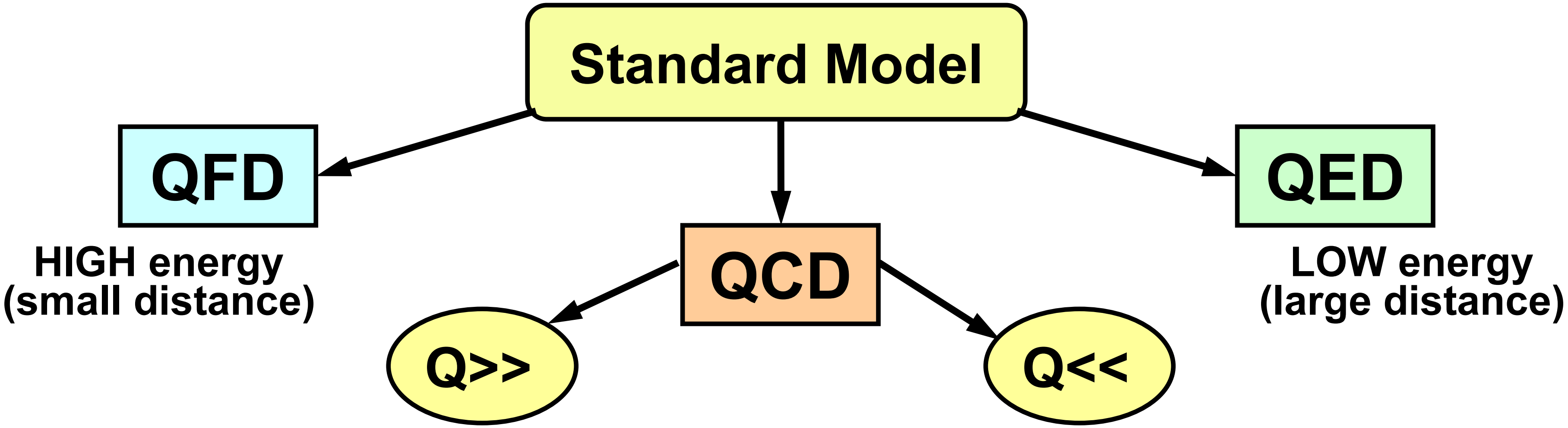
Creation of an intense source of $\pi\pi$, πK and other exotic atoms at SPS proton beam and using them for accurate measurements of $\pi\pi$, πK scattering length and other low energy parameters to check the precise low energy QCD predictions

VILLARS 2004 CERN SPS and PS Committee

L. Nemenov

25 September 2004

Theoretical motivation



Standard Model

QFD

HIGH energy
(small distance)

QCD

Q>>

Q<<

QED

LOW energy
(large distance)

perturbative QCD:

L_{QCD} **(q,g)**

interaction → „weak“ (asympt. freedom): expansion in coupling

Check only L_{sym}

chiral sym. & break:

L_{eff} **(GB: π,K,η)**

interaction → „strong“ (confinement) - **but**: expansion in energy

Check L_{sym} as well as

$L_{break\text{-}sym}$ ⟶ **q-condensate**

$\pi\pi$ scattering

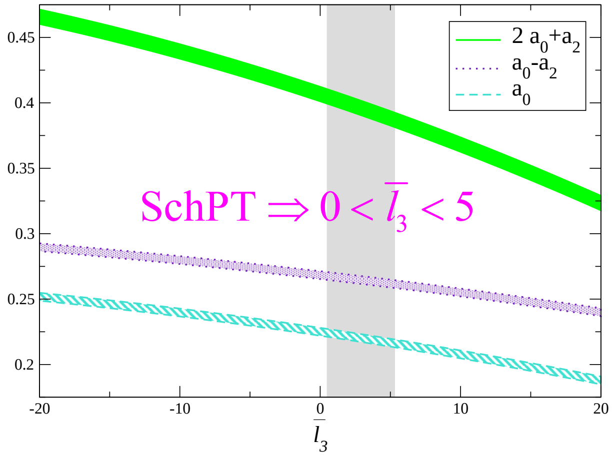
ChPT predicts s-wave scattering lengths:

$$\begin{aligned} a_0 &= 0.220 \pm 0.005\,(2.3\%) \\ a_2 &= -0.0444 \pm 0.0010\,(2.3\%) \\ a_0 - a_2 &= 0.265 \pm 0.004\,(1.5\%) \end{aligned}$$

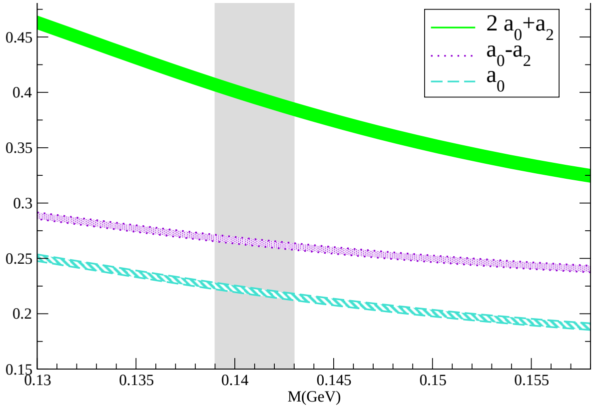
Chiral expansion of the π mass:

$$M_\pi^2 = (m_u + m_d)B - \left[(m_u + m_d)B\right]^2 \frac{\bar{l}_3}{32\pi^2 F^2} + O\left((m_u + m_d)^3\right)$$

where $BF_\pi^2 = |\langle 0 | \bar{u}u | 0 \rangle|$
is the *quark condensate*, reflecting
a property of the *QCD vacuum.*



SchPT $\Rightarrow 0 < \bar{l}_3 < 5$



Measurement of $\bar{l}_3 \overset{(1)}{\Rightarrow}$ estimate of $(m_u + m_d)|\langle 0 | \bar{u}u | 0 \rangle|$:

e.g.: $a_0 - a_2 = 0.260 \pm 3\% \Rightarrow \quad 1 < \bar{l}_3 < 11$ or $1.00 < M/M_\pi < 1.06$

E865: $\quad a_0 = 0.216 \pm 6\% \Rightarrow -4 < \bar{l}_3 < 12$ or $0.98 < M/M_\pi < 1.06$
(BNL)

Experimental Status

$$K^+ \rightarrow \pi^+\pi^-e^+\nu_e \ (K_{e4}) \Rightarrow \begin{cases}\text{measurement of the phase difference}\\ \delta(s) \equiv \delta_0^0(s)\text{-}\delta_1^1(s) \text{ for } 4M_\pi^2 < s < M_K^2\end{cases}$$

1) $a_0 = 0.26 \pm 0.05$, using Roy eq.

Rosselet et al. CERN, 1977

2a) $a_0 = 0.203 \pm 0.033$
$a_2 = -0.055 \pm 0.023$

2b) $a_0 = 0.216 \pm 0.013\ (\text{stat}) \pm 0.004\ (\text{sys}) \pm 0.002\ (\text{th})$,

using Roy eq. & $a_2 = f_{\text{ChPT}}(a_0)$

Pislak et al.
E865/BNL,
2001/03

DIRAC after analysis of all collected data

$$\delta(a_0 - a_2) = \pm 5\%(stat) \pm 3\%\,(syst) \pm 2\%\,(theor)$$

preliminary

πK scattering

I. ChPT predicts s-wave scattering lengths:

$$a_0^{1/2} = 0.19 \pm 0.2 \qquad a_0^{3/2} = -0.05 \pm 0.02$$

V. Bernard, N. Kaiser,
U. Meissner. – 1991

$\mathcal{L}^{(2)}, \mathcal{L}^{(4)}$ *and* $1-loop$

$$a_0^{1/2} - a_0^{3/2} = 0.23 \pm 0.01$$

A. Rossel. – 1999

J. Bijnens, P. Talaver. – April 2004

$\mathcal{L}^{(2)}, \mathcal{L}^{(4)}, \mathcal{L}^{(6)}$ *and* $2\text{-}loop$

II. Roy-Steiner equations:

$$a_0^{1/2} - a_0^{3/2} = 0.269 \pm 0.015$$

III. $A_{\pi K}$ lifetime:

$$A_{\pi^+K^-} \to \pi^0 \overline{K}^0 \quad (A_{K^+\pi^-} \to \pi^0 K^0)$$

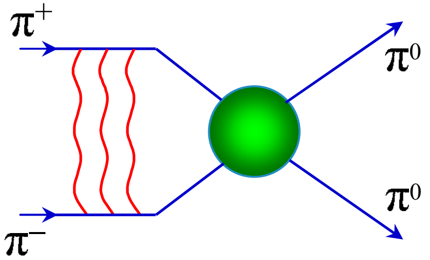
$\Gamma(\pi^0 \overline{K}^0) \sim | a_0^{1/2} - a_0^{3/2} |^2$ precission~1%

J. Schweizer. – 2004

$\tau = (3.7 \pm 0.4) \cdot 10^{-15}\ s$

1. $A_{2\pi}$ time of life

$$A_{2\pi} \rightarrow \pi^0 \pi^0$$



$$\frac{1}{\tau} = R_\pi (a_0 - a_2)^2$$

$$\frac{\Delta R_\pi}{R_\pi} \sim 1\%^{(*)}$$

(*) A. Gall et al. PR(2001)

DIRAC ADDENDUM

$$\frac{\Delta \tau}{\tau} = 4\% \, (stat), \quad \frac{\Delta |a_0 - a_2|}{a_0 - a_2} = 2\%(stat) \pm 1\%(syst) \pm 1\%(theor)$$

2. $A_{K^+\pi^-}$ and $A_{K^+\pi^-}$ time of life

$$A_{K^+\pi^-} \rightarrow \pi^0 K^0$$

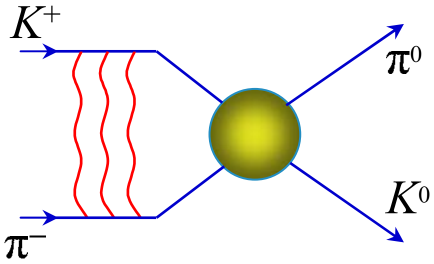
$$A_{\pi^+K^-} \rightarrow \pi^0 \overline{K}^0$$

$$\frac{1}{\tau} = R_K \, |a_{1/2} - a_{3/2}|^2$$

$$\frac{\Delta R_K}{R_K} \sim 2\%^{(**)}$$

(**) J. Schweizer (2004)

DIRAC ADDENDUM



$$\frac{\Delta \tau}{\tau} = 20\% \, (stat), \quad \frac{\Delta |a_{1/2} - a_{3/2}|}{a_{1/2} - a_{3/2}} = 10\%(stat) \pm 1\%(syst) \pm 1.5\%(theor$$

Energy Splitting between *np* - *ns* states in (π^+ - π^-) atom

$$\Delta E_n \equiv E_{ns} - E_{np}$$

$$\Delta E_n \approx \Delta E_n^{vac} + \Delta E_n^s \qquad \Delta E_n^s \sim 2a_0 + a_2$$

For $n = 2$

$$\Delta E_2^{vac} = -0.107\ eV \quad \textit{from QED calculations}$$

$$\Delta E_2^s \approx -0.45\ eV \quad \textit{numerical estimated value from ChPT}$$

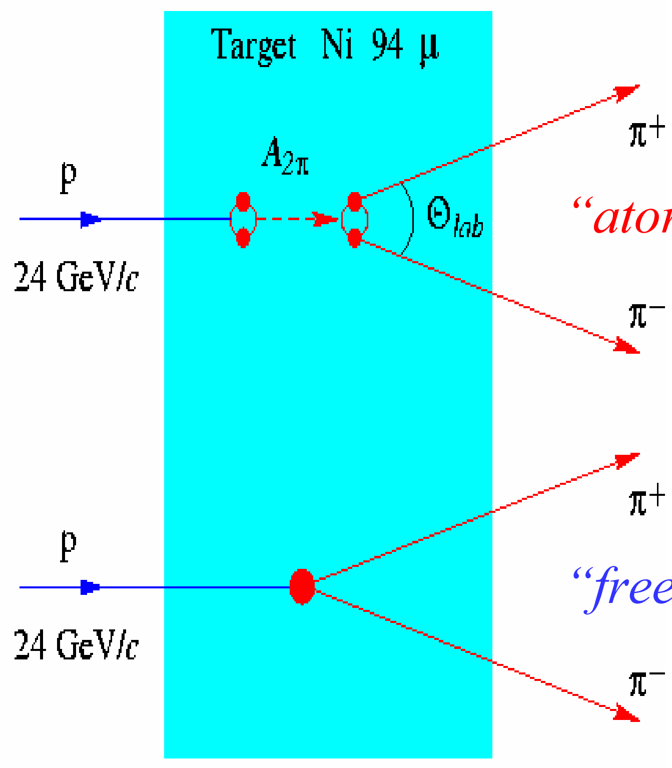
$$a_0 = 0.220 \pm 0.005$$
$$a_2 = -0.0444 \pm 0.0010$$

(2001) *G. Colangelo, J. Gasser and H. Leutwyler*

$$\Rightarrow \boxed{\Delta E_2 \approx -0.56\ \text{eV}}$$

(1979) A. Karimkhodzhaev and R. Faustov
(1983) G. Austen and J. de Swart
(1986) G. Efimov *et al.*
(1999) A. Gashi *et al.*
(2000) D. Eiras and J. Soto
A. Rusetsky, *priv. comm.*

Method of $A_{2\pi}$ Observation



"atomic pairs"

"free pairs"

$\tau(A_{2\pi})$ too small to be measured directly

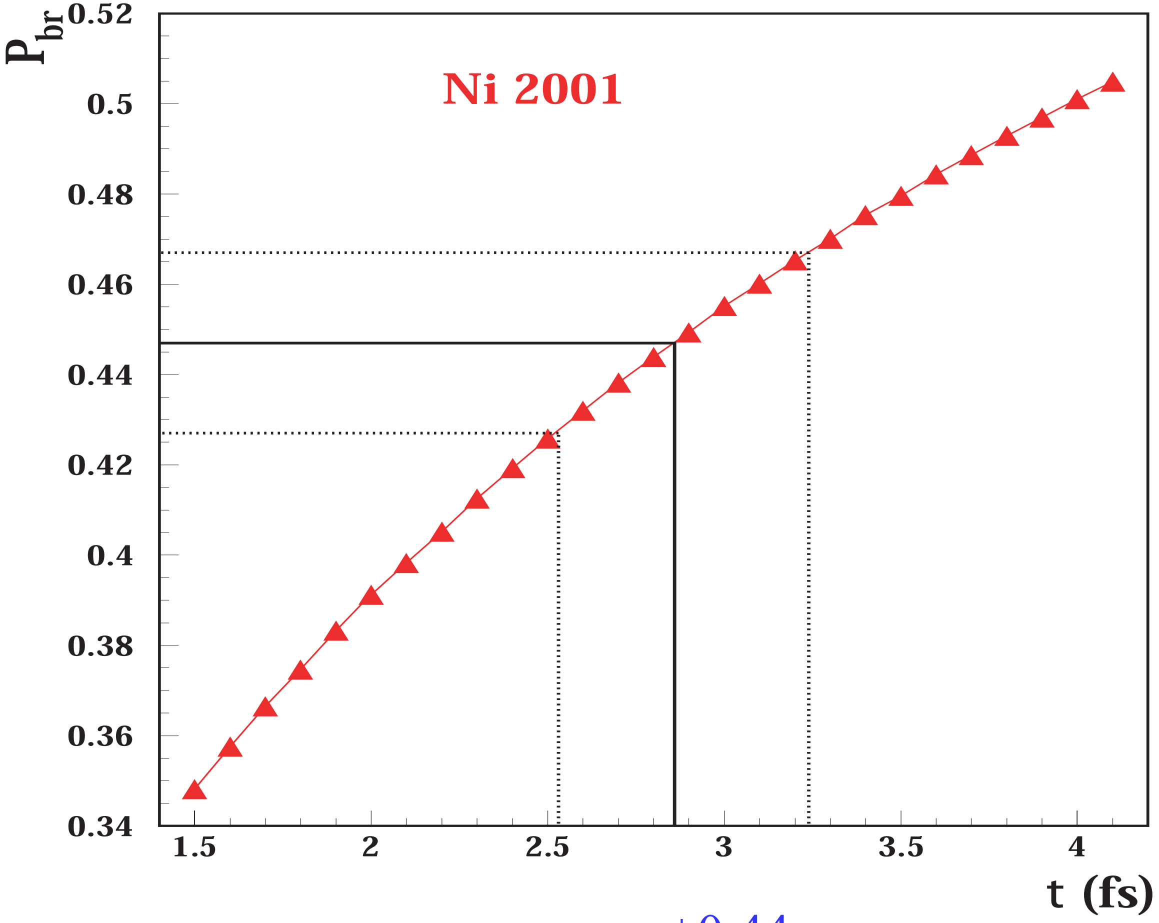
e.m. interation of $A_{2\pi}$ in the target

$A_{2\pi} \rightarrow \pi^+\pi^-$

$Q < 3 MeV/c \quad E_+ \approx E_-$

$\Theta_{lab} < 2.5$ *mrad*

Coulomb from short-lived sources
non- Coulomb from long-lived sources



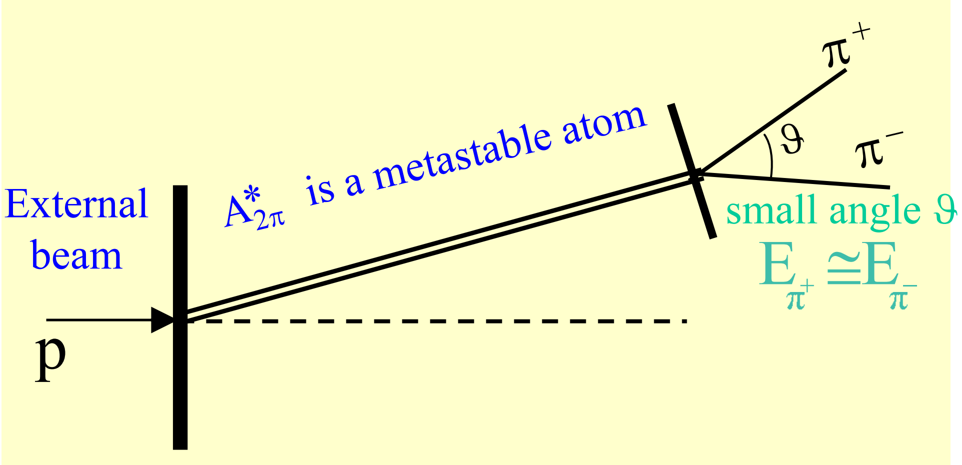
$P_{Br}=0.447\pm0.023_{stat}$

$\tau=2.85\,^{+0.44_{stat}}_{-0.38_{stat}}$ [fs]

Metastable atoms

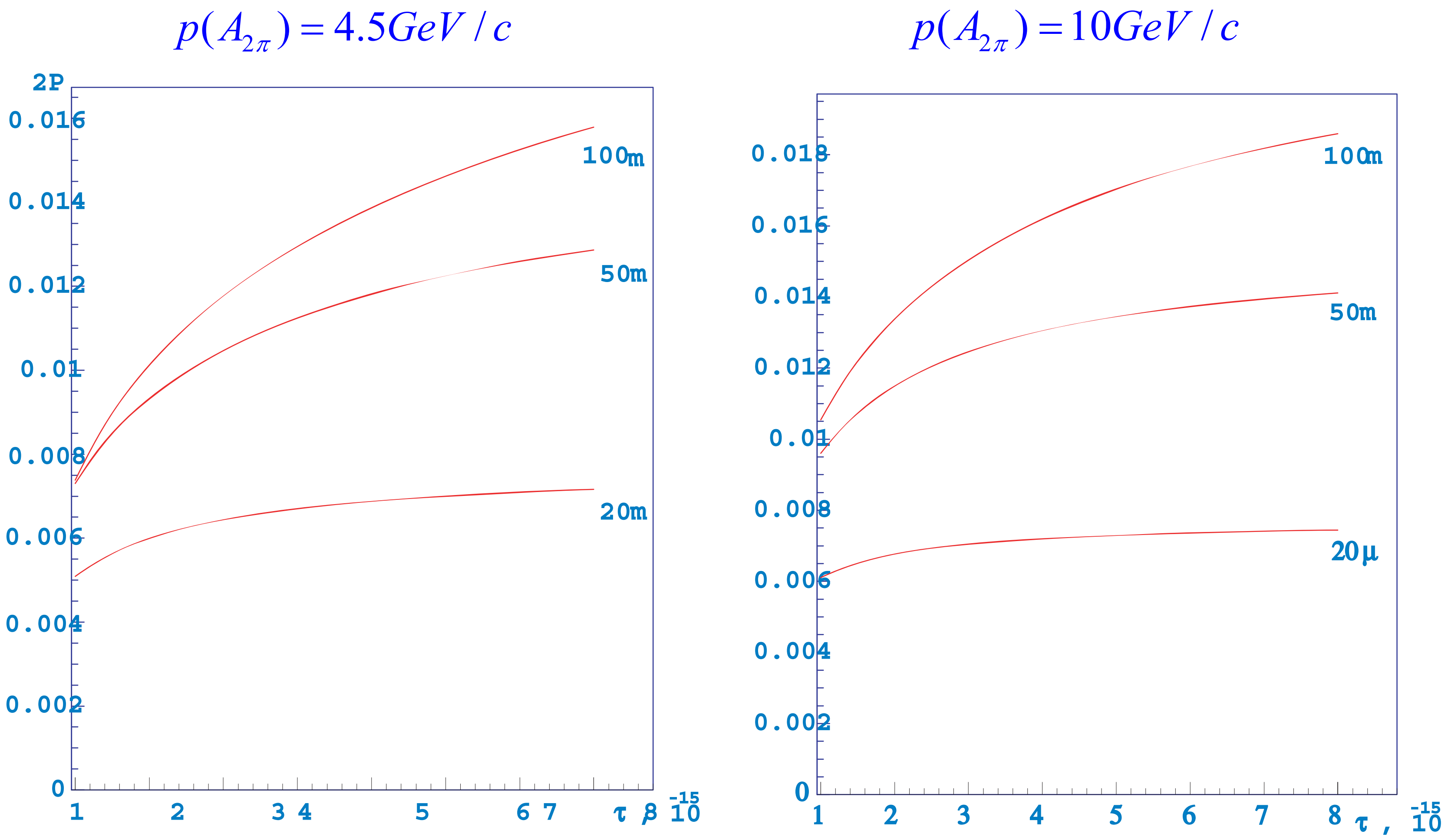
For $p_A = 5.6$ GeV/c and $\gamma = 20$

$$\begin{cases} \tau_{1s} = 2.9 \times 10^{-15}\ \mathrm{s}, & \lambda_{1s} = 1.7 \times 10^{-3}\ \mathrm{cm} \\ \tau_{2s} = 2.3 \times 10^{-14}\ \mathrm{s}, & \lambda_{2s} = 1.4 \times 10^{-2}\ \mathrm{cm} \\ \tau_{2p} = 1.17 \times 10^{-11}\ \mathrm{s}, & \lambda_{2p} = 7\ \mathrm{cm} \\ & \lambda_{3p} \approx 23\ \mathrm{cm} \\ & \lambda_{4p} \approx 54\ \mathrm{cm} \end{cases}$$



Probabilities of the $A_{2\pi}$ breakup (Br) and yields of the long-lived states for different targets provided the maximum yield of summed population of the long-lived states: **$\Sigma(l \geq 1)$**

Target Z	Thickness Mm	Br	Σ ($l \geq 1$)	$2p_0$	$3p_0$	$4p_0$	Σ ($l = 1$, $m = 0$)
04	100	4.45%	5.86%	1.05%	0.46%	0.15%	1.90%
06	50	5.00%	6.92%	1.46%	0.51%	0.16%	2.52%
13	20	5.28%	7.84%	1.75%	0.57%	0.18%	2.63%
28	5	9.42%	9.69%	2.40%	0.58%	0.18%	3.29%
78	2	18.8%	10.5%	2.70%	0.54%	0.16%	3.53%



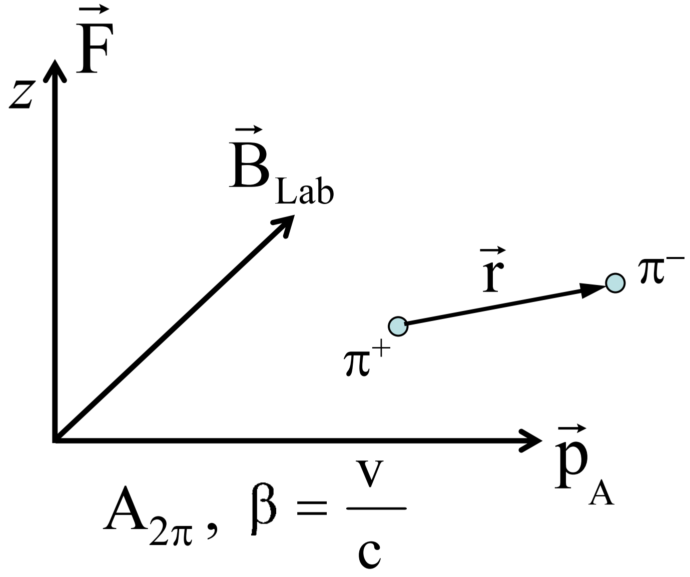
$p(A_{2\pi}) = 4.5 GeV/c$

$p(A_{2\pi}) = 10 GeV/c$

Yields of the long-lived states 2*p* ($m = 0$) as a function of the $A_{2\pi}$ lifetime for Beryllium targets ($Z = 04$). Target thicknesses are given in microns on the right side of the picture.

External magnetic and electric fields

Atoms in a beam are influenced by external magnetic field and the relativistic Lorentz factor



$\vec{r}$ ≡ relative distance between π^+ and π^- mesons in $A_{2\pi}$ atom

$\vec{B}_{Lab}$ ≡ laboratory magnetic field

$\vec{F}$ ≡ electric field in the CM system of an $A_{2\pi}$ atom

$$\boxed{F = \beta\gamma B_{Lab} \approx \gamma B_{Lab}}$$

The dependence of $A_{2\pi}$ life time in 2p-states τ_{eff} from a strength of the electric field F

$$\tau_{\text{eff}} = \frac{\tau_{2p}}{1 + \frac{|\xi|^2}{4} \frac{\tau_{2p}}{\tau_{2s}}} = \frac{\tau_{2p}}{1 + 120 |\xi|^2}$$

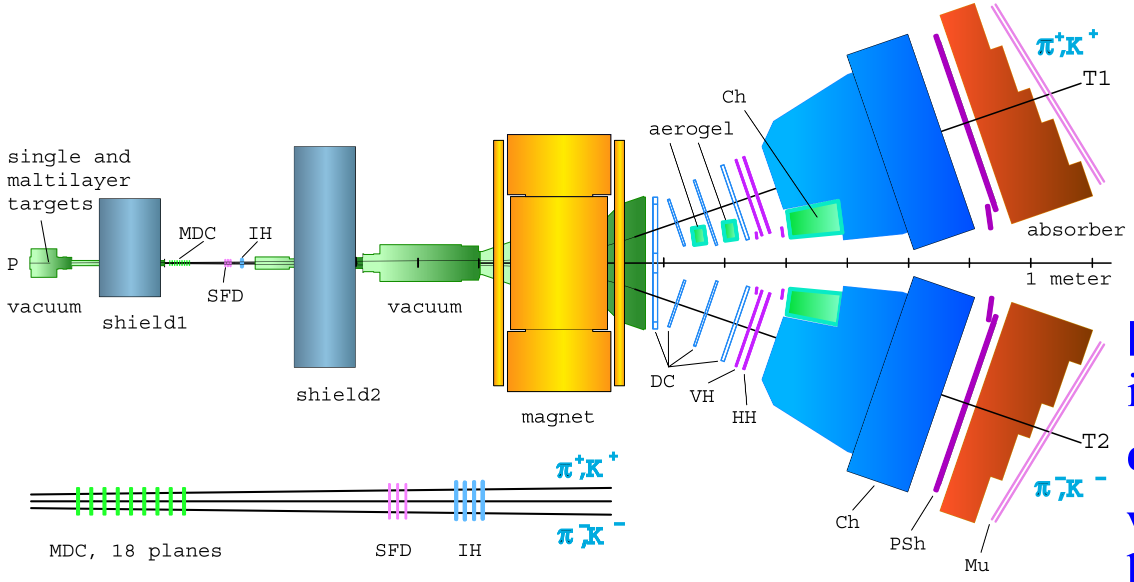
where: $$|\xi|^2 \approx \frac{F^2}{(E_{2p} - E_{2s})^2}$$

$$\boxed{B_{\text{Lab}} = 4 \text{ Tesla}}$$

$$\begin{cases} \gamma = 20 \;, & |\xi| = 0.1 \quad \Rightarrow \quad \tau_{\text{eff}} = \dfrac{\tau_{2p}}{2.2} \\ \gamma = 40 \;, & |\xi| = 0.2 \quad \Rightarrow \quad \tau_{\text{eff}} = \dfrac{\tau_{2p}}{6} \end{cases}$$

Upgraded DIRAC setup

(Addendum to DIRAC/PS212 (CERN) (13/04/04):
CERN-SPSC-2004-009 (SPSC-P-284 Add.4))



Schematic top view of the updated DIRAC spectrometer.

Upstream of the spectrometer magnet:

microdrift chambers (MDC) ,

scintillating fiber detectors (SFD) ,

ionization hodoscopes (IH).

Downstream of the magnet,
in each spectrometer arm:

drift chambers (DC),

vertical and **horizontal** scintillation hodoscopes (VH, HH),

gas **Cherenkov** counters (Ch),

preshower detector (PSh) and, behind the iron absorber,

muon detector (Mu).

In the left arm: Aerogel **Cherenkov** counters.

$A_{2\pi}$ and $A_{\pi K}$ production

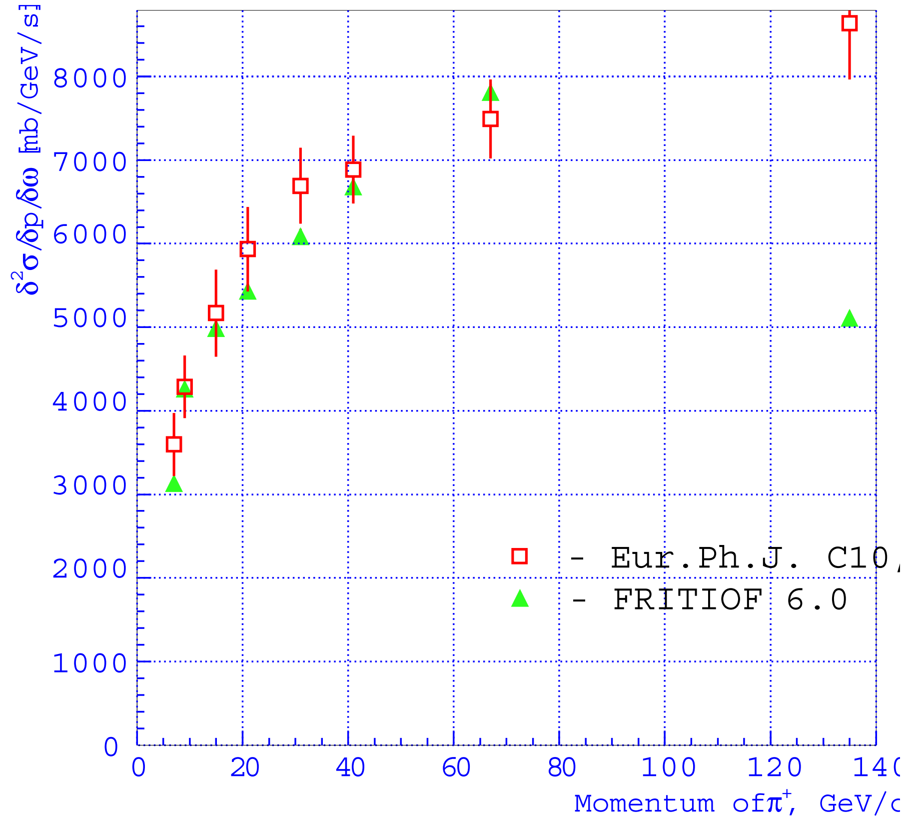
$$\frac{d\sigma^{A}_{nlm}}{d\vec{P}} = (2\pi)^3 \frac{E}{M} \left|\psi^{(C)}_{nlm}(0)\right|^2 \frac{d\sigma^{0}_{s}}{dp_1 dp_2} \sim \frac{d\sigma}{dp_1} \cdot \frac{d\sigma}{dp_2}$$

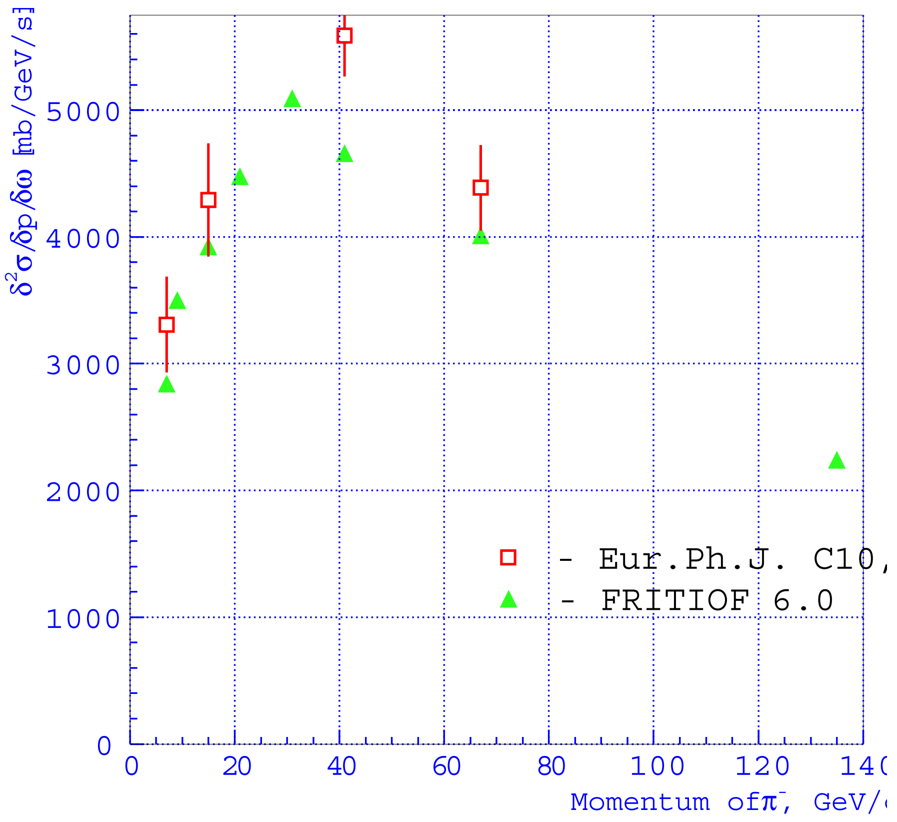
for atoms $\vec{\mathrm{v}}_1 = \vec{\mathrm{v}}_2$ *where* $\vec{\mathrm{v}}_1, \vec{\mathrm{v}}_2$ *– velocities of particles in the L.S. for all types of atoms*

for $A_{2\pi}$ *production* $\vec{p}_1 = \vec{p}_2$

for $A_{\pi K}$ *production* $\vec{p}_\pi = \dfrac{m_\pi}{m_K}\vec{p}_K$

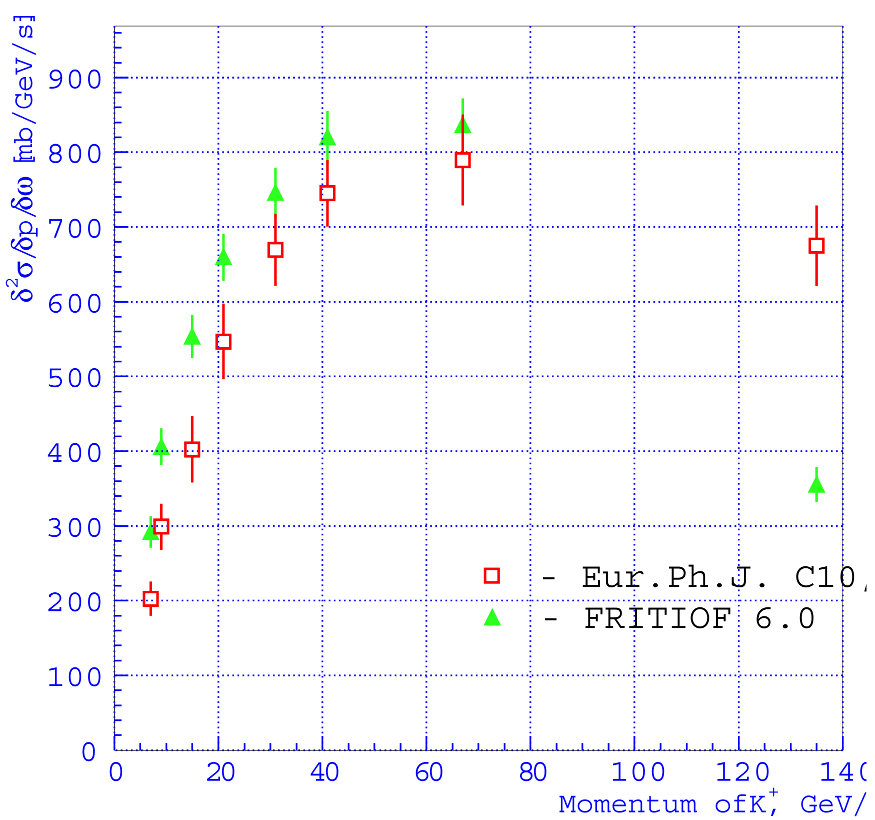
Inclusive cross-sections for π^+, π^- - mesons generation

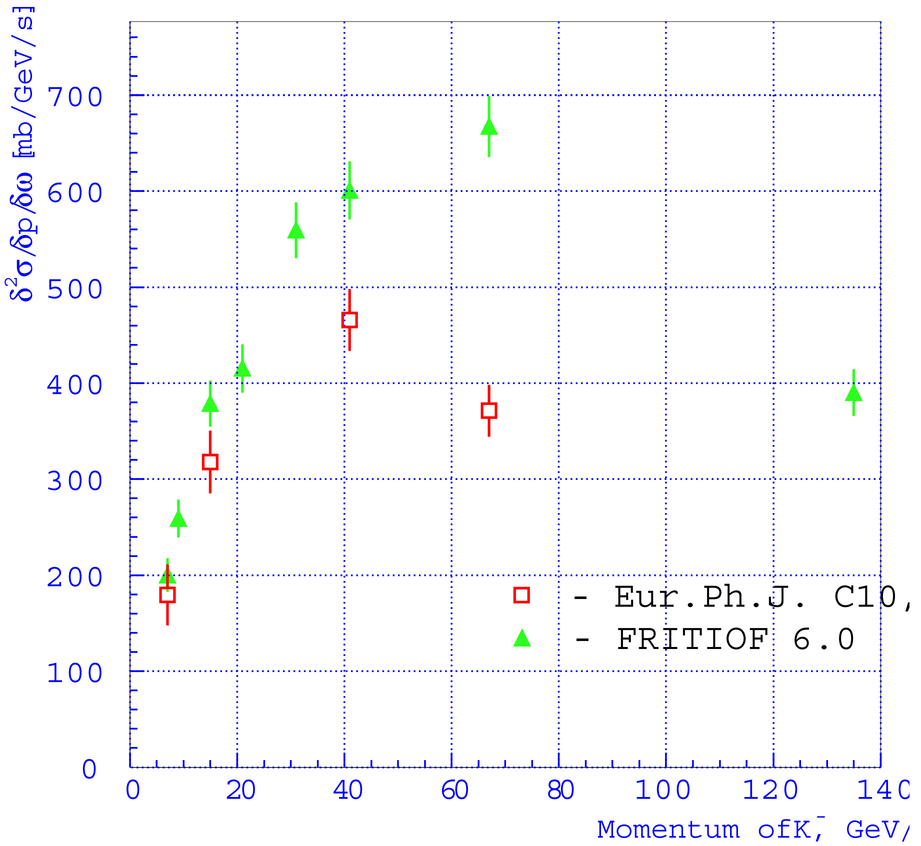




$$E_p = 450 GeV \quad \theta_L = 0°$$

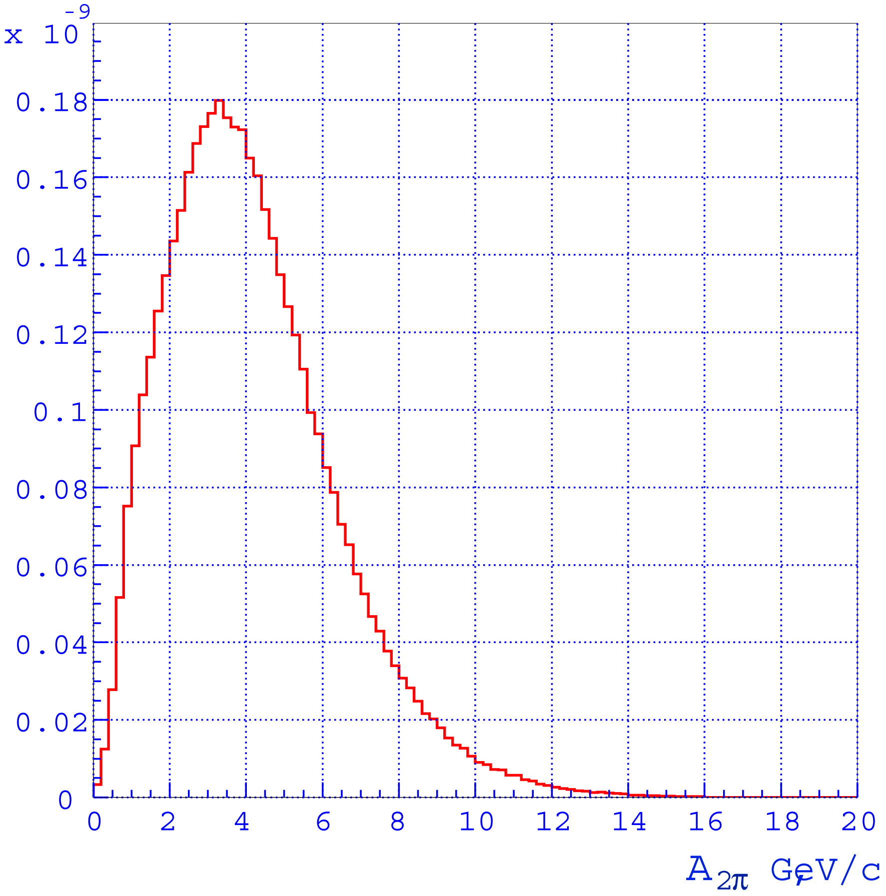
Inclusive cross-sections for K^{+}, K^{-} - mesons generation



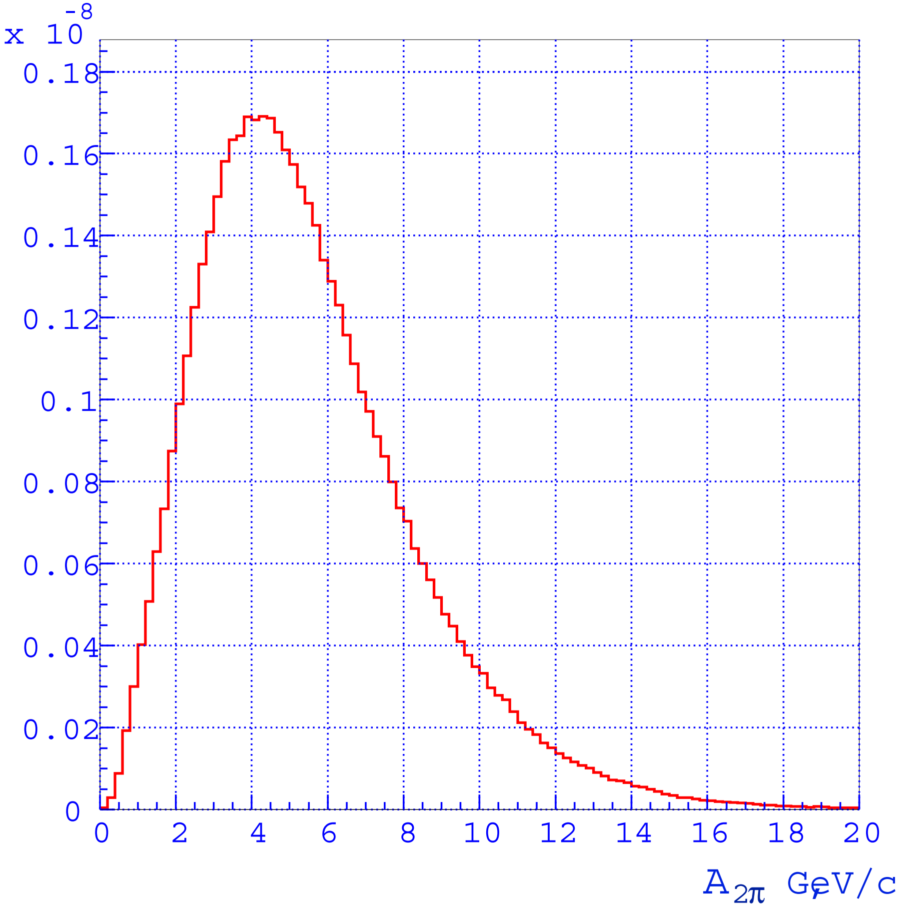


$$E_p = 450GeV \quad \theta_L = 0^\circ$$

$A_{2\pi}$ momentum distributions

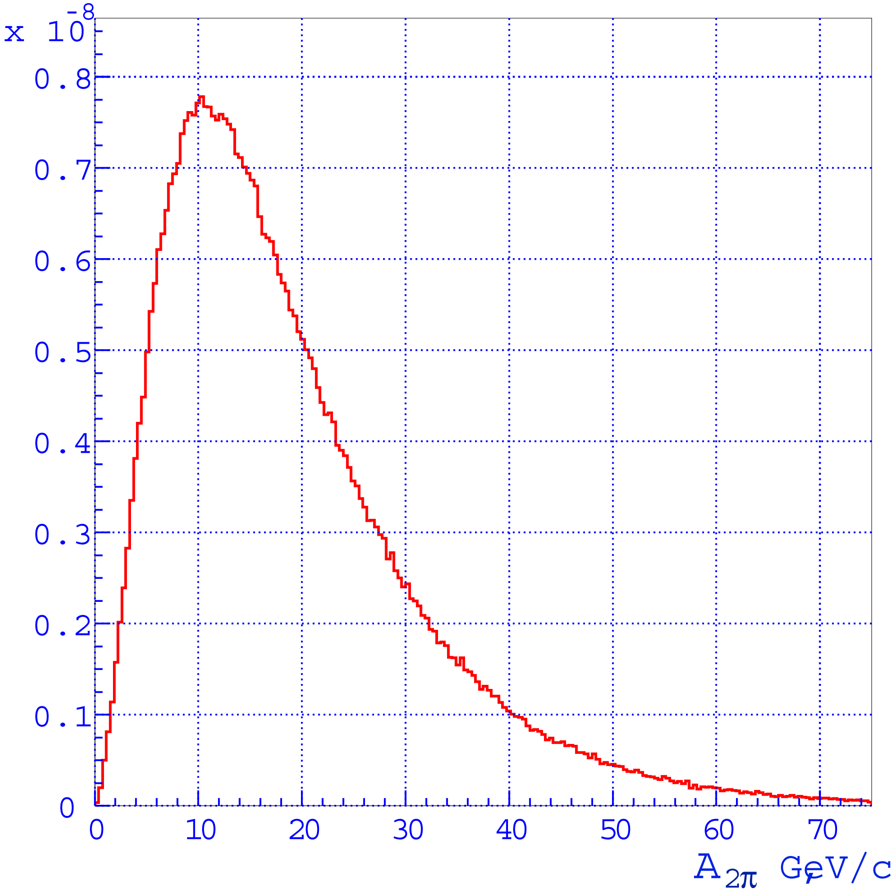


$$\theta_L = 5.7° \pm 1.3° \quad E_p = 24 GeV$$

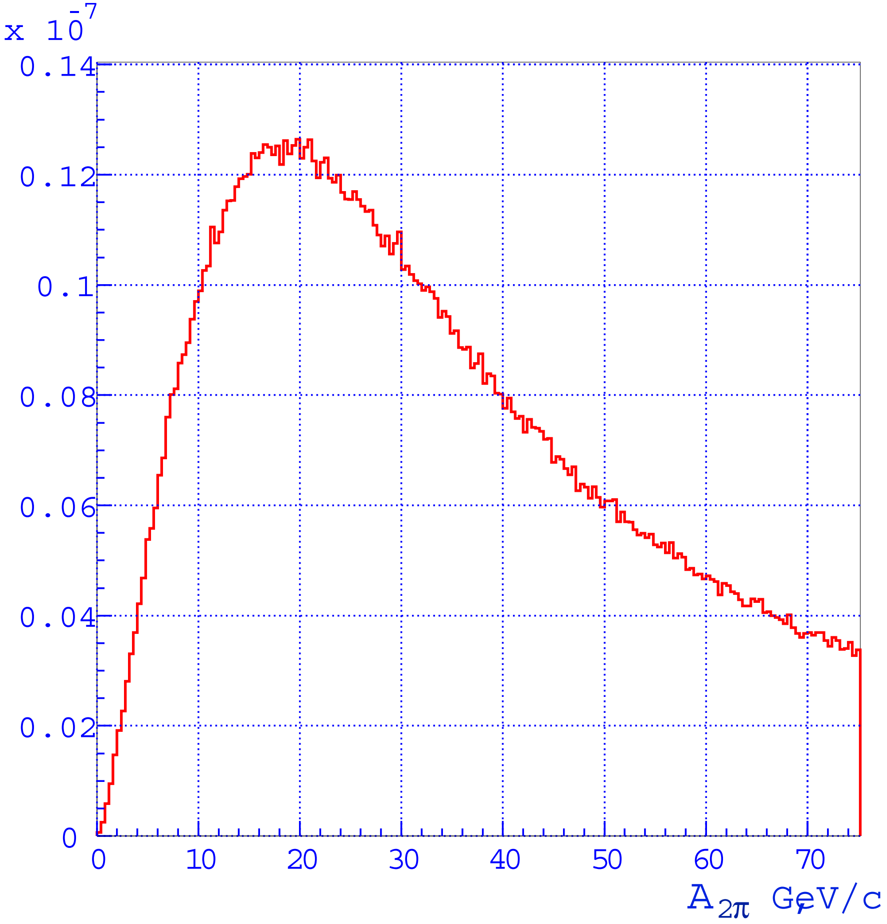


$$\theta_L = 5.7° \pm 1.3° \quad E_p = 450 GeV$$

$A_{2\pi}$ momentum distributions



$$\theta_L = 2° \pm 1.3° \quad E_p = 450 GeV$$

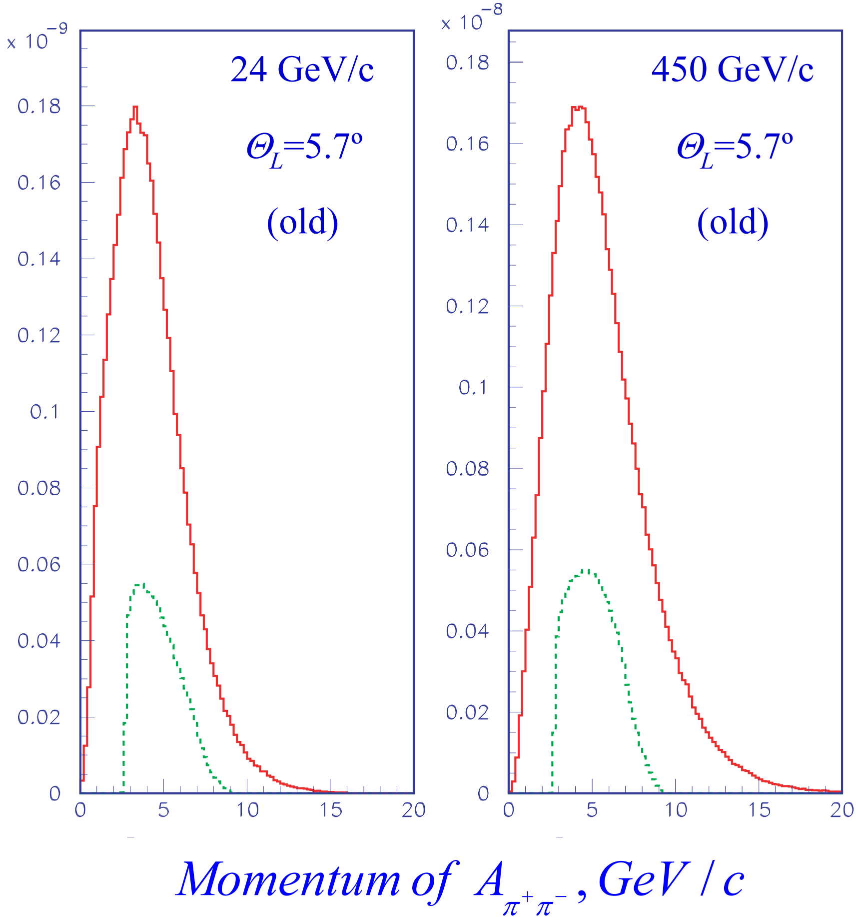


$$\theta_L = 0° \pm 1.3° \quad E_p = 450 GeV$$

Yield of $A_{2\pi}$ and $\pi^{\pm}$ into the aperture of $1.7\cdot10^{-3}$ sr per one proton interaction

Θ_L	$5.7^{o}\pm1.3^{o}$	$5.7^{o}\pm1.3^{o}$	$2^{o}\pm1.3^{o}$	$0^{o}\pm1.3^{o}$
E_p	**24 GeV**	**450 GeV**	**450 GeV**	**450 GeV**
W_A	$0.46\cdot10^{-8}$	$0.53\cdot10^{-7}$	$0.47\cdot10^{-6}$	$0.18\cdot10^{-5}$
W_A^N	1.	12.	103.	389.
W_π	0.032	0.1	1.1	6.6
W_π^N	1.	3.1	34.	206.
W_A/W_π	$1.4\cdot10^{-7}$	$5.3\cdot10^{-7}$	$4.3\cdot10^{-7}$	$2.7\cdot10^{-7}$
W_A^N/W_π^N	1.	3.7	3.	1.9

$A_{2\pi}$ momentum distributions



Yield of $A_{2\pi}$ per one proton interaction, detectable by DIRAC setup at Θ_L=5.7°

E_p	24 GeV (old)	450 GeV (old)	450 GeV (new)
W_A	$0.96\cdot10^{-9}$	$0.11\cdot10^{-7}$	$0.13\cdot10^{-7}$
W_A^N	1.	12.	13.
W_A/W_π	$3\cdot10^{-8}$	$1.1\cdot10^{-7}$	$1.3\cdot10^{-7}$
W_A^N/W_π^N	1.	3.7	4.3
		A multiplier due to different spill duration ~4	
Total gain	**1.**	**15.**	**17.**

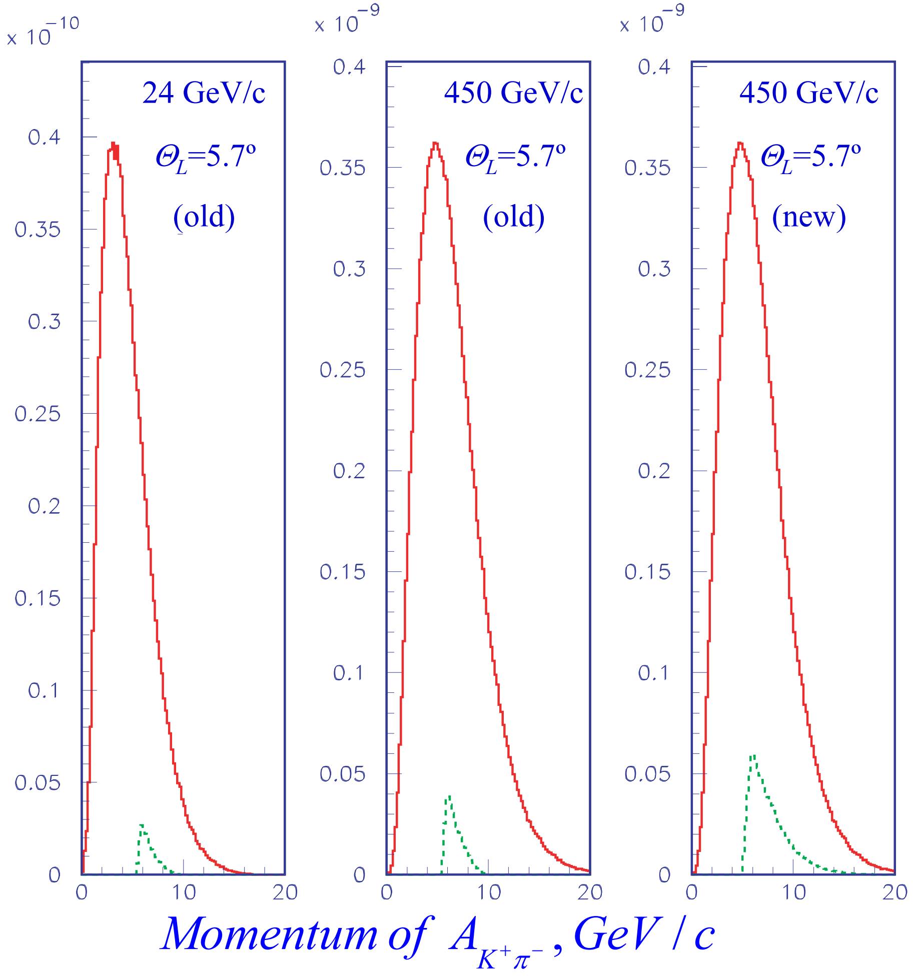
— - red curve $A_{2\pi}$ spectra in channel the aperture

••• - green curve $A_{2\pi}$ spectra registered by the set-up

Yield of $A_{K^+\pi^-}$ and $A_{K^-\pi^+}$ into the aperture of $1.7\cdot10^{-3}$ sr per one proton interaction

	$A_{K^+\pi^-}$				$A_{K^-\pi^+}$			
Θ_L	**5.7°±1.3°**	**5.7°±1.3°**	**2°±1.3°**	**0°±1.3°**	**5.7°±1.3°**	**5.7°±1.3°**	**2°±1.3°**	**0°±1.3°**
E_p	**24GeV**	**450GeV**	**450GeV**	**450GeV**	**24GeV**	**450GeV**	**450GeV**	**450GeV**
W_A	$0.11\cdot10^{-8}$	$0.13\cdot10^{-7}$	$0.16\cdot10^{-6}$	$0.65\cdot10^{-6}$	$0.60\cdot10^{-9}$	$0.10\cdot10^{-7}$	$0.11\cdot10^{-6}$	$0.54\cdot10^{-6}$
W_A^N	1.	12.	146.	610.	1.	17.	192.	897.
W_A/W_π	$3.4\cdot10^{-8}$	$1.3\cdot10^{-7}$	$1.5\cdot10^{-7}$	$9.8\cdot10^{-8}$	$1.9\cdot10^{-8}$	$1.\cdot10^{-7}$	$1.\cdot10^{-7}$	$8.2\cdot10^{-8}$
W_A^N/W_π^N	1.	3.8	4.2	2.9	1.	5.3	5.3	4.4

$A_{K^+\pi^-}$ momentum distributions



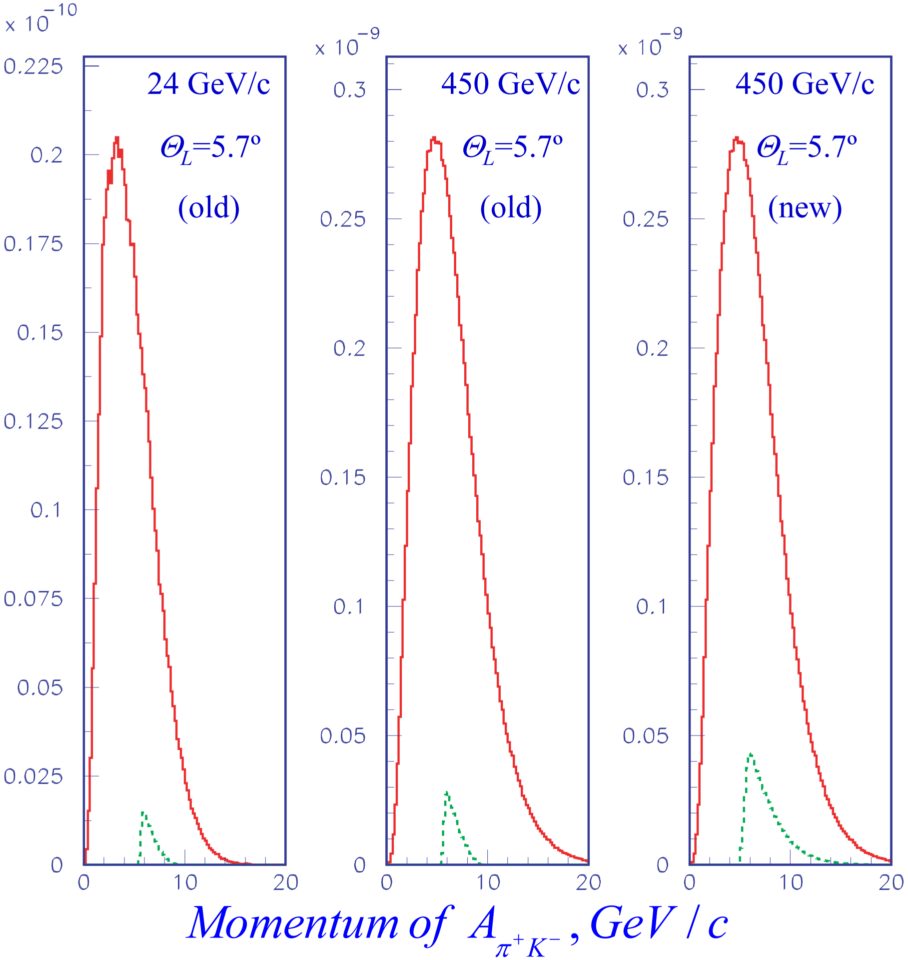
— - red curve $A_{2\pi}$ spectra in the channel aperture

••• - green curve $A_{2\pi}$ spectra registered by the set-up

Yield of $A_{K^+\pi^-}$ per one proton interaction, detectable by DIRAC setup at Θ_L=5.7°

E_p	24 GeV (old)	450 GeV (old)	450 GeV (new)
W_A	$0.20\cdot10^{-10}$	$0.34\cdot10^{-9}$	$0.10\cdot10^{-8}$
W_A^N	1.	17.	49.
W_A/W_π	$6.2\cdot10^{-10}$	$3.4\cdot10^{-9}$	$1\cdot10^{-8}$
W_A^N/W_π^N	1.	5.4	16
		A multiplier due to different spill duration ~4	
Total gain	**1.**	**22.**	**64.**

$A_{\pi^+K^-}$ momentum distributions



— - red curve $A_{2\pi}$ spectra in the channel aperture

••• - green curve $A_{2\pi}$ spectra registered by the set-up

Yield of $A_{\pi^+K^-}$ per one proton interaction, detectable by DIRAC setup at Θ_L=5.7º

E_p	**24 GeV (old)**	**450 GeV (old)**	**450 GeV (new)**
W_A	$0.11\cdot10^{-10}$	$0.23\cdot10^{-9}$	$0.71\cdot10^{-9}$
W_A^N	1.	21.	63.
W_A/W_π	$3.4\cdot10^{-10}$	$2.3\cdot10^{-9}$	$7.1\cdot10^{-9}$
W_A^N/W_π^N	1.	6.7	20.6
		A multiplier due to different spill duration ~4	
Total gain	**1.**	**27.**	**82.**

Conclusions

Present low energy ***QCD*** predictions for ***ππ*** and ***πK*** scattering lengths

$$\pi\pi \qquad \delta a_0 = 2.3\% \qquad \delta a_2 = 2.3\% \qquad \delta(a_0 - a_2) = 1.5\%$$

$$\pi K \qquad \delta(a_{1/2} - a_{3/2}) \approx 10\%$$

Expected results of DIRAC ADDENDUM at PS CERN

$$\tau(A_{2\pi}) \to \delta(a_0 - a_2) = \pm 2\% (stat) \pm 1\% \, (syst) \pm 1\% \, (theor)$$

$$\tau(A_{\pi K}) \to \delta(a_{1/2} - a_{3/2}) = \pm 10\% (stat) \pm ....\pm 1.5\% \, (theor)$$

Observation of metastable $A_{2\pi}$

DIRAC at SPS CERN

$$\tau(A_{2\pi}) \to \delta(a_0 - a_2) = \pm 0.5\% (stat)$$

$$\tau(A_{\pi K}) \to \delta(a_{1/2} - a_{3/2}) = \pm 2.5\% (stat)$$

$$(E_{np} - E_{ns})_{\pi\pi} \to \delta(2a_0 + a_2) \approx \pm 2.5\% (stat)$$

$$(E_{np} - E_{ns})_{\pi K} \to \delta(2a_{1/2} + a_{3/2})$$

Possibility of the observation $(\pi^{\pm}\mu^{\mp})$ *– atoms and* $(K^{+}K^{-})$ *– atoms will be studied.*