# Production rates for $\pi^{+} K^{-}, \pi^{-} K^{+}$and $\pi^{+} \pi^{-}$ atoms in $\mathrm{p}-\mathrm{Ni}$ interactions at proton momentum 24 and $450 \mathrm{GeV} / \mathrm{c}$. 

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#### Abstract

The results of performed analysis show that the yield of $\pi^{+} K^{-}, K^{+} \pi^{-}$and $\pi^{+} \pi^{-}$atoms in the p-nuclear interactions increases significantly at change of proton momentum $P_{p}$ from 24 up to $450 \mathrm{GeV} / \mathrm{c}$. The yield of these dimesoatoms at $P_{p}=450$ $\mathrm{GeV} / \mathrm{c}$ and $\theta_{l a b}=4^{\circ}$ is on the order more than the one at DIRAC experiment conditions: $P_{p}=24 \mathrm{GeV} / \mathrm{c}$ and $\theta_{\text {lab }}=5.7^{\circ}$. The large yield of dimesoatoms allows significant improvement of their lifetime measurement.


## 1 Introduction

Measurements of the annihilation probabilities for hadron atoms allow one to determine the threshold amplitudes of the transitions of the particles forming the atoms. The measurement of the annihilation probabilities for the channels

$$
\begin{align*}
A_{2 \pi} & \rightarrow \pi^{0}+\pi^{0}  \tag{1}\\
A_{\pi K} & \rightarrow \pi^{0}+K^{0} \tag{2}
\end{align*}
$$

allows one to determine the amplitudes of the processes

$$
\begin{align*}
\pi^{+}+\pi^{-} & \rightarrow \pi^{0}+\pi^{0}  \tag{3}\\
\pi^{+}+K^{-} & \rightarrow \pi^{0}+K^{0} \tag{4}
\end{align*}
$$

by the model-independent method practically at zero energy in the center-of-mass system of initial particles. The annihilation probabilities of the atoms formed by $\pi^{+}$- and $\pi^{-}$mesons $\left(A_{2 \pi}\right)$ were obtained in $[1,2,3]$ and by oppositely charged $\pi$ - and K-mesons $\left(A_{\pi K}\right)$ - in $[2,4]$.

Because the amplitudes of (3) and (4) at low energies can be expressed through the differences of scattering lengths $\pi \pi(\pi K)$ the measurement of the probabilities of processes (1) and (2) allows to determine the values

$$
\left.\begin{gather*}
\left|a_{0}-a_{2}\right|  \tag{5}\\
\mid b_{1 / 2} \tag{6}
\end{gather*}-b_{3 / 2} \right\rvert\,
$$

by the model-independent method. Here $a_{0}, a_{2}\left(b_{1 / 2}, b_{3 / 2}\right)$ are the $\pi \pi(\pi \mathrm{K}) \mathrm{S}$-wave scattering lengths in the states with isospins of $0,2(1 / 2,3 / 2)$.

The $\pi \pi$ scattering lengths $a_{0}, a_{2}$ have been calculated in the framework of Chiral Perturbation Theory [5] using an effective Lagrangian with a precision of better than $2.5 \%$ [6]. The model-independent measurement of the $\pi \pi$-scattering lengths allows one to
test the predictions of CHPT and, consequently, our understanding of chiral symmetry breaking [7] lying in the basis of the QCD Lagrangian, which describes interactions of quarks and gluons.

The $\pi \mathrm{K}$-scattering lengths were also calculated in CHPT [10, 8] and by using RayStiner equations [9]. The model-independent measurement of these values allows to test the concept of chiral symmetry breaking in the processes with strangeness.

In the paper [11] the relations were obtained that allows one to calculate the production cross-sections of $A_{2 \pi}, A_{\pi K}$ and any other atoms in the case if the inclusive production cross sections of the particles forming these bound states are known. In the same paper a method was also proposed for observation and lifetime measurement of the atoms. The estimates were given for the yields of $A_{2 \pi}, A_{\pi K}$ and other atoms in pp-collisions at a beam energy of 70 GeV and at atom emission angle of $8.4^{\circ}$ in the lab system. The observation of $A_{2 \pi}$ was carried out at the U-70 accelerator [12], the atoms produced in p-Ta collisions at $P_{p}=70 \mathrm{GeV} / \mathrm{c}$ were detected at $8.4^{\circ}$ in the lab system.

Later the measurement of the $A_{2 \pi}$ lifetime was done in [13, 14]. The atoms were generated in p-Ni interactions by proton beam of PS CERN with momentum $P_{p}=24$ $\mathrm{GeV} / \mathrm{c}$. The ionized $A_{2 \pi}$ (atomic pairs) were detected at laboratory angle $5.7^{\circ}$. The $\pi \pi$ scattering lengths were obtained with precision about $4 \%$ from the 21000 ionized $A_{2 \pi}$ analysis. At the same conditions there was performed the experiment on the search for $A_{\pi^{+} K^{-}}$and $A_{\pi^{-} K^{+}}$in the p-Pt interaction [15]. This experiment shows that expected number of $\pi K$ atomic pairs in p-Ni interactions can be on the level of 300-400 with statistical error which allows to achieve the measurement of the $\pi K$ scattering length with precision on the level around $25 \%$ only.

The investigation of $A_{2 \pi}$ allows to measure the Lamb shift in this atom [11, 16, 17] and to extract the another combination of $\pi \pi$ scattering lengths $2 a_{0}+a_{2}$. If the resonance method can be used then this combination will be extracted with precision on the order higher than accuracy of other methods $[18,19]$.

The increase of dimesoatom yield with growth of $P_{p}$ from 24 up to $1000 \mathrm{GeV} / \mathrm{c}$ was established in $[20,21]$. In the present work the yields and spectra of $A_{2 \pi}, A_{\pi^{+} K^{-}}$and $A_{\pi^{-} K^{+}}$generated in p-Ni interaction at $P_{p}=24$ and $450 \mathrm{GeV} / \mathrm{c}$ are calculated a bit precisely than in $[20,21]$. Because the yields of $A_{2 \pi}, A_{\pi^{+} K^{-}}$and $A_{\pi^{-} K^{+}}$at $P_{p}=24 \mathrm{GeV} / \mathrm{c}$ and $\theta_{\text {lab }}=5.7^{\circ}$ are known now, the ratio of these atom yields at $P_{p}=450 \mathrm{GeV} / \mathrm{c}, \theta_{\text {lab }}=0 \div 5.7^{\circ}$ and at $P_{p}=24 \mathrm{GeV} / \mathrm{c}$ were calculated. It allows to define the optimal value of $\theta_{\text {lab }}$ at 450 $\mathrm{GeV} / \mathrm{c}$ and expected statistics of $A_{2 \pi}$ and $A_{\pi K}$. It allows to extract the precision of $A_{\pi K}$ lifetime measurement and to obtain the possible accuracy of the Lamb shift of $A_{2 \pi}$ measurement.

## 2 Basic relations

The probability of atom production is proportional to the double inclusive cross section for generation of the two constituent particles of this atom with small relative momenta. Calculating the atom production cross section, one should exclude the contribution to the double cross section from those constituents that arise from the decays of long-lived particles and cannot form the atom. When one or both particles in the pair come from these decays, the typical range between them is much larger than the Bohr radius of the atom and the probability of atom production is negligible. The main long-lived sources of
pions are $\eta, \eta^{\prime}, \Lambda, K_{s}^{0}, \Sigma^{ \pm}$For the case of pion and kaons the short-lived sources constitute the main contribution.

The laboratory differential inclusive cross section for the atom production can be written in the form [11]

$$
\begin{equation*}
\frac{d \sigma_{n}^{A}}{d \vec{p}_{A}}=\left.(2 \pi)^{3} \frac{E_{A}}{M_{A}}\left|\Psi_{n}(0)\right|^{2} \frac{d \sigma_{s}^{0}}{d \vec{p}_{1} d \vec{p}_{2}}\right|_{\vec{p}_{1}=\frac{m_{1}}{m_{2}} \vec{p}_{2}=\frac{m_{1}}{M_{A}} \vec{p}_{A}} \tag{7}
\end{equation*}
$$

where $M_{A}$ is the atom mass, $\vec{p}_{A}$ and $E_{A}$ are the momentum and energy of the atom in the lab system, respectively, $\left|\Psi_{n}(0)\right|^{2}=p_{B}^{3} / \pi n^{3}$ is the atomic wave function(without regard for the strong interactions between the particles forming the atom, i.e. the pure Coulomb wave function) squared at the origin with the principal quantum number $n^{1}$ and the orbital momentum $l=0, p_{B}$ is the Bohr momentum of the particles in the atom, $d \sigma_{s}^{0} / d \vec{p}_{1} d \vec{p}_{2}$ is the double inclusive production cross section for the pairs from short-lived sources (hadronization processes, $\rho, \omega, \Delta, K^{*}, \Sigma^{*}$, etc.) without regard for the $\pi^{+} \pi^{-}$Coulomb interaction in the final state, $\vec{p}_{1}$ and $\vec{p}_{2}$ are the momenta of the particles forming the atom in the lab system. The momenta obey the relation $\vec{p}_{1}=\frac{m_{1}}{m_{2}} \vec{p}_{2}=\frac{m_{1}}{M_{A}} \vec{p}_{A}\left(m_{1}\right.$ and $m_{2}$ are the masses of the particles). The atoms are produced with the orbital momentum $l=0$, because $\left|\Psi_{n, l}(0)\right|^{2}=0$ when $l \neq 0$.

The double inclusive cross section without regard for the Coulomb interaction may be written in the form [22]:

$$
\begin{equation*}
\frac{d \sigma^{0}}{d \vec{p}_{1} d \vec{p}_{2}}=\frac{1}{\sigma_{i n}} \frac{d \sigma}{d \vec{p}_{1}} \frac{d \sigma}{d \vec{p}_{2}} R\left(\overrightarrow{p_{1}}, \overrightarrow{p_{2}}, S\right) \tag{8}
\end{equation*}
$$

where $d \sigma / d \vec{p}_{1}$ and $d \sigma / d \vec{p}_{2}$ are the single particle production inclusive cross sections, $\sigma_{i n}$ is the inelastic cross section of hadron production, $R$ is a correlation function due to strong interaction only and $S$ is the square of full c.m.s. energy of beam proton and target hadron.

The probability of particle production per interaction (yield) can be expressed through the differential cross section:

$$
\begin{equation*}
\frac{d N}{d \vec{p}}=\frac{d \sigma}{d \vec{p}} \frac{1}{\sigma_{i n}} \tag{9}
\end{equation*}
$$

From (7), (8) and (9), after substituting the expression for $\left|\Psi_{n}(0)\right|^{2}$ and summing over $n$, one can obtain an expression for the inclusive yield of atoms in all S-states through the inclusive yields of positive and negative hadrons

$$
\begin{equation*}
\frac{d^{2} N_{A}}{d p_{A} d \Omega}=1.202 \cdot 8 \pi^{2}(\mu \alpha)^{3} \frac{E_{A}}{M_{A}} \frac{p_{A}^{2}}{p_{1}^{2} p_{2}^{2}} \frac{d^{2} N_{1}}{d p_{1} d \Omega} \frac{d^{2} N_{2}}{d p_{2} d \Omega} R\left(\overrightarrow{p_{1}}, \overrightarrow{p_{2}}, S\right) \tag{10}
\end{equation*}
$$

where $\mu$ is the reduced mass of the atom $\left(\mu=\frac{m_{1} m_{2}}{m_{1}+m_{2}}\right), \alpha$ is the fine structure constant and $\Omega$ is a solid angle.

The yield and momentum distribution of $A_{2 \pi}$ atoms were measured at $P_{p}=24 \mathrm{GeV} / \mathrm{c}$ and $\theta_{\text {lab }}=5.7^{\circ}$, the yield of $\pi^{+} K^{-}$and $K^{+} \pi^{-}$atoms will be obtained during 2012. Hence

[^0]it is useful to present not only the absolute yields of atoms at different conditions but their yields relative to the known values.

To obtain the yields of $K^{ \pm}$- and $\pi^{ \pm}$-mesons we used the computer simulation programs Fritiof 6.0 and Jetset 7.3 [23] (CERN Program Library) based on the Lund string fragmentation model. Fritiof is a generator for hadron-hadron, hadron-nucleus and nucleus-nucleus collisions, which makes use of JETSET for fragmentation.

## 3 Experimental values of pion and kaon inclusive cross sections and their description by FRITIOF.

It is a important to know how well the inclusive cross sections obtained by FRITIOF coincide with corresponding experimental data.

In the case of of $A_{2 \pi}$ atoms the main interval of pion momentum is $1 \div 3 \mathrm{GeV} / \mathrm{c}$ and for kaons from $A_{\pi K}$ is $4 \div 7 \mathrm{GeV} / \mathrm{c}$. There are some data which cover partially the DIRAC setup momentum acceptance:

1. The inclusive cross sections of pions and kaons generated at $\theta_{\text {lab }}=17 \div 127 \mathrm{mrad}$ in p- $(\mathrm{Be}, \mathrm{Al}, \mathrm{Cu}, \mathrm{Pb})$ interaction at $P_{p}=24 \mathrm{GeV} / \mathrm{c}$ with secondary momentum from 4 $\mathrm{GeV} / \mathrm{c}[24]$.
2. The inclusive cross sections of pions and kaons generated at $\theta_{l a b}=0^{\circ}$ in p -Be interaction at $P_{p}=450 \mathrm{GeV} / \mathrm{c}$ with secondary momentum from $7 \mathrm{GeV} / \mathrm{c}[25]$.

It was shown that these experimental yields are described by FRITIOF with precision of $20 \div 50 \% ~([20,26])$.

We are going to use Ni target but there is no data for pion and kaon inclusive cross sections with such material. In this connection it is interesting for us the data [24] which reveals the weak dependence of soft pion and kaon yields on the nucleon atomic number A from Be up to Cu . Also there is the weak dependence of these yields on A in p nuclear $(\mathrm{C}, \mathrm{Al}, \mathrm{Cu})$ interactions at $P_{p}=100 \mathrm{GeV} / \mathrm{c}$ for the secondary particle momentum 30 $\mathrm{GeV} / \mathrm{c}[27]$. Therefore the values of soft dimesoatom yields calculated for $\mathrm{p}-\mathrm{Ni}$ interactions are a good estimation of the same yields in any p-Nucleus interactions also.

## 4 Results of calculations

The selection of particles from long-lived and short-lived sources was performed. Further, using yields only from the short-lived sources we obtained the distributions of the atom yields over the angle and momentum.

### 4.1 Calculations of $R\left(\overrightarrow{p_{1}}, \overrightarrow{p_{2}}, S\right)$ at proton momentum 24 and 450 GeV/c

The correlation factor $R$ was calculated using Fritiof generator according equation

$$
\begin{equation*}
R\left(\overrightarrow{p_{1}}, \overrightarrow{p_{2}}, S\right)=\frac{d N}{d \overrightarrow{p_{1}} d \overrightarrow{p_{2}}} /\left(\frac{d N}{d \vec{p}_{1}} \frac{d N}{d \vec{p}_{2}}\right)=\frac{d N}{d p_{1} d p_{2}} /\left(\frac{d N}{d p_{1}} \frac{d N}{d p_{2}}\right) . \tag{11}
\end{equation*}
$$

The values of $R$ were calculated for the momentum interval of setup acceptance.
The results for $R$ are shown on Fig. $1,2,3,4,5$ and 6 . It follows that the factor R for $\pi^{+} \pi^{-}$pairs at $P_{p}=24 \mathrm{GeV} / \mathrm{c}$ decreases from 1 to 0.5 when $P_{A}$ grows up(practically it's the same for $\pi K$-pairs). It is due to conservation laws constrains. At $P_{p}=450 \mathrm{GeV} / \mathrm{c}$ the value of R depends slightly on $P_{A}$ and $\theta_{l a b}$. The value of $R$ for $\pi K$ pairs practically does not depend on $P_{A}$ but the dependence on $\theta_{\text {lab }}$ is enough strong. We found that the values of $R$ for $\pi K$-pairs at $\theta_{\text {lab }}=0^{\circ}$ depend on the value of taken solid angle $\Delta \Omega$ and presented dependence is approximation of $R_{\Delta \Omega \rightarrow 0}$ behavior. For the $\theta_{\text {lab }} \geq 0^{\circ}$ the value of $R$ doesn't depend on $\Delta \Omega$.

The yields were calculated for the two cases: 1) solid angle of $10^{-3} \mathrm{sr}$ without taking into account the setup acceptance and setting the correlation factor to $R=1$ and 2) with taking into account the acceptance and using the calculated values of $R$.

### 4.2 The calculations of inclusive cross sections of pions, $\pi^{+} \pi^{-}$ and $\pi K$ atoms.

These calculations were performed with $R=1$, solid angle of $10^{-3} \mathrm{sr}$ and without taking into account the setup acceptance. The main results of calculations are presented on Fig. $7,8,9$ and 10 and in Tab.1.

Table 1: The total yield of charged pions $W_{\pi^{ \pm}}, \pi^{+} \pi^{-}, \pi^{+} K^{-}$and $K^{+} \pi^{-}$atoms $W_{A}$ into the aperture of $10^{-3}$ sr per one p-Ni interaction at the proton momenta $P_{p}=24$ and $450 \mathrm{GeV} / \mathrm{c}$ versus emission angle $\theta_{\text {lab }} . W_{\pi^{ \pm}}^{N}=W_{\pi^{ \pm}} / W_{\pi^{ \pm}}\left(5.7^{\circ}, 24 \mathrm{GeV} / \mathrm{c}\right)$. The acceptance is not taken into account.

| $\theta_{\text {lab }}$ | $5.7^{\circ}$ | $5.7^{\circ}$ | $4^{\circ}$ | $2^{\circ}$ | $0^{\circ}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $P_{p}$ | $24 \mathrm{GeV} / \mathrm{c}$ | $450 \mathrm{GeV} /$ | $450 \mathrm{GeV} / \mathrm{c}$ | $450 \mathrm{GeV} / \mathrm{c}$ | $450 \mathrm{GeV} / \mathrm{c}$ |
| The yield of charged pions |  |  |  |  |  |
| $W_{\pi^{ \pm}}$ | 0.0152 | 0.0445 | 0.0898 | 0.328 | 2.0 |
| $W_{\pi^{ \pm}}^{N}$ | 1 | 2.93 | 5.92 | 21.6 | 131.4 |
| The yield of $\pi^{+} \pi^{-}$atoms |  |  |  |  |  |
| $W_{A}$ | $2.26 \mathrm{E}-9$ | $1.41 \mathrm{E}-8$ | $2.79 \mathrm{E}-8$ | $8.73 \mathrm{E}-8$ | $3.40 \mathrm{E}-7$ |
| $W_{A}^{N}$ | 1 | 6.2 | 12.3 | 38.6 | 150.2 |
| The yield of $\pi^{+} K^{-}$atoms |  |  |  |  |  |
| $W_{A}$ | $5.10 \mathrm{e}-10$ | $4.55 \mathrm{e}-9$ | $9.75 \mathrm{e}-9$ | $3.53 \mathrm{e}-8$ | $1.68 \mathrm{e}-7$ |
| $W_{A}^{N}$ | 1 | 8.9 | 19.1 | 69.3 | 330 |
| The yield of $K^{+} \pi^{-}$atoms |  |  |  |  |  |
| $W_{A}$ | $9.48 \mathrm{e}-10$ | $5.71 \mathrm{e}-9$ | $1.26 \mathrm{e}-8$ | $4.80 \mathrm{e}-8$ | $2.19 \mathrm{e}-7$ |
| $W_{A}^{N}$ | 1 | 6.0 | 13.3 | 50.6 | 231. |

On Fig. 7 the $\pi^{ \pm}$total yields $W_{\pi^{ \pm}}$per one p-Ni interaction at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=5.7^{\circ}$ as a function of the pion momenta $P_{\pi^{ \pm}}$are shown. In the Tab. 1 these yields integrated over $P_{\pi^{ \pm}}$are presented.

On Fig. 8,9 and 10 the yields of $A_{2 \pi}, A_{\pi^{+} K^{-}}$and $A_{K^{+} \pi^{-}}$per one p-Ni interaction at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{l a b}=5.7^{\circ}$ as a function of these atoms momentum $P_{A}$ are presented. In the Tab. 1 these yields integrated over $P_{A}$ are shown.

At $\theta_{\text {lab }}=0^{\circ}$ and $P_{p}=450 \mathrm{GeV} / \mathrm{c}$ the yields of $\pi^{+} \pi^{-}, \pi^{+} K^{-}$and $K^{+} \pi^{-}$atoms are in 150,330 and 230 times more respectively, than at $\theta_{\text {lab }}=5.7^{\circ}$ and $P_{p}=24 \mathrm{GeV} / \mathrm{c}$.

### 4.3 Calculations of inclusive cross-sections of $\pi^{+} \pi^{-}$- and $\pi K$ atoms detected by setup

The previous values deal with the the atom flux into the solid angle $10^{-3} \mathrm{sr}$. To have the possibility to obtain the number of atoms which are detected by upgraded DIRAC setup its acceptance was determined( Fig.11,12 and 13). The maximum values of acceptances are less than 1 due to pion and kaon decays in the setup.

Table 2: The yield of $\pi^{+} \pi^{-}-, \pi^{+} K^{-}$- and $K^{+} \pi^{-}$- atoms $W_{A}$ into the aperture of $10^{-3} \mathrm{sr}$ taking into account the setup acceptance per one p-Ni interaction at the proton momenta $P_{p}=24$ and $450 \mathrm{GeV} / \mathrm{c}$ versus emission angle $\theta_{\text {lab }} . \quad W_{A}^{N}=W_{A} / W_{A}\left(5.7^{\circ}, 24 \mathrm{GeV} / \mathrm{c}\right)$ and $\left(W_{A} / W_{\pi}\right)^{N}=\left(W_{A} / W_{\pi}\right) /\left(\left(W_{A} / W_{\pi}\right)\left(5.7^{\circ}, 24 G e V / c\right)\right)$.

| $\theta_{\text {lab }}$ | $5.7^{\circ}$ | $5.7^{\circ}$ | $4^{\circ}$ | $2^{\circ}$ | $0^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $E_{p}$ | $24 \mathrm{GeV} / \mathrm{c}$ | $450 \mathrm{GeV} / \mathrm{c}$ | $450 \mathrm{GeV} / \mathrm{c}$ | $450 \mathrm{GeV} / \mathrm{c}$ | 450 GeV |  |
| The yield of $\pi^{+} \pi^{-}$atoms |  |  |  |  |  |  |
| $W_{A}$ | $0.11 \mathrm{E}-8$ | $0.10 \mathrm{E}-7$ | $0.18 \mathrm{E}-7$ | $0.27 \mathrm{E}-7$ | $0.29 \mathrm{E}-7$ |  |
| $W_{A}^{N}$ | 1 | 9.74 | 17.17 | 25.82 | 27.57 |  |
| $W_{A} / W_{\pi}$ | $0.70 \mathrm{E}-7$ | $0.23 \mathrm{E}-6$ | $0.20 \mathrm{E}-6$ | $0.83 \mathrm{E}-7$ | $0.15 \mathrm{E}-7$ |  |
| $\left(W_{A} / W_{\pi}\right)^{N}$ | 1 | 3.32 | 2.90 | 1.20 | 0.21 |  |
| The yield of $\pi^{+} K^{-}$atoms |  |  |  |  |  |  |
| $W_{A}$ | $0.26 \mathrm{E}-10$ | $0.20 \mathrm{E}-9$ | $0.93 \mathrm{E}-9$ | $0.23 \mathrm{E}-8$ | $0.25 \mathrm{E}-8$ |  |
| $W_{A}^{N}$ | 1 | 7.48 | 35.41 | 86.85 | 95.99 |  |
| $W_{A} / W_{\pi}$ | $0.17 \mathrm{E}-8$ | $0.44 \mathrm{E}-8$ | $0.10 \mathrm{E}-7$ | $0.70 \mathrm{E}-8$ | $0.13 \mathrm{E}-8$ |  |
| $\left(W_{A} / W_{\pi}\right)^{N}$ | 1 | 2.55 | 5.98 | 4.02 | 0.73 |  |
| The yield of $K^{+} \pi^{-}$atoms |  |  |  |  |  |  |
| $W_{A}$ | $0.44 \mathrm{E}-10$ | $0.21 \mathrm{E}-9$ | $0.12 \mathrm{E}-8$ | $0.30 \mathrm{E}-8$ | $0.34 \mathrm{E}-8$ |  |
| $W_{A}^{N}$ | 1 | 4.71 | 27.17 | 68.73 | 78.15 |  |
| $W_{A} / W_{\pi}$ | $0.29 \mathrm{E}-8$ | $0.47 \mathrm{E}-8$ | $0.13 \mathrm{E}-7$ | $0.92 \mathrm{E}-8$ | $0.17 \mathrm{E}-8$ |  |
| $\left(W_{A} / W_{\pi}\right)^{N}$ | 1. | 1.61 | 4.59 | 3.18 | 0.59 |  |

On Fig. $14,15,16$ the yields of $A_{2 \pi}, A_{\pi^{+} K^{-}}$and $A_{K^{+} \pi^{-}}$per one p-Ni interaction at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$, emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{l a b}=5.7^{\circ}$ as a function of these atoms momentum with taking into account the acceptance of setup are presented. In the Tab. 2 these yields integrated over $P_{A}$ are shown. There are shown the absolute values of atomic yields, their relative values when their yield at $24 \mathrm{GeV} / \mathrm{c}$ and $5.7^{\circ}$ is set to 1 and their
values relative to the flux of charged pion in the DIRAC channel. The last values are important as in the channel there are the forward detectors which should operate at this flux of charged pions. Also this ratio is less sensitive to the accuracy of the meson production inclusive cross sections than the atomic absolute yield. Therefore it is better to use this ratio for the dimesoatom yield comparison at $P_{p}=24$ and $450 \mathrm{GeV} / \mathrm{c}$.

At $\theta_{\text {lab }}=4^{\circ}$ and $P_{p}=450 \mathrm{GeV} / \mathrm{c}$ the yields of $\pi^{+} \pi^{-}, \pi^{+} K^{-}$and $K^{+} \pi^{-}$atoms are in 17,35 and 27 times more respectively, than at $\theta_{\text {lab }}=5.7^{\circ}$ and $P_{p}=24 \mathrm{GeV} / \mathrm{c}$. It means that it will be possible to decrease the proton beam intensity in several times to obtain the reduction of trigger events with accidental coincidences. The additional increasing of the atom production is connecting with beam time during supercycle on PS and SPS. At standard condition the DIRAC has on PS the 4 spills with duration 0.5 s (full time 2 s ). On SPS during the same supercycle the beam time is $4.6^{*} 2=9.6 \mathrm{~s}$ which gives the increasing for the atom production per time unit more than 4.

We can notice that at $P_{p}=450 \mathrm{GeV} / \mathrm{c}$ the soft proton background is more than an order less than at $24 \mathrm{GeV} / \mathrm{c}[24,25]$ making it easier to pion and kaon separation from protons.

## 5 Conclusion.

1. The performed analysis shows that the atom production in the p-nuclear interactions is significantly increasing if the momentum of proton $P_{p}$ will change from 24 up to $450 \mathrm{GeV} / \mathrm{c}$.
2. At $\theta_{\text {lab }}=0^{\circ}$ and $P_{p}=450 \mathrm{GeV} / \mathrm{c}$ the yields of $\pi^{+} \pi^{-}, \pi^{+} K^{-}$and $K^{+} \pi^{-}$atoms are in 150,330 and 230 times more respectively, than at $\theta_{\text {lab }}=5.7^{\circ}$ and $P_{p}=24 \mathrm{GeV} / \mathrm{c}$ in the same solid angle.
3. If we take into account the acceptance of setup like DIRAC(PS212) then at $\theta_{l a b}=4^{\circ}$ and $P_{p}=450 \mathrm{GeV} / \mathrm{c}$ the yields of $\pi^{+} \pi^{-}, \pi^{+} K^{-}$and $K^{+} \pi^{-}$atoms are in 17,35 and 27 times more respectively, than at $\theta_{l a b}=5.7^{\circ}$ and $P_{p}=24 \mathrm{GeV} / \mathrm{c}$.
4. Taking in to account the duty factor of PS and SPS the previous numbers will be increased to 4 times more.
5. The large yield of dimesoatoms at $P_{p}=450 \mathrm{GeV} / \mathrm{c}$ allows to use the primary proton beam with lower intensity and decrease the trigger events connecting with accidentals.


R. $P=450 \mathrm{GeV} / \mathrm{c} . \quad \Theta=4^{\circ}$




Figure 1: The dependence of correlation factor $R$ for $\pi^{+} \pi^{-}$pairs in the DIRAC setup on $\pi^{+}$and $\pi^{-}$momentum for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{l a b}=$ $5.7^{\circ}$.


Figure 2: The dependence of correlation factor $R$ for $\pi^{+} K^{-}$pairs in the DIRAC setup on $\pi^{+}$and $K^{-}$momentum for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=$ $5.7^{\circ}$.



Figure 3: The dependence of correlation factor $R$ for $\pi^{-} K^{+}$pairs in the DIRAC setup on $\pi^{-}$and $K^{+}$momentum for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=$ $5.7^{\circ}$.

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\pi}\mp@subsup{\pi}{}{+}\mp@subsup{\pi}{}{-}\mathrm{ : R, correlation factor
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Figure 4: The dependence of correlation factor $R$ for $\pi^{+} \pi^{-}$pairs in the DIRAC setup on total momentum of $\pi^{+} \pi^{-}$-pair for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=$ $5.7^{\circ}$.

## $\pi^{+} K^{-}$: R, correlation factor



Figure 5: The dependence of correlation factor $R$ for $\pi^{+} K^{-}$pairs in the DIRAC setup on total momentum of $\pi^{+} K^{-}$-pair for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=$ $5.7^{\circ}$.

## $\mathbf{K}^{+} \pi$ : $\mathbf{R}$, correlation factor



Figure 6: The dependence of correlation factor $R$ for $\pi^{-} K^{+}$pairs in the DIRAC setup on total momentum of $\pi^{-} K^{+}$-pair for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=$ $5.7^{\circ}$.

Yield of $\pi^{ \pm}$per one p-Ni interaction, bin=7.5MeV/c


Figure 7: The total yield $W_{\pi^{ \pm}}$of charged pions per one p-Ni interaction at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=5.7^{\circ}$ as a function of the pions momentum.

$$
\text { Yield of } \mathrm{A}_{\pi^{+} \pi} \text { per one p-Ni interaction, bin=15MeV/c }
$$



Figure 8: Yields of $A_{2 \pi}$ per one p-Ni interaction at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{l a b}=5.7^{\circ}$ as a function of the $A_{2 \pi}$ momentum. The acceptance is not taken into account and $R=1$.


Figure 9: Yields of $A_{\pi^{+} K^{-}}$per one p-Ni interaction at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=5.7^{\circ}$ as a function of the $A_{\pi^{+} K^{-}}$momentum. The acceptance is not taken into account and $R=1$.


Figure 10: Yields of $A_{K^{+} \pi^{-}}$per one p-Ni interaction at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=5.7^{\circ}$ as a function of the $A_{K^{+} \pi^{-}}$momentum. The acceptance is not taken into account and $R=1$.

## Acceptance for $\mathrm{A}_{2 \pi}$



Figure 11: The acceptance for $A_{2 \pi}$ atoms in the DIRAC setup on their momentum for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{l a b}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=5.7^{\circ}$.

## Acceptance for $\mathrm{A}_{\pi^{+} \mathrm{K}^{-}}$



Figure 12: The acceptance for $A_{\pi^{+} k^{-}}$atoms in the DIRAC setup on their momentum for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{l a b}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{l a b}=5.7^{\circ}$.

## Acceptance for $\mathrm{A}_{\mathrm{K}^{-}{ }^{+}}$



Figure 13: The acceptance for $A_{k^{+} \pi^{-}}$atoms in the DIRAC setup on their momentum for the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{l a b}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{l a b}=5.7^{\circ}$.

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Yield of A }\mp@subsup{}{2\pi}{}\mathrm{ per one p-Ni interaction, bin=15MeV/c
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Figure 14: Yields of $A_{2 \pi}$ atoms per one $\mathrm{p}-\mathrm{Ni}$ interaction with taking into account their acceptance value and calculated correlation factor $R$ at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{\text {lab }}=5.7^{\circ}$ as a function of the $A_{2 \pi}$ momentum.


Figure 15: Yields of $A_{\pi^{+} K^{-}}$atoms per one p-Ni interaction with taking into account their acceptance value and calculated correlation factor $R$ at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{\text {lab }}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{l a b}=5.7^{\circ}$ as a function of the $A_{\pi^{+} K^{-}}$momentum.


Figure 16: Yields of $A_{K^{+} \pi^{-}}$atoms per one p-Ni interaction with taking into account their acceptance value and calculated correlation factor $R$ at the proton momentum $450 \mathrm{GeV} / \mathrm{c}$ and emission angles $\theta_{l a b}=0^{\circ}, 2^{\circ}, 4^{\circ}, 5.7^{\circ}$ and at the proton momentum $24 \mathrm{GeV} / \mathrm{c}$ and emission angle $\theta_{l a b}=5.7^{\circ}$ as a function of the $A_{K^{+} \pi^{-}}$momentum.

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[^0]:    ${ }^{1}$ The atoms are distributed over $n$ as $n^{-3}: W_{1}=83 \%, W_{2}=10.4 \%, W_{3}=3.1 \%, W_{n \geq 4}=3.5 \%$. Note that $\sum_{n=1}^{\infty}\left|\Psi_{n}(0)\right|^{2}=1.202\left|\Psi_{1}(0)\right|^{2}$.

