A permanent Magnet to search for long-lived $\pi \cdot \pi^{-}$atoms

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Dirac collaboration
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## 1. Introduction

The DIRAC experiment [1] has been running at CERN PS measuring lifetimes of $\pi \cdot \pi$, $K \cdot \pi-$ and $\pi \cdot \mathrm{K}$ atoms by a two arm spectrometer at CERN PS as shown in Fig. 1. Up to now more than $21,000 \pi \cdot \pi^{-}$pairs originated from the $\pi^{\cdot} \pi^{-}$atom $\left(\mathrm{A}_{2^{*}}\right)$ breakup were identified and the overall accuracy of the $\mathrm{A}_{2 *}$ lifetime is about $9 \%$ in accordance with the DIRAC proposal.


Figure 1: Dirac setup: MDC are microdrift gas chambers, SFD is a scintillating fiber detector and IH is a scintillation ionization hodoscope. Downstream the spectrometer magnet there are drift chambers (DC), vertical (VH) and horizontal (HH) scintillation hodoscopes, Cherenkov detectors containing nitrogen $(\mathrm{CH})$, heavy gas $C_{4} F_{10}$ and aerogel radiatiors, shower detectors (PSh) and scintillation muon detectors (MU).

A new DIRAC proposal [2] was presented to measure long-lived $\pi \cdot \pi$ - atoms at CERN PS. This measurement allows to extract the difference $\left|a_{0}-a_{2}\right|$ of $s$-wave $\pi \pi$-scattering length with accuracy of $4.3 \%$. In addition, the observation of long-lived (metastable) $A_{2^{\pi}}$ states will be performed with the same setup. This observation opens a possibility to measure the energy difference between $n s$ and $n p$ states and to determine the value of another combination $2 a_{0}+a_{2}$ of $\pi \pi$ scattering length in a model-independent way. In combination with the first measurement it allows to get $a_{0}$ and $a_{2}$ separately. The long long-lived $\pi^{\cdot} \pi^{-}$atoms are generated at the Be target and broken up at the the Pt foil as shown in Fig. 2. The distance between the Be target and the Pt foil was chosen to be 100 mm
for excluding interaction of the primary beam halo with the Pt foil.


Figure 2: The long long-lived $\pi \cdot \pi$ - atoms are generated at the Be target and broken up at the the Pt foil.

Simulation of "experimental data" has been performed adding the contributions of simulated : atomic pairs from long-lived atoms produced in the Platinum foil, atomic pairs from the Beryllium target, "Coulomb pairs", "non-Coulomb pairs" and accidentals. Fig. 3 presents the distribution of simulated data over the $Y$ projection of relative momentum $Q$. The cuts on $\left|\mathrm{Q}_{\mathrm{x}}\right|<1 \mathrm{MeV} / \mathrm{c}$ and $\left|\mathrm{Q}_{\mathrm{L}}\right|<1 \mathrm{MeV} / \mathrm{c}$ have been applied. The quantity $\mathrm{Q}_{\mathrm{T}}, \mathrm{Q}_{\mathrm{L}}, \mathrm{Q}_{\mathrm{X}}$ and $\mathrm{Q}_{\mathrm{Y}}$ is the transverse, longitudinal, X and Y components of the relative momentum Q in the atomic pair c.m.s. Simulation shows that in each projection such criterion selects more than $90 \%$ of "atomic pairs" from long-lived atoms. Hatched area is a sum of all pairs produced in Beryllium target and light area corresponds to "atomic pairs" from long-lived atoms broken in the Platinum foil.


Figure 3: Simulated distribution of $\pi \cdot \pi$ pairs over $Q_{Y}$ with criteria: $\left|Q_{X}\right|<1 \mathrm{MeV} / \mathrm{c},\left|Q_{L}\right|<1 \mathrm{MeV} / \mathrm{c}$. "Atomic pairs" from long-lived atoms (light area) above background produced in Beryllium target (hatched area)

The signal-to-background ratio is small. It can be improved by installation of an additional retractable magnet which induces the horizontal magnetic field in the gap between Be target and Pt foil. A magnet with the bending power of 0.01 Tm (Tesla*meter) would shift the $Q_{Y}$ value by 6 $\mathrm{MeV} / \mathrm{c}$ only for the pairs produced in the Be target leaving unchanged the $Q_{Y}$ distribution of the pairs produced in the Pt foil. The magnet with such characteristics is installed (see Fig. 4).


Figure 4: Permanent magnet dimensions and arrangement of the Beryllium target, permanent magnet and Platinum foil.

Fig. 5 shows the resulting $Q_{Y}$ distribution with the improved signal-to-background ratio.
We could discriminate the atomic pairs from the background easier than in Fig. 3.


Figure 5: Simulated distribution of $\pi \cdot \pi$ pairs over $Q_{Y}$ with criteria: $\left|Q_{X}\right|<1 \mathrm{MeV} / \mathrm{c},\left|Q_{L}\right|<1 \mathrm{MeV} / \mathrm{c}$. Additional magnet is implemented. "Atomic pairs" from long-lived atoms (light area) above

A new sample of simulated data with the additional magnet has been fitted with the sum of the distributions of atomic pairs from long-lived atoms, "Coulomb pairs" and "non-Coulomb pairs". The atomic pairs produced in the Be target with the $Q_{Y}$ about $6 \mathrm{MeV} / c$ are absent in the fit region.

The fit results for distribution over $Q_{L}$ (with cut $Q_{T}<1 \mathrm{MeV} / c$ ) are presented in Fig. 6.


Figure 6: Simulated distribution of $\pi \cdot \pi$ pairs over $Q_{L}$, with criterion $Q_{T}<1 \mathrm{MeV} / \mathrm{c}$. "Experimental" data (points with error bars) are fitted by a sum of "atomic pairs" from longlived states (dashed line), "Coulomb pairs" (by dotted-dashed line), "non-Coulomb pairs" (dotted line). The background sum is shown by the solid line.

Number of atomic pairs are found to be:

$$
\begin{equation*}
n_{A}^{l}=281 \pm 48 \tag{1}
\end{equation*}
$$

Analysis of the experimental data accounting widths of the atomic pairs distribution over different components of the relative momentum $Q$ allows to find the variable $F$ which provides the distribution of atomic pairs with the best signal-to-background ratio:

$$
\begin{equation*}
F=\sqrt{\left(\frac{Q_{x}}{0.50}\right)^{2}+\left(\frac{Q_{Y}}{0.32}\right)^{2}+\left(\frac{Q_{L}}{0.56}\right)^{2}} \tag{2}
\end{equation*}
$$

Here $0.50,0.32$ and 0.56 in units of $\mathrm{Mev} / c$ are RMSs of the atomic pairs distribution over corresponding
components of the relative momentum $Q$.
Fig. 7 presents results of analysis for distribution of $\pi \cdot \pi$ - pairs over $F$. We find excess events of the atomic pairs over the solid line at $F<3 \mathrm{MeV} / c$. It provides a greater number of found atomic pairs due to the weaker cut on $Q_{T}<2 \mathrm{MeV} / \mathrm{c}$ and a better signal-to-error ratio:

$$
\begin{align*}
& n_{A}^{l}=327 \pm 37  \tag{3}\\
& \frac{n_{A}}{\partial_{n_{A}}}=8.8 \tag{4}
\end{align*}
$$

It is worth noting that the simulated number $n_{A}^{l}$ is 310 .


Figure 7: Simulated distribution of $\pi \cdot \pi$ pairs over $F$, with criterion $Q_{T}<2 \mathrm{MeV} / \mathrm{c}$.
"Experimental" data (points with error bars) are fitted by a sum of "atomic pairs" from longlived states (dashed line), "Coulomb pairs" (by dotted-dashed line), "non-Coulomb pairs" (dotted line). The background sum is shown by the solid line.

In order to declare an observation of long-lived $\pi \cdot \pi$ atoms it is needed to achieve a ratio between signal and error greater than 5. The current simulation provides the ratio of 8.8. This means that probability to observe long-lived $\pi \cdot \pi$ atoms is close to $100 \%$.

In the approach without the additional permanent magnet the accuracy of effect
separation will be worse. At equal conditions the simulated number of reconstructed atomic pairs is

$$
\begin{equation*}
n_{A}^{l}=334 \pm 89 \tag{5}
\end{equation*}
$$

To achieve the value of 5 in the signal-to-error ratio, required for the observation, the needed statistic should be increased by 1.8 that can be accomplished with a high efficiency of the event reconstruction and/or a longer run time.

## 2. Permanent magnet

We had experienced to fabricate a small analyzing magnet consist of neodymium [3] to get better separation of $\pi^{+}$and $\pi^{-}$- tracks broken from an $\pi^{+} \pi^{-}$atom in scintillation fiber detectors [4] set between the production target and the main analyzing magnet in the Dirac spectrometer [5]. We utilized a permanent magnet to realize the required-measuring conditions. The magnet should be installed in the vacuum target station just after the production target made of Beryllium in Fig. 1 where the primary-proton beam delivered from CERN PS passing through. The beam with the halos should pass through without hit the magnet. The magnet is set between the production target and the Platinum foil where the atomic pairs are broken as in Fig.4. The distance is determined to be 10 cm so that most of the long-lived atomic pairs are alive. The magnetic field strength should not much higher than 0.1 T not to quench the p or d states of the atoms to the s state since the quench effect is proportional to the square of the field strength as long as the enough bending power of BL of 0.01 Tm to move aside the peak of the short-lived $\pi \cdot \pi$ - pairs in $\mathrm{Q}_{\mathrm{Y}}$ as shown in Fig. 5. At and after the breaking target the magnetic field should be low enough not to bend the charged pions broken from $\pi^{\prime} \pi^{-}$atoms.

We utilize a permanent magnet to realize the required conditions. Permanent magnet has advantages over electromagnets. It does not need an electric current to magnetize which generates heat in the vacuum setting. The magnetic field is stable since there is no thermal and mechanical change due to the electric current. But it is reported that the magnetic field strength would decrease due to strong irradiations from protons and neutrons, on the other hand the degradation seems rather weak for gamma rays [6-8]. We should keep a watch on it in the radiation environment.

We made a permanent magnet as shown in Figs. 8 and 9. A couple of neodymium pieces of $70 \times 60 \mathrm{~mm}^{2}$ with the thickness of 5 mm were employed. The gap is 60 mm and the integrated magnetic field along the beam is around 0.01 Tm . The external sizes of the magnet are 150 mm wide, 80 mm high and 60 mm deep. Two permanent magnet poles are glued on the inner facing surfaces of the iron frame. The dimensions of the permanent magnets are $70 \mathrm{~mm} \times 60 \mathrm{~mm} \times 5 \mathrm{~mm}$. The
gap distance between the two magnets is 60 mm . Each permanent magnet is composed of a piece of $70 \mathrm{~mm} \times 40 \mathrm{~mm}$, two pieces of $30 \mathrm{~mm} \times 20 \mathrm{~mm}$, and a piece of $20 \mathrm{~mm} \times 10 \mathrm{~mm}$, as shown in Fig. 9. The permanent magnet material of the largest plates is N35 and that of the rest is N40, where the remanent field Br of N 40 and N 35 are specified as $1.22 \sim 1.25 \mathrm{~T}$ and $1.17 \sim 1.22 \mathrm{~T}$, respectively. The leg thickness of the iron frame is 20 mm and the thickness of the top and bottom plate is 10 mm . The iron frame is made of structural mild steel, which should correspond to SS 400. Because the magnetic field in the gap is not generated by the iron part but the permanent magnets, the field should not be much affected by the difference of irons, as long as it has enough high permeability.


Figure 8: A drawing of the permanent magnet with the axes of the coordinates


Figure 9: A picture of the permanent magnet. The directions of + and - for the $X$ coordinate labeled on the surface should be reversed. The right-handed coordinate system is employed as shown in Fig. 8.

## 3. Magnetic field simulation

RADIA version 4.29[9-11] is used for the magnetic field calculation. The calculated geometry is shown in Fig. 10, where the permanent magnet plate dimensions are 5 mm in thickness, 60 mm in length and 70 mm in width. The coordinate system is shown in the figure, where the origin is coincides with the centroid of the magnet. In the calculation, three symmetries are used: $x-y$ plane, y-z plane, and $z-x$ plane, which reduces the system $1 / 8$ comparing with the full system. The remanent flux density of the magnet is assumed to be 1.22 T and the recoil permeability of 1.05 . The return yoke material is "steel37" equipped in RADIA, which is an inexpensive steel with $\mathrm{C}<0.13 \%$.

The resulted field map of By is shown in Table 1. Since the assumption may not reflect the real situation, the result is scaled to match the measured value at the origin $(0,0,0)$, where the factor was 0.983 . This value seems good for this kind of calculation.

Figure 11 shows the $y$-component of the magnetic flux density along $z$ on $x=y=0$ line and $x=y=10$. The integrated values on these lines are as follows:

$$
\begin{equation*}
\int_{0 m m}^{100 m m} B y_{x=0, y=0}(z) d z=0.00490[\mathrm{~T} . \mathrm{m}], \quad \int_{0 m m}^{100 m m} B y_{x=10, y=10}(z) d z=0.00492[\mathrm{~T} . \mathrm{m}], \tag{6}
\end{equation*}
$$

where only the half of the region is counted along the path. The $x$-component and $y$-component along $x=y=10 \mathrm{~mm}$ line are shown in Fig. 12,

$$
\begin{equation*}
\int_{0 m m}^{100 m m} B x_{x=10, y=10}(z) d z=0.316[\mathrm{mT} . \mathrm{m}], \quad \int_{0 m m}^{100 m m} B z_{x=10, y=10}(z) d z=1.297[\mathrm{mT} . \mathrm{m}] \tag{7}
\end{equation*}
$$



Fig. 10: Calculated geometry, the unit is in mm: Top View, Front view and side view.


Fig. 11: Calculated By distributions on lines of $x=y=0$ and $x=y=10 \mathrm{~mm}$.


Fig. 12: Calculated $B x$ and $B z$ distributions on line $x=y=10 \mathrm{~mm}$.

Table 1: The field map calculated by RADIA. The values are scaled to match the measured value,
where the factor was 0.983 . Units are in Gauss and mm.


|  |  |
| :---: | :---: |
| $\mathrm{z}=0$ | 13271325132013121300128412641240121111771137 |
|  |  |
| $\mathrm{z}=10$ |  |
| $\mathrm{z}=15$ | 10 |
| $\mathrm{z}=20$ | 105410381018 |
|  | 52948942934922 |
|  | 7 |
|  | $\begin{array}{llllllllllllllllllll}639 & 638 & 635 & 631 & 625 & 617 & 607 & 595 & 581 & 565\end{array}$ |
|  | $\begin{array}{lllllllllllllll}3 & 492 & 490 & 486 & 482 & 475 & 468 & 458 & 44\end{array}$ |
| $\mathrm{z}=45$ | $\begin{array}{llllllllll} & 369 & 368 & 365 & 361 & 356 & 350 & 343 & 335\end{array}$ |
|  | 73 273271269267 |
|  | $\begin{array}{llllllllll}199 & 198 & 196 & 193 & 190 & 186 & 182 & 177\end{array}$ |
|  | 42139137 |
|  | $\begin{array}{lllllllllll}109 & 109 & 108 & 107 & 106 & 105 & 103 & 101 & 9996 .\end{array}$ |
|  | 81.18180 .780 .179 .478 .477 .275 .874 .272 .470. |
|  | 61.160 .9 |
|  | 46.746 .746 .546 .245. |
|  | 36.136 .13635 .735 .435 |
|  | 28.328 .228 .12827 .827 .527 .126 .826 .3 |
|  | . 2 |
|  |  |


| $\mathrm{y}=4$ | $\mathrm{x}=0 \quad \mathrm{x}=2 \quad \mathrm{x}=4 \quad \mathrm{x}=6 \mathrm{x}=8 \quad \mathrm{x}=10 \mathrm{x}=12 \mathrm{x}=14 \mathrm{x}=16 \mathrm{x}=18 \mathrm{x}=20$ |
| :---: | :---: |
| $\mathrm{z}=0$ | 13381337133213241312129612771253122411901151 |
| $\mathrm{z}=5$ | 13251323131813101299128312641240121211781139 |
| $\mathrm{z}=10$ | 12821281127612681257124212241201117311411103 |
| $\mathrm{z}=15$ | 12081207120311951185117111531132110610751040 |
| $\mathrm{z}=20$ | 110110991095108910791066105010311007979947 |
| $\mathrm{z}=25$ | $\begin{array}{llllllllllllllllllllll}962 & 960 & 957 & 951 & 942 & 931 & 917 & 900 & 879 & 855 & 827\end{array}$ |
| $z=30$ |  |
| $\mathrm{z}=35$ | $\begin{array}{llllllllllllllllllll}638 & 637 & 635 & 631 & 625 & 617 & 607 & 595 & 581 & 565 & 546\end{array}$ |
| $\mathrm{z}=40$ | $\begin{array}{llllllllllllllllllll}489 & 488 & 486 & 483 & 478 & 472 & 464 & 455 & 444 & 432 & 418\end{array}$ |
| $\mathrm{z}=45$ | $\begin{array}{lllllllllllllllllll}365 & 364 & 363 & 360 & 357 & 352 & 346 & 339 & 331 & 322 & 311\end{array}$ |
| $\mathrm{z}=50$ | $\begin{array}{lllllllllllllllllll}268 & 268 & 267 & 265 & 262 & 259 & 254 & 249 & 243 & 236 & 229\end{array}$ |
| $\mathrm{z}=55$ | $\begin{array}{lllllllllllllll}196 & 196 & 195 & 194 & 192 & 189 & 186 & 182 & 178 & 173 & 168\end{array}$ |
| $\mathrm{z}=60$ | $\begin{array}{llllllllllllllllll}144 & 144 & 143 & 142 & 141 & 139 & 137 & 134 & 131 & 128 & 124\end{array}$ |
| $\mathrm{z}=65$ | 10710610610510410310199.397 .194 .792 |
| $\mathrm{z}=70$ | 79.679 .579 .278 .777 .976 .975 .874 .472 .871 .169 .1 |
| $\mathrm{z}=75$ | 60.260 .159 .859 .458 .958 .257 .456 .455 .25452 .6 |
| $\mathrm{z}=80$ | 4645.945 .845 .545 .144 .64443 .242 .441 .540 .5 |
| $\mathrm{z}=85$ | 35.635 .635 .435 .234 .934 .634 .133 .63332 .331 .6 |
| $\mathrm{z}=90$ | 27.927 .827 .827 .627 .427 .126 .826 .42625 .525 |
| $\mathrm{z}=95$ | $22.122 .12221 .921 .721 .521 .3 \quad 2120.720 .319 .9$ |
| $z=100$ | 17.717 .717 .617 .517 .417 .317 .116 .916 .616 .416 .1 |



| $\mathrm{y}=8$ | $\mathrm{x}=0 \mathrm{x}=2 \quad \mathrm{x}=4 \quad \mathrm{x}=6 \mathrm{x}=8 \quad \mathrm{x}=10 \mathrm{x}=12 \mathrm{x}=14 \mathrm{x}=16 \mathrm{x}=18 \mathrm{x}=20$ |
| :---: | :---: |
| $\mathrm{z}=0$ | 13841382137813711360134613281306127912461207 |
| $\mathrm{z}=5$ | 13711370136513581348133413161294126712351196 |
| $\mathrm{z}=10$ | 13311330132613191308129512781256123011991162 |
| $\mathrm{z}=15$ | 12581257125312471237122412081188116311341099 |
| $\mathrm{z}=20$ | 11471146114211361128111611011083106010331001 |
| $\mathrm{z}=25$ | 996995992987979969956939920896868 |
| $\mathrm{z}=30$ | $\begin{array}{llllllllllllllll}817 & 816 & 813 & 809 & 802 & 793 & 782 & 769 & 752 & 733 & 710\end{array}$ |
| $\mathrm{z}=35$ | $\begin{array}{llllllllllllllllllllll}634 & 633 & 631 & 627 & 622 & 615 & 606 & 595 & 582 & 566 & 548\end{array}$ |
| $\mathrm{z}=40$ | $\begin{array}{lllllllllllllllllllllll}472 & 472 & 470 & 467 & 463 & 457 & 450 & 442 & 432 & 420 & 406\end{array}$ |
| $\mathrm{z}=45$ | $\begin{array}{lllllllllllllllllll}344 & 344 & 342 & 340 & 337 & 333 & 327 & 321 & 313 & 305 & 295\end{array}$ |
| $\mathrm{z}=50$ | $\begin{array}{lllllllllllllllll}249 & 249 & 248 & 246 & 244 & 240 & 237 & 232 & 226 & 220 & 213\end{array}$ |
| $\mathrm{z}=55$ | $\begin{array}{lllllllllllllllllll}181 & 181 & 180 & 179 & 177 & 175 & 172 & 168 & 164 & 160 & 155\end{array}$ |
| $\mathrm{z}=60$ | $\begin{array}{lllllllllllllllllllllll}133 & 133 & 132 & 131 & 130 & 128 & 126 & 123 & 121 & 117 & 114\end{array}$ |
| $\mathrm{z}=65$ | 98.398 .297 .897 .196 .194 .993 .491 .789 .687 .484 .9 |
| $\mathrm{z}=70$ | 73.873 .773 .472 .972 .271 .370 .268 .967 .565 .964 .1 |
| $\mathrm{z}=75$ | 5655.955 .755 .454 .954 .253 .452 .551 .550 .349 |
| $\mathrm{z}=80$ | 43.14342 .942 .642 .241 .841 .240 .539 .838 .938 |
| $\mathrm{z}=85$ | 33.533 .533 .433 .232 .932 .632 .231 .731 .130 .529 .8 |
| $\mathrm{z}=90$ | $26.426 .426 .326 .1 \quad 2625.725 .4 \quad 2524.624 .223 .7$ |
| $z=95$ | $21 \quad 2120.920 .820 .720 .520 .3 \quad 2019.719 .419$ |
| $\mathrm{z}=10$ | 16.916 .916 .916 .816 .716 .516 .416 .215 .915 .715 .4 |


| $\mathrm{y}=10$ | $\mathrm{x}=0 \quad \mathrm{x}=2 \mathrm{x}=4 \quad \mathrm{x}=6 \mathrm{x}=8 \quad \mathrm{x}=10 \mathrm{x}=12 \mathrm{x}=14 \mathrm{x}=16 \mathrm{x}=18 \mathrm{x}=20$ |
| :---: | :---: |
| $\mathrm{z}=0$ | 14171416141214051396138313661345132012881250 |
| $\mathrm{z}=5$ | 14051404140013931384137113551334130912771240 |
| $\mathrm{z}=10$ | 13671366136213561347133513191299127412441207 |
| $\mathrm{z}=15$ | 12961295129212861277126612511232120811801145 |
| $\mathrm{z}=20$ | 11831182117911741166115511411124110310771045 |
| $\mathrm{z}=25$ | 10231022101910151008998986971952929902 |
| $\mathrm{z}=30$ | $\begin{array}{llllllllllllllllll}828 & 827 & 825 & 821 & 815 & 807 & 797 & 784 & 768 & 749 & 727\end{array}$ |
| $\mathrm{z}=35$ | $\begin{array}{lllllllllllllllll}629 & 629 & 626 & 623 & 618 & 612 & 603 & 593 & 581 & 566 & 548\end{array}$ |
| $\mathrm{z}=40$ | $\begin{array}{lllllllllllllllllll}458 & 457 & 456 & 453 & 449 & 444 & 437 & 430 & 420 & 409 & 396\end{array}$ |
| $\mathrm{z}=45$ | $\begin{array}{llllllllllllllllllllll}328 & 327 & 326 & 324 & 321 & 317 & 312 & 306 & 299 & 291 & 282\end{array}$ |
| $\mathrm{z}=50$ | $\begin{array}{lllllllllllllllllll}234 & 234 & 233 & 232 & 229 & 226 & 223 & 218 & 213 & 207 & 201\end{array}$ |
| $\mathrm{z}=55$ | $\begin{array}{lllllllllllllll}169 & 169 & 169 & 167 & 166 & 163 & 161 & 158 & 154 & 150 & 145\end{array}$ |
| $\mathrm{z}=60$ | $\begin{array}{llllllllllllll}124 & 124 & 123 & 123 & 121 & 120 & 118 & 116 & 113 & 110 & 107\end{array}$ |
| $\mathrm{z}=65$ | 92.392 .191 .791 .190 .289 .187 .78684 .18279 .7 |
| $\mathrm{z}=70$ | $69.569 .469 .168 .66867 .266 .1 \quad 6563.662 .160 .4$ |
| $\mathrm{z}=75$ | $53 \quad 5352.752 .451 .951 .350 .649 .748 .747 .646 .4$ |
| $\mathrm{z}=80$ | 4140.940 .840 .540 .239 .739 .238 .637 .83736 .2 |
| $\mathrm{z}=85$ | $32 \quad 3231.931 .731 .531 .130 .730 .329 .729 .228 .5$ |
| $z=90$ | 25.325 .325 .225 .124 .924 .724 .42423 .623 .222 .7 |
| $\mathrm{z}=95$ | 20.320 .220 .220 .119 .919 .819 .519 .31918 .718 .3 |
| $\mathrm{z}=100$ | 16.416 .416 .316 .216 .11615 .815 .615 .415 .214 .9 |

## 4. Magnetic field measurement

In this measurement we used 3MH3: 3-Axis Digital Teslameter [12]. This tesla meter has a three-axis Hall probe, which is fixed on an $x$-y stage. The probe head has 4 mm width, 2 mm thickness, and 15 mm length. The field sensitive area is $0.15 \mathrm{~mm} \times 0.01 \mathrm{~mm} \times 0.15 \mathrm{~mm}$. The measurement range along $z$-axis is from -100 mm to 100 mm with 4 mm steps. X and Y-coordinates of the measurement lines are from -6 mm to 6 mm with 2 mm steps. We also measured additional points on four lines of $(x, y)=\{(-10,-10),(-10,10),(10,-10),(10,10)\}$. The alignment error on the magnet against the $x-y$ stage axis is less than 0.1 mm .

Results of the magnetic field By is shown in Table 2. Those of Bx and Bz are shown in Table 3 and 4 , respectively. These values are calibrated using a NMR probe. And in this measurement, the room temperature is changed from 19.0 to 19.3 degree C. The values are also calibrated using with temperature coefficient (TC). A TC of the probe is $1 * 10^{-3} /$ degree C. A result of BLy integrated along z-axis is shown in Table 5. Those of BLx and BLz are shown in Table 6 and 7, respectively. These BL values are calculated by Simpson's formula. Figure 13 shows the $y$-component of the magnetic flux density along $z$-axis on $x=y=0$ and $x=y=10$. Figure 14 shows the same information as Fig. 13 on $\mathrm{x}=\mathrm{y}=10$ and $\mathrm{x}=\mathrm{y}=-10$. Figure 15 and 16 show the x and $y$ component of the magnetic flux density along $z$-axis on $x=y=0$ and $x=y=10$, respectively.

Differences between experimental data and calculated data are shown in Figs. 17a, 18a, 19a, 20a, 21a, and 22a following Eq. (8).

$$
\begin{equation*}
\text { Difference }=(\text { Calculated Value })-(\text { Measured Value }) \tag{8}
\end{equation*}
$$

The differences are plotted in $\%$ at Figs. 17b, 18b, 19b, 20b, 21b, and 22b following Eq. (9).

$$
\begin{equation*}
\text { Difference in } \%=\frac{(\text { Calculated Value })-(\text { Measured Value })}{(\text { Calculated Value })} \times 100 \tag{9}
\end{equation*}
$$

The difference and difference in $\%$ of By along z -axis on $\mathrm{x}=\mathrm{y}=0, \mathrm{x}=\mathrm{y}=10$, and $\mathrm{x}=\mathrm{y}=-10$ are shown in Fig. 17, 18, and 19, respectively. Those of Bx on $\mathrm{x}=\mathrm{y}=10$ and $\mathrm{x}=\mathrm{y}=-10$ are shown in Fig. 20 and Fig. 21, respectively. The difference of Bz on $\mathrm{x}=\mathrm{y}=10$ is shown in Fig. 22.

The calculated data of $B z$ has 0 value at $z=0$. Differences of $B L$ values are also calculated. The results of these are shown in Table 8.

The positioning angle accuracy of the Hall probe may have some errors, since the Bx and Bz component should be zero on the axis because of the symmetry, while the measured values have none zeros. According to the $(\mathrm{Bx}, \mathrm{By}, \mathrm{Bz})$ values $(15,-32,1323)$ Gauss at the center, the rolling and the tilt angles are estimated as 0.65 degree and 1.4 degree, respectively. The angle positioning of such a small device is not easy in the measurement.

## 5. Summary

We fabricated a permanent magnet employing neodymium pieces to get better signal to noise ratio to find long-lived $\pi \cdot \pi$ - atoms. The maximum magnetic field strength at the center was 1323 Gauss with the gap of 60 mm which was requied by the beam halo interference to the magnet. The maximum field strength is within the allowed value not to quench the long-lived excited $\pi \cdot \pi$ - atoms to the ground state. The magnetic field along the tracks is diminished enough at 10 cm from the production target of Be , where the platinum foil is set not to bend the $\pi^{*}$ and $\pi^{-}$- tracks after the break-up of the long-lived $\pi \cdot \pi$ atoms, while keeping the bending power BLy of 0.01T.m.

There are small differences between the simulated and the realized field strength as shown in Figs. $17-22$, but the differences are not large problem in the bending function. The maximum field strength at the center and the bending power of BLy are consistent within $1.7 \%$ and $0.02 \%$, respectively. In conclusion the magnet was fabricated to fulfill the requirements in Dirac experiment to find the long-lived $\pi \cdot \pi$ - atoms.

Table 2: Magnetic field map of By measured by 3MH3: 3-Axis Digial Teslameter. Units are in Gauss and mm.
$(x, y)=(0,0)$ and $(10,10)$

| $\mathrm{z}, \mathrm{x}=\mathrm{y}=0$ | By | $\mathrm{z}, \mathrm{x}=\mathrm{y}=0$ | By | $\mathrm{z}, \mathrm{x}=\mathrm{y}=10$ | By | $\mathrm{z}, \mathrm{x}=\mathrm{y}=10$ | By |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -100 | 18 | 4 | 1310 | -100 | 13 | 4 | 1369 |
| -96 | 22 | 8 | 1280 | -96 | 20 | 8 | 1343 |
| -92 | 25 | 12 | 1229 | -92 | 23 | 12 | 1297 |
| -88 | 32 | 16 | 1161 | -88 | 26 | 16 | 1226 |
| -84 | 38 | 20 | 1073 | -84 | 31 | 20 | 1132 |
| -80 | 45 | 24 | 967 | -80 | 39 | 24 | 1005 |
| -76 | 59 | 28 | 846 | -76 | 51 | 28 | 857 |
| -72 | 75 | 32 | 721 | -72 | 62 | 32 | 701 |
| -68 | 93 | 36 | 595 | -68 | 77 | 36 | 546 |
| -64 | 119 | 40 | 483 | -64 | 95 | 40 | 418 |
| -60 | 152 | 44 | 382 | -60 | 122 | 44 | 317 |
| -56 | 194 | 48 | 300 | -56 | 156 | 48 | 238 |
| -52 | 247 | 52 | 234 | -52 | 204 | 52 | 183 |
| -48 | 319 | 56 | 182 | -48 | 266 | 56 | 140 |
| -44 | 405 | 60 | 140 | -44 | 351 | 60 | 112 |
| -40 | 509 | 64 | 109 | -40 | 465 | 64 | 86 |
| -36 | 628 | 68 | 87 | -36 | 606 | 68 | 68 |
| -32 | 756 | 72 | 68 | -32 | 771 | 72 | 55 |
| -28 | 883 | 76 | 55 | -28 | 936 | 76 | 45 |
| -24 | 1006 | 80 | 44 | -24 | 1094 | 80 | 36 |
| -20 | 1112 | 84 | 36 | -20 | 1213 | 84 | 28 |
| -16 | 1195 | 88 | 26 | -16 | 1296 | 88 | 26 |
| -12 | 1257 | 92 | 25 | -12 | 1349 | 92 | 21 |
| -8 | 1299 | 96 | 20 | -8 | 1375 | 96 | 16 |
| -4 | 1320 | 100 | 15 | -4 | 1385 | 100 | 13 |
| 0 | 1323 |  |  | 0 | 1384 |  |  |

Table 3: Magnetic field map of Bx measured by 3MH3: 3-Axis Digial Teslameter. Units are in Gauss and mm.
$(x, y)=(0,0)$ and $(10,10)$

| $\mathrm{z}, \mathrm{x}=\mathrm{y}=0$ | Bx | $\mathrm{z}, \mathrm{x}=\mathrm{y}=0$ | Bx | $\mathrm{z}, \mathrm{x}=\mathrm{y}=10$ | Bx | $z, x=y=10$ | Bx |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -100 | -1 | 4 | 15 | -100 | -2 | 4 | 102 |
| -96 | -2 | 8 | 15 | -96 | 0 | 8 | 101 |
| -92 | 0 | 12 | 13 | -92 | 0 | 12 | 96 |
| -88 | -2 | 16 | 11 | -88 | 2 | 16 | 89 |
| -84 | 0 | 20 | 10 | -84 | 3 | 20 | 82 |
| -80 | -1 | 24 | 11 | -80 | 3 | 24 | 73 |
| -76 | -2 | 28 | 9 | -76 | 3 | 28 | 62 |
| -72 | 1 | 32 | 8 | -72 | 5 | 32 | 54 |
| -68 | 2 | 36 | 4 | -68 | 7 | 36 | 43 |
| -64 | 0 | 40 | 3 | -64 | 10 | 40 | 33 |
| -60 | 1 | 44 | 3 | -60 | 12 | 44 | 26 |
| -56 | 1 | 48 | 2 | -56 | 18 | 48 | 22 |
| -52 | 3 | 52 | 2 | -52 | 21 | 52 | 17 |
| -48 | 3 | 56 | 2 | -48 | 26 | 56 | 12 |
| -44 | 5 | 60 | 0 | -44 | 35 | 60 | 8 |
| -40 | 6 | 64 | 0 | -40 | 44 | 64 | 7 |
| -36 | 8 | 68 | 0 | -36 | 52 | 68 | 5 |
| -32 | 8 | 72 | -1 | -32 | 62 | 72 | 3 |
| -28 | 11 | 76 | -1 | -28 | 71 | 76 | 3 |
| -24 | 11 | 80 | -1 | -24 | 79 | 80 | 0 |
| -20 | 13 | 84 | -2 | -20 | 88 | 84 | 0 |
| -16 | 13 | 88 | 0 | -16 | 94 | 88 | 0 |
| -12 | 13 | 92 | -13 | -12 | 99 | 92 | 0 |
| -8 | 13 | 96 | -1 | -8 | 103 | 96 | 0 |
| -4 | 16 | 100 | -1 | -4 | 105 | 100 | -2 |
| 0 | 15 |  |  | 0 | 105 |  |  |

Table 4: Magnetic field map of Bz measured by 3MH3: 3-Axis Digial Teslameter. Units are in Gauss and mm.
$(x, y)=(0,0)$ and $(10,10)$

| $\mathrm{z}, \mathrm{x}=\mathrm{y}=0$ | Bz | $\mathrm{z}, \mathrm{x}=\mathrm{y}=0$ | Bz | $\mathrm{z}, \mathrm{x}=\mathrm{y}=10$ | Bz | $\mathrm{z}, \mathrm{x}=\mathrm{y}=10$ | Bz |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -100 | -1 | 4 | -35 | -100 | 18 | 4 | -88 |
| -96 | 11 | 8 | -33 | -96 | 18 | 8 | -129 |
| -92 | -1 | 12 | -33 | -92 | 21 | 12 | -177 |
| -88 | 0 | 16 | -32 | -88 | 26 | 16 | -230 |
| -84 | -1 | 20 | -30 | -84 | 28 | 20 | -281 |
| -80 | 0 | 24 | -27 | -80 | 35 | 24 | -331 |
| -76 | 0 | 28 | -24 | -76 | 40 | 28 | -355 |
| -72 | 11 | 32 | -20 | -72 | 46 | 32 | -355 |
| -68 | 0 | 36 | -15 | -68 | 62 | 36 | -328 |
| -64 | 11 | 40 | -12 | -64 | 76 | 40 | -282 |
| -60 | 11 | 44 | -9 | -60 | 95 | 44 | -230 |
| -56 | 12 | 48 | -6 | -56 | 124 | 48 | -180 |
| -52 | 12 | 52 | -3 | -52 | 158 | 52 | -140 |
| -48 | 11 | 56 | -2 | -48 | 200 | 56 | -106 |
| -44 | 12 | 60 | -2 | -44 | 250 | 60 | -81 |
| -40 | 13 | 64 | -1 | -40 | 298 | 64 | -62 |
| -36 | 0 | 68 | -2 | -36 | 341 | 68 | -48 |
| -32 | -1 | 72 | -2 | -32 | 360 | 72 | -38 |
| -28 | -8 | 76 | 11 | -28 | 347 | 76 | -28 |
| -24 | -12 | 80 | -2 | -24 | 300 | 80 | -23 |
| -20 | -16 | 84 | -1 | -20 | 232 | 84 | -17 |
| -16 | -20 | 88 | 0 | -16 | 161 | 88 | -14 |
| -12 | -25 | 92 | -1 | -12 | 99 | 92 | -10 |
| -8 | -29 | 96 | 0 | -8 | 45 | 96 | -10 |
| -4 | -30 | 100 | 0 | -4 | -10 | 100 | -7 |
| 0 | -32 |  |  | 0 | -49 |  |  |

Table 5: Magnetic field map of Bly which is integrated along z-axis. Units of BL is in Tesla and m. Unit of coordinates is mm .

| $\mathrm{x}(\mathrm{mm})$ | $\mathrm{y}(\mathrm{mm})$ | BLy (T.m) | $\mathrm{x}(\mathrm{mm})$ | $\mathrm{y}(\mathrm{mm})$ | BLy (T.m) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -10 | -10 | 0.0098700 | 2 | 0 | 0.0097874 |
| 10 | -10 | 0.0098543 | 4 | 0 | 0.0097500 |
| -6 | -6 | 0.0098087 | 6 | 0 | 0.0096791 |
| -4 | -6 | 0.0098636 | -6 | 2 | 0.0097238 |
| -2 | -6 | 0.0098908 | -4 | 2 | 0.0097888 |
| 0 | -6 | 0.0099019 | -2 | 2 | 0.0098071 |
| 2 | -6 | 0.0098869 | 0 | 2 | 0.0098212 |
| 4 | -6 | 0.0098577 | 2 | 2 | 0.0098024 |
| 6 | -6 | 0.0097936 | 4 | 2 | 0.0097560 |
| -6 | -4 | 0.0097475 | 6 | 2 | 0.0096922 |
| -4 | -4 | 0.0098075 | -6 | 4 | 0.0097704 |
| -2 | -4 | 0.0098402 | -4 | 4 | 0.0098284 |
| 0 | -4 | 0.0098409 | -2 | 4 | 0.0098530 |
| 2 | -4 | 0.0098303 | 0 | 4 | 0.0098605 |
| 4 | -4 | 0.0097848 | 2 | 4 | 0.0098465 |
| 6 | -4 | 0.0097168 | 4 | 4 | 0.0098019 |
| -6 | -2 | 0.0097066 | 6 | 4 | 0.0097372 |
| -4 | -2 | 0.0097674 | -6 | 6 | 0.0098352 |
| -2 | -2 | 0.0097961 | -4 | 6 | 0.0098907 |
| 0 | -2 | 0.0098067 | -2 | 6 | 0.0099185 |
| 2 | -2 | 0.0097931 | 0 | 6 | 0.0099274 |
| 4 | -2 | 0.0097516 | 2 | 6 | 0.0099092 |
| 6 | -2 | 0.0096845 | 4 | 6 | 0.0098760 |
| -6 | 0 | 0.0097041 | 6 | 6 | 0.0098114 |
| -4 | 0 | 0.0097665 | -10 | 10 | 0.0099217 |
| -2 | 0 | 0.0097922 | 10 | 10 | 0.0098679 |
| 0 | 0 | 0.0098022 |  |  |  |

Table 6: Magnetic field map of BLx which is integrated along z-axis. Units of BLx is in milli-Tesla and $m$. Unit of coordinates is mm .

| $\mathrm{x}(\mathrm{mm})$ | $\mathrm{y}(\mathrm{mm})$ | BLx (mT.m) | $\mathrm{x}(\mathrm{mm})$ | $\mathrm{y}(\mathrm{mm})$ | BLx (mT.m) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -10 | -10 | 0.6722 | 2 | 0 | 0.0907 |
| 10 | -10 | -0.7531 | 4 | 0 | 0.0843 |
| -6 | -6 | 0.2815 | 6 | 0 | 0.0873 |
| -4 | -6 | 0.2071 | -6 | 2 | -0.0046 |
| -2 | -6 | 0.1379 | -4 | 2 | 0.0268 |
| 0 | -6 | 0.0680 | -2 | 2 | 0.0532 |
| 2 | -6 | 0.0009 | 0 | 2 | 0.0859 |
| 4 | -6 | -0.2029 | 2 | 2 | 0.1138 |
| 6 | -6 | -0.3117 | 4 | 2 | 0.1339 |
| -6 | -4 | 0.2142 | 6 | 2 | 0.1509 |
| -4 | -4 | 0.1702 | -6 | 4 | -0.1864 |
| -2 | -4 | 0.1131 | -4 | 4 | -0.0450 |
| 0 | -4 | 0.0717 | -2 | 4 | 0.0320 |
| 2 | -4 | 0.0166 | 0 | 4 | 0.0931 |
| 4 | -4 | -0.0243 | 2 | 4 | 0.1417 |
| 6 | -4 | -0.1791 | 4 | 4 | 0.1829 |
| -6 | -2 | 0.1352 | 6 | 4 | 0.2506 |
| -4 | -2 | 0.1179 | -6 | 6 | -0.2754 |
| -2 | -2 | 0.1043 | -4 | 6 | -0.1438 |
| 0 | -2 | 0.0846 | -2 | 6 | 0.0186 |
| 2 | -2 | 0.0607 | 0 | 6 | 0.1008 |
| 4 | -2 | 0.0118 | 2 | 6 | 0.1705 |
| 6 | -2 | -0.0050 | 4 | 6 | 0.2480 |
| -6 | 0 | 0.0672 | 6 | 6 | 0.3189 |
| -4 | 0 | 0.0731 | -10 | 10 | -0.6962 |
| -2 | 0 | 0.0845 | 10 | 10 | 0.7536 |
| 0 | 0 | 0.0886 |  |  |  |

Table 7: Magnetic field map of BLz which is integrated along z-axis. Units of BLz is in milli-Tesla and $m$. Unit of coordinates is mm .

| $\mathrm{x}(\mathrm{mm})$ | $\mathrm{y}(\mathrm{mm})$ | BLz (mT.m) | $\mathrm{x}(\mathrm{mm})$ | $\mathrm{y}(\mathrm{mm})$ | BLz (mT.m) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| -10 | -10 | -0.0901 | 2 | 0 | -0.1559 |
| 10 | -10 | -0.0775 | 4 | 0 | -0.1358 |
| -6 | -6 | -0.0892 | 6 | 0 | -0.1400 |
| -4 | -6 | -0.0860 | -6 | 2 | -0.0883 |
| -2 | -6 | -0.0880 | -4 | 2 | -0.0923 |
| 0 | -6 | -0.0886 | -2 | 2 | -0.0911 |
| 2 | -6 | -0.0842 | 0 | 2 | -0.0937 |
| 4 | -6 | -0.0817 | 2 | 2 | -0.0930 |
| 6 | -6 | -0.0730 | 4 | 2 | -0.0923 |
| -6 | -4 | -0.0770 | 6 | 2 | -0.0877 |
| -4 | -4 | -0.0822 | -6 | 4 | -0.0770 |
| -2 | -4 | -0.0896 | -4 | 4 | -0.0805 |
| 0 | -4 | -0.0842 | -2 | 4 | -0.0854 |
| 2 | -4 | -0.0799 | 0 | 4 | -0.0906 |
| 4 | -4 | -0.0883 | 2 | 4 | -0.0846 |
| 6 | -4 | -0.0818 | 4 | 4 | -0.0852 |
| -6 | -2 | -0.0918 | 6 | 4 | -0.0822 |
| -4 | -2 | -0.1021 | -6 | 6 | -0.0817 |
| -2 | -2 | -0.0947 | -4 | 6 | -0.0825 |
| 0 | -2 | -0.0937 | -2 | 6 | -0.0905 |
| 2 | -2 | -0.0872 | 0 | 6 | -0.0945 |
| 4 | -2 | -0.0974 | 2 | 6 | -0.0909 |
| 6 | -2 | -0.0939 | 4 | 6 | -0.0860 |
| -6 | 0 | -0.1144 | 6 | 6 | -0.0875 |
| -4 | 0 | -0.1009 | -10 | 10 | -0.0781 |
| -2 | 0 | -0.1168 | 10 | 10 | -0.0951 |
| 0 | 0 | -0.1385 |  |  |  |

Table 8: Differences of BL between calculated data and experimental data.

|  | $\begin{aligned} & \text { BLx } @ x=y \\ & =10(\mathrm{mT} \cdot \mathrm{~m}) \\ & -100<=\mathrm{z}<= \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { BLx @ } \mathrm{x}=\mathrm{y} \\ & =10(\mathrm{mT} \cdot \mathrm{~m}) \\ & -100<=\mathrm{z}<= \\ & 100 \end{aligned}$ | $\begin{aligned} & \mathrm{BLz} @ \mathrm{x}=\mathrm{y}= \\ & 10(\mathrm{mT} . \mathrm{m}) \\ & -100<=\mathrm{z}<=0 \end{aligned}$ | $\begin{aligned} & \text { BLy@ } \mathrm{x}=\mathrm{y} \\ & =0 \text { (T.m) } \\ & -100<=\mathrm{z}<= \end{aligned}$ | $\begin{gathered} \text { BLy @ } \mathrm{x}=\mathrm{y} \\ =0 \text { (T.m) } \\ -100<=\mathrm{z}<= \\ 100 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Calculated <br> Data | 0.3052 | 0.6329 | -1.2946 | 0.0047160 | 0.0097848 |
| Experimental <br> Data | 0.3837 | 0.7536 | -1.3397 | 0.0048083 | 0.0098021 |
| Difference (\%) | -27.4 | -19.1 | -3.5 | -2.0 | -0.2 |


|  | $\begin{aligned} & \text { BLy@ } \mathrm{x}=\mathrm{y} \\ & =10 \text { (T.m) } \\ & -100<=\mathrm{z}<= \end{aligned}$ <br> 0 | $\begin{gathered} \text { BLy@ } \mathrm{x}=\mathrm{y} \\ =10 \text { (T.m) } \\ -100<=\mathrm{z}<= \\ 100 \end{gathered}$ | $\begin{aligned} & \text { BLy@ } \mathrm{x}=\mathrm{y} \\ & =-10 \text { (T.m) } \\ & -100<=\mathrm{z}<= \end{aligned}$ | $\begin{gathered} \text { BLy @ } \mathrm{x}=\mathrm{y} \\ =-10 \text { (T.m) } \\ -100<=\mathrm{z}<= \\ 100 \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| Calculated <br> Data | 0.00473365 | 0.00983605 | 0.00473365 | 0.00983605 |
| Experimental <br> Data | 0.00489866 | 0.00986792 | 0.00477692 | 0.00987001 |
| Difference <br> (\%) | -3.5 | -0.3 | -0.9 | -0.3 |



Fig. 13: Graph of By at $(x, y)=(0,0)$ and $(x, y)=(10,10) . \quad$ The black line is at $x=y=0$. The red line is at $x=y=10$.


Fig. 14: Graph of By at $(x, y)=(-10,-10)$ and $(x, y)=(10,10)$. The blue line is at $x=y=-10$. The red line is at $x=y=10$.


Fig. 15: Graph of Bx at $(x, y)=(0,0)$ and $(x, y)=(10,10)$. The black line is at $x=y=0$. The red line is at $x=y=10$.


Fig. 16: Graph of Bz at $(x, y)=(0,0)$ and $(x, y)=(10,10)$. The black line is at $x=y=0$. The red line is at $x=y=10$.


Fig. 17a: Differences between the calculated By and measured By at $x=y=0$.


Fig. 17b: Differences in $\%$ between the calculated By and measured By at $x=y=0$.


Fig. 18a: Differences between the calculated By and the measured By at $x=y=10$.


Fig. 18b Difference in \% between the calculated By and the measured By at $x=y=10$.


Fig. 19a: Differences between the calculated By and the measured By at $x=y=-10$.


Fig. 19b: Difference in \% between the calculated By and the measured By at $x=y=-10$.


Fig. 20a: Differences between the calculated Bx and the measured Bx at $x=y=10$.


Fig. 20b: Ddifferences in \% between the calculated Bx and the measured Bx at $x=y=10$.


Fig. 21a: Differences between the calculated $B x$ and the measured $B x$ at $x=y=-10$.


Fig. 21b: Differencea between the calculated Bx and the measured Bx at $x=y=-10$.


Fig. 22a: Differences between the calculated Bz and the measured Bz at $x=y=10$.


Fig. 22b: Differences between the calculated Bz and the measured Bz at $x=y=10$.

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