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Angular Resolution and Multiple Scattering in ‘downstream’ tracking.

Y. Allkofer, A. Benelli, L.Tauscher

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1 Introduction

The reconstruction of tracks using only the drift chambers[1] ('downstream') fixes the momentum of a track by extrapolating it in x-direction through the magnetic field back to the target. The momentum resolution thus is limited by multiple scattering in the Al-window at the exit of the magnet and, to a lesser extent, in the bulk of up-stream detectors [2]. In y-direction the extrapolation through the magnet is uncertain because of the multiple scattering the track undergoes in the Al-window and because of uncertainties in defining the track by the tracking procedure, especially the magnetic deflection in vertical direction.

Using measured data (Ni2001) and events that were reconstructed by 'downstream' tracking and the usual tracking involving up-stream detectors ('full') we may extract the deviation produced by multiple scattering in the Al-window. This is done by comparing the extrapolated track inclination in y-direction from 'downstream' tracking (which does not include multiple scattering) with the inclination of the real track from 'full' tracking, which includes multiple scattering. We may as well investigate multiple scattering in the up-stream detectors by measuring the kinck the track receives in the up-stream detectors while propagating from the target to the Al-window. Finally we test the accuracy by which the vertical magnetic deflection is understood in the simulation.

The Monte Carlo simulation was using the new event generators and the new version of DIRAC-GEANT (2.66)[3]. Pions were as usual allowed to decay. In the analysis muons were vetoed. No pions from weak decays (distant from the target) were considered due to lack of corresponding event generators. No cuts were applied whatsoever for tracking and reconstruction.

The aim of the study is to test multiple scattering in the Al-window and in the bulk of up-stream detectors and to assess the angular uncertainty from the 'downstream' tracking.

2 Procedure

Multiple scattering is approximately described by the Gaussian approximation, where the Gaussian has a sigma, that is inversely proportional to the momentum [4]. The angular distribution

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[1] see DIRAC Note 07-07

[2] Since the magnetic deflection angle and the multiple scattering angle are both inversely proportional to the momentum there is no way to disentangle the two.

[3] New multiple scattering, new generators etc.

[4] Particle data group, Physics Lett. B592 (2004) 1

produced by multiple scattering, multiplied with the momentum of the scattered particle has a sigma of

$$\sigma_{\theta p} = \frac{0.0136[GeV/c]}{\beta} \times \sqrt{\frac{x}{X_0}} \times [1 + 0.038\ ln(\frac{x}{X_0})] \tag{1}$$

with θ the scattering angle, p the track momentum in GeV/c, β the corresponding velocity of the particle with unit charge, x the thickness and X_0 the radiation length of the scatterer.

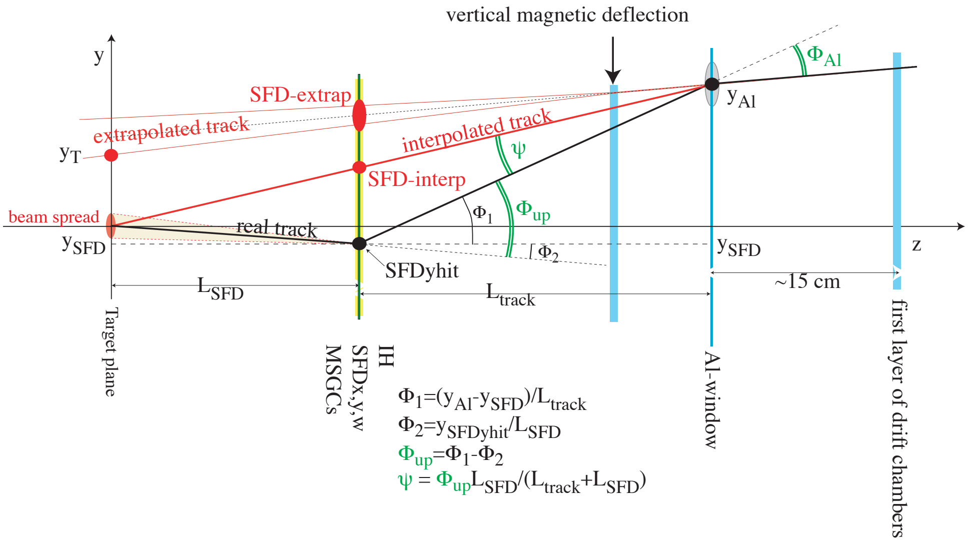

Figure 1: *Schematics for the vertical track propagation with definitions used later.*

The scattering angle in vertical direction was determined by the difference in vertical position of the ‘downstream’ track at the y-plane of the SFD and the identified SFD-y hit position from ‘full’ reconstruction, divided by the projection of the effective track length from the SFD-y plane to the Al-window onto the z-x-plane (see Fig. 1).

2.1 Track length

The effective length is obtained from the known momentum p of the track and from the measurements of its x-position (x_{Al}) and inclination (φ_{Al}) in the plane of the Al-window using the drift chambers only (see Fig. 2). Moreover, we use the magnet parameters $B = 1.65[T]$ and $BL_{eff} = 2.21[Tm]$, resulting in $L_{eff} = 1.34[m]$, and the relation $p[GeV/c] = 0.29979B[T]R[m]$. Finally we need the geometrical distances from the target to the SFD-y-plane, $L_{SFD} = 2.878[m]$, the distances from the SFD-y-plane to the middle of the magnet $L_{Magnet} = 5.422[m]$ and to the Al-window $L_{Al} = 6.860[m]$. The distance between the magnet exit and the Al-window is $L_{M-W} = L_{Al} - L_{Magnet} - 0.5\ L_{eff} = 0.770[m]$. The effective track length L_{track} then is composed of the upstream straight line L_{up}, the circular part L_{curv} and the down stream straight part L_{down} (see Fig. 2):

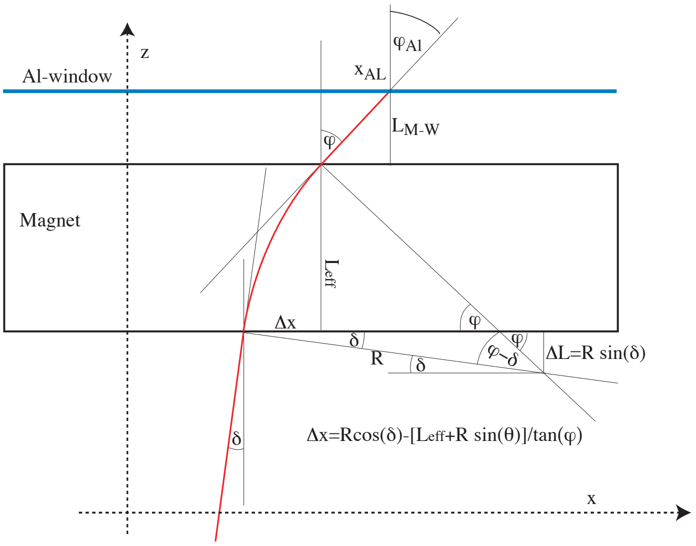

Figure 2: *Horizontal magnetic deflection and definitions.*

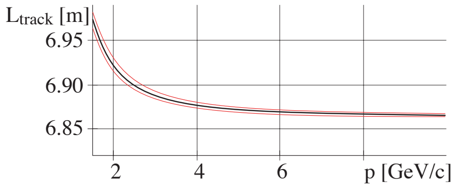

Figure 3: *Track length as a function of momentum. The red lines represent the limits given by the maximum incident angles $\delta_{max} \approx \pm 1^o$.*

$$
\begin{aligned}
L_{track} &= L_{up} + L_{curv} + L_{down} \\
L_{up} &= L_{Magnet} - 0.5\ L_{eff} \\
L_{down} &= L_{M-W}/cos(\varphi_{Al}) \\
L_{curv} &= (\varphi - \delta)R, \text{ with } R = \frac{p}{0.2997\ B}
\end{aligned}
$$

with δ the incident angle to the magnet, and φ the exit angle from the magnet. With the help of $x_{out} = x_{Al} - L_{M-W}\ tan(\varphi_{Al})$ we obtain semi-empirically and assuming $\varphi_{Al} \approx \varphi$

$$
\begin{aligned}
\varphi &= \frac{15}{4}\frac{1}{(p-0.5)^{1.05}}\left[[-1 + \sqrt{1 + \frac{4}{15}(p-0.5)^{1.05}(2\frac{x_{out}}{L_{up}+L_{SFD}} + 0.59958B\frac{L_{eff}}{p})} \right] \\
\delta &= -0.29979B\frac{L_{eff}}{p} + sin(\varphi)
\end{aligned}
$$

The track length as a function of momentum is shown in Fig. 3. The accuracy of the track-length determination is estimated to be at the level of centimeter as compared to a total length of about 7 meters, hence of the order of permille.

2.2 Vertical magnetic deflection and magnet centre

Vertical magnetic deflection is caused by the z-components of the fringe fields of the magnet at its downstream exit, as there the track momenta have large components transverse to the fringe field (angles in horizontal direction $\varphi_{Al} > 7^o$), while at the up-stream fringe fields the track inclinations δ are negligible in both directions (c.f. Fig. 2).

Vertical magnetic deflection does not occur close to the vertical center of the magnet, as the fringe fields there have no z-component[5].

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[5] This is not true anymore if the middle plane of the magnet is inclined with respect to the spectrometer axis.

We concentrate our analysis onto the central part of the magnet in vertical direction. In order to find the centre we have analyzed the data in 2 cm intervals in the y_{Al} coordinate and measured the angle between the backward extrapolated track and the interpolated track (c.f. Fig. 1):

$$\Delta\Phi_{extrap-interp} \quad = \quad \frac{p_y}{p_z} - \frac{y_{Al}}{L_{track} + L_{SFD}}$$

where $p_{y,z}$ are the p components obtained from the extrapolated track.

A scan in intervals of 2 cm shows, how the angular distribution changes as a function of vertical position of the interval. The result is shown Fig. 4.

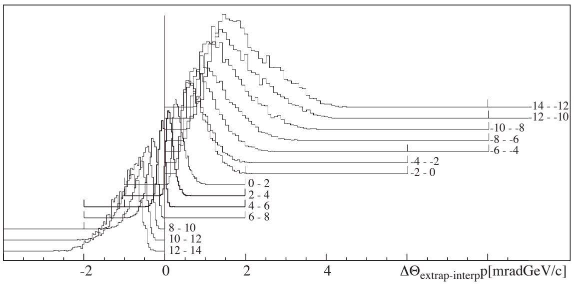

Figure 4: *Vertical distributions of the difference of angle between the extrapolated track and and the interpolated track* $\Delta\Phi_{extrap-interp}$*, multiplied with the track momentum p, as a function of* y_{Al} *interval.*

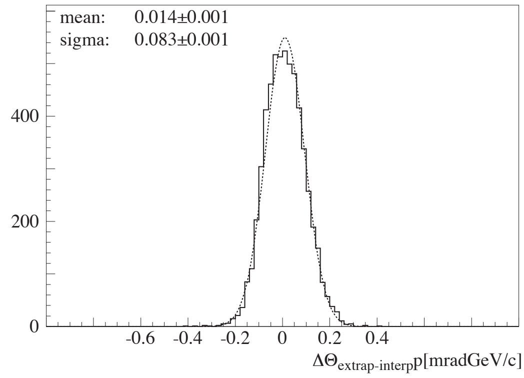

Figure 5: *Same as Fig 4, but for the centre at* $y_{Al} = 4.21$ *cm.*

In the interval around the magnet centre the distribution for $\Delta\Phi_{extrap-interp}p$ should be centered around $\Delta\Phi_{extrap-interp}p = 0$. We find the centre to be at $y_{Al}^{centre} = 4.21$ cm with a width of $\sigma_{\Delta\Phi p} = 0.083$ mradGeV/c. The corresponding distribution is shown in Fig. 5. The width is due to residual vertical magnetic deflection and intrinsic accuracy of the transport algorithm through the magnetic field.

The following investigations will thus use the interval $3.21 \leq y_{Al} \leq 5.2$ for the center part of the magnet.

2.3 Intrinsic directional resolution of DC-tracking

The track definition by the drift chambers alone was studied by comparing the track inclination in horizontal and vertical direction from the track fit with the true track inclination, using Monte Carlo events. In Fig. 6 we show the results.

The intrinsic resolution for determining the direction of a track by the drift chambers is roughly equal in both directions and of the order of 0.3 mrad or 0.6 mradGeV/c. This includes multiple scattering in the drift chambers as well as uncertainties from the fitting algorithm.

3 Multiple scattering in the Al-window using track extrapolation

In this section we present the results using the track extrapolation as described above. In doing so we assume the incident track inclinations on the Al-window to be the same as the outgoing

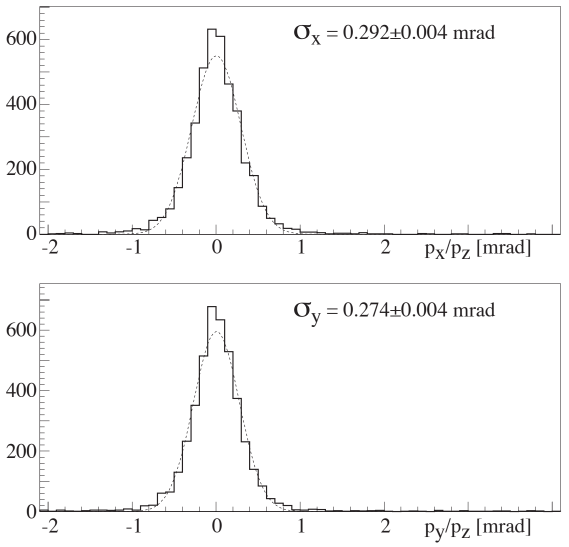

Figure 6: *Difference of track inclinations (track fit minus true) at the exit of the Al window for Monte Carlo events. Ni-2001 data. The hatched curve is a Gaussian, fitted to the data, with parameters as shown in the figure.*

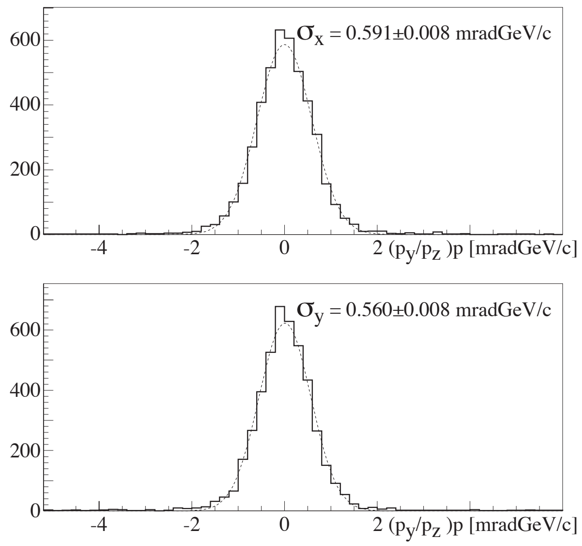

Figure 7: *Same as Fig 6, but the angle multiplied with the momentum p of the track.*

ones (see also Fig. 1). Moreover we assume the track direction to be the one from the drift chamber track fit. Both assumptions lead to a track that does not contain multiple scattering in the Al-window and in the first drift chamber set. Finally, by extrapolating the track through the magnet it is vertically deflected while the comparison track from the track intersect with the Al-window y_{Al} to the SFD-y hit ignores this deflection partly, but includes the effect of multiple scattering in the Al-window. Selecting only the central part of the magnet, magnetic deflection is largely eliminated.

The angle investigated here is (see Fig. 1)

$$\begin{aligned}\Phi_{Al} &= \arctan\left(\frac{y_{Al} - SFD_{extrap}}{L_{track}}\right) - \arctan\left(\frac{y_{Al} - SFD_{hit}}{L_{track}}\right) \\ &\approx \frac{SFD_{extrap} - SFD_{hit}}{L_{track}}\end{aligned}$$

where we accept an uncertainty of the order of 2 permille.

The coordinate SFD_{extrap} is obtained from the extrapolated track's y-coordinate at the target (y_T) and the track's slope, obtained from p_y/p_z:

$$SFD_{extrap} = y_T + \frac{p_y}{p_z} L_{SFD}$$

In Figures 8 and 9 the angular distributions are shown for data and Monte Carlo simulation, for the central part. For comparison, we also show the distributions for full y-acceptance. Only events were used which were reconstructed with 'downstream' and 'full' tracking. In Table 1 we show the fitted values for the distributions. They should directly reflect the multiple scattering in the Al window.

We observe:

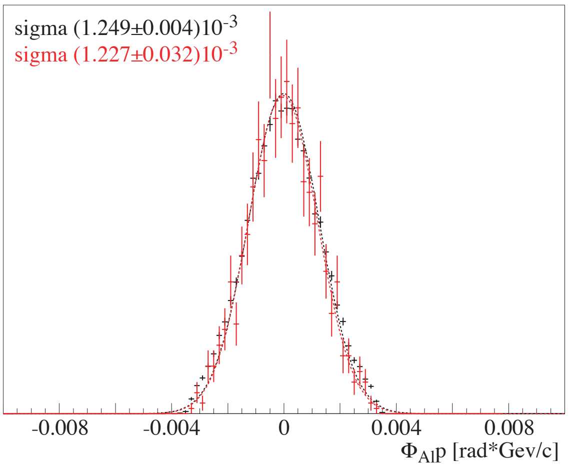

Figure 8: $\Phi_{Al}P$*-distribution in vertical direction of tracks from both arms of the spectrometer using events that were reconstructed with "full" and "downstream' tracking (extrapolation method). Central part of the magnet: red, full y-acceptance: black. Ni-2001 data. The hatched curve is a Gaussian, fitted to the data, with parameters as shown in the figure.*

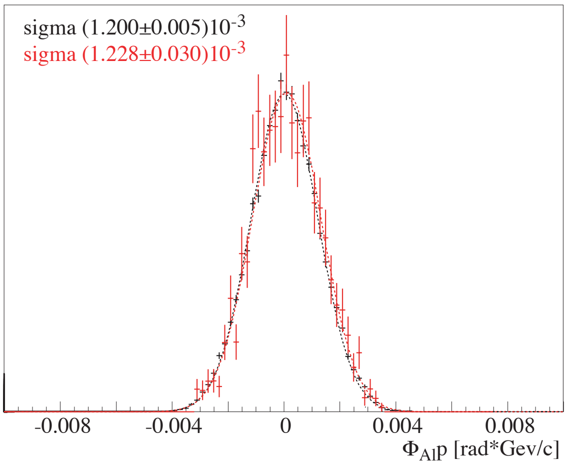

Figure 9: *Same as Fig 8, but for Monte Carlo. Ni-2001 Monte Carlo.*

Table 1: *Results from the fits to the angular distributions* $\Phi_{Al}p$ *for data and Monte Carlo using the extrapolated track, for the central part of the magnet.*

Type	$\sigma_{\Phi_{Al}P}$ [mrad GeV/c] data	$\sigma_{\Phi_{Al}P}$ [mrad GeV/c] Monte Carlo
central part	1.23±0.03	1.23±0.03

- the Gaussians fitted to the distributions from data and Monte Carlo reproduce the distributions well.
- data and Monte Carlo agree within errors for the central part of the magnet.
- widths for the central part and for full y-acceptance agree within errors for data and Monte Carlo.
- The width is larger than expected.

The expected multiple scattering in the Al-window, based on the measured thickness of 680 μm [6], should result in $\sigma_{\Phi_{Al}P} = 0.963 \pm 0.001$ mrad*GeV/c [7] [8]. The measured width of Table 1 corresponds to a thickness of $x/X_0 = 0.0118 \pm 0.0005$, hence 0.0043 larger than the Aluminum thickness. The additional width is $\sigma_{add} = 0.706$ mradGeV/c.

The intrinsic resolution of the drift chambers of 0.29 mrad or 0.591 mradGeV/s explains a large part of the "additional" width, but leaves us still with a thickness of the Al-window of 0.00875, roughly 16 % more than the true thickness.

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[6] L. Nemenov, private communication.

[7] We have used $X_0^{Al} = 8.9943$ cm and obtain $x/X_0 = 0.00756$.

[8] the multiple scattering measurement had an error of $\approx 1\%$.

We conclude that the extrapolation method confirms the expected multiple scattering in the Al window, but is heavily biased by the intrinsic angular resolution of the drift chamber tracking.

4 Multiple scattering in the up-stream detectors using track interpolation

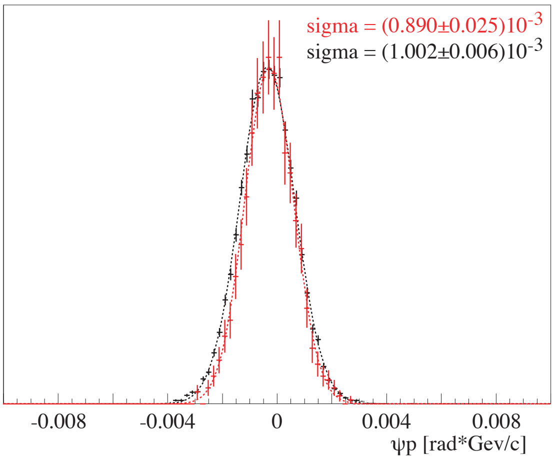

Figure 10: *ψp-distribution in vertical direction of tracks from both arms of the spectrometer using events that were reconstructed with "full" and "downstream' (interpolation method) tracking. Black: full y-acceptance, red: central part of magnet. Ni-2001 data. The hatched curve is a Gaussian, fitted to the data, with sigmas as shown in the figure.*

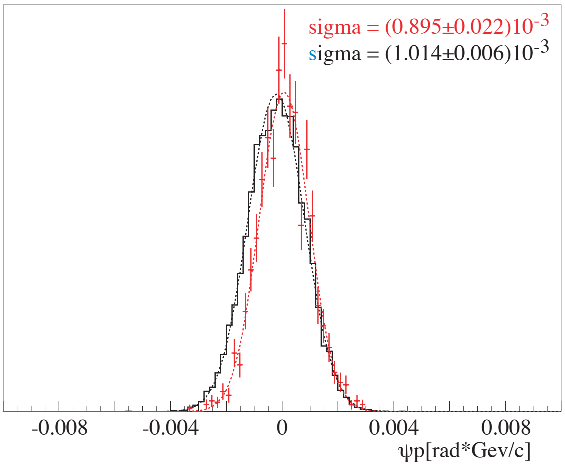

Figure 11: *Same as Fig 8, but for Monte Carlo. Ni-2001 Monte Carlo.*

In order to further study effects from the tracking procedure, alignement we now abandon the extrapolation of the track through the magnet and instead use, for the vertical direction, the interpolation between the track position at the Al-window and the track position at the SFD, reconstructed from an assumed track origin at the centre of the target (SFD-interp in Fig. 1). This procedure provides the angle ψ in Fig. 1, which is related to the scattering angle Φ_{up}, produced by the bulk of all up-stream detectors, by $\psi \approx \Phi_{up} L_{SFD}/(L_{track} + L_{SFD})$. A common error in both, data and Monte Carlo is due to the neglect of vertical magnetic deflection in both, the interpolated track and the "true" track, which partly cancels in the comparison. Restricting the event selection to the central part of the magnet eliminates this uncertainty.

From the dedicated measurement on multiple scattering we expect[9] a total sigma of $\sigma_{\Phi_{up}P} = 3.084 \pm 0.017$mrad*GeV/c. With an average track length $L_{track} = 6.92$ m we expect $\sigma_{\psi P} \approx 0.906 \pm 0.005$ mrad*GeV/c.

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[9] The $\sigma_{\theta P}$ of all scatterers were not added in quadrature but the effective thicknesses in units of radiation length, as suggested by the PDG, were added to obtain a total thickness of 0.06411 ± 0.000643.

Table 2: *Results from the fits to the angular distributions ψP for data and Monte Carlo using the interpolated track, for the central part of the magnet and for full y-acceptance.*

Type	data $\sigma_{\psi P}$ [mradGeV/c]	Monte Carlo $\sigma_{\psi P}$ [mradGeV/c]	expected $\sigma_{\psi P}$ [mradGeV/c]
central part	0.890 ± 0.025	0.894 ± 0.022	0.906 ± 0.005
full y-acceptance	1.002 ± 0.006	1.014 ± 0.006	

The angle investigated here is (see Fig. 1)

$$\psi \quad \approx \quad \frac{SFD_{interp} - SFD_{hit}}{L_{track}}$$

.

In Figures 10 and 11 the angular distributions obtained with the interpolation method are shown for data and Monte Carlo simulation. The fit results are summarized in Table 2.

- the Gaussians fitted to the data from experiment and Monte Carlo reproduce the distributions very well.
- data and Monte Carlo agree within errors
- the central part reproduces the expected multiple scattering within the errors.

We therefore conclude, that multiple scattering as described in Monte Carlo for the up-stream detectors agrees with data. We furthermore conclude that the interpolation method is a valuable method for determining the up-stream part of the track in ‘downstream’ tracking, as it is independent of the multiple scattering in the Al-window and on the intrinsic drift chamber resolution.

5 Direct measurement of multiple scattering in the up-stream detectors

In order to verify this finding, we measure the integral multiple scattering in the upstream detectors directly (see Fig. 1) through the angle Φ_{up}. We assume the real track to originate from the target, producing the hit in SFD-y and then propagating linearly to the measured intersect with the Al-window in the central part of the magnet.

The angle investigated here is (see Fig. 1)

$$\Phi_{up} \quad \approx \quad \frac{y_{Al} - SFD_{hit}}{L_{track}} - \frac{SFD_{hit}}{L_{SFD}}$$

.

The result is shown in Figs. 12 and 13 and summarized in Table 3. The Monte Carlo values agree to within 1.0% or within 2σ with the expected value.

The measured sigma $\sigma^{data}_{\Phi_{up}p} = 3.00 \pm 0.08$ mradGeV/c agrees with the expected width of $\sigma_{\Phi_{up}p} = 3.084 \pm 0.017$mradGeV/c from section 4. This method has made explicit use of the measured y-coordinate at the Aluminum window, y_{Al} and establishes the correctness of this measurement.

The measured width corresponds to a total thickness of all up-stream detectors of 0.061 ± 0.004, hence in agreement of the ‘true’ thickness of 0.0641 in units of radiation length.

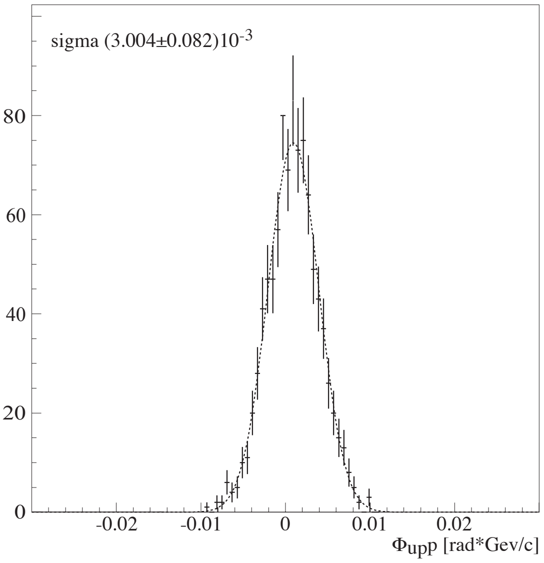

Figure 12: *Multiple scattering in the bulk of upstream detectors. $\Phi_{up}p$-distribution in vertical direction of tracks from both arms of the spectrometer using events that were reconstructed with "full" and "downstream' tracking. Ni-2001 data. The hatched curve is a Gaussian, fitted to the data, with parameters as shown in the figure.*

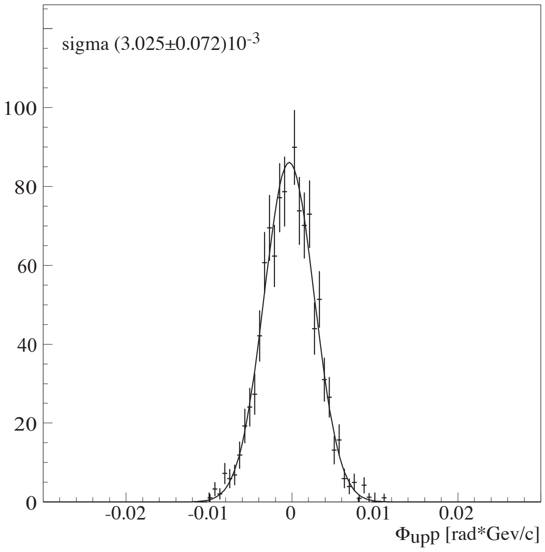

Figure 13: *Same as Fig 8, but for Monte Carlo. Ni-2001 Monte Carlo.*

Table 3: *Results from the fits to the angular distributions θp for data and Monte Carlo for the direct measurement of multiple scattering in th up-stream detectors, for the central part of the magnet.*

Type	data $\sigma_{\psi P}$ [mradGeV/c]	Monte Carlo $\sigma_{\psi P}$ [mradGeV/c]	expected $\sigma_{\psi P}$ [mradGeV/c]
central part	3.00 ± 0.08	3.02 ± 0.07	3.084 ± 0.017

6 Conclusion

Using events from Ni2001, which were reconstructed with 'downstream' and 'full' tracking, we have tested experimentally the following features:

- The alignment of the spectrometer with the magnet in vertical direction.
- The intrinsic angular resolution of the track reconstruction by the drift chambers
- the multiple scattering in the Al-window of the magnet exit.
- the multiple scattering in the up-stream detectors, using the track interpolation method, and using the direct determination method.

We conclude that

- Monte Carlo describes the data on the 3% level
- Multiple scattering in the up-stream detectors as obtained from Monte Carlo is established on the level of 3 percent using the interpolation and the direct method.
- Multiple scattering in the Al-window of the magnet exit is confirmed, but heavily biased by the intrinsic resolution of the drift chambers.
- The interpolation method for reconstructing the up-stream part of the track as obtained from the ‘downstrem’ tracking is independent of multiple scattering in the Al-window and of the intrinsic angular resolution of the drift chambers and therefore should be used in the general ‘downstrem’ tracking procedure.