_{CC}} - \dots\right\}^2}{N_p^i + \left(\frac{\beta\alpha_{CC}}{N_{CC}}\right)^2 N_{CC}^i + \dots}$$

Formula 1 of the note (see formula below) is numerically correct for $\beta = N_p$, **but fails to give correct errors if β is a variable parameter:**

$$\chi^2 = \sum_i \frac{\left\{N_p^i - \beta\alpha_{CC}\frac{N_{CC}^i}{N_{CC}} - \dots\right\}^2}{\beta\left[\frac{N_p^i}{N_p} + \frac{\alpha_{CC}{}^2}{\frac{N_{CC}}{N_p}}\frac{N_{CC}^i}{N_{CC}} + \dots\right]}$$

$$= \sum_i \frac{\left\{N_p^i - \beta\alpha_{CC}\frac{N_{CC}^i}{N_{CC}} - \dots\right\}^2}{\frac{\beta}{N_p}N_p^i + \beta N_p\left(\frac{\alpha_{CC}}{N_{CC}}\right)^2 N_{CC}^i + \dots} \xrightarrow[\beta = N_p]{} \sum_i \frac{\left\{N_p^i - \beta\alpha_{CC}\frac{N_{CC}^i}{N_{CC}} - \dots\right\}^2}{N_p^i + \beta^2\left(\frac{\alpha_{CC}}{N_{CC}}\right)^2 N_{CC}^i + \dots}$$

As it stands, Formula 1 is formally incorrect since β is a parameter.

Statistical Errors

The statistical errors given in the note Table 6 are very small, despite the fact, that the total amount of atomic pair events from the DIRAC lifetime paper and the note are very similar. The larger CC background of the note originates from a Q_T-cut at 5 instead of 4 MeV/c:

	Dirac	Note	Note/DIRAC (errors)
N_A full range fit including MC shape	6530±294 (4.5%)	6738±190 (2.8%)	0.62
N_A residual	6518±373 (Q) 6509±330 (Q_L)	$6700 \pm 350_{min}$ (estimated from CC-background)	About 1
N_{CC}	106549±1014 (0.95%)	120611±812 (0.67%)	0.70

We estimate the residual atomic signal of the note (not given there) to have an error of at least $\sqrt{120611} \approx 350$ (CC-background), and from Fig 18 to be approximately 380. We can not understand the reduction of error from 350-380 to 190, a factor of more than two, in a multivariate fit procedure over the full range. The reduction of 10-20% in the DIRAC lifetime paper has been studied in detail, and a similarly strong error reduction was only found for a one-parameter fit.

We suggest a comprehensive error discussion in the note, with clear mention of what the errors are and how they were obtained (e.g. MINOS errors are usually not symmetric, also mention the errors from SIMPLEX, HESSE or MIGRAD, fit ranges, free parameters etc.). It also would help to know how many MonteCarlo events were used.

From the contour-plot Fig 24 we estimate $\sigma_{NA} = \pm 240$ and $\sigma_{NC} = \pm 660$. This seems inconsistent with Table 6.

Fit results (plots)

1. The Q_T residual in Fig. 17 is not well reproduced by Monte Carlo (12 points above, 5 points below the atomic shape distribution, of which 8 by more than 2σ). This seems to contradict the small χ^2 of Table 7 for Q_L<2 MeV/c.
2. The errors in the Q_L residual Fig 18 should be biggest for smallest Q_L and then become gradually smaller (see Q_L total) with increasing Q_L, while in the plot they are small for Q_L<2MeV/c, and from there on are large.

Lifetimes

The lifetimes given in the summary are difficult to understand in what concerns their values and their statistical errors. It is unclear which P_{br}-τ relation was used.

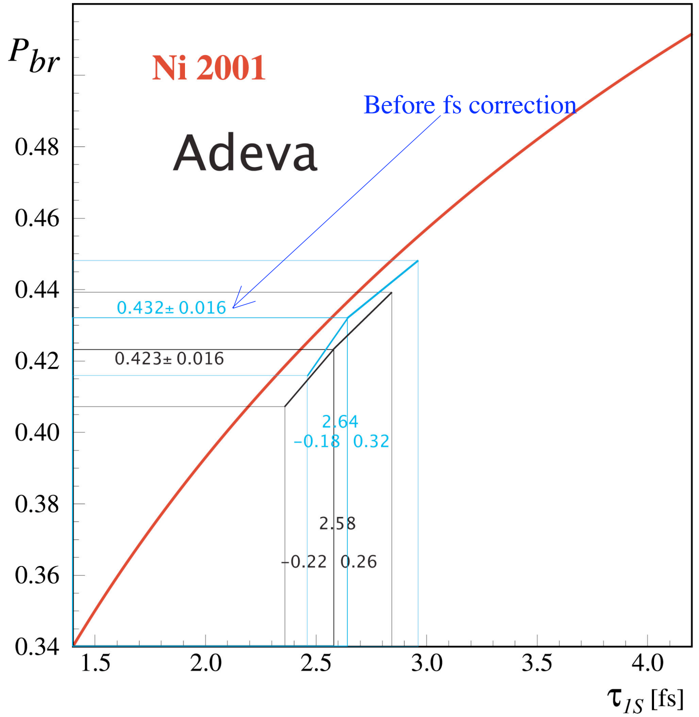
Using the P_{br}-τ relation from the DIRAC lifetime paper, the lifetime without finite size correction as quoted in the conclusion of the note (page 36) τ_{1S} = 2.64 +0.32 -0.18 (stat) yields a Break-up probability P_{br}= 0.436 +0.018 -0.011 (stat). The lifetime after correcting P_{br} for finite size (ΔP_{br}= -0.009), namely τ_{1S} = 2.58 +0.30 -0.26 (stat), leads to a Break-up probability of P_{br}= 0.432 +0.016 -0.014 (stat). We observe:

1. In the note P_{br}= 0.432 ±0.016 is quoted before correction for finite size (page 35). That would correctly provide τ_{1S} = 2.58, but in the note this lifetime is quoted as the value after finite size correction.
2. The difference in P_{br} without finite size correction and after correction is ΔP_{br} = -0.004 and not the quoted -0.009.
3. The statistical errors on P_{br} from the uncorrected lifetime value is extremely asymmetric, while it should be fully symmetric.

P_{br}-τ relations from the note

In the plot (right) we show the published DIRAC P_{br}-τ curve and the P_{br}- and τ-values with statistical errors from the note. We observe:

1. The P_{br}-τ relation for τ_{1S} = 2.64 (P_{br}= 0.432±0.016) is clearly inconsistent with the one for τ_{1S} = 2.58 (P_{br}= 0.432-0.009=0.423±0.016)
2. Both are inconsistent with the published P_{br}-τ relation.



Multiple scattering

Multiple scattering was used (see page 4 of the note) as found in ref 9 of the note. It is not clear whether the 15% increase in multiple scattering was applied everywhere in the set-up, in all upstream components including the target, or only in the MSGCs.

Given the recent findings of DIRAC Note 06-02, the effective thicknesses of the MSGCs and the SFDs were severely underestimated in the GEANT-DIRAC detector description. New GEANT MonteCarlos will be necessary to estimate selectively the influence of the upstream multiple scattering on P_{br}. The error due to multiple scattering in the DIRAC lifetime publication is based on an over-all (upstream and downstream including target) error in multiple scattering. With the above note 06-02 this error source has disappeared.

The statement of the note (page 34) that the radiation length of the upstream detectors is known to ±1.5% is apparently wrong.

Trigger acceptance correction

The procedure is based on the assumption, that accidentals and prompt are equivalent as far as the apparatus is concerned.

Comments:

1. The conditions for accidentals and prompt events are not really identical, since the gates fluctuate for accidentals while they are well defined for prompt events. This is also true for the trigger. Moreover, the momentum distributions are not the same for accidentals and prompt, especially at low momenta.
2. The slopes suggest that the ratio plotted is data/MC and not MC/data
3. The study should have been done on Q_L and not on $|Q_L|$, since an overall slope could be hidden by the constraint $|Q_L|$.
4. The slope corrections should be given explicitely. Monte Carlo and Data should be shown individually.
5. Table 4 is only meaningful if the asymmetry is given also before correction.
6. An empirical correction on the slopes is inappropriate since the reasons for them may be physics or acceptance or physics conditioned acceptance. The presence of slopes demonstrates, that the Monte Carlo contains an incomplete description of the experiment (e.g. trigger-simulation) or physics (e.g. different momentum distributions for π^+ and π^-) The procedure to follow is to find the reason for the slopes and correct the MonteCarlo.
7. Identifying the trigger as the source of the slope irregularities is an ad-hoc assumption and remains to be proven.
8. Non-understood slopes in Q_L may indicate that there are slopes also in $Q_{X,Y}$. This was not investigated. Thus the Q_L slopes may depend on Q_T.
9. The assumption, that a slope found with accidentals is the same also for prompt background, needs verification. A similar study comparing prompt with an appropriate mixture of CC, NC,ACC MonteCarlo is missing.
10. Fig. 22 compares P_{br} as function of the cut limit in Q_L and shows independence. This is rather meaningless as all Q_L dependent irregularities have been smoothed away by the slope corrections. The only thing to conclude is that the slopes for prompt events are similar to those for accidentals. This conclusion was, however, not done.
11. The scientific approach to the problem would be to investigate the sensitivity of the break-up probability on the slopes by not correcting but determining P_{br} by leaving out the first and last momentum band (which show the slopes most pronouncedly) and/or studying the dependence from fit range. Deforming the data by arbitrary corrections for getting higher statistics introduces new systematics, that have not been studied.

Systematic errors

The discussion on systematics must be quantitative. The note gives the impression that the errors were sometimes guessed (trigger acceptance, MSGC noise, SFD simulation).

Correction for the Ni-purity: This correction (see DIRAC lifetime paper) was not done. It should therefore have been added to the systematics.

Finite size correction: The model for correlations is based upon assumptions, which can be checked only at $Q>20$ MeV/c (bulk of data). This is also where the ω, η contributions were fitted to. The model has never been checked at $Q<20$MeV/c, relevant to DIRAC. In fact the DIRAC correlation data deviate from the model for low Q. This is why in the DIRAC lifetime publication a maximum error was given. Nothing has changed since.

K^+K^- and pp_{bar} contamination: This contribution to systematics, the largest in the DIRAC lifetime publication, was not even discussed in the note (strangely enough it shows up in the bibliography). The situation has not changed since writing of the DIRAC lifetime paper.

Concluding remarks

These comments are not exhaustive but address the most evident weaknesses of the note. Once these have been clarified, the reading will be more easy and efficient.

Appendix (D. Drijard)

Standard method

define the function $M(\mu) = \sum_k \frac{U_k^2}{D_k}$

where the numerator is (with i=1,4 channels)

$U_k = N_p^k - \sum_i \mu_i N_i^k$

the denominator D should be the variance of the numerator U

sum of variances of terms (because uncorrelated)

$D_k = V(U_k) = V(N_p^k) + \sum_i \mu_i V(N_i^k)$

numbers N follow Binomial law, hence

$D_k = N_p^k(1 - N_p^k / N_p) + \sum_i \mu_i^2 N_i^k (1 - N_i^k / N_i)$

one may neglect the factors introduced by Binomial law (hence restrict to Poisson)

because they are very close to 1

BUT one could keep them !

$D_k = N_p^k + \sum_i \mu_i^2 N_i^k$

then minimise function M with respect to the set of parameters μ

to check equation (1) of note from Santiago, use change of variable:

$n_i^k = N_i^k / N_i$

$x_i = N_i / N_p$

apply to U_k ==> $\beta\alpha_i n_i^k = \mu_i N_i^k$ equivalently $\beta\alpha_i = \mu_i N_i$

then to D_k ==> $D_k = \beta[(N_p/\beta) n_p^k + \sum_i (\beta/N_p)\alpha_i^2 n_i^k / x_i]$

there are so far 4 parameters $\beta\alpha_i$, not 5 parameters β and α_i. Setting however $\sum_i \alpha_i = 1$ brings back to 4 parameters. It remains to have a correct expression of D_k:

<u>when compared to formula (1), it is NECESSARY to set</u> $\beta = N_p$

and finally $\sum_i \alpha_i = 1$ is an added constraint, imposed to force the total fitted number of Monte Carlo events equal to that of prompt events. Indeed, using $\beta = N_p$, it follows that $\sum_i \alpha_i = \sum \mu_i N_i / N_p$

Properties of parameters from minimisation

Here I use the standard representation (parameters μ).

Bias

If D_k does not depend on parameters, then one can prove that the fit garantees that they are bias-free (mathematical demonstration, not feeling). More generally, this statement is right if in addition the D_k are all positive (but otherwise arbitrary) ... only the precision will depend on this choice.

In the case treated here, D_k is evidently dependent on parameters. One may use the trick of not using this explicit dependence. Using a local algorithm for the minimisation (i.e. not Minuit), then one can drop the contribution of the derivatives of D_k with respect to the parameters. In the case studied so far, U_k is linear with respect to the parameters and thus the minimisation would be solved directly. However, transferring to D_k the parameters μ obtained from the fit will require iterations.

If one needs Minuit for a more difficult case, one may profit of the option to feed the program with derivates and there providing only the derivates of U_k. I do not know if Minuit will detect this trick (it has enough information to find it!).

Uncertainties

If D_k is correctly the variance of U_k, the shape of the function M(μ) in the vicinity of its minimum provides information on the uncertainties on the parameters. Otherwise one could not estimate these precisions by this method (see Monte Carlo simulation at the end). It follows that D_k must be <u>correctly</u> defined. In addition, correct D_k will give rise to minimal uncertainties, independently of the minimisation algorithm.

There should be no problem with local minimisation because U_k is linear with respect to parameters. I do not know how Minuit would behave in this case.

Constraints

The note from Santiago considers cases of additional requirements on parameters:

There is a discussion on β which could be fixed or left adjustable. From the beginning of this note no choice on this is allowed (otherwise the calculation of uncertainties would be wrong).

In another case, some α_i parameters are fixed. No problem for this, the number of free parameters is simply decreased.

The case of imposing $\sum_i \alpha_i = 1$ gives rise to more complex equations. It may again be solved directly. Thus one eliminates one parameter and U_k follows :

$U_k = N_p^k - \sum_{i=1,3} \mu_i N_i^k - (1 - \sum_{i=1,3} \mu_i N_i / N_p) N_4^k$

and D_k is a bit more complicated.

Monte Carlo check of uncertainty

The safest way to check the estimation of uncertainties is to generate a set "experiments" similar to ours and compare calculation directly to estimation from width of the fitted parameters. It is enough to have a simple representation of the data, without all fancy corrections due to the detector acceptance.