# Estimation of beam time needed for measurement of pion-kaon s-wave scattering length combination $a_{0}^{-}$with a statistical accuracy $5 \%$. 

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#### Abstract

Measurement of s-wave pion-kaon scattering length combination $a_{0}^{-}=\frac{1}{3}\left(a_{0}^{1 / 2}-a_{0}^{3 / 2}\right)$ with an accuracy $5 \%$ provides check of prediction made by Chiral Perturbation theory and Lattice QCD. Estimation of beam time for 450 GeV proton beam, needed for achievement this accuracy, has been done on a base of accuracy of $\left|a_{0}^{-}\right|$measurement, performed in DIRAC experiment.


## 1 Introduction

The $\pi K$ atom $\left(A_{\pi K}\right)$ decay width $\Gamma_{\pi K}$ in the ground state (1S) is given by the relation [1, 2]:

$$
\begin{equation*}
\Gamma_{\pi K}=\frac{1}{\tau} \simeq \Gamma\left(A_{K \pi} \rightarrow \pi^{0} K^{0} \text { or } \pi^{0} \bar{K}^{0}\right)=8 \alpha^{3} \mu^{2} p^{*}\left(a_{0}^{-}\right)^{2}\left(1+\delta_{K}\right) . \tag{1}
\end{equation*}
$$

The basic s-wave isospin-odd $\pi K$ scattering length $a_{0}^{-}=\frac{1}{3}\left(a_{0}^{1 / 2}-a_{0}^{3 / 2}\right)\left(a_{0}^{I}\right.$ is s-wave scattering length for isospin $I$ ) is defined in pure QCD for quark masses $m_{u}=m_{d}, \alpha$ is the fine structure constant, $\mu=109 \mathrm{MeV} / c^{2}$ is the reduced mass of the $\pi^{\mp} K^{ \pm}$system and $p^{*}=11.8 \mathrm{MeV} / \mathrm{c}$ is the outgoing $\pi^{0}$ or $K^{0}\left(\bar{K}^{0}\right)$ momentum in the $\pi K$ atom system. The last term $\delta_{K}=0.040 \pm 0.022$ [2] accounts for corrections, due to isospin breaking, at order $\alpha$ and quark mass difference $\left(m_{u}-m_{d}\right)$.

The scattering length $M_{\pi} a_{0}^{-}$( $M_{\pi}$ is charged pion mass) has been studied and calculated in ChPT $[3,4,5]$, in the dispersive framework of the Roy-Steiner equations [6] and in lattice QCD (see e.g. [7]).

Experimental measurement of $a_{0}^{-}$with an accuracy $5 \%$ check our understanding of the simplest hadron-hadron processes with participation of strange quark.

## 2 Present experimental result

First measurement of $\pi K$ atom lifetime and $\pi K$ scattering length measurement was presented by DIRAC experiment [8]. After improvement of SFD detector response [9], the setup description in simulation and off-line analysis [10, 11], tuning criteria for $\pi K$ pair selection, the new measurement of $\pi K$ atom lifetime has been obtained [12]:

$$
\begin{equation*}
\tau_{A_{\pi K}}=\left.3.52_{-2.10}^{+3.40}\right|_{s t a t} f s \tag{2}
\end{equation*}
$$

This measurement is obtained, using data collected with Nickel target in 2008-2010 years. Total amount of spills is $2.7 \cdot 10^{7}$. Spill length is 0.45 s and an average number of protons is $1.2 \cdot 10^{11}$ protons per spill. In another terms, total beam time is $1.215 \cdot 10^{6} \mathrm{~s}$ at intensity $2.67 \cdot 10^{11} \mathrm{~s}^{-1}$. Systematic effects are much lower and total error is close to statistical one [12]:

$$
\begin{equation*}
\tau_{A_{\pi K}}=\left.3.52_{-2.11}^{+3.42}\right|_{t o t} f s \tag{3}
\end{equation*}
$$

It is possible to calculate values of break up probabilities, which correspond to values $\tau_{A_{\pi K}}$, $\tau_{A_{\pi K}}-\sigma$ and $\tau_{A_{\pi K}}+\sigma$ :

$$
\begin{equation*}
P_{b r}^{1}=0.276, P_{b r}^{2}=0.198, P_{b r}^{3}=0.353, \tag{4}
\end{equation*}
$$

These values have been calculated with dependence of breakup probability $P_{b r}=f\left(\tau_{A_{\pi K}}, P_{\text {Lab }}\right)$ [13] on $A_{\pi K}$ lifetime and atom laboratory momentum $P_{L a b}$ for average momentum $6.31 \mathrm{GeV} / c$.

This result is used as a base for estimation of data set which is needed for measurement of $A_{\pi K}$ lifetime with accuracy $10 \%$ and $a_{0}^{-}$pion-kaon scattering length combination with accuracy $5 \%$.

## 3 Simulation procedure

Distributions of $\pi K$ pair, detected by DIRAC setup have been prepared by simplified simulation code of DIRAC experiment (sdistr5ns2_chi2.f). This program prepares distributions of pairs over relative momentum $Q$ and its projections similar to experimental ones. Also it produced distributions analogous to simulated distributions of "atomic", "Coulomb" and "non-Coulomb" pairs which are used for description of simulated experimental distribution with the same procedure that used for data analysis in experiment DIRAC.

Spectrometer magnet is described by box-like magnetic field approximation. Reference systems of program have origin in the centre of magnetic field. Secondary particle channel is oriented along Z-axis of principal reference system. For upstream part of the setup Z-coordinates are negative, for downstream part Z-coordinates are positive.

Detector and partitions planes are described by Z coordinate in local reference system (Zaxis is perpendicular to the plane), X - and Y- coordinates of of plane edges, average angle of multiple scattering and spatial and time resolution (for detector planes only).

At the first stage program simulates distribution of "Coulomb", "atomic", "non-Coulomb" and "accidental" pairs which present experimental distribution in defined range of initial relative momentum in pair CM system $\vec{q}\left(\left|q_{X}\right|<10 \mathrm{MeV} / c,\left|q_{Y}\right|<10 \mathrm{MeV} / c,\left|q_{L}\right|<25 \mathrm{MeV} / c\right)$.

For "Coulomb" pairs a distribution over $q$ is modified in accordance with Sommerfeld-Gamov-Sakharov factor:

$$
\begin{equation*}
A_{c}(q)=\frac{4 \pi \mu \alpha / q}{1-\exp (-4 \pi \mu \alpha / q)} \tag{5}
\end{equation*}
$$

Distribution of "atomic pairs" over $\vec{q}$ is described by Gaussian distribution with widths: $\sigma_{q_{X}}=0.43 \mathrm{MeV} / c, \sigma_{q_{Y}}=0.43 \mathrm{MeV} / c, \sigma_{q_{L}}=0.40 \mathrm{MeV} / c$. Number of atomic pairs is defined as a number of "Coulomb pairs" with $q<3.12 \mathrm{MeV} / c$ multiplied by factor 0.61 [14] and expected value of breakup probability.

Distribution of "non-Coulomb" and accidental pairs have uniform distributions over $q_{X}$, $q_{Y}, q_{L}$. Accidental pairs (particle are generated in different proton-nucleon interaction) are separated into two sample. Distribution of "prompt accidental", which have measured time difference between positive and negative particle in a range defined for "real pairs" (both particles are generated in one proton-nucleon interactions) is added to simulated distributions of "Coulomb", "non-Coulomb" and "atomic pairs". Distribution of "accidental pairs" outside this region is saved separately and used for subtraction of "accidental pairs" from distribution of "prompt pairs" as it is done for analysis of experimental data.

Total laboratory momentum of pair $P_{t o t}$ is simulated in accordance with spectra obtained in [ $15,16,17]$. Angles between pair momentum and axes of secondary particle momentum (Z-axes of principal reference system) are simulated uniformly in X- and Y- projection in ranges defined by collimator. These angles and value $P_{t o t}$ define total pair momentum vector $\vec{P}_{t o t}$.

For each generated pair, multiple scattering in the target is simulated as smearing of X- and Y-components of initial relative momentum $\vec{q}$, using information about matter and thickness of the target. It is assumed that points of generation are uniformly distributed in the target depth.

Laboratory momentum vector $\vec{P}_{\pi}\left(\vec{P}_{K}\right)$ is calculated for each particle from $\vec{P}_{\text {tot }}$ and $\vec{q}$, modified by multiple scattering in the target. Trajectory of particles is propagated as straight line up to magnetic field of spectrometer magnet, then as part of circle with radius defined by magnetic field $B$ and momentum of a particle. After magnetic field region a track is again propagated as straight line. Coordinates of all intersections of particle trajectory with detector and partition planes are stored. If any plane is missed then event is rejected. Program calculates length of trajectory from the target to the last plane of the setup and calculates probability of a particle decay on this path. Decay is simulated and event is rejected if any of the particles is decayed.

Information about coordinates of track in the setup planes, average angles of multiple scattering scattering (including multiple scattering in air) and coordinate resolution of detector planes are used for global fit of particle tracks, using procedure which is equivalent to procedure in off-line analysis of experimental data in DIRAC experiment.

Reconstructed values of particle coordinates, vector components and laboratory momentum are practically coincide with initial (only difference is due to rounding), because a track has been defined without distortion, induced by multiple scattering and final resolution of coordinate detectors. But information about detector resolution and multiple scattering allows to define error matrix for track parameters.

For each pair of particles the procedure performs vertex fit, using reconstructed values of track parameters at the target and their error matrices. It gives values of relative momentum in CM of pair system and estimation of error matrix for its components. Values of projections
of relative momentum in CM pair system are smeared in accordance with their error estimations and provide values of reconstructed relative momentum $Q$. Distribution over $Q$ and its projection are filled as for experimental data.

At the second stage program simulates distributions of "Coulomb", "atomic", "non-Coulomb" pairs, which present Monte-Carlo distributions are used for description of experimental data. Statistic of these pairs as rule few times more than statistics of experimental data. Program allows to obtain distributions of MC data with the setup parameters (detector resolutions, multiple scattering, particle spectra) other than for "experimental" data in order to investigate sensitivity of results to errors in description of the setup in simulation and to estimate induced systematic errors. But for current investigation all parameters are taken equal for "experimental" and MC events, because an aim of this job is analysis of statistical errors only.

## 4 Simulation of reference data set at 24 GeV beam

Reconstruction of simulated event is done in simplified conditions: there are no background hits and misidentifying of sub-tracks upstream and downstream the magnet. Therefore comparison of experimental result obtained at 24 GeV and simulated results at 450 GeV is not correct. It is needed to prepare simulated set of data with statistic which is equivalent to obtained at experiment.

Geometry and properties of the setup have been defined in accordance with property of the DIRAC setup [18].

Table 1 presents number of "Coulomb", "non-Coulomb", "prompt accidental", "accidental" and background (wrong identified $\pi^{+} \pi^{-}, \mathrm{p} \pi^{-}$and $\pi^{+} \overline{\mathrm{p}}$ pairs) for experimental and simulated data.

Table 1: Numbers of experimentally measured and simulated "experimental" $K^{+} \pi^{-}$and $\pi^{+} K^{-}$ events of different types for experiment with 24 GeV proton beam. Events are selected with criteria: $\left|Q_{L}\right|<15 \mathrm{MeV} / c, Q_{T}<4 \mathrm{MeV} / c$.

| "Coulomb" "non-Coulomb" | background | "prompt <br> accidental" | "accidental" |  |
| :---: | :---: | :---: | :---: | :---: |
| Experimental $K^{+} \pi^{-}$pairs |  |  |  |  |
| 6231 | 2803 | 4254 | 1542 | 106236 |
| Simulated "experimental" $K^{+} \pi^{-}$pairs |  |  |  |  |
| 6287 | 7026 | 0 | 1534 | 105721 |
| 2890 | 356 | 972 | 411 | 16072 |
| Experimental $\pi^{+} K^{-}$pairs |  |  |  |  |
| 2900 | 1302 | 0 | 410 | 16022 |

Analysis of simulated "experimental" data for experiment at $24 \mathrm{GeV} / c$ beam gives estimations of $P_{b r}$ measurement accuracy:

$$
\begin{equation*}
\sigma_{K^{+} \pi^{-}}^{24}=0.0721, \sigma_{\pi^{+} K^{-}}^{24}=0.0891, \sigma_{\pi K}^{24}=0.0560 \tag{6}
\end{equation*}
$$

## 5 Simulation of data set with 450 GeV beam at intensity 1. $\cdot 10^{11}$ proton per second

Simulation of spectra $K^{+} \pi^{-}, \pi^{+} K^{-}$pairs and inclusive spectra of charged particles $[15,16,17]$ shows that optimal angle between axis of secondary particle channel and proton beam is $4^{\circ}$. At this angle ratio of produced atoms to flux of single particles (which limits intensity of beam) is maximal.

Geometry is defined for magnet of DIRAC experiment [19]. It leads to increasing distance between the target and the centre of magnet from 830 to 1190 cm to provide the same distance between proton beam and secondary particle channel in region of magnet pole. Also solid angle of secondary particle channel is decreased from $2^{\circ} \times 2^{\circ}$ to $1.4^{\circ} \times 1.4^{\circ}$.

Table 2 presents lists of partitions and detector planes on the path for positive particles. Negative particles are detected by right arm which is symmetrical to left arm.

Figure 1 presents flux of particles per $1 \mathrm{~cm}^{2}$ for the first X-plane of drift chambers in experiment DIRAC at 24 GeV proton beam with intensity $2.67 \cdot 10^{11}$ proton per second $\left(1.2 \cdot 10^{11}\right.$ protons in 0.45 s spill) and for proposed experiment with 450 GeV proton beam at intensity $1 \cdot \cdot 10^{11}$, using spectra from $[15,16,17]$. It is seen that flux of particles with 450 GeV beam is not higher than for DIRAC experiment. Therefore drift chambers could be used as coordinates detectors in spectrometer arms and an efficiency is to be not less than efficiency achieved at DIRAC.

For upstream detectors flux of charged particles is $1.45 \cdot 10^{7} \mathrm{~s}^{-1}$ at DIRAC experiment conditions and $8.48 \cdot 10^{6} \mathrm{~s}^{-1}$ for experiment with 450 GeV beam. For upstream detector it is also essential to know size of region ("particle safe region") in detector planes which correspond to particles generated at beam position at the target and detected by downstream detectors. This region is defined by multiple scattering in upstream part of the setup, in aluminum membrane after the spectrometer magnet and accuracy of coordinates measurement in detectors upstream and downstream a spectrometer magnet. The region size is proportional to an error of track coordinate at the target and a ratio of distance between upstream detector plane and plane of downstream detector, closest to magnet, to a distance between the target and plane of downstream detector, closest to magnet. Pass of background particle through one region with any of particle of pair provides unambiguity in connection of track upstream and downstream parts of spectrometer and events could be lost.

For DIRAC experiment accuracy of reconstruction of track coordinate at the target described by equation: $\sigma_{X(Y)}=0.8 / P \mathrm{~cm}$ (here $P$ is particle momentum in $\mathrm{GeV} / c$ ). For experiment with 450 GeV beam with list of detectors, described in Table 2 accuracy of track coordinate at the target is $\sigma_{X(Y)}=0.43 / P$. Taking into account sizes and position of vertex detector at DIRAC experiment and experiment with 450 GeV beam, one obtains that ratio of "particle safe region" to detector plane size is less by $20 \%$ for experiment with 450 GeV beam than it is for DIRAC experiment. Together with lower particle flux, it means that probability


Figure 1: A value of particle flux per $\mathrm{cm}^{2}$ per second as function of X - coordinate in the first X-plane of drift chambers for: a) DIRAC experiment at 24 GeV proton beam; b) experiment with 450 GeV proton beam at intensity $1 . \cdot 10^{11}$
of event lost due to unambiguity in downstream and upstream parts of tracks approximately 2.2 times less for experiment with 450 GeV proton beam with intensity $1 . \cdot 10^{11} \mathrm{~s}^{-1}$ than for DIRAC experiment at 24 GeV proton beam.

For simulation of "experimental" data, which could be collected in one run by experiment with 450 GeV proton beam it is assumed that setup would obtain two 4.5 s spills in supercycle ( 48 seconds length), 20 hours per day in 150 day run. It means $4.5 \cdot 10^{5}$ spills or $2.025 \cdot 10^{6}$ seconds of beam time. Using this estimation and difference of spectra for proton-nuclear interactions at 24 and 450 GeV , number of $K^{+} \pi^{-}, \pi^{+} K^{-}, \pi^{+} \pi^{-}, \mathrm{p} \pi^{-}, \pi^{+} \overline{\mathrm{p}}$ and accidental pairs (the last four items are components of background) have been estimated. Analysis has been performed as for experimental data of DIRAC experiment, including recognition of background particles, using heavy gas Cherenkov counter to distinguish pions from kaons, aerogel Cherenkov counter to distinguish kaons from protons and analysis of event distribution over difference of particle generation times. To obtain particle generation time, the time measured by Vertical Hodoscope $(\mathrm{VH})$ is subtracted by time-of-flight from the target to the VH plane for particle of expected mass and measured momentum. For true $\pi K$ pairs distribution is to be centered at 0 , and for background pairs distribution is to be shifted (see example in Fig. 2). Procedure introduces criterion of time difference, as function of kaon momentum, in order to minimize admixture of background pairs for certain efficiency of $\pi K$ pair accepting. The last criterion was selected to provide the same $K^{+} \pi^{-}\left(\pi^{+} K^{-}\right)$pair efficiency as for DIRAC experiment. Efficiency of
aerogel counters and time resolution of VH have been assumed to be equal to ones in DIRAC experiment.


Figure 2: For experiment with 450 GeV proton beam at intensity $1 . \cdot 10^{11} \mathrm{~s}^{-1}$, simulated distributions over difference of particle generation times are presented for events with positively charged particle momenta in the interval $5.4 \div 5.5 \mathrm{GeV} / c$. Simulated "experimental" data (histogram) are fitted by the sum of the distributions: $K^{+} \pi^{-}$(red, dashed), $\pi^{+} \pi^{-}$(blue, dotted), $\mathrm{p} \pi^{-}$(magenta, dotted-dashed) and accidental pairs (green, constant). The sum of all the fractions is shown as black solid line.

Fig. 3 presents simulated distributions of "atomic pairs" over total pair momentum $P_{t o t}$ for 24 and 450 GeV beam. Table 3 presents efficiency of criterion on generation time difference $\epsilon^{\Delta T}$ and fraction of $\pi K$ pairs among event which fit this criterion $F^{\pi K}$.

It is seen that a number of "atomic pairs" is much higher for experiment with 450 GeV beam. Reasons are higher cross section of $K^{+} \pi^{-}$and especially $\pi^{+} K^{-}$atom production, such absolute as comparing with background pairs (excluding $\pi^{+} \bar{p}$ ), increasing beam time by $67 \%$, and improvement of pair identification for momenta in a range $7.5 \div 11.5 \mathrm{GeV} / c$ due to increasing length of flight from the target to VH planes ( 1157.5 cm for DIRAC experiment and 1517.5 cm for experiment with 450 GeV beam) at the same time resolution of Vertical Hodoscope.

Accuracy of simulated breakup probability measurement for experiment with 450 GeV beam are found to be:

$$
\begin{equation*}
\sigma_{K^{+} \pi^{-}}^{450}=0.0187, \sigma_{\pi^{+} K^{-}}^{450}=0.0176, \sigma_{\pi K}^{450}=0.0128 \tag{7}
\end{equation*}
$$



Figure 3: Simulated distribution of "atomic pairs" over total pair momentum $P_{\text {tot }}$ for 450 GeV (solid) and 24 GeV (dashed) proton energies. Distributions are normalized by number of simulated "experimental" atomic pairs for: a) $K^{+} \pi^{-}$pairs; b) $\pi^{+} K^{-}$pairs

Experimental accuracy for this experiment is estimated to be, taking into account Eqs. $(4,6,7)$ :

$$
\begin{equation*}
\sigma_{\mathrm{Exp}}^{450}=\frac{\sigma_{\pi K}^{450}\left(P_{b r}^{3}-P_{b r}^{2}\right)}{2 \sigma_{\pi K}^{24}} \tag{8}
\end{equation*}
$$

It allows to estimate relative accuracy of $\pi K$ atom lifetime measurement $\delta_{\tau}$, taking into account difference of average momenta of distribution in Fig. 3 and dependence $P_{b r}=f\left(\tau_{A_{\pi K}}, P_{L a b}\right):$

$$
\begin{equation*}
\delta_{\tau}=16.5 \% . \tag{9}
\end{equation*}
$$

It leads to measurement of pion-kaon s-wave scattering length combination $a_{0}^{-}$with a statistical accuracy $8.2 \%$. For achivement of statistical accuracy $5 \%$ at this experimental conditions it is needed to increase beam time up to $5.5 \cdot 10^{6} \mathrm{~s}$, or 13.6 months of run time at assumptions about number of spills made above.

Expected number of $\pi K$ "atomic pairs" is about 14000. In parallel it would be possible to collect about $450000 \pi^{+} \pi^{-}$"atomic pairs" which could be used for measurement of difference of pion-pion s-wave scattering length with isospin 0 and 2 with statistical accuracy $\sim 0.7 \%$.

## 6 Simulation of data set with 450 GeV beam at intensity 1. $\cdot 10^{12}$ proton per second

One of the possible way to decrease run time of experiment is to increase intensity of proton beam. This section describe results of simulation for experiment with 450 GeV proton beam at intensity $1 . \cdot 10^{12} \mathrm{~s}^{-1}$. Table 4 presents list of detectors are used for this simulation.

Comparison of Tables 2 and 4 shows that for high intensity beam it is needed to have vertex detector with high time and spatial resolution (at least for planes closest to the target) and low radiation thickness. At present, detectors with similar properties are designed and are under testing, for example [20]. For spectrometer arms, Drift Chambers probably are to be replaced detectors with higher counting rate (MSGC or SFD detectors). Number of planes is decreased, because this detectors have no problem of left-right unambiguity. From another side their radiation thickness is higher than for DC , and big number of planes would not give positive effect for track reconstruction accuracy.

Simulation has been done like for data set, described in section 5. Only numbers of events was increased by factor 10 for "real pairs" and by factor 100 for accidental pairs. It leads to increasing background, mainly due to "accidental pairs". Comparison of Figs. 2 and 4 illustrates this. Table 5 presents $\epsilon^{\Delta T}$ and $F^{\pi K}$ for experiment with 450 GeV beam at intensity 1. • $10^{12} \mathrm{~s}^{-1}$.

Analysis of simulated data for experiment with 450 GeV proton beam at intensity $1 . \cdot 10^{12}$ $\mathrm{s}^{-1}$ gives accuracy of breakup probability measurement to be:

$$
\begin{equation*}
\sigma_{K^{+} \pi^{-}}^{450,2}=0.0065, \sigma_{\pi+K^{-}}^{450,2}=0.0060, \sigma_{\pi K}^{450,2}=0.0044 \tag{10}
\end{equation*}
$$

On the base of value $\sigma_{\pi K}^{450,2}$ it is possible to obtain (see section 5) relative accuracy of measurement of $a_{0}^{-}$to be $2.8 \%$. It means that for statistical accuracy $5 \%$ it is sufficient to have $6.5 \cdot 10^{5} \mathrm{~s}$ of beam time or 1.6 months of run time (see section 5).

Expected number of $\pi K$ "atomic pairs" is about 16000 . In parallel it would be possible to collect about $500000 \pi^{+} \pi^{-}$"atomic pairs". Statistical accuracy for measurement of difference of pion-pion s-wave scattering length with isospin 0 and 2 with statistical accuracy $\sim 0.7 \%$ as in section 5 .

## 7 Conclusion

Simulations shows that measurement of pion-kaon s-wave scattering length combination $a_{0}^{-}$ with a statistical accuracy $5 \%$ could be achieved in DIRAC style experiment with 450 GeV proton beam at intensity $1 . \cdot 10^{11} \mathrm{~s}^{-1}$ in $5.5 \cdot 10^{6} \mathrm{~s}$ of beam time. If the setup would be able to work at intensity $1 . \cdot 10^{12} \mathrm{~s}^{-1}$, then beam time would be decreased to $6.5 \cdot 10^{5} \mathrm{~s}$. If the setup would obtain two 4.5 s spills in supercycle ( 48 seconds length), 20 hours per day, then it means 13.6 and 1.6 months of run time, correspondingly. Even fourteen months is not dramatically big for experiment. And result with high intensity beam ( 1.6 months) is sufficiently short to use, if it would be needed, non-symmetric setup ( $K^{+} \pi^{-}$or $\pi^{+} K^{-}$detection only) in order to decrease geometrical size of the setup, if it would be needed at conditions of available experimental zone.


Figure 4: For experiment with 450 GeV proton beam at intensity $1 . \cdot 10^{12} \mathrm{~s}^{-1}$, simulated distributions over difference of particle generation times are presented for events with positively charged particle momenta in the interval $5.4 \div 5.5 \mathrm{GeV} / c$. Simulated "experimental" data (histogram) are fitted by the sum of the distributions: $K^{+} \pi^{-}$(red, dashed), $\pi^{+} \pi^{-}$(blue, dotted), $\mathrm{p} \pi^{-}$(magenta, dotted-dashed) and accidental pairs (green, constant). The sum of all the fractions is shown as black solid line.

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Table 2: List of detector and partitions for the setup, designed for experiment with 450 GeV proton beam at intensity $1 . \cdot 10^{11}$. Sizes, distances from planes to a target (upstream part only) and to centre of magnet, thickness in radiation length and resolution (for detector planes only) over coordinate in local reference system $\left(\sigma_{X_{l c}}\right)$ and (or) over time $\left(\sigma_{t}\right)$ are presented

| Plane | $\begin{gathered} \text { Distance to } \\ \text { the target } \\ \mathrm{cm} \end{gathered}$ | $\begin{gathered} \hline \text { Distance to } \\ \text { magnet } \\ \mathrm{cm} \end{gathered}$ | $\begin{aligned} & \text { Size } \\ & \mathrm{cm}^{2} \end{aligned}$ | Radiation thickness X0 | $\begin{gathered} \sigma_{X_{l c}} \\ \mathrm{~cm} \end{gathered}$ | $\sigma_{t}$ <br> ns |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Upstream part |  |  |  |  |  |  |
| Target | 0. | 1190.0 | 4.x4. | $7.5 \cdot 10^{-3}$ |  |  |
| Mylar membrane | 268.0 | 921.0 | 6.52x6.52 | $8.7 \cdot 10^{-4}$ |  |  |
| Detector Y-plane | 301.1 | 888.9 | 7.6x7.6 | $3.0 \cdot 10^{-3}$ | 0.005 | 0.4 |
| Detector X-plane | 302.0 | 888.0 | 7.6x7.6 | $3.0 \cdot 10^{-3}$ | 0.005 | 0.4 |
| Particle identification (RICH ?) | 352.0 | 838.0 | $9 . \mathrm{x} 9$. | $1.5 \cdot 10^{-2}$ |  |  |
| SFD Y-plane | 402.5 | 787.5 | 9.8 x 9.8 | $1.0 \cdot 10^{-2}$ | 0.006 | 0.6 |
| SFD X-plane | 403.6 | 786.4 | 9.8 x 9.8 | $1.0 \cdot 10^{-2}$ | 0.006 | 0.4 |
| SFD W-plane | 404.6 | 785.4 | 9.8 x 9.8 | $0.9 \cdot 10^{-2}$ | 0.013 | 0.6 |
| Mylar membrane | 479.5 | 710.5 | 11.x11. | $8.7 \cdot 10^{-4}$ |  |  |
| Left arm |  |  |  |  |  |  |
| Aluminum membrane |  | 143.3 | 100x40 | $7.6 \cdot 10^{-3}$ |  |  |
| DC L1X |  | 154.2 | 80x40 | $4.1 \cdot 10^{-4}$ | 0.008 |  |
| DC L1Y |  | 155.2 | $80 \times 40$ | $1.6 \cdot 10^{-4}$ | 0.008 |  |
| DC L1W |  | 156.1 | 80x40 | $1.6 \cdot 10^{-4}$ | 0.008 |  |
| DC L2X |  | 157.1 | 80x40 | $1.6 \cdot 10^{-4}$ | 0.008 |  |
| DC L2Y |  | 158.1 | 80x40 | $1.6 \cdot 10^{-4}$ | 0.008 |  |
| DC L2W |  | 159.1 | 80x40 | $3.4 \cdot 10^{-4}$ | 0.008 |  |
| DC L3X |  | 201.6 | 80x40 | $4.1 \cdot 10^{-4}$ | 0.008 |  |
| DC L3Y |  | 202.6 | 80x40 | $3.4 \cdot 10^{-4}$ | 0.008 |  |
| DC L4X |  | 251.6 | 112x40 | $4.1 \cdot 10^{-4}$ | 0.008 |  |
| DC L4Y |  | 252.6 | 112x40 | $3.4 \cdot 10^{-4}$ | 0.008 |  |
| DC L5X |  | 301.6 | 128x40 | $4.1 \cdot 10^{-4}$ | 0.008 |  |
| DC L5Y |  | 302.6 | 128x40 | $1.6 \cdot 10^{-4}$ | 0.008 |  |
| DC L6X |  | 303.6 | 128x40 | $1.6 \cdot 10^{-4}$ | 0.008 |  |
| DC L6Y |  | 304.6 | 128x40 | $3.4 \cdot 10^{-4}$ | 0.008 |  |
| Vertical Hodoscope |  | 327.5 | 140x40 | $8.7 \cdot 10^{-4}$ |  | 0.10 |
| Horisontal Hodoscope |  | 341.0 | 150x40 | $8.7 \cdot 10^{-4}$ |  | 0.22 |

Table 3: Efficiency of criterion on generation time difference $\left(\epsilon^{\Delta T}\right)$ and fraction of $\pi K$ pairs among event which fit this criterion for $K^{+} \pi^{-}$and $\pi^{+} K^{-}\left(F^{\pi K}\right)$, collected with proton beam with energy $\left(E_{\text {beam }}\right) 24\left(2.67 \cdot 10^{11} \mathrm{~s}^{-1}\right)$ and $450 \mathrm{GeV}\left(1 \cdot \cdot 10^{11} \mathrm{~s}^{-1}\right)$.

| Pairs | Beam energy <br> GeV | $\epsilon^{\Delta T}$ <br> $\%$ | $F^{\pi K}$ <br> $\%$ |
| :---: | ---: | ---: | ---: |
| $K^{+} \pi^{-}$ | 24 | 75.9 | 68.0 |
| $K^{+} \pi^{-}$ | 450 | 76.0 | 78.3 |
| $\pi^{+} K^{-}$ | 24 | 89.1 | 77.0 |
| $\pi^{+} K^{-}$ | 450 | 89.3 | 85.2 |

Table 4: List of detector and partitions for the setup, designed for work with 450 GeV proton beam at intensity $1 \cdot \cdot 10^{12}$. Sizes, distances from planes to a target (upstream part only) and to a centre of magnet, thickness in radiation length and resolution (for detector planes only) over coordinate in local reference system ( $\sigma_{X_{l c}}$ ) and (or) over time ( $\sigma_{t}$ ) are presented

| Plane | Distance to the target cm | Distance to magnet cm | Size $\mathrm{cm}^{2}$ | Radiation thickness X0 | $\begin{gathered} \sigma_{X_{l c}} \\ \mathrm{~cm} \end{gathered}$ | $\overline{\sigma_{t}}$ <br> ns |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Upstream part |  |  |  |  |  |  |
| Target | 0. | 1190.0 | $4 . x 4$. | $7.5 \cdot 10^{-3}$ |  |  |
| Mylar membrane | 268.0 | 921.0 | 6.52x6.52 | $8.7 \cdot 10^{-4}$ |  |  |
| Detector Y-plane | 301.1 | 888.9 | 7.6x7.6 | $3.0 \cdot 10^{-3}$ | 0.005 | 0.2 |
| Detector X-plane | 302.0 | 888.0 | 7.6x7.6 | $3.0 \cdot 10^{-3}$ | 0.005 | 0.2 |
| Particle identification (RICH ?) | 352.0 | 838.0 | $9 . \mathrm{x} 9$. | $1.5 \cdot 10^{-2}$ |  |  |
| SFD Y-plane | 402.5 | 787.5 | 9.8 x 9.8 | $1.0 \cdot 10^{-2}$ | 0.006 | 0.6 |
| SFD X-plane | 403.6 | 786.4 | 9.8 x 9.8 | $1.0 \cdot 10^{-2}$ | 0.006 | 0.4 |
| SFD W-plane | 404.6 | 785.4 | 9.8 x 9.8 | $0.9 \cdot 10^{-2}$ | 0.013 | 0.6 |
| Mylar membrane | 479.5 | 710.5 | 11.x11. | $8.7 \cdot 10^{-4}$ |  |  |
| Left arm |  |  |  |  |  |  |
| Aluminum membrane |  | 143.3 | 100x40 | $7.6 \cdot 10^{-3}$ |  |  |
| L1X |  | 154.2 | 80x40 | $0.5 \cdot 10^{-2}$ | 0.006 |  |
| L1Y |  | 155.2 | 80x40 | $0.5 \cdot 10^{-2}$ | 0.006 |  |
| L2X |  | 201.6 | 80x40 | $0.5 \cdot 10^{-2}$ | 0.006 |  |
| L2Y |  | 202.6 | $80 \times 40$ | $0.5 \cdot 10^{-2}$ | 0.006 |  |
| L3X |  | 251.6 | 112x40 | $0.5 \cdot 10^{-2}$ | 0.006 |  |
| L1W |  | 252.6 | 112x40 | $0.5 \cdot 10^{-2}$ | 0.006 |  |
| L3Y |  | 301.6 | 128x40 | $0.5 \cdot 10^{-2}$ | 0.006 |  |
| L2W |  | 302.6 | 128x40 | $0.5 \cdot 10^{-2}$ | 0.006 |  |
| Vertical Hodoscope |  | 327.5 | 140x40 | $8.7 \cdot 10^{-4}$ |  | 0.10 |
| Horisontal Hodoscope |  | 341.0 | 150x40 | $8.7 \cdot 10^{-4}$ |  | 0.22 |

Table 5: Efficiency of criterion on generation time difference $\left(\epsilon^{\Delta T}\right)$ and fraction of $\pi K$ pairs among event which fit this criterion for $K^{+} \pi^{-}$and $\pi^{+} K^{-}\left(F^{\pi K}\right)$, collected with 450 GeV proton beam at intensity $1 . \cdot 10^{12} \mathrm{~s}^{-1}$.

| Pairs | $\epsilon^{\Delta T}$ $\%$ | $F^{\pi K}$ $\%$ |
| :---: | :---: | :---: |
| $K^{+} \pi^{-}$ | 75.2 | 39.9 |
| $\pi^{+} K^{-}$ | 89.2 | 72.2 |

