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Multiplicity Fluctuations in One- and Two-Dimensional Angular Intervals Compared with Analytic QCD Calculations

DELPHI Collaboration

Abstract

Multiplicity fluctuations in rings around the jet axis and in off-axis cones have been measured by the DELPHI collaboration in e^+e^- annihilations into hadrons at LEP energies. The measurements are compared with analytical perturbative QCD calculations for the corresponding multiparton system, using the concept of Local Parton Hadron Duality. Some qualitative features are confirmed by the data but substantial quantitative deviations are observed.

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1 Introduction

To describe multiplicity fluctuations in angular regions by analytical calculations using perturbative QCD is a challenge. It could help to improve our understanding of the parton cascading mechanism and might lead to a simple description of multiparticle correlations by QCD alone. The idea that QCD jets might exhibit a self-similar (or fractal) structure was brought up already in 1979 by R.P.Feynman [1], A.Giovannini [2] and G.Veneziano [3]. In recent years this conception has been confirmed by various groups [4–6], giving detailed predictions on variables and phase space regions where fractality is expected to show up. A simple predicted dependence of the fractal dimensions on α_s stimulated further interest in measuring them experimentally.

The analytical calculations are performed in the Double Log Approximation (DLA) [7,8], neglecting energy-momentum conservation, and concern only idealized jets. They provide leading order predictions applicable quantitatively at very high energies ($\geq$ 1 TeV) [4]. At LEP energies, non-perturbative effects may be important. Also, they refer to multiparton states, whereas only multihadron states can be measured. It has been suggested that the parton evolution should be extended from the perturbative regime down to a lower mass scale (if possible to the mass scale of light hadrons) to be able to compare the partonic states directly with the hadronic states. This concept of Local Parton Hadron Duality (LPHD) [9] is quite successful for single particle distributions and for global moments of multiplicity distributions. It remains questionable in the case of the more refined variables used here, namely factorial moments and cumulants in phase space bins. First experimental measurements [10–14] revealed, indeed, substantial deviations.

On the other hand it can be expected that these calculations will improve in the future. This would provide us with a better understanding of the internal structure of jets in terms of analytical expressions than can be obtained by Monte Carlo calculations with many parameters. In fact, the analytical predictions considered in this paper involve only one adjustable parameter, namely the QCD scale Λ.

The aim of this study is to use DELPHI data to measure multiplicity fluctuations in one- and two-dimensional angular intervals and compare them with the available theoretical predictions. It is hoped that such a study may show how to approach nearer to a satisfying theory based on QCD and LPHD which describes high energy multiparticle phenomena.

In section 2 the theoretical framework is sketched, section 3 contains information about the experimental data and the Monte Carlo comparisons and in section 4 the comparison with the analytical calculations is presented. Section 5 contains the final discussion and the summary.

2 Theoretical framework

The theoretical calculations treat correlations between partons emitted within an angular window defined by two angles ϑ and Θ. The parton and particle density correlations (fluctuations) in this window are described by normalized factorial moments of order n:

$$F^{(n)}(\Theta,\vartheta) = \frac{\int \rho^{(n)}(\Omega_1,\ldots,\Omega_n)d\Omega_1\ldots d\Omega_n}{\int \rho^{(1)}(\Omega_1)\ldots\rho^{(1)}(\Omega_n)d\Omega_1\ldots d\Omega_n} \tag{1}$$

where $\rho^{(n)}(\Omega_1,\ldots,\Omega_n)$ are the n-parton/particle density correlation functions which depend on the spherical angles Ω_k. The integrals extend over the window chosen.

The angular windows considered here are either rings around the jet axis with mean opening angle $\Theta = 25^\circ$ and half width ϑ in the case of 1 dimension ($D = 1$), or cones with half opening angle ϑ around a direction (Θ, Φ) with respect to the jet axis in the case of 2 dimensions ($D = 2$). At sufficiently large jet energies, the parton flow in these angular windows is dominated by parton avalanches caused by gluon bremsstrahlung off the initial quark.

The cumulants $C^{(n)}$ are obtained from the moments $F^{(n)}$ by simple algebraic equations [15], e.g. $C^{(2)} = F^{(2)} - 1$, $C^{(3)} = F^{(3)} - 3(F^{(2)} - 1) - 1$.

The theoretical scheme for deriving the moments described above is based on the generating functional techniques [8,16] in the DLA of perturbative QCD. The probability of radiating a gluon with momentum k at an emission angle Θ_g and azimuthal angle Φ_g from an initial parton a has been approximated by

$$M(k)d^3k = c_a \gamma_0^2 \frac{dk}{k} \frac{d\Theta_g}{\Theta_g} \frac{d\Phi_g}{2\pi} \tag{2}$$

$$\gamma_0^2 = 6\alpha_S/\pi \tag{3}$$

with $c_a = 1$ if a is a gluon and $c_a = 4/9$ if a is a quark.

Ref. [4] derived their predictions explicitly for cumulant moments $C^{(n)}$, whereas [5] and [6] obtained similar expressions for the factorial moments $F^{(n)}$. It has been shown [4] by Monte Carlo calculations that, at very high energy ($\sqrt{s} \geq 1800$ GeV), the values of $F^{(n)}$ and $C^{(n)}$ converge to each other. At LEP energies, however, the cumulants are still far away from the asymptotic predictions (see section 4).

For the normalized cumulant moments $C^{(n)}$ [4] and the factorial moments $F^{(n)}$ [5,6], the following prediction has been made:

$$C^{(n)}(\Theta, \vartheta) \text{ or } F^{(n)}(\Theta, \vartheta) \sim \left(\frac{\Theta}{\vartheta}\right)^{\phi_n} \tag{4}$$

All 3 references [4–6] give in the high energy limit and for large ring regions ($\vartheta \leq \Theta$) the same linear approximation for the exponents ϕ_n:

$$\phi_n \approx (n-1)D - \left(n - \frac{1}{n}\right)\gamma_0 \tag{5}$$

where D is a dimensional factor, 1 for ring regions and 2 for cones. If α_s is kept constant along the parton shower, eq. 5 is asymptotically valid for <u>all</u> angles representing a power low. In this case the fractal (Renyi-) dimensions D_n [17] can be obtained [18] from ϕ_n (eq. 5) via:

$$D_n = D - \frac{\phi_n}{n-1} \tag{6}$$

$$D_n = \frac{n+1}{n}\gamma_0 \tag{7}$$

When the running of α_S with ϑ in the parton cascade is taken into account, in [4] the following was obtained

$$\phi_n \approx (n-1)D - 2\gamma_0(n - \omega(\epsilon, n))/\epsilon \tag{8}$$

$$\omega(\epsilon, n) = n\sqrt{1-\epsilon}\left(1 - \frac{1}{2n^2}\ln(1-\epsilon)\right) \tag{9}$$

and

$$\epsilon = \frac{\ln(\Theta/\vartheta)}{\ln(P\Theta/\Lambda)} \tag{10}$$

where $P \approx \sqrt{s}/2$ is the momentum of the initial parton. The dependence on the QCD parameters α_s or Λ enters in the above equations via γ_0 and ϵ that are determined by the scale $Q \approx P\Theta$. In the present study it is about 20 GeV for $\sqrt{s}$=91.1 GeV.

The corresponding predictions of refs. [5] (eq. 11) and [6] (eq. 12) are analytically different, but numerically similar:

$$\phi_n = (n-1)D - \frac{2\gamma_0}{\epsilon} \cdot \frac{n^2-1}{n}\left(1 - \sqrt{1-\epsilon}\right) \tag{11}$$

$$\phi_n = (n-1)D - \frac{n^2-1}{n}\gamma_0\left(1 + \frac{n^2+1}{4n^2}\epsilon\right) \tag{12}$$

It should be noted that all three papers [4–6] the lowest order QCD relation (13) between the coupling α_s and the QCD scale Λ, which is also used in the present analysis:

$$\alpha_s = \frac{\pi\beta^2}{6}\frac{1}{\ln(Q/\Lambda)} \tag{13}$$

$$\beta^2 = 12\left(\frac{11}{3}n_c - \frac{2}{3}n_f\right)^{-1} \tag{14}$$

where n_c=3 (number of colours). These relations depend also on the number of flavours (n_f). Since eq. 13 emerges only from "one loop" calculations, the parameter Λ is not the universal $\Lambda_{\overline{MS}}$, but only an effective parameter Λ_{eff}. But also in this approximation α_s runs, having a scale dependence $1/\ln(Q^2/\Lambda^2)$.

The running of α_s during the process of jet cascading is implicitly taken into account in (8), (11) and (12) by the dependence of ϕ_n on ϵ (or ϑ). In theory this causes a deviation from a potential behaviour (eqs. 4 and 5) of $F^{(n)}$ when approaching smaller values of ϑ (larger ϵ).

All theoretical predictions concern the partonic states. The corresponding experimental measurements, however, are of hadronic states. When comparing them, the hypothesis of LPHD has to be used.

It may be noted that the factorial moments $F^{(n)}$ measured in the present study (see also eq. 15 below) are very similar to the well known and previously measured moments in rapidity space. Here the angle ϑ is used (translated by constant factors into ϵ), because this is the natural variable in the QCD calculations.

3 Experimental data and comparison with the Monte Carlo calculations

The normalized factorial moments (1) are determined experimentally by counting n_m, the number of charged particles in the respective windows of phase space, for each event:

$$F^{(n)}(\Theta,\vartheta) = \frac{\langle n_m(n_m-1)\ldots(n_m-n+1)\rangle}{\langle n_m\rangle^n} \tag{15}$$

where the brackets <> denote averages over the whole event sample.

The data sample used contains about 600000 e^+e^- interactions (after cuts) collected by DELPHI at $\sqrt{s}$ = 91.1 GeV in 1994. A sample of about 1200 high energy events at $\sqrt{s}$=183 GeV incident energy collected in 1997 is used to investigate the energy dependence. The calculated hadron energy was required to be greater than 162 GeV (corresponding to a mean energy of 175 GeV). The standard cuts as in [19] for hadronic events and track quality were applied by demanding a minimum charged multiplicity, enough visible charged energy and events well contained within the detector volume. In the present study all charged particles (except identified electrons and muons) with momentum larger than 0.1 GeV have been considered. The special procedures for selecting high energy events are described in [20]. WW-events have been excluded. Detailed Monte Carlo studies were done using the JETSET 7.4 PS model [21]. The corrections were determined using events from a JETSET Monte Carlo simulation which had been tuned (Λ=0.346 GeV and Q_0=2.25 GeV) to reproduce general event characteristics [22], which included variables different from those referred to in section 2. These events were examined at

* **Generator level**
  where all charged final–state particles (except electrons and muons) with a mean lifetime longer than 10^{-9} seconds have been considered;
* **Detector level**
  which includes distortions due to particle decays and interactions with the detector material, other imperfections such as limited resolution, multi–track separation and detector acceptance, and the event selection procedures.

Using these events, the factorial moments and cumulants introduced in section 2 of order n, A_n, studied below were corrected (for each ϵ interval considered) by

$$A_n^{cor} \;=\; g_n A_n^{raw}\,, \quad g_n \;=\; \frac{A_n^{gen}}{A_n^{det}} \tag{16}$$

where the superscript "raw" indicates the quantities calculated directly from the data, and "gen" and "det" denote those obtained from the Monte Carlo events at generator and detector level respectively. The simulated data at detector level were found to agree satisfactorily with the experimental data. The measurement error on the relative angle ϑ_{12} between two outgoing particles was determined to be of order 0.5° (if both tracks had good Vertex Detector hits, even as small as 0.1°). The jet axis is chosen to be the sphericity axis. To increase statistics in the case of the high energy sample the moments (15) have been calculated in both sphericity hemispheres and averaged.

In addition, all phenomena which were not included in the analytical calculations had to be corrected for, namely (i) initial state photon radiation, (ii) Dalitz decays of the π^0, (iii) residual K_s^0 and Λ^0 decays near the vertex, and (iv) the effect of Bose–Einstein correlations. The corrections were estimated, for each ϵ interval, like g_n in eq. 16, by switching the effects on and off. Each of these correction factors were found to be below 10% in the case of factorial moments. The largest corrections have been found in the case of cumulants of higher orders and amounted to 16–25%, depending on the analysis angle.

The total correction factor including all effects is denoted by g_n^{tot} and is the product of the individual factors. Systematic errors have been calculated from g_n^{tot} according to $\Delta A_n^{corr} = \pm|A_n^{raw}(g_n^{tot}-1)/2|$. Due to uncertainties in measuring multiple tracks at very small separation angles, an additional systematic error was added for small ϑ values for $F^{(4)}$ and $F^{(5)}$.

Fig. 1 shows a comparison at $\sqrt{s}$=91.1 GeV of the measured 1-dimensionel cumulants and 1- and 2-dimensional factorial moments with JETSET 7.4 tuned as described above.

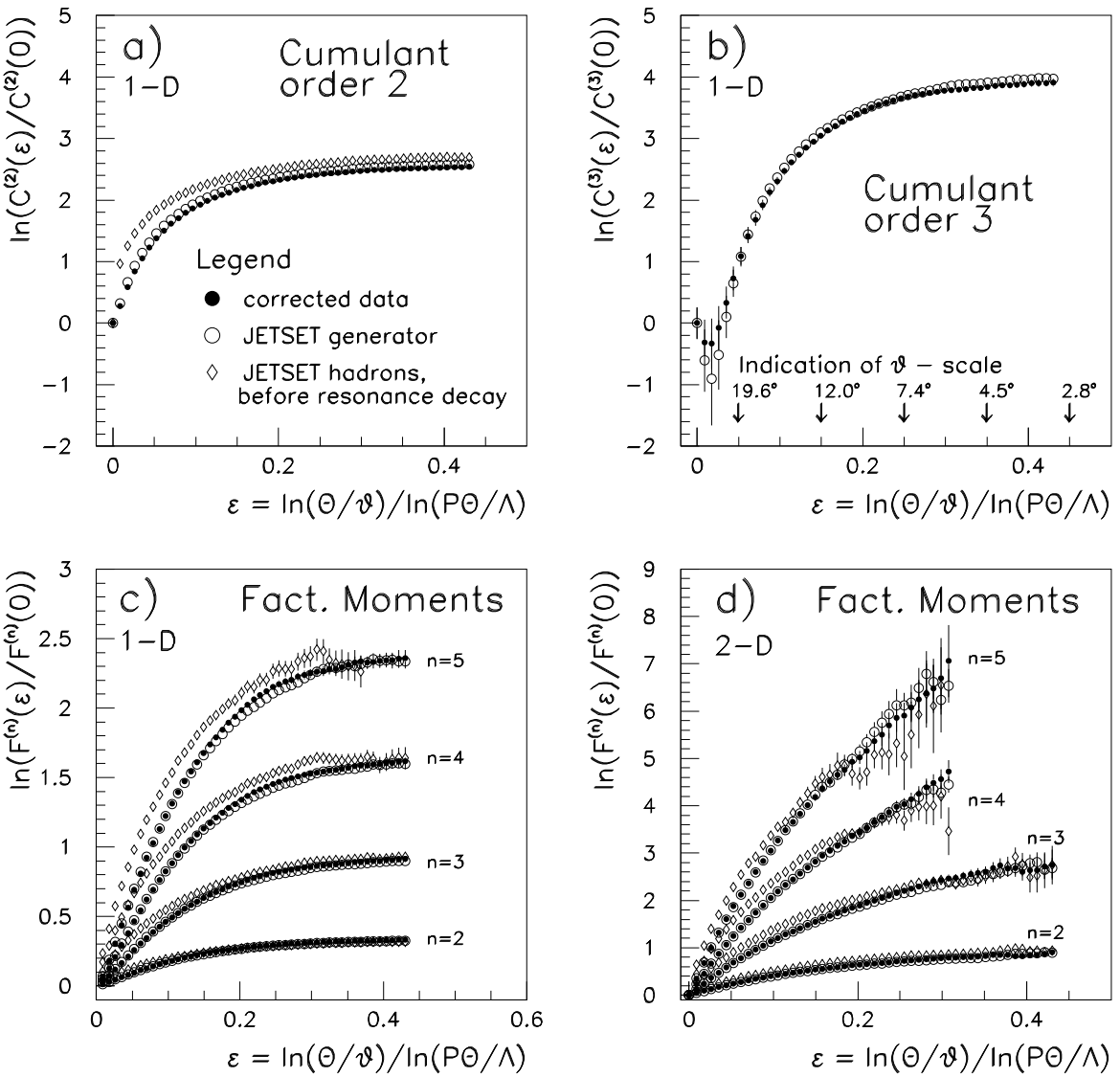

Figure 1: **a)** The measured (•) cumulants of order $n = 2$ in 1-D rings, **b)** those of order $n = 3$, **c)** the factorial moments of orders $n = 2$ to 5 in 1-D rings, and **d)** those in 2-D off-axis cones are compared with JETSET 7.4 before ($\diamond$) and after ($\circ$) resonance decay. The polar angle $\Theta = 25°$. Only charged hadrons have been considered. Because of a negative $C^3(0)$ value in JETSET before resonance decay, no normalization was possible in this case and the corresponding points have therefore been omitted in **b)**. Values of ϑ which correspond to the respective values of ϵ, with $\Lambda = 0.15$ GeV, are also indicated in **b)**.

The cumulants and factorial moments are normalized by $C^{(n)}(0)$ and $F^{(n)}(0)$ for easy comparison of the measured shapes with the analytical predictions. There is generally good agreement between the Monte Carlo simulation (open circles) and the corrected data (full circles). The study of the influence of the resonance decay shown in Fig. 1 reveals significant effects.

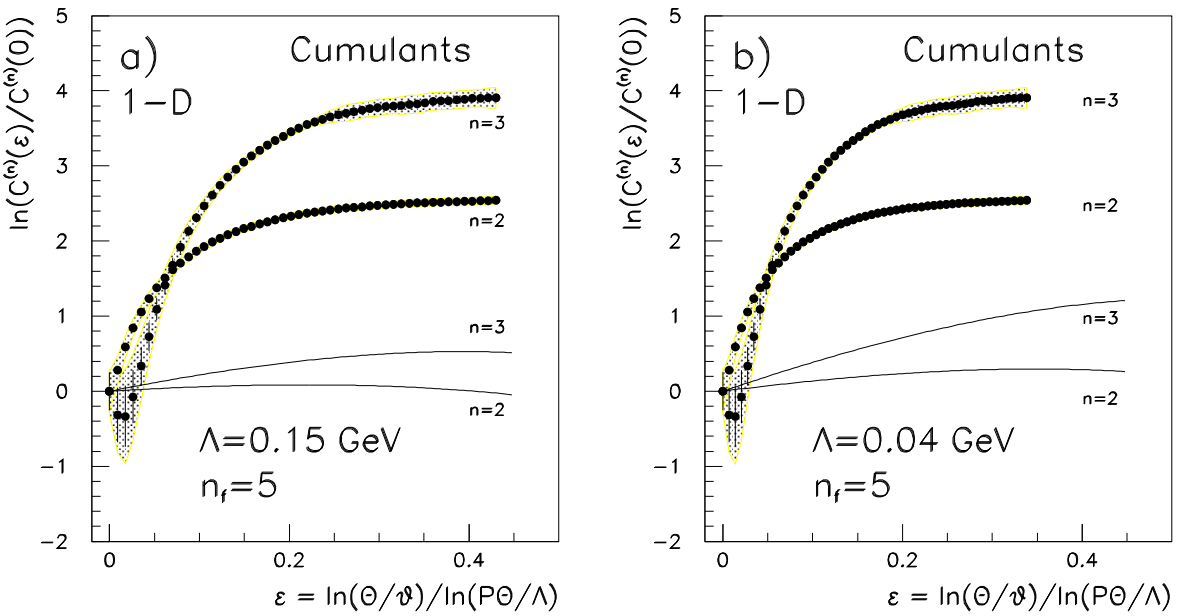

Figure 2: The second and third order cumulants (full circles) are compared with the predictions [4], eq. 8 (solid lines) with $n_f = 5$ for *a)* $\Lambda = 0.15$ GeV, *b)* $\Lambda = 0.04$ GeV. The statistical errors are shown by the error bars, the systematic errors by the shaded regions.

4 Comparison with the analytical calculations

4.1 Quantitative comparison at $\sqrt{s} = 91.1$ GeV

Fig. 2 shows the cumulants of orders $n = 2$ and $n = 3$ in one-dimensional rings around jet cones normalized by $C^{(n)}(0)$ and compared with the predictions of ref. [4].

* A clear disagreement is observed: the predictions lie well below the data and differ in shape (Fig. 2a). Using a lower value of Λ (i.e. $\Lambda = 0.04$ GeV instead of 0.15 GeV) does not help, as can be seen in Fig. 2b (neither does a smaller value of n_f, not shown here).

Fig. 3 shows the factorial moments of orders 2, 3, 4 and 5 normalized by $F^{(n)}(0)$, together with the predictions of refs. [4–6], in one- and two- dimensional angular intervals (i.e. rings and side cones) for various numerical values of Λ and n_f.

* The correlations in one-dimensional rings around jets, expressed by factorial moments, are not described well by the theoretical predictions [4–6] using the QCD parameters $\Lambda = 0.15$ GeV and $n_f = 5$ (Fig. 3a). The predictions lie below the data for not too large ϵ, differing also in shape.
* Choosing $n_f = 3$ (Fig. 3b) instead of $n_f = 5$ as in Fig. 3a reduces the discrepancies.

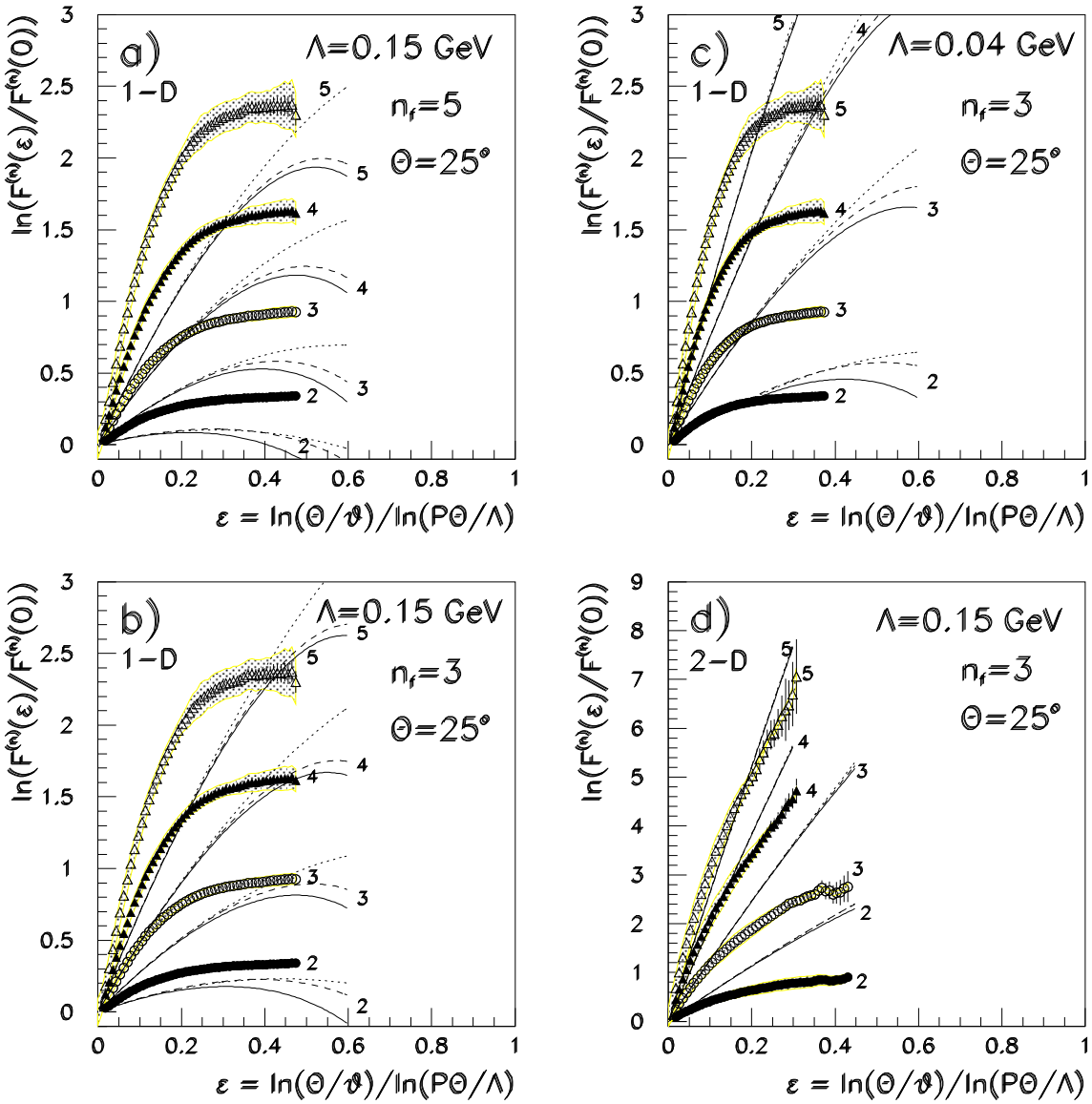

Figure 3: Factorial moments in 1-dimensional rings are compared with the analytical calculations of refs [4–6], eqs. 8 (solid lines), 11 (dashed lines), and 12 (dotted lines). The dependences on n_f and Λ are shown in a), b) and c). As a consistency test, 1- and 2-dimensional factorial moments are compared in b) and d) with same QCD parameters: note the different vertical scales. The orders 2 to 5 are indicated in all figures, the data are also distinguished by different symbols. The statistical errors are shown by the error bars, the shaded regions indicate the systematic errors. The 1-dimensional factorial moments agree very well with those measured by L3 [13].

* Choosing in addition the smaller value of $\Lambda = 0.04$ GeV (Fig. 3c), $F^{(2)}$ is well predicted for smaller values of ϵ, the higher orders ($n > 2$) still deviate considerably.
* The factorial moments in 1 and 2 dimensions show different behaviour for the lower order moments $n < 4$: choosing the same set of parameters ($\Lambda = 0.15$ GeV, $n_f = 3$), $F^{(2)}$ and $F^{(3)}$ lie above the predictions in the 1-dimensional case (Fig. 3b), but below them in the 2-dimensional case (Fig. 3d).
* The higher moments $F^{(4)}$ and $F^{(5)}$ have similar features in the 1- and 2-dimensional case (Figs. 3b,3d).
* In Fig. 3 the slopes at small ϵ are generally steeper than predicted (with the exception of $F^{(2)}$ and $F^{(3)}$ in Fig. 3d) and the "bending" begins at smaller values of ϵ.
* It is not possible to find one set of QCD parameters Λ and n_f which simultaneously minimize the discrepancies between data and predictions for moments of all orders 2,3,4 and 5 in both the 1- and 2-dimensional cases.

4.2 Energy dependence

Fig.4 shows a comparison with high energy data at $\sqrt{s}$=183 GeV (with a mean energy of $\sqrt{s}$=175 GeV) and the corresponding predictions according to eq. 8, where the energy dependence enters via the parameter γ_0. It can be seen that for small values of ϵ there is no improvement of agreement at high energy. For larger values of ϵ the statistical errors of the high energy data are substantial. The relative increase of the predicted moments agrees qualitatively with that of the JETSET model. As shown in Fig. 1, JETSET agrees with the measurement at $\sqrt{s}$=91.1 GeV. Similar conclusions can be found from the predictions based on eqs. 11 and 12.

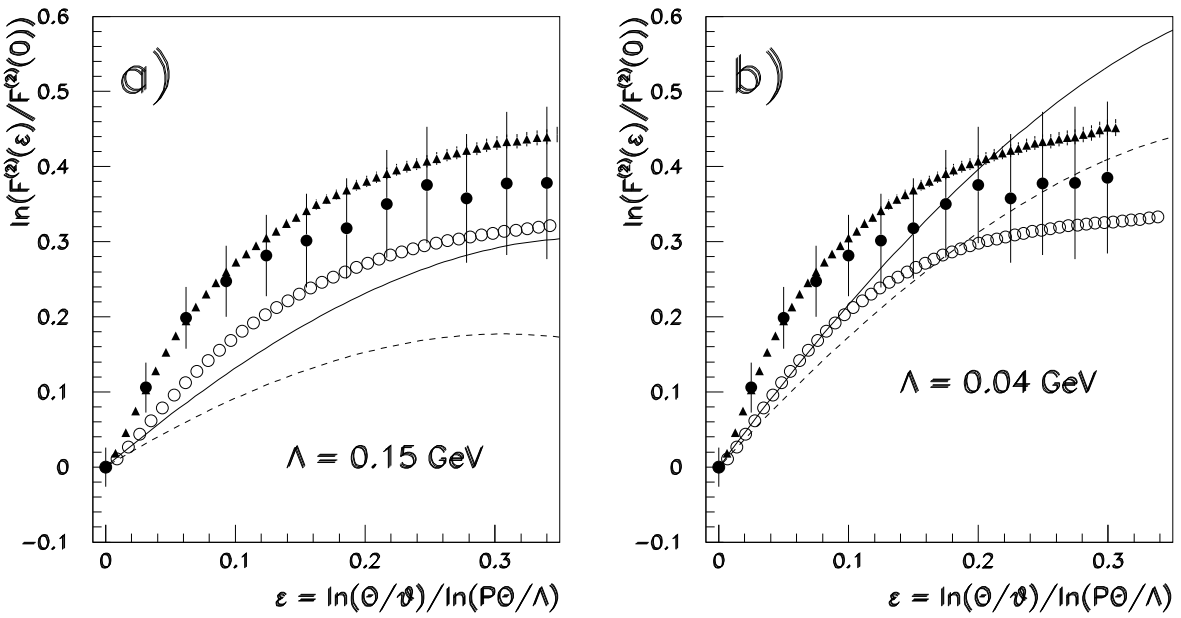

Figure 4: The energy dependence of the normalized factorial moment of order 2. $\sqrt{s}$**=91.1 GeV**: data (open circles) and prediction ref.[4] dashed lines and $\sqrt{s}$**=175 GeV**: data (full circles) and prediction ref.[4] solid lines, for the QCD-parameter combinations **a)** ($n_f = 3$, Λ=0.15) and **b)** (n_f=3, Λ=0.04). The full triangles denote the high energy JETSET simulation.

4.3 Qualitative features

In the introduction, arguments have been given that the DLA might not be accurate enough for a quantitative description of experiments. Some disagreement with the measurement could be expected considering the asymptotic nature of the calculations, but nevertheless an overall qualitative description of the data should be provided. Indeed the data (see Figs. 3,4) show some general qualitative features that are predicted well by the analytical calculations:

* The factorial moments rise linearly at small ϵ exhibiting a fractal structure as predicted in eqs. 4,5 for the parton cascade and saturate at higher values.
* The factorial moments increase from $\sqrt{s}$=91.1 GeV to $\sqrt{s}$=175 GeV.
* The 2-dimensional moments rise much more steeply than the 1-dimensional moments (Figs 3b, 3d).
* The values of ϕ_n obtained by fitting eq. 4 to the data in the region of small ϵ ($\epsilon < 0.1$) follow the predictions eq. 5 qualitatively, as can be seen in Table 1.
* In Fig. 3 it is shown that the analytically calculated factorial moments depend sensitively on Λ. It should be noted that a similar dependence (although weaker because of the Λ-independent fragmentation) is observed in JETSET when varying Λ and keeping all other parameters constant.

4.4 Discussion of the QCD parameter γ_0

The first term in the perturbative formula eq. 5 involves the phase space volume, the second one depends explicitly on the parameter γ_0 (eq. 3), i.e. the QCD coupling α_s. Fig. 5a summarizes the behaviour at small ϵ, where the numerical values of γ_0^{eff} derived from the measured slopes ϕ_n are given for the orders $n = 2, 3, 4, 5$.

Table 1: Comparison of measured and predicted slopes ϕ_n
the errors were obtained by adding statistical and systematic errors quadratically

1-dimensional case	**n=2**	**n=3**	**n=4**	**n=5**
data	0.38 ± 0.006	1.04 ± 0.02	1.87 ± 0.02	2.78 ± 0.03
$\Lambda = 0.15$ GeV, $n_f = 5$	0.15	0.49	0.88	1.28
$\Lambda = 0.04$ GeV, $n_f = 5$	0.25	0.66	1.12	1.59
$\Lambda = 0.15$ GeV, $n_f = 3$	0.22	0.61	1.04	1.49
$\Lambda = 0.04$ GeV, $n_f = 3$	0.30	0.76	1.26	1.77
$\Lambda = 0.005$ GeV, $n_f = 3$	0.40	0.93	1.50	2.07
2-dimensional case	**n=2**	**n=3**	**n=4**	**n=5**
data	0.93 ± 0.02	2.62 ± 0.04	4.77 ± 0.05	7.15 ± 0.06
$\Lambda = 0.15$ GeV, $n_f = 5$	1.15	2.49	3.88	5.28
$\Lambda = 0.04$ GeV, $n_f = 5$	1.25	2.66	4.12	5.59
$\Lambda = 0.15$ GeV, $n_f = 3$	1.22	2.61	4.04	5.49
$\Lambda = 0.04$ GeV, $n_f = 3$	1.30	2.76	4.26	5.77
$\Lambda = 0.005$ GeV, $n_f = 3$	1.40	2.93	4.50	6.07

From the present theoretical understanding, γ_0 is expected to be independent of n. For example, for Λ=0.15 GeV and n_f=3 ($\Theta = 25^o$, $Q \sim P\Theta$) eqs. 13 and 14 give the numerical

value α_s=0.143 and hence from eq. 3 the value γ_0=0.523. This is indicated as horizontal line in Fig. 5, where also the lines for Λ=0.01 GeV and Λ=0.8 GeV are given for comparison. The average measured values of γ_0^{eff} are of the same order as the expectation. The n-dependence observed, however, is not described by the calculations. The measured values of γ_0^{eff} agree, however, extremely well with the corresponding values obtained from JETSET, as can be seen in Fig. 5a.

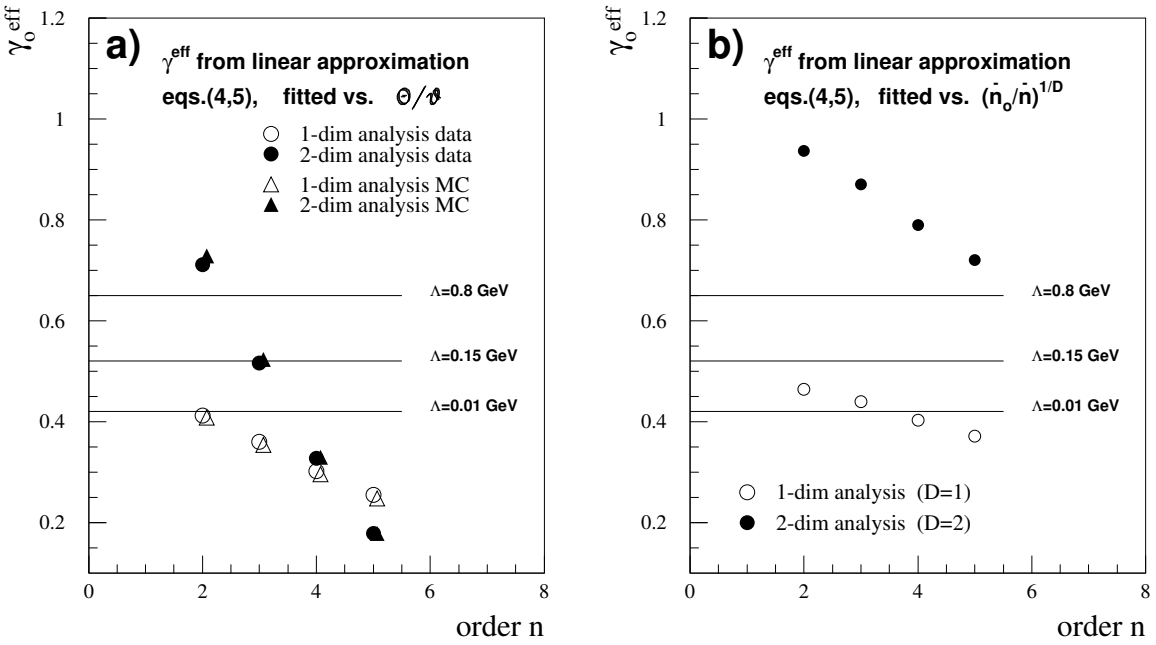

Figure 5: Values of γ_0^{eff} obtained from fitting the linear approximation eq. 5 **a)** vs. $\frac{\Theta}{\vartheta}$ and **b)** vs. $\frac{\bar{n}_0}{\bar{n}}$ (explanation in section 4.5) the 1-dimensional case (open circles) and the 2-dimensional case (full circles), for orders $n = 2,3,4,5$. In **a)**, the measured values of γ_0^{eff} are also compared with those obtained from JETSET at generator level: open triangles (1-dimension) and full triangles (2-dimensions).

4.5 Attempts for improvement

One of the shortcomings of the present calculations is the lack of energy-momentum conservation. There exist two attempts for improvement.

Firstly, in ref. [6], Modified Leading Log Approximation (MLLA) corrections have been calculated for the intermittency exponents ϕ_n. An order dependent correction for γ_0 has been proposed, leading to a correction to γ_0 amounting to only a few percent for all orders $n = 2$ to 5. The deviations observed in Fig. 5 are much larger.

In a second attempt, Meunier and Peschanski [23] introduced energy conservation terms explicitly. This leads, however, to even smaller predicted slopes ϕ_n and consequently larger values of γ_0, increasing the discrepancies shown in Table 1 and Fig. 5. No angular recoil effects were included in these calculations.

Recently Meunier [24] proposed to use, instead of the evolution variable $\epsilon = \ln(\Theta/\vartheta)/\ln(P\Theta/\Lambda)$, the variable $\epsilon = \ln((\bar{n}_0/\bar{n})^{1/D})/\ln(P\Theta/\Lambda)$, where $\bar{n}_o$ and $\bar{n}$ are the mean multiplicities in the first ϵ bin ($\vartheta = \Theta$) and in the $\epsilon(\vartheta)$ bins respectively. Using this new variable, the discrepancies of the 1-dimensional factorial moments observed so far are reduced by almost a factor 2 – see Fig. 5b – and the n-dependence is less strong. The discrepancy between the 1- and 2-dimensional moments, however, is increased (Fig. 5b).

Whether the use of the evolution variable $\frac{\bar{n}_o}{\bar{n}}$ is more suitable than the angular evolution variable Θ/ϑ, which is indicated only in the 1-dimensional case, must still remain open.

Another question concerns the range of validity of the LPHD hypothesis, which can be studied only by using Monte Carlo simulations at both partonic and hadronic levels. Different Monte Carlo models [13] or different choices of the cut-off parameter Q_o at which the parton cascade is "terminated", even in a moderate interval (0.3 - 0.6 GeV), lead to different answers [4,12,25]. In the strict sense LPHD demands a low cut-off scale ($Q_0 \approx$ 0.2-0.3 GeV) [9,26]. In a JETSET study of the partonic state with Λ=0.15 GeV and Q_0=0.33 GeV a steeper rise of the moments than that of the hadron state is observed at small ϵ thus even increasing the discrepancy with the analytical predictions. These studies and the results of [13] indicate that even a possible violation of LPHD might not be the reason for the observed discrepancies.

Fig. 1 also shows that shape distortions due to resonance decay, although significant, are much smaller than the discrepancies between data and theoretical predictions. Similarily a slightly steeper rise of moments is also observed in Monte Carlo studies when replacing the sphericity axis by the "true" $q\bar{q}$ axis and excluding initial heavy flavour production. These effects, however, are smaller than that caused by inhibiting resonance decay (see Fig. 1c,d).

This discussion suggest that the analytical calculations need to be improved beyond the above attempts. Only after improving the perturbative calculations does one have a better handle to estimate how far nonperturbative effects are spoiling the agreement with the data. The importance of including angular recoil effects into the parton cascade, as it is also stressed in [4], is intuitively evident when analysing angular dependent functions.

5 Summary and outlook

Experimental data on multiplicity fluctuations in one- and two- dimensional angular intervals in e^+e^- annihilations into hadrons at $\sqrt{s} = 91.1$ GeV and $\sqrt{s} \sim 175$ GeV collected by the DELPHI detector have been compared with first order analytical calculations of the DLA and MLLA of perturbative QCD. Some general features of the calculations are confirmed by the data: the factorial moments rise approximately linearly for large angles (as expected from the multifractal nature of the parton shower) and level off at smaller angles; the dimensional-, order- and energy dependences are met qualitatively.

At the quantitative level, however, large deviations are observed: the cumulants are far off the predictions; the factorial moments level off with substantially smaller radii; even by reducing the QCD parameters Λ and/or n_f, the analytical calculations are not able to describe simultaneously the factorial moments at all orders $n = 2, 3, 4, 5$ and at different dimensionalities (1- and 2-dimensions). Thus an evaluation of QCD parameters from the data is not possible at present. From Monte Carlo studies there are indications that possible violations of LPHD are not responsible for these discrepancies.

Therefore these shortcomings are probably mainly due to the high energy approximation inherent in the DLA (which is most responsible for the extreme failure of calculations using cumulants). Available MLLA calculations cannot substantially improve on the DLA. To match the data at presently available energies, improvements such as the inclusion of full energy-momentum conservation are needed.

Similar conclusions have been obtained by a parallel one-dimensional study [13]. More checks on refined predictions are desirable in the future.

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