Multiple Scattering Studies

Dirac Note 2012-04

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The aim of this work is to present the multiple scatting analysis on the 2009 data and simulation.

1 Introduction

The knowledge about multiple scattering is extremely important in DIRAC due to the nature of the events we are studying. The tracks of the atomic and Coulomb pairs are characterised by a very small opening angle, and the signal is extracted by comparing data and simulated events, thus a very detailed knowledge of the muliple scattering in data and simulation is needed.

As a first approximation we can consider the two tracks of a prompt $\pi\pi$ pair as originating from a single space point inside the target. The tracks are firstly reconstructed using only the Downstream part of the detector and extrapolated to the target plane, in a second step the tracks are reconstructed in the upstream part of the detector, and a matching of the parameters upstream-downstream is performed.

Call $x_2(x_1)$ the $\pi^+(\pi^-)$ final track extrapolation to the target plane in the X and Y coordinates. The experimental error in the measurement of the tracks determines the width of the $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ which we call vertex resolution. Δx and Δy mainly depend on

- the SFD space resolution (momentum independent)
- the detailed material description for the upstream detectors and target (multiple scattering MS $\alpha\frac{1}{p}$
- the distance between target and upstream detecor (very detailed knowledge with sufficient precision)

2 Event selection

The events selected for this analysis are $\pi\pi$ data and MC simulated data, the cuts applied on them are listed below:

- $|\Delta(t)_{VH}| < 0.3ns$
- 1 track per arm (DC)
- $4MeV < |Q_x| < 8MeV$
- $3MeV < |Q_y| < 6MeV$
- $|Q_l| < 22MeV$

- No electrons, cut applied on the Ni Cherenkov amplitudes are $Ampl_{N1} < 62$ $Ampl_{N2} < 75$, this cut need to be studied further.

- No muons, cut applied $AND(Muonflag, 3) \neq 0$, this means that there is no hit in muon detector which corresponds to the reconstructed trak. If this cut is not applied, Δx and Δy on average have an increase of of $0.510^{-2}cm$ for both data and MC.

- Events where the two tracks share the same SFD(x) or SFD(y) hits are eliminated.

- For the MonteCarlo events, the simulation of the background has been added, this has anyway a negligible influence on the ΔX and ΔY. Events have been weighted in order to reproduce the momentum $(p_1 + p_2)$ distribution shape in experimental data.

- For the Experimental data, accidentals events in the prompt window have been subtracted.

3 First method for the comparison between data and MC of the vertex resolution Δx and Δy

Our first approch is to measure Δx and Δy for different momentum bins (see Table 1 for the experimental data and Table 2 for the simulated data) and compare the results with the simulation, in order to do so we fit the vertex resultion histograms with a gaussian and we extract the parameter $\sigma_{\Delta x(y)}$. As we expect the resolution improves for increasing momentum $p = P_+ + P_-$.

Table 1: Momentum bins for Experimental data.

Range Name	Momentum Range p	Fraction of statistics	Number of events
	(GeV/c)	%	
R1	2.6-4.2	53.4	67834
R2	4.2-5.6	30.5	38729
R3	5.6-8	16.1	20457

Table 2: Momentum bins for MC data.

Range Name	Momentum Range p	Fraction of statistics	Number of events
	(GeV/c) %		
R1	2.6-4.2	53.1	434446
R2	4.2-5.6	30.7	251726
R3	5.6-8	16.2	132314

A detailed study of the Multiple scattering was already been performed on the 2001-2003 data, and the results are reported in a Dirac Note [DIRAC Note 2007-04]. Since then, two of the upstream detectors have been replaced by new ones with different fiber diameter and different number of layers. We refer to them as SFDX and SFDY. These two detectors are thus under investigation in this work.

To perform this study we have generated three samples of simulated data with different conditions of the Multiple Scattering angle for SFDX and SFDY

- MS 1.0) Unchanged condition in the simulation of the SFDX and SFDY detector, the materials are kept as the 2001-2003 ones, and the multiple scattering angle is described by the measured quantities reported in the Dirac note [DIRAC Note 2007-04].

- MS 0.95) Changed condition in the simulation of the SFDX and SFDY detector, the materials are kept as the 2001-2003 ones but the multiple scattering angle is reduced of 5%.

- MS 0.90) Changed condition in the simulation of the SFDX and SFDY detector, the materials are kept as the 2001-2003 ones but the multiple scattering angle is reduced of 10%.

At the first stage of global tracking, ARIANE finds the tracks in DC. After it is needed to merge DC tracks with hits in upstream detectors. The first of them is the SFD, which is upstream coordinate detector, closest to DC. Planes of fiber detector are located along the setup axis in the direction from the target to the magnet in the next order: Y, X, W. W-plane is not used in the current version of the tracking to exclude correlation between X- and Y- projection. ARIANE extrapolates track to the X-plane of fiber detector (SFDX) in some assumption about momentum of a particle. Varying the momentum value (and the angle of rotation in the field of spectrometer magnet), ARIANE looks for the coincidence of the propagated coordinate with the coordinate of the hit in SFDX. It fixes the momentum value, the coordinate and the angle in XZ-plane for downstream track. It is needed to pointed out, that these track parameters are obtained for pieces of track between SFDX and DC and does not take into account multiple scattering in SFDX itself.

After the program propagates track to SFDY plane, finds the hit in vicinity to the propagated coordinate, and (if proper hit exists) updates track parameters (Y-coordinate and angle in YZ-plane), using Kalman filter algorithm. Also the updated parameters do not take into account the multiple scattering in SFDY itself. At the last step ARIANE propagates the tracks to the target level. The accuracy of the propagation is defined by the coordinate accuracy of SFD and DC detectors, and by the multiple scattering.

Multiple scattering in SFDX, SFDY and MDC has slightly higher weight in the reconstruction of scattering the X-coordinate at the target comparing with other upstream detectors. Because SFDW and IH detectors are located between SFDX, SFDY plane from one side and DC from another side. As result, variation of track parameters partly is taken into account. The same is valid for SFDY and MDC for the reconstruction of the Y-coordinate.

3.1 Δx

In the Tables 3 below the values of the RMS of the Δx distribution are summarised, and in Table 4 is shown $\sigma_{\Delta x}$ from the gaussian fit of the Vertex resolution histogram for the different momentum bins for the X projection.

The values for the $\sigma_{\Delta x}$ are slightly smaller compared to the RMSs, this is due to the tails of the distributions that are not completely described by a Gaussian fit. Especially it is essential for total momentum range, where even central part are different from normal distribution (because it is a sum of few Gaussian distributions with different widths). As an estimation of the overall Multiple Scattering effect we concentrate on the main part of the distribution, thus we restrict our study on the $\sigma_{\Delta x}$.

In order to give an unique number that can estimate how well we reproduce the MS in the Simulation, we introduce a weighted average of the $\sigma_{\Delta x}$ of the measurements: $\widehat{S_{\Delta x}}$ and the error associated to it

$$\widehat{S_{\Delta x}} = \sqrt{\sum_{i=1}^{3} \sigma^2_{\Delta x}(i) \cdot w(i)}$$

$$Er(\widehat{S_{\Delta x}}) = \frac{\sqrt{\sum (2 \cdot \sigma_{\Delta x}(i) \cdot w(i) \cdot Er_{\sigma_{\Delta x}(i)})^2}}{2 \cdot \widehat{S_{\Delta x}}}$$

where $w(i)$ are the weights defined as the proportion of events in the three momentum bins, $w(1) = N_1/N, w(2) = N_2/N, w(3) = N_3/N$ and $N_1 + N_2 + N_3 = N$ where N is the total number of events, (see Tables 1 and 2).

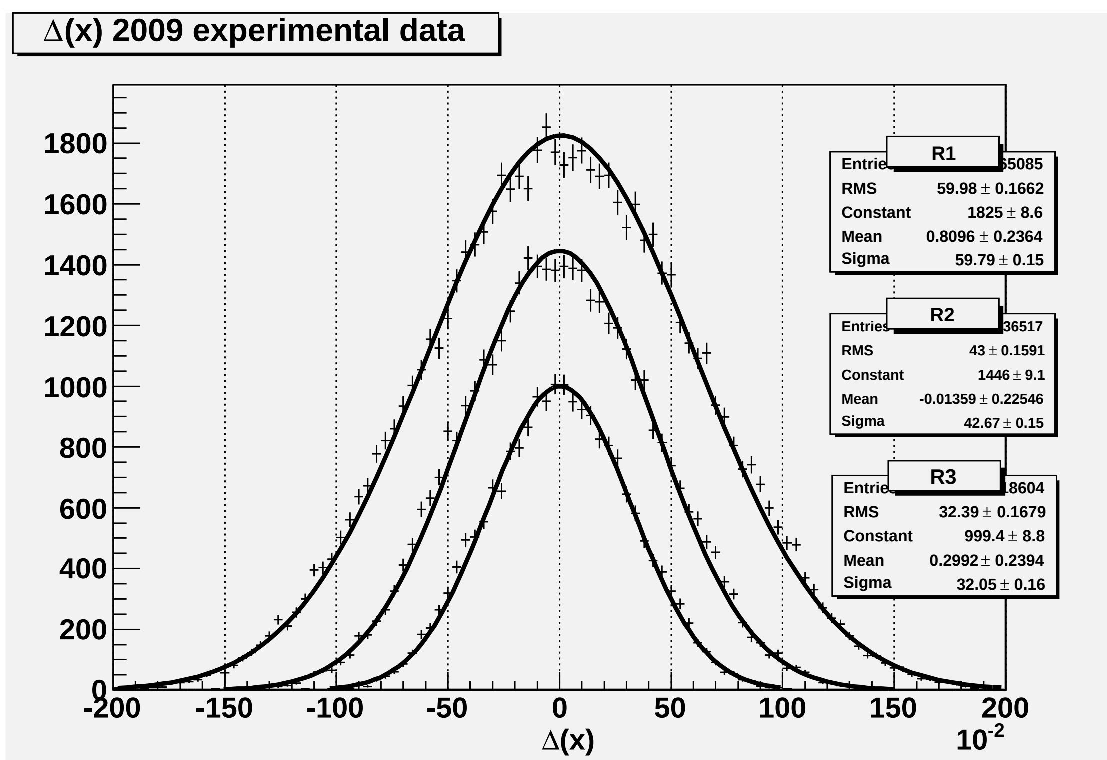

Figure 1: (Dx).

Table 3: RMS of the Vertex resolution histogram of Δx for the three momentum bins.

	Data	MS 1.0	MS 0.95	MS 0.90
	10^{-2} cm	10^{-2} cm	10^{-2} cm	10^{-2} cm
R1	(59.98 ± 0.16)	(60.41 ± 0.06)	(59.55 ± 0.06)	(58.80 ± 0.06)
R2	(43.0 ± 0.16)	(44.98 ± 0.06)	(44.26 ± 0.06)	(43.42 ± 0.06)
R3	(33.39 ± 0.17)	(34.86 ± 0.07)	(34.10 ± 0.07)	(33.52 ± 0.07)

Table 4: $\sigma_{\Delta x}$ from the gaussian fit of the Vertex resolution histogram.

	Data	MS 1.0	MS 0.95	MS 0.90
	10^{-2} cm	10^{-2} cm	10^{-2} cm	10^{-2} cm
R1	(59.78 ± 0.15)	(60.74 ± 0.07)	(59.84 ± 0.06)	(59.04 ± 0.06)
R2	(42.67 ± 0.15)	(44.67 ± 0.06)	(43.65 ± 0.06)	(42.82 ± 0.06)
R3	(32.02 ± 0.16)	(34.15 ± 0.07)	(33.16 ± 0.08)	(32.63 ± 0.07)
$\widehat{S_{\Delta x}}$	(51.35 ± 0.10)	(52.54 ± 0.05)	(51.63 ± 0.05)	(50.86 ± 0.05)

The last line of the Table [4] shows the value of the variable $\widehat{S_{\Delta x}}$ that gives us an average of the Vertex resolution in the X plane in data and Monte Carlo simulation. The best agreement is obtained with the reduction of the scattering angle of 10%.

3.2 Δy

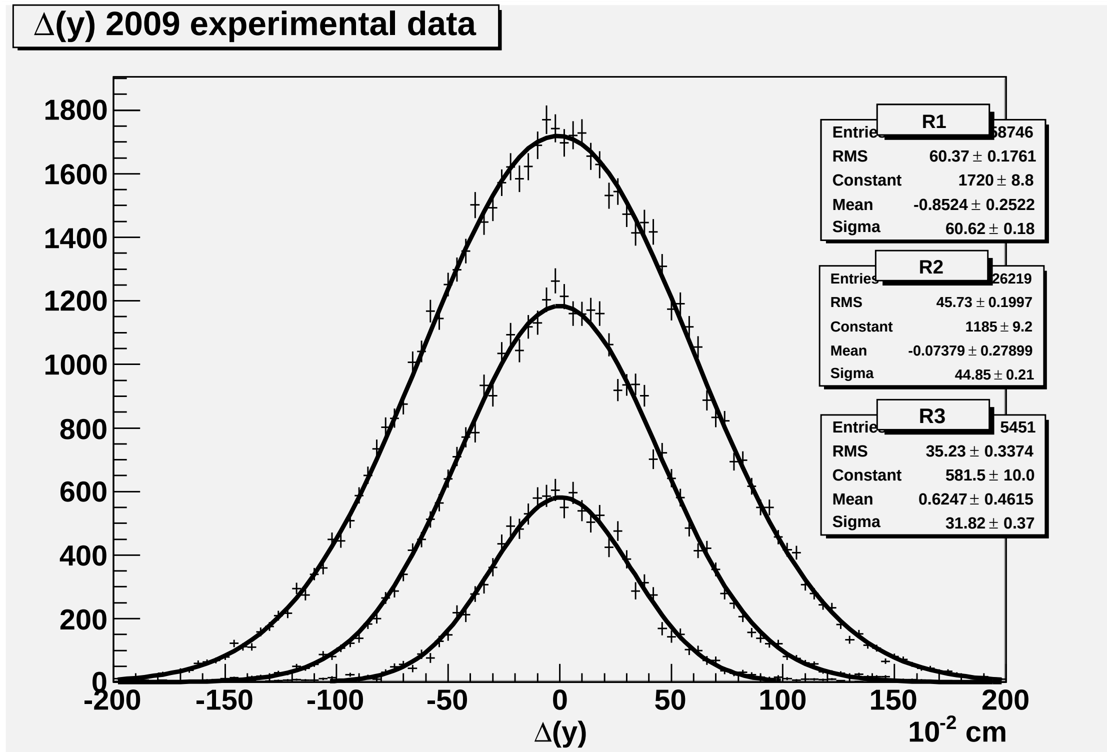

Figure 2: (Dy).

Table 5: RMS of the Vertex resolution histograms of Δy for the three momentum bins.

	Data	MS 1.0	MS 0.95	MS 0.90
	10^{-2} cm	10^{-2} cm	10^{-2} cm	10^{-2} cm
R1	(60.37 ± 0.18)	(60.10 ± 0.06)	(59.48 ± 0.06)	(58.56 ± 0.06)
R2	(45.73 ± 0.20)	(46.28 ± 0.06)	(45.64 ± 0.06)	(45.00 ± 0.06)
R3	(35.23 ± 0.33)	(37.23 ± 0.08)	(36.75 ± 0.08)	(36.29 ± 0.08)

The last line of Table 6 shows the value of the variable $\widehat{S_{\Delta y}}$ that gives us an average of the Vetex resolution in the Y plane in data and Monte Carlo simulation. The best agreement is obtained with the standard parameters of the scattering angle. Even in the best case, MS 1.0, the parameters in the MC are still smaller than in the exprerimental data, for this reason we are going to make a test with an increase of the scatterin angle of 5%.

Table 6: $\sigma_{\Delta y}$ from the gaussian fit of the Vertex resolution histogram.

	Data	MS 1.0	MS 0.95	MS 0.90
	10^{-2} cm	10^{-2} cm	10^{-2} cm	10^{-2} cm
R1	(60.62 ± 0.18)	(60.07 ± 0.07)	(59.37 ± 0.07)	(58.31 ± 0.07)
R2	(44.85 ± 0.21)	(44.03 ± 0.06)	(42.77 ± 0.07)	(42.13 ± 0.06)
R3	(31.82 ± 0.40)	(33.66 ± 0.08)	(32.70 ± 0.08)	(32.22 ± 0.08)
$\widehat{S_{\Delta y}}$	(52.42 ± 0.13)	(51.91 ± 0.05)	(51.06 ± 0.05)	(50.19 ± 0.05)

3.3 X and Y with different MS

Different trials have been done in order to find a best value for the Multiple Scattering, here are reported the two set of data that have been generated with a different multiple scattering angle for the X plane and Y plane.

Table 7: $\sigma_{\Delta x}$ from the gaussian fit of the Vertex resolution histogram with different multiple scattering in X and Y plane.

	Data	MS 0.90 x 1.05 y	MS 0.90 x 1.0 y
	10^{-2} cm	10^{-2} cm	10^{-2} cm
R1	(59.78 ± 0.15)	(60.28 ± 0.06)	(59.85 ± 0.07)
R2	(42.67 ± 0.15)	(44.11 ± 0.06)	(43.54 ± 0.06)
R3	(32.02 ± 0.16)	(33.58 ± 0.08)	(33.26 ± 0.08)
$\widehat{S_{\Delta x}}$	(50.90 ± 0.17)	(52.05 ± 0.05)	(51.61 ± 0.05)

Table 8: $\sigma_{\Delta y}$ from the gaussian fit of the Vertex resolution histogram with different multiple scattering in X and Y plane..

	Data	MS 0.90 x 1.05 y	MS 0.90 x 1.0 y
	10^{-2} cm	10^{-2} cm	10^{-2} cm
R1	(60.62 ± 0.18)	(59.67 ± 0.07)	(59.42 ± 0.07)
R2	(44.85 ± 0.21)	(43.26 ± 0.07)	(43.28 ± 0.07)
R3	(31.82 ± 0.40)	(33.02 ± 0.08)	(32.74 ± 0.08)
$\widehat{S_{\Delta y}}$	(52.42 ± 0.13)	(51.40 ± 0.05)	(51.22 ± 0.04)

4 Second method for the comparison between data and MC of the vertex resultion Δx and Δy

In this second approach we study the Vertex resolution as function of the total momentum of the track-pair. We can parametrise the distribution width of the two track impact points (in the target) as

$$\sigma^2_{\Delta x} = c_1^2 + \frac{ms_1^2}{P_-^2} + c_2^2 + \frac{ms_2^2}{P_+^2}$$

here c_1 is the sigma (width) of the distribution of the x_1 coordinate for the part of the contribution that is momentum independent, $ms1$ is the sigma of the part of the distribution that is momentum dependent.

c_2 and ms_2 have the same meaning but for positive particle. Let assume that $c_1 = c_2 = c$ and $ms_1 = ms_2 = m$,

$$\sigma^2_{\Delta x} = 2 \cdot c^2 + (\frac{1}{P_-^2} + \frac{1}{P_+^2}) \cdot ms^2$$

Therefore it is reasonable to use

$$Z = \frac{1}{(P_- \cdot \beta -)^2} + \frac{1}{(P_+ \cdot \beta +)^2}$$

and to fit $\sigma^2_{\Delta x}$ or the square of RMS of the Δx distribution for different value of the variable Z.

The error associated to this value is

$$Er(RMS^2) = \frac{RMS^4}{(N_b - 2)}$$

where N_b is the number of events in the momentum bin b which were used for the corresponding RMS. For the error to associate to $\sigma^2_{\Delta x}$, extracted from a Gaussian fit, the situation is simpler: the fit procedure gives an estimation of error on $\sigma_{\Delta x}$ that we write as $Er(\sigma_{\Delta x})$ thus we can write:

$$Er(\sigma^2_{\Delta x}) = 4 \cdot \sigma^2_{\Delta x} \cdot Er^2(\sigma_{\Delta x})$$

We divide the space covered by the variable Z in 11 bins of width 0.1, this covers the space between 0.1 to 1.2 of the Z variable for our experimental data.

4.1 X projection

In the plot 3 are the experimental data and the best approximation of the Monte Carlo for the X projection, the result of the fit as a second degree polinomial is reported in Table 9.

Table 9: $(Dx)^2$ as function of Z, Dx is the sigma resulting from a gaussian fit,P_0, P_1andP_2 are the three parameters of a polynomial of second degree.

	Data	MS 1.0	MS 0.95	MS 0.90
P_0	(0.007 ± 0.002)	(0.012 ± 0.001)	(0.012 ± 0.001)	(0.013 ± 0.001)
P_1	(0.51 ± 0.02)	(0.545 ± 0.006)	(0.517 ± 0.006)	(0.492 ± 0.006)
P_2	(-0.03 ± 0.02)	(-0.056 ± 0.007)	(-0.04 ± 0.007)	(-0.03 ± 0.006)
Chi^2	24/7	67/7	61/7	85/7
$\widehat{S^2_{\Delta x}}$	0.287 ± 0.002	0.299 ± 0.0006	0.290 ± 0.0006	0.282 ± 0.0006

Table 10: $(Dx)^2$ as function of Z, Dx is the sigma resulting from a gaussian fit,P_0, P_1andP_2 are the three parameters of a polynomial of second degree.

	Data	MS 0.90 x 1.05 y	MS 0.90 x 1.0 y
P_0	(0.007 ± 0.002)	(0.014 ± 0.001)	(0.012 ± 0.001)
P_1	(0.51 ± 0.02)	(0.518 ± 0.006)	(0.518 ± 0.006)
P_2	(-0.03 ± 0.02)	(-0.035 ± 0.006)	(-0.044 ± 0.006)
$(Chi)^2$	24/7	98/7	42/7
$\widehat{S^2_{\Delta x}}$	0.287 ± 0.002	0.295 ± 0.0006	0.2899 ± 0.0006

4.2 Y projection

The formulaes are just the same as for the X projection, just we replace Δx with Δy.

In the plot 4 are the experimental data and the best approximation of the Monte Carlo for the Y projection, the result of the fit as a second degree polinomial is reported in Table 11..

Table 11: $(Dy)^2$ as function of Z, Dy is the sigma resulting from a gaussian fit,P_0, P_1andP_2 are the three parameters of a polynomial of second degree.

	Data	MS 1.0	MS 0.95	MS 0.90
P_0	(0.001 ± 0.003)	(0.02 ± 0.001)	(0.013 ± 0.001)	(0.014 ± 0.001)
P_1	(0.609 ± 0.02)	(0.501 ± 0.006)	(0.496 ± 0.006)	(0.471 ± 0.006)
P_2	(-0.144 ± 0.02)	(-0.03 ± 0.007)	(-0.03 ± 0.006)	(-0.02 ± 0.006)
$(Chi)^2$	14/7	58/7	75/7	98/7
$\widehat{S^2_{\Delta y}}$	0.293 ± 0.002	0.2908 ± 0.0006	0.2837 ± 0.0006	0.2736 ± 0.0006

Due to the difficulty to compare fit parameters (with a not so good $(Chi)^2$) we have reproduced the parameter $\widehat{S_{\Delta x}}$ and $\widehat{S_{\Delta y}}$ built as above but on the 11 bins as in the figure 3.

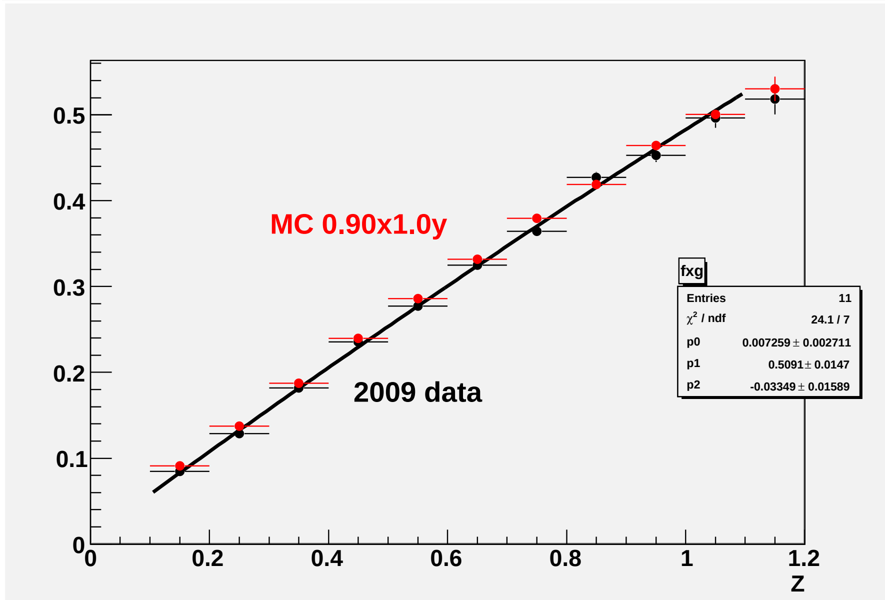

Figure 3: $(Dx)^2$ as function of $Z = \frac{1}{(P_- \cdot \beta -)^2} + \frac{1}{(P_+ \cdot \beta +)^2}$.

Table 12: $(Dy)^2$ as function of Z, Dy is the sigma resulting from a gaussian fit,P_0, P_1andP_2 are the three parameters of a polynomial of second degree.

	Data	MS 0.90 x 1.05 y	MS 0.90 x 1.0 y
P_0	(0.001 ± 0.003)	(0.012 ± 0.001)	(0.013 ± 0.001)
P_1	(0.609 ± 0.02)	(0.517 ± 0.006)	(0.498 ± 0.006)
P_2	(-0.144 ± 0.02)	(-0.048 ± 0.006)	(-0.031 ± 0.006)
$(Chi)^2$	14/7	94/7	94/7
$\widehat{S^2_{\Delta y}}$	0.293 ± 0.002	0.2867 ± 0.0006	0.2838 ± 0.0006

5 Cross assignement ..

Since in our analysis we are mainly interested in the transvers component $\delta T = \sqrt{\delta(x)^2 + \delta(y)^2}$ and not separately in the $\delta(x)$,$\delta(y)$ as a first approximation we have decided to choose the MC simulation that provides the best cross agreement since we do not see how to tune the single projection. This means that we look for the possibility to compare:

$$\widehat{S^2_{\Delta x}}(data) \simeq \widehat{S^2_{\Delta y}}(MC)$$
$$\widehat{S^2_{\Delta y}}(data) \simeq \widehat{S^2_{\Delta x}}(MC)$$

the MC0.90x1.0y corresponds exactly at this, having an agreement at best of 3.1×10^{-4} for X projection and 5.5×10^{-4} for Y, see figures 5, 6.

$$\widehat{S^2_{\Delta x}}(data) = 0.2867 \simeq \widehat{S^2_{\Delta y}}(MC) = 0.2868$$
$$\widehat{S^2_{\Delta y}}(data) = 0.2933 \simeq \widehat{S^2_{\Delta x}}(MC) = 0.2949$$

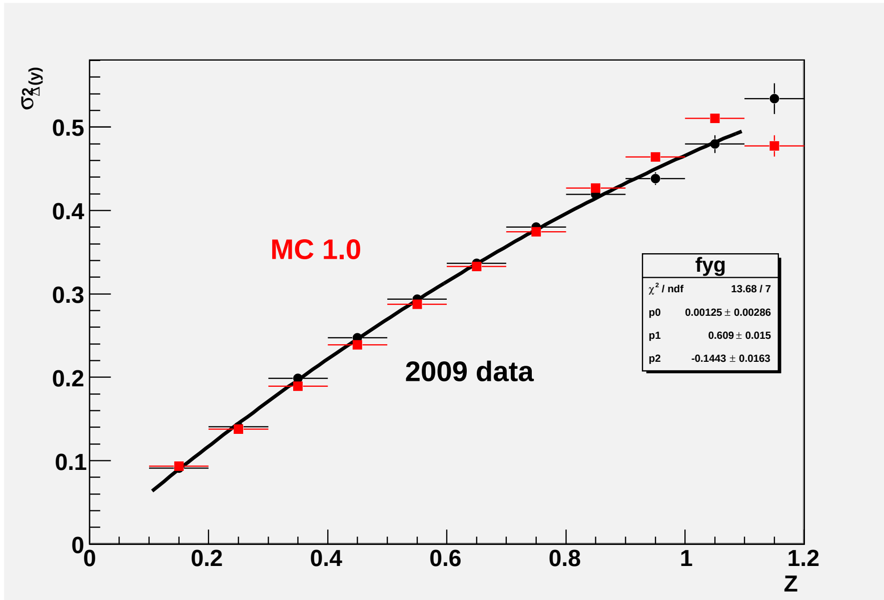

Figure 4: $(Dy)^2$ as function of $Z = \frac{1}{(P_- \cdot \beta -)^2} + \frac{1}{(P_+ \cdot \beta +)^2}$

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5.1 Special studies

In order to undestand a bit better the results we did generate MonteCarlo events with special conditions :

- NoMS) No Multiple Scattering in any detector.
- NoMSH) No Multiple Scattering and No Hadron interactions.
- allOFF) all interaction are switched OFF.

In Figure 7 are the superimposed histograms of the $\sigma^2_{\Delta x}$ and $\sigma^2_{\Delta y}$ as function of Z when in the simulation the Multiple scattering is swithched OFF.

Table 13: $(Dx)^2$ as function of Z, Dx is the sigma resulting from a gaussian fit,P_0, P_1andP_2 are the three parameters of a polynomial of second degree.

	Data	noMS	no MSH	all OFF
P_0	(0.007 ± 0.002)	(0.0003 ± 4.0 $\cdot 10^{-6}$)	(0.0003 ± 4.0 $\cdot 10^{-6}$)	(0.0003 ± 3.0 $\cdot 10^{-6}$)
P_1	(0.51 ± 0.02)	(0.0011 ± 1.8 $\cdot 10^{-5}$)	(0.00109 ± 1.7 $\cdot 10^{-5}$)	(0.00102 ± 1.4 $\cdot 10^{-5}$)
P_2	(-0.03 ± 0.02)	(-0.0005 ± 1.7 $\cdot 10^{-5}$)	(-0.0004 ± 1.6 $\cdot 10^{-5}$)	(-0.0004 ± 1.3 $\cdot 10^{-5}$)
Chi^2	24/7	3647/7	3300/7	4372/7
$\widehat{S^2_{\Delta x}}$	0.287 ± 0.002	7.85 $\cdot 10^{-4}$ ± 1.$\cdot 10^{-6}$	7.85 $\cdot 10^{-4}$ ± 1.$\cdot 10^{-6}$	7.80 $\cdot 10^{-4}$ ± 1.$\cdot 10^{-6}$

Table 14: $(Dy)^2$ as function of Z, Dy is the sigma resulting from a gaussian fit,P_0, P_1andP_2 are the three parameters of a polynomial of second degree.

	Data	no MS	no MSH	allOFF
P_0	(0.001 ± 0.003)	$(0.0004 \pm 2.0 \cdot 10^{-6})$	$(0.0004 \pm 2.0 \cdot 10^{-6})$	$(0.0004 \pm 2.0 \cdot 10^{-6})$
P_1	(0.609 ± 0.02)	$(-7. \cdot 10^{-5} \pm 7.0 \cdot 10^{-6})$	$(-8.4 \cdot 10^{-5} \pm 8. \cdot 10^{-6})$	$(-0.0001 \cdot 10^{-5} \pm 8. \cdot 10^{-6})$
P_2	(-0.144 ± 0.02)	$(-1. \cdot 10^{-5} \pm 7.0 \cdot 10^{-6})$	$(9. \cdot 10^{-6} \pm 7. \cdot 10^{-6})$	$(2. \cdot 10^{-5} \pm 7. \cdot 10^{-6})$
$(Chi)^2$	13/7	1329/7	1351/7	1947/7
$\widehat{S^2_{\Delta y}}$	0.293 ± 0.002	$3.36 \cdot 10^{-4} \pm 4. \cdot 10^{-7}$	$3.36 \cdot 10^{-4} \pm 4. \cdot 10^{-7}$	$3.32 \cdot 10^{-4} \pm 4. \cdot 10^{-7}$

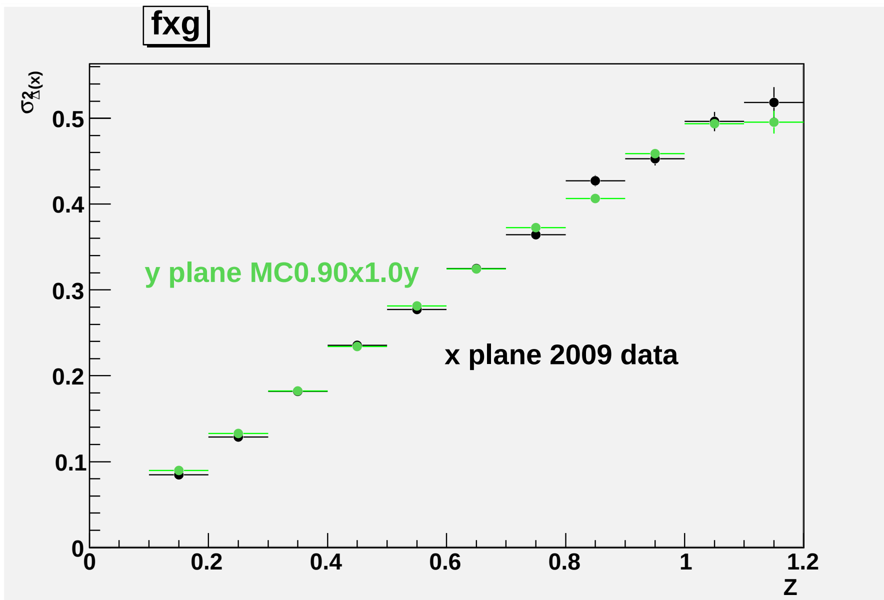

Figure 5: Comparison of the MS for the X plane of data with the Y plane from MC0.90x1.0y

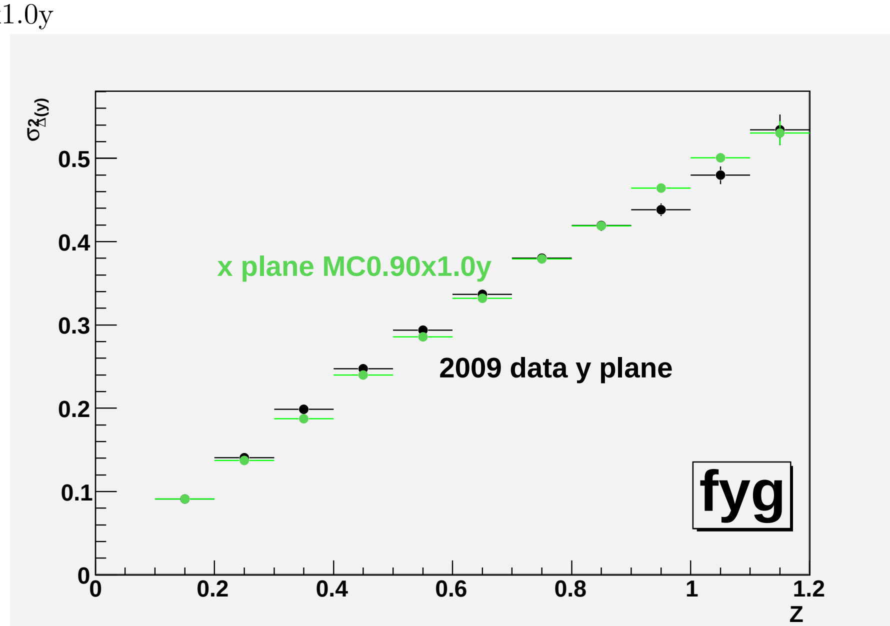

Figure 6: Comparison of the MS for the Y plane of data with the X plane from MC0.90x1.0y

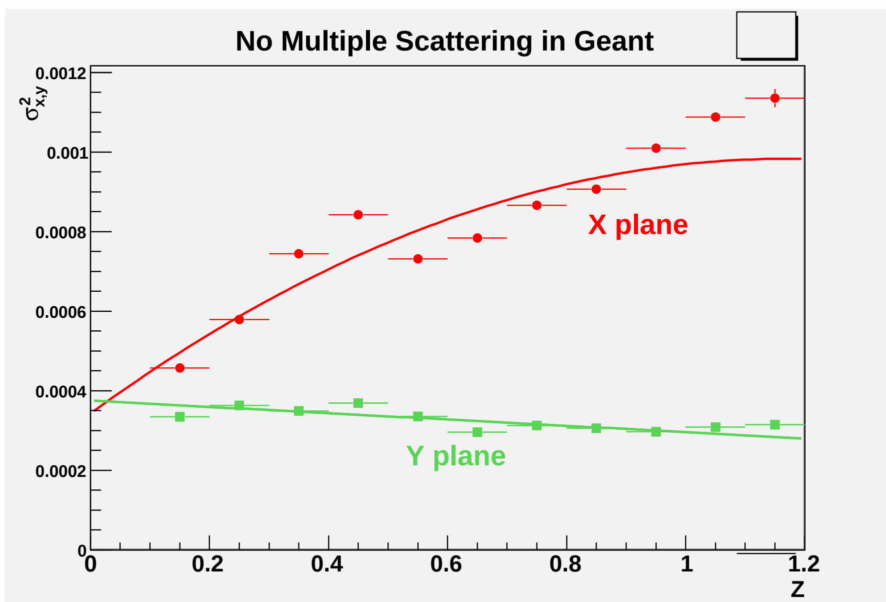

Figure 7: $\sigma^2_{\Delta x}$ and $\sigma^2_{\Delta y}$ as function of Z when in the simulation the Multiple scattering is swithched OFF.