DIRAC note 2012-05
16 March 2012

On the separation r^*-distribution for pion-kaon pairs in the experiment DIRAC

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The method of the lifetime measurement of hadronic $(\pi^+\pi^-, \pi^+K^-, \ldots)$ atoms in the DIRAC experiment at CERN is based on the production of the atoms in a thin target and subsequent detection of highly correlated hadronic pairs leaving the target as a result of the breakup of a part of the atoms which did not decay within the target [1]. Clearly, the breakup probability is a unique function of the target geometry and material, the Lorentz factor and the ground-state lifetime of the atom, thus allowing one to determine the latter.

The breakup probability is defined as the ratio of the number $N_A^{\rm br}$ of breakup atoms to the number N_A of the atoms produced in the target: $P_{\rm br} = N_A^{\rm br}/N_A$. To determine $N_A^{\rm br}$, one has to measure the excess of highly correlated hadronic pairs at very small relative momenta Q of a few MeV$/c$ in the pair center-of-mass system above the pairs correlated due to the Coulomb and strong final-state interaction (FSI). As for the number of produced atoms N_A, it can be calculated based on the FSI theory in discrete and continuous spectrum.

The FSI effect on the production of hadronic pairs is sensitive to the space-time extent of the production region mainly through the distance r^* between the production points of the hadrons in the pair rest frame. In [1], only the Coulomb FSI was considered and the r^*-dependence was treated in an approximate way, dividing the pion emitters into short-lived and long-lived ones. It was assumed that $r^* = 0$ for pion pairs arising solely from the short-lived emitters and characterized by the distances r^* much smaller than the Bohr radius of the dihadron system, otherwise $r^* = \infty$.

Following [1], in the analysis of the data from DIRAC experiment, the nonrelativistic point-like Coulomb wave functions in continuous and discrete spectrum were respectively used in the simulation of correlated dihadron background and calculation of the number of produced hadronic atoms. Due to large Bohr radii of $\pi^+\pi^-$ or π^+K^- systems of 387.5 fm or 248.6 fm respectively, the point-like treatment of Coulomb final state interaction is valid for the hadronic pairs composed of directly produced hadrons or hadrons from the decays of short-lived resonances. This treatment should be however modified for the pairs containing hadrons originating from decays of narrow resonances, like ω and η' with the decay lengths in a decay pion rest frame $(l = \langle p_{\rm dec}\rangle(m_\pi\Gamma)^{-1})$ of 30 and 900 fm respectively, that are significant or comparable with the pair Bohr radii. One should also account for the strong final state interaction including, besides the elastic transitions $\pi^+\pi^- \to \pi^+\pi^-$ or $\pi^+K^- \to \pi^+K^-$ (driven at $Q \to 0$ by the s-wave scattering lengths of 0.186 fm or

0.137 fm respectively), also the inelastic ones $\pi^0\pi^0 \to \pi^+\pi^-$ or $\pi^0\bar{K}^0 \to \pi^+K^-$ (with the respective scattering lengths of -0.176 fm or -0.147 fm).

The corrections to the point-like Coulomb final state interaction can be summarized in correction factors $1+\delta(Q)$ and $1+\delta_n$ respectively multiplying the calculated point-like production cross sections of Coulomb correlated hadronic pairs and s-wave hadronic atoms (with the main quantum number n). These correction factors for $\pi^+\pi^-$ system have been studied with the help of UrQMD transport code simulations with the following results [2, 3, 4]:

(i) The global shifts of the correction factors from unity are determined with rather large uncertainty of several percent due to the uncertainty in the treatment of short-lived emitters. However, being practically the same and nearly constant, the short-distance contribution to the shifts $\delta(Q)$ and δ_n practically cancel in the calculated break-up probability.

(ii) The finite-size correction factors $1+\delta(Q)$ and $1+\delta_n$ depend on the fractions of $\pi^+\pi^-$ pairs containing pions from ω- or η'-decays. Besides leading to slightly different global shifts of the correction factors, the ω and η' decay pions also noticeably affect their Q- and n-dependence. The correction factors are basically determined by the fraction f_ω of $\pi^+\pi^-$ pairs containing a pion from ω-decay and the other pion from a short-lived emitter or from another ω-decay. The simulated fraction f_ω increases with decreasing Q and, in the analysis region of $Q < 15$ MeV$/c$, composes ~ 0.15, Depending on the analysis details, it leads to $1.5-3\%$ (4-8%) correction in the breakup probability (pionium lifetime).

Similarly, since the $\pi^\pm K^\mp$ Bohr radius is comparable with the $\pi^+\pi^-$ one, one may expect also a significant effect of the decay pions from ω- and η'-resonances on the extraction of the lifetime of $\pi^\pm K^\mp$ atoms. Using the pion, kaon and resonance production rates in pp collisions at 24 GeV/c [5, 6], one may roughly estimate the fractions of the $\pi^\pm K^\mp$ pairs containing the pions from ω and η' decays as $f(\pi^\pm_\omega K^\mp) \approx \langle\omega \to 3\pi + 2\pi\rangle/\langle\pi^+\rangle = 0.05$ and $f(\pi^\pm_{\eta'} K^\mp) \approx \langle\eta' \to \pi^+\pi^- + X\rangle/\langle\pi^+\rangle = 0.006$. They represent about half of the corresponding inclusive fractions contributing to the $\pi^+\pi^-$ pairs in pp interactions at 24 GeV/c [5]. For pNi interactions in the experiment DIRAC the fraction $f(\pi^\pm_\omega \pi^\mp)$ at small Q is estimated by $\sim 50\%$ larger than the inclusive one in pp interactions [5] in agreement with the UrQMD simulations [3]. Assuming a similar enhancement of $\pi^\pm_\omega K^\mp$ pairs in pNi interactions at small Q, one may expect $f(\pi^\pm_\omega K^\mp) \approx 0.07$. One has to account also for the kaons from the ϕ-meson decays [6] with the decay length $l_\phi = p_{\rm dec}/(m_K\Gamma_\phi)^{-1} = 11.9$ fm. A rough estimate of the fraction of $\pi^\pm K^\mp$ pairs containing such decay kaons composes $f(\pi^\pm K^\mp_\phi) \approx \langle\phi \to K^+K^-\rangle/\langle K^-\rangle = 0.12$ [6] (this estimate may be considered as an upper limit since $\langle K^+\rangle \doteq 3.5\langle K^-\rangle$; on the other hand, a smaller fraction of the channels with a K^+ in the final state contributes to the π^-K^+ pairs as compared with the contribution of the channels with a K^- in the final state to the π^+K^- pairs [6]). This additional contribution, together with $\sim 36\%$ smaller Bohr radius may at least partially compensate for a smaller contribution of ω- and η'-resonances to the $\pi^\pm K^\mp$ pairs and, as a result, lead to similar finite-size effects on the extracted lifetimes of $\pi^\pm K^\mp$ and $\pi^+\pi^-$ atoms.

Following [2, 3, 4], one may employ a simplified simulation of the respective contributions to the r^*-separation distribution for $\pi^\pm K^\mp$ pairs. For the pairs containing any kaon and a pion from η' decay ($\sim 0.6\%$) and, a kaon from ϕ decay and any pion except those from η' and ω decays ($\sim 12\%$), one can use an exponential-like formula

$$\mathcal{F}(r^*; r_R, l_R) = \frac{x^2}{2.2}\left\{1 - \exp\left[-\frac{2.2}{x^2}\left(1 + 0.2x^2\frac{1+0.15x^2y}{1+x^5/125}\right)\right]\right\} \mathrm{e}^{-y}$$
$$x = \frac{r^*}{r_R} \qquad y = \frac{r^*}{l_R} \qquad , \tag{1}$$

where $r_{\eta'} \approx r_\phi = 2$ fm, $l_{\eta'} = 790$ fm and $l_\phi = 11.9$ fm. For $\sim 7\%$ pairs containing a pion from ω decay and any kaon from a short-lived emitter (including kaons from ϕ decays, since $l_\phi \ll l_\omega$), a superposition of two exponential-like expressions is required:

$$\sum_i \frac{dN(\pi_\omega K_i)}{dr^*} \doteq n_{1\omega}\mathcal{F}(r^*; r_{1\omega}, l_{1\omega}) + n_{2\omega}\mathcal{F}(r^*; r_{2\omega}, l_{2\omega}). \tag{2}$$

The parameters $r_{1\omega} = 1.07$ fm, $l_{1\omega} = 43.0$ fm, $r_{2\omega} = 2.65$ fm, $l_{2\omega} = 25.5$ fm, $n_{1\omega}/n_{2\omega} = 0.991$. As for the $\pi^\pm K^\mp$ pairs containing pions from ϕ decays (branching ratio of $\sim 15\%$) with the corresponding ϕ decay length of ~ 100 fm, one may estimate their fraction as $f(\pi^\pm_\phi K^\mp) \approx \langle \phi \to 3\pi\rangle / \langle \pi^+ \rangle = 0.0014$. It appears to be quite small and can be neglected. The remaining short-distance part of the r^*-distribution may be approximated by a Gaussian one:

$$\mathcal{G}(r^*; r_\mathcal{G}) = r^{*2} \exp\left(-\frac{r^{*2}}{4r_\mathcal{G}^2}\right) \tag{3}$$

with $r_\mathcal{G} \approx 3$ fm. In later stage, one may refit the parameters in these formulae and get a more reliable estimates of the corresponding pair fractions based on the transport code simulations.

References

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