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# Combined results on *b*-hadron production rates, lifetimes, oscillations and semileptonic decays.

**ALEPH, CDF, DELPHI, L3, OPAL, SLD**

**Abstract**

Combined results on $b$-hadrons are obtained from published and preliminary measurements available in Summer 99 from the ALEPH, CDF, DELPHI, L3, OPAL and SLD Collaborations.

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# 1 Introduction.

Accurate determinations of $b$-hadron decay properties provide constraints on the values of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1]. The $|V_{cb}|$ and $|V_{ub}|$ elements can be obtained from semileptonic decay rates into charmed and non-charmed hadrons and measurements of the oscillation frequency in $B^0_d - \overline{B^0_d}$ and $B^0_s - \overline{B^0_s}$ systems give access to $|V_{td}|$ and $|V_{ts}|$.

Elements of the CKM matrix govern weak transitions between quarks whereas experimental results are obtained from processes involving $b$-hadrons. Effects from strong interactions have thus to be controlled and, in this respect, $b$-hadrons are also a good laboratory. Lifetime differences between the different weakly decaying states can be related to interactions between the heavy quark and the light system inside hadrons. The polarization of $\Lambda_b^0$ baryons produced in $b$-jets, emitted from Z decays at LEP or SLC, indicates how polarization is transmitted from the heavy quark to the baryon(s) along the hadronization chain. Rates and decay properties of excited $D^{**}$ states[1], produced in $b$-hadron semileptonic decays, are necessary inputs to obtain accurate determinations of $|V_{cb}|$. Decays of $D^{**}$ states are also an important source of background in other channels and their properties have to be monitored. Finally, it is mandatory to measure, as precisely as possible, production rates of the different weakly decaying $b$-hadrons, produced during the hadronization of $b$-jets created in collisions at high energies, because, all these states having different properties, the study of one of them requires the control of the others.

Results obtained on $b$-hadron lifetimes, $b$-hadron production rates, $B^0_d - \overline{B^0_d}$ and $B^0_s - \overline{B^0_s}$ oscillations, and on the values of the CKM matrix elements, $|V_{cb}|$ and $|V_{ub}|$ made available during Summer 1999, are presented here. A limit on the decay width difference between the mass eigenstates of the $B^0_s - \overline{B^0_s}$ system is also given. These quantities have been obtained by averaging published and preliminary measurements released publicly by the experiments ALEPH, CDF, DELPHI, L3, OPAL and SLD. Whenever possible, the input parameters used in the various analyses have been adjusted to common values and all known correlations have been taken into account. Close contacts have been established, between representatives from the experiments and members of the different working groups in charge of the averages, to ensure that data are prepared in a form suitable for combinations. Working group activities are coordinated by a steering group[2].

In Section 2, the values of the input parameters that contribute to the systematic uncertainties common to all measurements presented in this note are given. Some of these values have been taken from averages obtained by the LEP ElectroWeak Working Group (LEPEWWG) [2, 3] or the PDG [4]. Studies on the production rates of weakly decaying $b$-hadrons, on some characteristics of $D^{**}$ mesons in semileptonic $b$-decays and on the $\Lambda_b^0$ polarization are the results of the present work. As measured quantities can be considered also as necessary inputs in the evaluation of systematic uncertainties in other measurements, an iterative procedure has been adopted to obtain stable results.

Section 3 describes the averaging of the $b$-hadron lifetimes measurements. The com-

[1]The notation $D^{**}$ includes all charm-meson and non-resonant charmed final states which are not simply D or $D^*$ mesons.

[2]The members of the Heavy Flavour Steering Group are: D. Abbaneo, J. Alcaraz, E. Barberio, M. Battaglia, S. Blyth, D. Su, P. Gagnon, R. Hawkings, S. Mele, D. Pallin, M. Paulini, P. Roudeau, O. Schneider, F. Simonetto, A. Stocchi, P. Wells, B. Wicklund and S. Willocq.

bined values are compared with expectations from theory. The inclusive $b$-hadron and $\overline{\mathrm{B^0_d}}$ meson[3] lifetimes are necessary inputs in the determination of $|\mathrm{V}_{ub}|$ and $|\mathrm{V}_{cb}|$ to convert measured branching fractions into partial widths that can be then compared with theory.

In Section 4, oscillations of neutral B mesons are studied. Production rates of the different $b$-hadrons, in a jet induced by a $b$-quark, are also determined using direct measurements together with the constraints provided by B mixing. The $\overline{\mathrm{B^0_d}}$ production rate is another important input for the $|\mathrm{V}_{cb}|$ measurement, obtained from samples of $\overline{\mathrm{B^0_d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}$ decays, and the sensitivity to $\mathrm{B^0_s}$-$\overline{\mathrm{B^0_s}}$ oscillations depends on the assumed rate for strange $b$-meson production.

In Section 5, a limit on the decay width difference between mass eigenstates of the $\mathrm{B^0_s}$-$\overline{\mathrm{B^0_s}}$ system is given. As this average is obtained for the first time, more details on the adopted procedure have been given than for lifetimes or oscillations measurements.

The determination of $|\mathrm{V}_{cb}|$ and $|\mathrm{V}_{ub}|$ presented in Sections 6 and 7 include only LEP results. Such averages are released officially for the first time. The analysis to determine $|\mathrm{V}_{ub}|$ is based on a novel technique using information from the lepton momentum and the mass of the hadronic system produced in these transitions. The accuracy of these results (especially on $|\mathrm{V}_{cb}|$) depends mostly on theoretical uncertainties. In Appendix E, details on the theoretical inputs are given.

A summary of all results obtained by the different working groups is given in Section 8. In addition, Appendix A-D contain, respectively, measurements of the production rates of narrow $\mathrm{D}^{**}$ states, $\Lambda^0_b$ polarization, $b$-hadron lifetimes and direct measurements of $b$-meson and $b$-baryon production rates, used in the present analyses.

# 2 Common input parameters.

Properties of $b$-hadrons, used as inputs in the derivation of the quantities studied in this paper, are given in Table 1. The measurements of quantities that have not been the subject of dedicated studies from the working groups have been taken from results obtained by the LEPEWWG [2, 3] or quoted by the PDG [4]. The others, which concern the production rates and decay properties of $\mathrm{D}^{**}$ mesons in $b$-hadron semileptonic decays and the value of the $\Lambda^0_b$ polarization, are explained.

Note the following:

- $\mathrm{R}_b$, $\mathrm{R}_c$ *and* $< x_E >$

  These results apply only to $b$-hadrons produced in Z decays.

- *the b-quark fragmentation function.*

  The mean fraction of the beam energy taken by a weakly decaying $b$-hadron at LEP or SLC, $< x_E >$, assuming the Peterson function [5], is given in Table 1. To evaluate the corresponding systematic uncertainty, parameter(s) governing $b$-quark fragmentation functions, taken from models, have been varied in accordance with the uncertainty quoted for $< x_E >$. Fragmentation functions taken from two models [6, 7] have been chosen to estimate the systematic uncertainties coming from the shape of the function. These models typically yield results on either side of those obtained using the Peterson function which is commonly used by the experiments.

[3]Throughout the paper charge conjugate states are implicitly included unless stated otherwise.

| Quantity | Symbol | Value | Reference |
|---|---|---|---|
| Fraction of $b$ events | $R_b$ | $0.21643 \pm 0.00073$ | [2] |
| Fraction of $c$ events | $R_c$ | $0.1694 \pm 0.0038$ | [2] |
| Beam energy fraction | $< x_E >$ | $0.702 \pm 0.008$ | [3] |
| $b$-had. sl. BR | $BR(b \to \ell^- \overline{\nu_\ell} X)$ | $0.1058 \pm 0.0018$ | [2] |
| Casc. $b$ sl. decay (r.s.) | $BR(b \to \overline{c} \to \ell^- \overline{\nu_\ell} X)$ | $0.0162 \pm^{0.0044}_{0.0036}$ | [3] |
| Casc. $b$ sl. decay (w.s.) | $BR(b \to c \to \ell^+ \nu_\ell X)$ | $0.0807 \pm 0.0025$ | [2] |
| $c$-had. sl. BR | $BR(c \to \ell^+ \nu_\ell X)$ | $0.0985 \pm 0.0032$ | [2] |
| $b$ quarks from gluons | $P(g \to b\overline{b})$ | $0.00251 \pm 0.00063$ | [2] |
| $c$ quarks from gluons | $P(g \to c\overline{c})$ | $0.0319 \pm 0.0046$ | [2] |
| $b$ dec. charged mult. | $n^b_{ch}$ | $4.955 \pm 0.062$ | [3] |
| $b$-had. mixing | $\overline{\chi}$ | $0.1186 \pm 0.0043$ | [2] |
| $B^0_d$ mixing from $\Upsilon(4S)$ | $\chi_d(\Upsilon(4S))$ | $0.156 \pm 0.024$ | [4] |
| $D^{**}$ in sl. $b$ dec. | $BR(\overline{B^0_d} \to D^{**+} \ell^- \overline{\nu_\ell})$ | $0.0304 \pm 0.0044$ | 2.1 |
| $D^{*+}$ in $D^{**}$ sl. $b$ dec. | $BR(B^- \to D^{*+} \pi^- \ell^- \overline{\nu_\ell})$ | $0.0125 \pm 0.0016$ | 2.3 |
| $D^{*+}$ in $\tau$ sl. $b$ dec. | $BR(\overline{B^0_d} \to D^{*+} \tau^- \overline{\nu_\tau} X)$ | $0.0127 \pm 0.0021$ | 2.3 |
| $D^{*+}$ in dble. charm | $BR(b \to D^{*+} X_{\overline{c}}(\to \ell^- X))$ | $0.008 \pm 0.003$ | 2.3 |
| $\Lambda_b^0$ polarization | $\mathcal{P}(\Lambda_b^0)$ | $-0.45 \pm^{0.19}_{0.17}$ | 2.4 |
| $\tau$ in sl. $b$ decays | $BR(b \to \tau^- \overline{\nu_\tau} X)$ | $0.026 \pm 0.004$ | [4] |
| $J/\psi$ in $b$ decays | $BR(b \to J/\psi X$ | $0.0116 \pm 0.0010$ | [4] |
| $D^0$ branching fraction | $BR(D^0 \to K^- \pi^+)$ | $0.0385 \pm 0.0009$ | [4] |
| $D^+$ branching fraction | $BR(D^+ \to K^- \pi^+ \pi^+)$ | $0.091 \pm 0.006$ | [4] |
| $D_s^+$ branching fraction | $BR(D_s^+ \to \phi \pi^+)$ | $0.036 \pm 0.009$ | [4] |
| $\Lambda_c$ branching fraction | $BR(\Lambda_c \to pK^- \pi^+)$ | $0.050 \pm 0.013$ | [4] |
| $D^{*+}$ branching fraction | $BR(D^{*+} \to D^0 \pi^+)$ | $0.683 \pm 0.014$ | [4] |

Table 1: *Common set of input parameters used for the derivation of the various measurements presented in this paper. The first set of results has been taken from those obtained by the LEPEWWG [2, 3] or quoted by the PDG [4], the second set corresponds to averages obtained in the present report and, for the sake of completeness, in the last set, branching fractions in charm decays needed for the production rates of these particles, are listed as well.*

For analyses which are rather insensitive to the fragmentation uncertainty, it is considered adequate to only use the Peterson model and to inflate the uncertainty on $< x_E >$ to $\pm 0.02$.

- *the inclusive semileptonic branching fraction of b-hadrons.*

  The average LEP value for $BR(b \to \ell^- \overline{\nu_\ell} X) = (10.58 \pm 0.07(\text{stat.}) \pm 0.17(\text{syst.}))\%$ is taken from the global LEPEWWG fit which combines the heavy flavour measurements performed at the Z [2].

  The dominant error on the branching ratio comes from the uncertainty on the semileptonic decay model. This modelling error is reduced from $\pm 0.084 \times 10^{-2}$ to $\pm 0.065 \times 10^{-2}$ when the fit is performed including the forward-backward asymmetry measurements in addition to all heavy flavour measurements performed at the Z. Extensive studies have been done to understand the origin of this reduction.

It comes from including various asymmetry measurements which were obtained using different methods. Asymmetry measurements obtained using leptons depend on the semileptonic decay model while measurements using a lifetime tag combined with jet-charge or D-meson reconstruction do not. To achieve consistency between these measurements, the fitting procedure effectively constrains the size of the error attributed to the semileptonic decay model, reducing the systematic error from this source assigned to the semileptonic branching ratio. If asymmetry measurements are included, a compatible value for the inclusive $b$-hadron semileptonic branching fraction of $(10.62 \pm 0.17)\%$ is obtained. Hence, to be conservative, we have not included the asymmetry measurements in the fit for the purpose of extracting the semileptonic branching ratio. The average value was obtained assuming the average $\mathrm{B}^0 - \overline{\mathrm{B}^0}$ mixing parameter and $\mathrm{R}_b = \Gamma_{bb}/\Gamma_{had}$ given in Table 1.

In the absence of direct measurements of the semileptonic branching fractions for the different B meson states, it has been assumed, when needed in the following analyses, that all $b$-hadron semileptonic widths are equal. This hypothesis is strictly valid for $\mathrm{B}^-$, $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{c}\ell^-\overline{\nu_\ell}$ decays because of isospin invariance originating from the $b \to c$ transition which is $\Delta\mathrm{I} = 0$ (similar considerations apply also to $\mathrm{D}^{0,+} \to s\ell^+\nu_\ell$ decays). Such an equality is not valid in $\mathrm{B}^-$, $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{u}\ell^-\overline{\nu_\ell}$ decays but the induced difference between $\mathrm{B}^-$ and $\overline{\mathrm{B}^0_\mathrm{d}}$ total semileptonic decay rates can be neglected. Its validity has been assumed also for $\overline{\mathrm{B}^0_\mathrm{s}}$ mesons and, for baryons, an uncertainty of 15% has been added, as measured by comparing the lifetime and semileptonic branching fractions ratios for $b$-mesons and $b$-baryons [8]. Exclusive semileptonic branching fractions averages, given in the following, have been obtained using this hypothesis and quoted for the $\overline{\mathrm{B}^0_\mathrm{d}}$ meson. Results for other $b$-hadron flavours can be obtained using, in addition, the corresponding lifetimes ratio. In these averages, $b$-hadron lifetimes values obtained in Section 3 have been used.

- *gluon splitting to heavy quarks.*

  The quantities $\mathrm{P}(g \to c\overline{c})$ and $\mathrm{P}(g \to b\overline{b})$ are equal to the ratios $\frac{\mathrm{BR}(\mathrm{Z}\to q\overline{q}g,\ g\to Q\overline{Q})}{\mathrm{BR}(\mathrm{Z}\to\mathrm{hadrons})}$ in which, respectively, $Q$ is a $c$ or a $b$ quark.

- *b-hadron decay multiplicity.*

  The value given in Table 1 is an average of DELPHI [9] and OPAL [10] measurements which does not include charged decay products of strange particles.

## 2.1 Inclusive $\mathrm{D}^{**}$ production rate in $b$-hadron semileptonic decays.

The inclusive $b$-hadron semileptonic branching fraction into $\mathrm{D}^{**}$ mesons has been measured in three different ways:

- by subtracting the contributions of $\overline{\mathrm{B}^0_\mathrm{d}} \to (\mathrm{D}^+ + \mathrm{D}^{*+})\ell^-\overline{\nu_\ell}$ from the total semileptonic branching fraction of $\overline{\mathrm{B}^0_\mathrm{d}}$ mesons:

$$\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) \quad = \quad (3.93 \pm 0.44)\% \qquad (1)$$

  This value has been obtained using the inclusive semileptonic branching fraction given in Table 1 and published values for the exclusive rates [4].

- from a direct measurement, at the $\Upsilon(4S)$, of final states with a $D^{*+}$ and using models to account for the other channels [11]:

$$\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) \;=\; (2.7 \pm 0.7)\% \tag{2}$$

- from an inclusive measurement of final states in which a D or a $D^{*+}$ meson is accompanied by a charged hadron and assuming that non-strange $D^{**}$ decay channels involve only $D\pi$ and $D^*\pi$ final states:

$$\begin{aligned} \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) &= (2.16 \pm 0.30 \pm 0.30)\% \text{ [12]} \\ &= (3.40 \pm 0.52 \pm 0.32)\% \text{ [13]} \end{aligned} \tag{3}$$

As the $\chi^2$ per degree of freedom of the average of these four measurements is equal to 3, the uncertainty on the average has been increased accordingly[4], giving:

$$\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) \;=\; (3.04 \pm 0.44)\% \tag{4}$$

## 2.2 $D^{**}$ decays involving $D^*$ mesons in semileptonic *b*-decays.

Semileptonic decays to excited charm states which subsequently decay to a $D^{*+}$ are a source of correlated (physics) background in studies of $\overline{\mathrm{B}^0_\mathrm{d}}$ meson properties. It is appropriate to express the different measurements in terms of the parameter $b^{**}$ which is equal to the branching fraction of $\overline{\mathrm{B}^0_\mathrm{d}}$ semileptonic decays involving $D^{**}$ final states in which a $D^*$, charged or neutral, is produced:

$$\mathrm{b}^{**} = \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) \times \mathrm{BR}(\mathrm{D}^{**+} \to \mathrm{D}^*\mathrm{X}) \tag{5}$$

This is to differenciate such $D^{**}$ decays from those in which a D meson, not coming from a $D^*$, is emitted. Throughout this section and as there is, at present, no information on multi-pion final states, it is assumed that decays of $D^{**}$ mesons involve at most one pion (or one kaon for strange states).

In the following, existing measurements have been expressed in terms of the quantity $b^{**}$, production fractions ($f_{\mathrm{B}_i}$) and lifetimes ($\tau(\mathrm{B}_i)$) of the different types of weakly decaying *b*-hadrons. These measurements have then been combined to obtain the value for $b^{**}$:

- semi-inclusive measurements of semileptonic decays in which a $D^{*+}$ and a charged pion have been isolated:

$$\begin{aligned} \mathrm{BR}(b \to \mathrm{D}^{*+}\pi^-\mathrm{X}\ell^-\overline{\nu_\ell}) &= (4.73 \pm 0.77 \pm 0.55) \times 10^{-3} \text{ [12]} \\ &= (4.8 \pm 0.9 \pm 0.5) \times 10^{-3} \text{ [13]} \\ &= f_{\mathrm{B}^+}\frac{2}{3}\mathrm{b}^{**}\frac{\tau(\mathrm{B}^-)}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})} \end{aligned} \tag{6}$$

[4] It has been multiplied by $\sqrt{3}$.

- the already mentioned ARGUS measurement [11]:

$$\begin{aligned} \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) &= (2.7 \pm 0.7)\% \\ &= \frac{\mathrm{b}^{**}}{0.77} \end{aligned} \tag{7}$$

  The value of 0.77 corresponds to the modelling used for $\mathrm{D}^{**}$ decays in that analysis.

- the inclusive production rate of charged $\mathrm{D}^*$ mesons in semileptonic $b$ decays:

$$\begin{aligned} \mathrm{BR}(b \to \mathrm{D}^{*+}\mathrm{X}\ell^-\overline{\nu_\ell}) &= (2.75 \pm 0.17 \pm 0.16)\% \text{ [13]} \\ &= (2.86 \pm 0.18 \pm 0.23)\% \text{ [14]} \\ &= f_{\mathrm{B_d}}\mathrm{b}^* + f_{\mathrm{B_d}}\frac{1}{3}\mathrm{b}^{**} + f_{\mathrm{B}^+}\frac{2}{3}\mathrm{b}^{**}\frac{\tau(\mathrm{B}^-)}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})} \\ &\quad + f_{\mathrm{B_s}}\frac{\alpha}{2}\mathrm{b}^{**}\frac{\tau(\overline{\mathrm{B}^0_\mathrm{s}})}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})} \end{aligned} \tag{8}$$

  The quantity $\mathrm{b}^*$ is the exclusive semileptonic branching fraction:

$$\mathrm{b}^* = \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}) = (4.64 \pm 0.25)\% \tag{9}$$

  which has been obtained by averaging results quoted in PDG [4] for corresponding decays of $\overline{\mathrm{B}^0_\mathrm{d}}$ and $\mathrm{B}^-$ mesons. The scaling factor $\alpha = 0.75 \pm 0.25$ is introduced to account for a possible SU(3) flavour violation, when comparing $\mathrm{D}^{**+}_\mathrm{s} \to \mathrm{D}^{*+}\mathrm{K}^0$ and $\mathrm{D}^{**0} \to \mathrm{D}^{*+}\pi^-$ decays.

These measurements form a consistent set of results and their average amounts to:

$$\mathrm{b}^{**} = \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu_\ell}) \times \mathrm{BR}(\mathrm{D}^{**+} \to \mathrm{D}^*\mathrm{X}) = (1.75 \pm 0.21 \pm 0.08)\% \tag{10}$$

From this result, and using the same hypotheses, it is possible to derive other quantities, of interest in several analyses presented in this paper, as:

$$\mathrm{BR}(\mathrm{B}^- \to \mathrm{D}^{**0}(\to \mathrm{D}^{*+}\pi^-)\ell^-\overline{\nu_\ell}) = \frac{2}{3}\mathrm{b}^{**}\frac{\tau(\mathrm{B}^-)}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})} = (1.25 \pm 0.16)\% \tag{11}$$

$$\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\pi^0\ell^-\overline{\nu_\ell}) = \frac{1}{3}b^{**} = (0.58 \pm 0.08)\% \tag{12}$$

$$\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{s}} \to \mathrm{D}^{*+}\mathrm{K}^0\ell^-\overline{\nu_\ell}) = \frac{1}{2}b^{**}\frac{\tau(\overline{\mathrm{B}^0_\mathrm{s}})}{\tau(\overline{\mathrm{B}^0_\mathrm{d}})}\alpha = (0.61 \pm 0.22)\%. \tag{13}$$

It is also possible to determine the fraction of $\overline{\mathrm{B}^0_\mathrm{d}}$ semileptonic decays which contain a $\mathrm{D}^{*+}$:

$$\frac{\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}) + \mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\pi\ell^-\overline{\nu_\ell})}{\mathrm{BR}(\overline{\mathrm{B}^0_\mathrm{d}} \to \ell^-\overline{\nu_\ell}\mathrm{X})} = 0.49 \pm 0.03 \tag{14}$$

and the fraction of events in which the $\mathrm{D}^{*+}$ comes from a $\mathrm{D}^{**}$ decay, in a $b$-quark jet:

$$\frac{\mathrm{BR}(b \to \mathrm{D}^{*+}\pi\ell^-\overline{\nu_\ell})}{\mathrm{BR}(b \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}) + \mathrm{BR}(b \to \mathrm{D}^{*+}\pi\ell^-\overline{\nu_\ell})} = 0.28 \pm 0.04 \tag{15}$$

Other production mechanisms in which the $D^{*+}$ is accompanied by a lepton have been also considered as they are sources of background for the exclusive channel $\overline{B^0_d} \to D^{*+}\ell^-\overline{\nu_\ell}$.

### 2.2.1 Charged D* production in semileptonic decays of *b*-hadrons involving $\tau$ leptons.

The value of $BR(\overline{B^0_d} \to D^{*+}\tau^-\overline{\nu_\tau}X)$ is obtained from the average measured value of $BR(b \to \tau\overline{\nu_\tau}X)$ taken from PDG [4] and assuming that a $D^{*+}$ is produced in $(50 \pm 10)\%$ of the cases, as measured in semileptonic decays involving light leptons (Equation (14)). The uncertainty on this last number has been increased, as compared to the value obtained in Equation (14), to account for a possible different behaviour of decays when a $\tau$ is produced instead of a light lepton because of differences in the masses involved in the final state.

### 2.2.2 Charged D* production in double charm semileptonic ($b \to \overline{c} \to \ell$) decays.

The double charm production rate in $b$-decays was measured by the ALEPH [15] and CLEO [16] Collaborations, isolating the different contributions of D meson species. From these measurements the inclusive semileptonic branching fraction, involving a wrong sign charmed meson, has been obtained [3]:

$$BR(b \to \overline{c} \to \ell) \;=\; 0.0162^{+0.0044}_{-0.0036}. \tag{16}$$

The product $BR(b \to D^{*+}X_{\overline{c}}Y) \times BR(X_{\overline{c}} \to \ell^-X)$ was then determined; once again it was assumed that a $D^{*+}$ is produced in $(50 \pm 10)\%$ of the cases.

## 2.3 Production of narrow D** states in *b*-hadron semileptonic decays.

A reliable modelling of $b$-hadron semileptonic decays requires detailed rates measurements of the different produced states which can be resonant or non-resonant $D^{(*)}n\pi$ systems[5], each having characteristic decay properties. As an example, there are four orbitally excited states with L=1. They can be distributed in two pairs according to the value of the spin of the light system $j = L + 1/2$ (L=1). States with $j = 3/2$ can have $J^P = 1^+$ and $2^+$. The $1^+$ state decays only through $D^*\pi$ and the $2^+$ through $D\pi$ or $D^*\pi$, always within the hypothesis of decays involving a single pion. Because of parity and angular momentum conservation, for the $2^+$, there is a D wave between the pion and the charmed particle. For the $1^+$, S and D waves are possible but, if one assumes that the heavy quark spin is decoupled, the conservation of $j(= 3/2)$ implies that S waves are not possible. An important D wave component and the fact that masses of these states are not far away from threshold, imply that $j = 3/2$ states are narrow. These states have been observed with a typical width

[5] In the following the study has been limited to $n$=1.

of 20 MeV/c$^2$, in accordance with expectations. On the contrary, $j = 1/2$ states can have $J^P = 0^+$ and $1^+$ and are expected to decay mainly through an S wave and to be broad resonances with typical widths of several hundred MeV/c$^2$. The first experimental evidence for broad $1^+$ states has been obtained by CLEO [17].

At present informations are rather scarce on the composition of this fraction of $b$-hadron semileptonic decays, involving $D^{**}$, which corresponds to 30% of the total rate (Section 2.1). The ALEPH [12] and CLEO [22] Collaborations have reported evidence for the production of narrow resonant states, which can be summarised as follows:

$$\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D}_1 \ell^- \overline{\nu_\ell}) = (0.63 \pm 0.11)\% \quad (17)$$

$$\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D}_2^* \ell^- \overline{\nu_\ell}) = (0.23 \pm 0.09)\% \text{ or } < 0.4\% \text{ at the 95\% CL.} \quad (18)$$

$$\mathrm{R}^{**} = \frac{\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D}_2^* \ell^- \overline{\nu_\ell})}{\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D}_1 \ell^- \overline{\nu_\ell})} = 0.37 \pm 0.16 \text{ or } < 0.6 \text{ at the 95\% CL.} \quad (19)$$

$\mathrm{R}^{**}$ is the ratio between the production rates of $\mathrm{D}_2^*$ and $\mathrm{D}_1$ in $b$-meson semileptonic decays. Original measurements have been given in Appendix A. Absolute rates for the $\mathrm{D}_1$ and $\mathrm{D}_2^*$ mesons have been obtained assuming that the former decays always into $\mathrm{D}^*\pi$ and the latter decays into $\mathrm{D}\pi$ in $(71 \pm 7)\%$ of the cases as measured by ARGUS [23] and CLEO [24] experiments.

It is of interest to note that these results are very different from naive expectations from HQET in which $\mathrm{R}^{**} \simeq 1.6$ [18]. This implies large $1/m_c$ corrections in these models as was noted for instance in [19]. These corrections do not only reduce the expected value for $\mathrm{R}^{**}$, they also enhance the expected production rate of $j = 1/2$ states which can become of similar importance as the $j = 3/2$ rate.

Using Equations (17-18) it can be deduced that narrow $\mathrm{D}^{**}$ states account for less than one third of the total $\mathrm{D}^{**}$ rate. If, in addition, model expectations for the production rates of $j = 1/2$ states are used, it can be concluded that non resonant production of $\mathrm{D}^{(*)}\pi$ states is as important or even dominates over the production of resonant states. As a consequence, models like GISW [20, 21], which contain only resonant charmed states, do not provide a complete description of $b$-hadron semileptonic decays.

## 2.4 $\Lambda_b^0$ polarization.

$b$-quarks, emitted at the Z pole during collisions of unpolarized $e^\pm$ beams, possess, according to the Standard Model, a high longitudinal polarization: $\mathcal{P}_b = -0.94$. Hard gluon emission and quark mass effects are expected to change this polarization by 3% [25]. However, during the hadronization process of the heavy quark, part or all of the initial $b$-quark polarization may be lost by the final weakly decaying $b$-hadron state. The $b$-mesons always cascade down to spin zero pseudoscalar states which do not retain any polarization information. In contrast, $\Lambda_b^0$ baryons are expected to carry the initial $b$-quark polarization since the light quarks are arranged in a spin-0 and isospin-0 singlet according to the constituent quark model. However, $b$-quark fragmentation into $\Sigma_b^{(*)}$ states can lead to a depolarization of the heavy quark [26].

The $\Lambda\ell$ final state is used to select events samples enriched in $\Lambda_b^0$ semileptonic decays and the polarization is measured using the average values of lepton and neutrino energies. Samples of events, enriched in $b$-mesons, which carry no polarization information are used to calibrate the measurements.

Measurements from ALEPH [27], DELPHI [28] and OPAL [29], collected in Appendix B, have been averaged, assuming systematic uncertainties to be completely correlated:

$$\mathcal{P}(\Lambda_b^0) \;=\; -0.45^{+0.17}_{-0.15} \pm 0.08 \tag{20}$$

Uncertainties of present results are dominated by the registered statistics. The following sources of systematic uncertainties, common to all measurements, have been considered:

- $b$-quark fragmentation,
- $\Lambda$ production in semileptonic and inclusive decays of the $\Lambda_c$,
- $\Lambda_c$ polarization,
- theoretical uncertainties in the modelling of polarized $\Lambda_b^0$ semileptonic decays.

# 3 Averages of $b$-hadron lifetimes.

Best estimates for the various $b$-hadron lifetimes, for the ratio of the $B^+$ and $B_d^0$ lifetimes and for the average lifetime of a sample of $b$-hadrons produced in $b$-jets, have been obtained by the B lifetime working group[6]. Details on the procedure used to combine the different measurements can be found in [30].

The measurements, used in the averages presented here are listed in Appendix C. Possible biases can originate from the averaging procedure. They depend on the statistics and time resolution of each measurement and in the way systematic uncertainties have been included. These effects have been studied in detail using simulations and have been measured to be at a level which can be neglected as compared to the statistical accuracy.

## 3.1 Dominant sources of systematic uncertainties.

Dominant sources of systematic uncertainties leading to correlations between measurements are briefly reviewed (more details can be found in [30]). They include the estimation of the background contamination, the evaluation of the $b$-hadron momentum and the decay length reconstruction.

The background contamination can be due either to physics processes leading to the same final state used to tag the signal or to accidental combinations of tracks which simulate the decay of interest. When "physics" background is present, the experimental uncertainties on the branching ratios of the background processes and on the lifetime of the background particles lead to a systematic error which is correlated between different experiments. When the background is combinatorial, the amount and/or the lifetime of the background "particles" is normally extracted from the data using the sidebands of

[6]The members of the B lifetime working group are: J. Alcaraz, L. Di Ciaccio, T. Hessing, I.J. Kroll, H.G. Moser and C. Shepherd-Themistocleous.

mass plots or wrong sign combinations. In this case the related systematic uncertainty is usually not correlated between experiments. But, in the measurement of the $b$-baryon lifetime using $\Lambda\ell$ correlations, the amount of accidental background is obtained from the wrong sign combinations and a correction, common to all analyses, has to be applied to this number to take into account the production asymmetry of accidental $\Lambda\ell$ pairs.

Most of the exclusive B lifetime measurements are based on the reconstruction of B decay length and momentum. In most analyses the B particles are only partially reconstructed and their energies are estimated from the energies of the detected decay products. In all cases, systematic uncertainties have been evaluated for the following effects:

- determination of the $b$-quark fragmentation function,
- branching fractions of B and charmed hadrons,
- $b$-hadron masses,
- $b$-baryon polarization and
- modelling of neutral hadronic energy.

Finally there are uncertainties correlated within an experiment. They are due to primary and secondary vertex reconstruction procedures, detector resolution, tracking errors, B flight direction reconstruction and detector alignment.

## 3.2 Measurements of $\mathbf{B_d^0}$ and $\mathbf{B^+}$ lifetimes.

Apart from CDF which has collected a large sample of exclusive $B_d^0 \to J/\psi K^{(*)0}$ and $B^+ \to J/\psi K^+$ decays, the most precise measurements of $B_d^0$ and $B^+$ lifetimes originate from two classes of partially reconstructed decays (see Figure 3.2). In the first class the decay $B \to \overline{D^{(*)}}\ell^+\nu_\ell X$ is used in which the charge of the charmed meson allows to distinguish between neutral and charged B mesons. In the second class the charge attached to the B decay vertex is used to achieve this separation.

The following sources of correlated systematic uncertainties have been considered: background composition (includes $D^{**}$ branching ratio uncertainties as obtained in Section 2), momentum estimation, lifetime of $B_s^0$ and $b$-baryons (as obtained in Sections 3.3 and 3.4), fractions of $B_s^0$ and $b$-baryons (as measured in Section 4.1) produced in Z decays.

The world average lifetimes of $B_d^0$ and $B^+$ mesons are:

$$\tau(B_d^0) = (1.562 \pm 0.029) \text{ ps} \quad (21)$$

$$\tau(B^+) = (1.656 \pm 0.025) \text{ ps}. \quad (22)$$

## 3.3 $\mathbf{B_s^0}$ lifetime measurements.

The most precise measurements of the $B_s^0$ lifetime originate from partially reconstructed decays in which a $D_s^-$ meson has been completely reconstructed (see Figure 3.3-left).

The following sources of correlated systematic uncertainties have been considered: average $b$-hadron lifetime used in backgrounds, $B_s^0$ decay multiplicity, branching ratios of B and charmed hadrons.

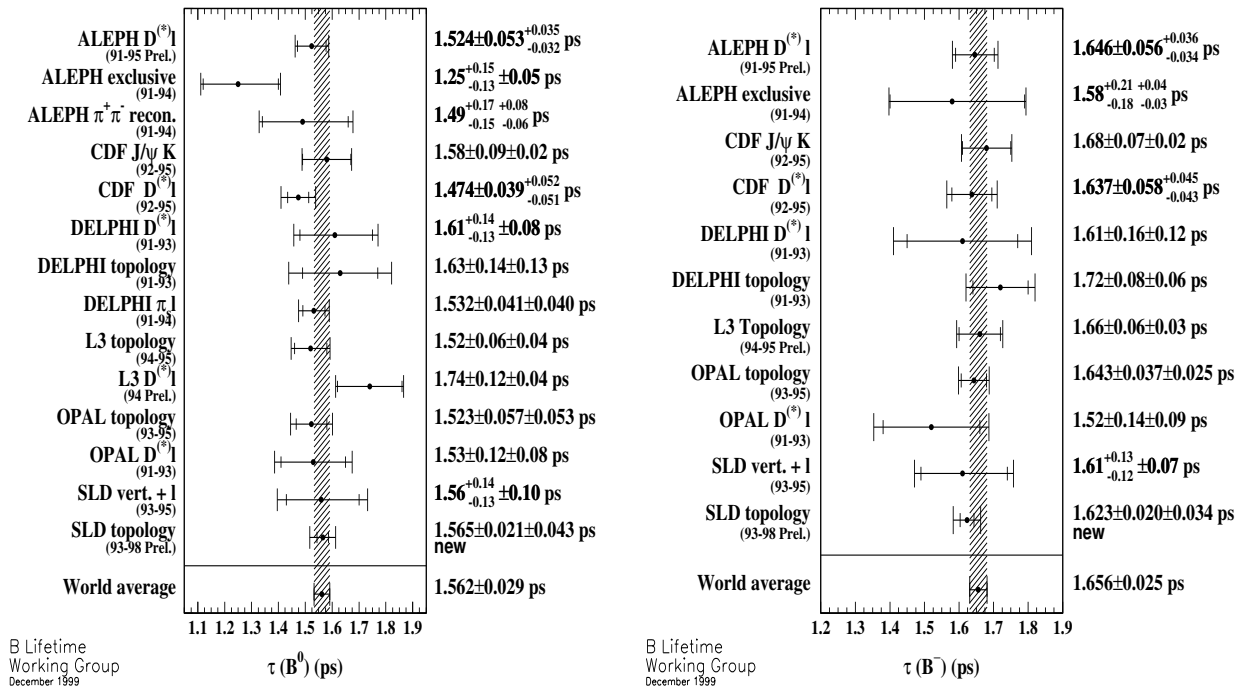


Figure 1: *Left:* $B_d^0$ *lifetime measurements. Right:* $B^+$ *lifetime measurements. Hatched areas correspond to present averages of the measurements given in the corresponding figures. Internal error bars correspond to statistical uncertainties and full error bars include systematics.*

The world average lifetime of $B_s^0$ mesons is equal to:

$$\tau(B_s^0) \;=\; (1.464 \pm 0.057)\ \text{ps} \tag{23}$$

## 3.4 *b*-baryon lifetime measurements.

The most precise measurements of the $b$-baryon lifetime originate from two classes of partially reconstructed decays (see Figure 3.3-right). In the first class, decays with an exclusively reconstructed $\Lambda_c$ baryon and a lepton of opposite charge are used. In the second class, more inclusive final states with a baryon (p, $\overline{\text{p}}$, $\Lambda$ or $\overline{\Lambda}$) and a lepton have been used.

The following sources of correlated systematic uncertainties have been considered: experimental time resolution within a given experiment, $b$-quark fragmentation distribution into weakly decaying $b$-baryons, $\Lambda_b^0$ polarization, decay model and evaluation of the $b$-baryon purity in the selected events samples. In computing the averages, the central values of the masses are scaled to $m(\Lambda_b) = (5624 \pm 9)$ MeV/c$^2$ and m($b$-baryon) $= (5670 \pm 100)$ MeV/c$^2$. Uncertainties related to the decay model include mostly assumptions on the fraction of $n$-body decays. To be conservative it is assumed that they are correlated whenever given as an error. Furthermore, in computing the average, results have been corrected for the effect of the measured value of the $\Lambda_b^0$ polarization and it has

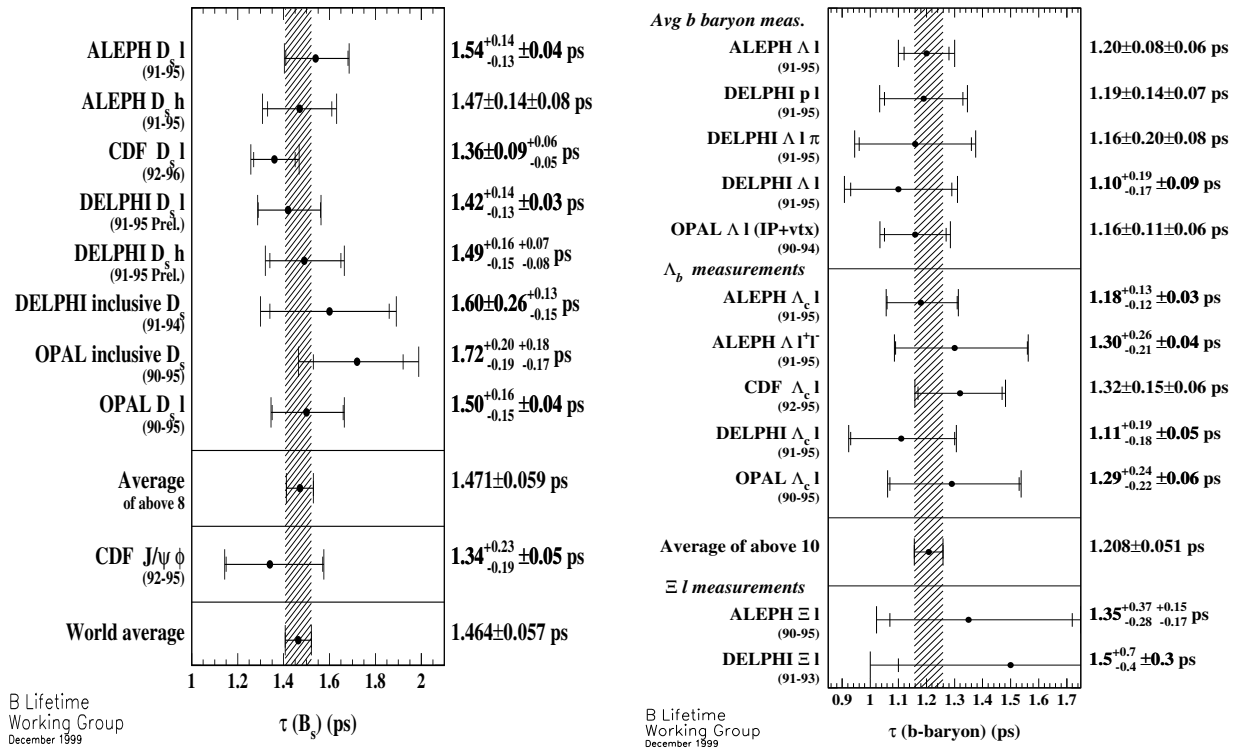


Figure 2: *Left:* $B_s^0$ *lifetime measurements, Right: b-baryon lifetime measurements. Hatched areas correspond to present averages of the measurements given in the corresponding figures. Internal error bars correspond to statistical uncertainties and full error bars include systematics.*

been assumed that $b$-baryons have the same fragmentation distribution as all $b$-hadrons (Section 2).

The world average lifetime of $b$-baryons is then:

$$\tau(b-\text{baryon}) \;=\; (1.208^{+0.051}_{-0.050})\ \text{ps} \tag{24}$$

Keeping only $\Lambda_c^{\pm}\ell^{\mp}$ and $\Lambda\ell^-\ell^+$ final states, as representative of the $\Lambda_b^0$ baryon, the following lifetime is obtained:

$$\tau(\Lambda_b^0) \;=\; (1.229^{+0.081}_{-0.079})\ \text{ps} \tag{25}$$

Averaging the measurements based on the $\Xi^{\mp}\ell^{\mp}$ final states gives a lifetime value for a sample of events containing $\Xi_b^0$ and $\Xi_b^-$ baryons:

$$\tau(\Xi_b) \;=\; (1.39^{+0.34}_{-0.28})\ \text{ps} \tag{26}$$

## 3.5 Average *b*-hadron lifetime.

Available results have been divided into three sets (see Figure 3.5-left):

- measurements at LEP and SLD which accept any $b$-hadron decay:

$$\tau_b^{incl.} \;=\; (1.577 \pm 0.016)\ \text{ps} \tag{27}$$

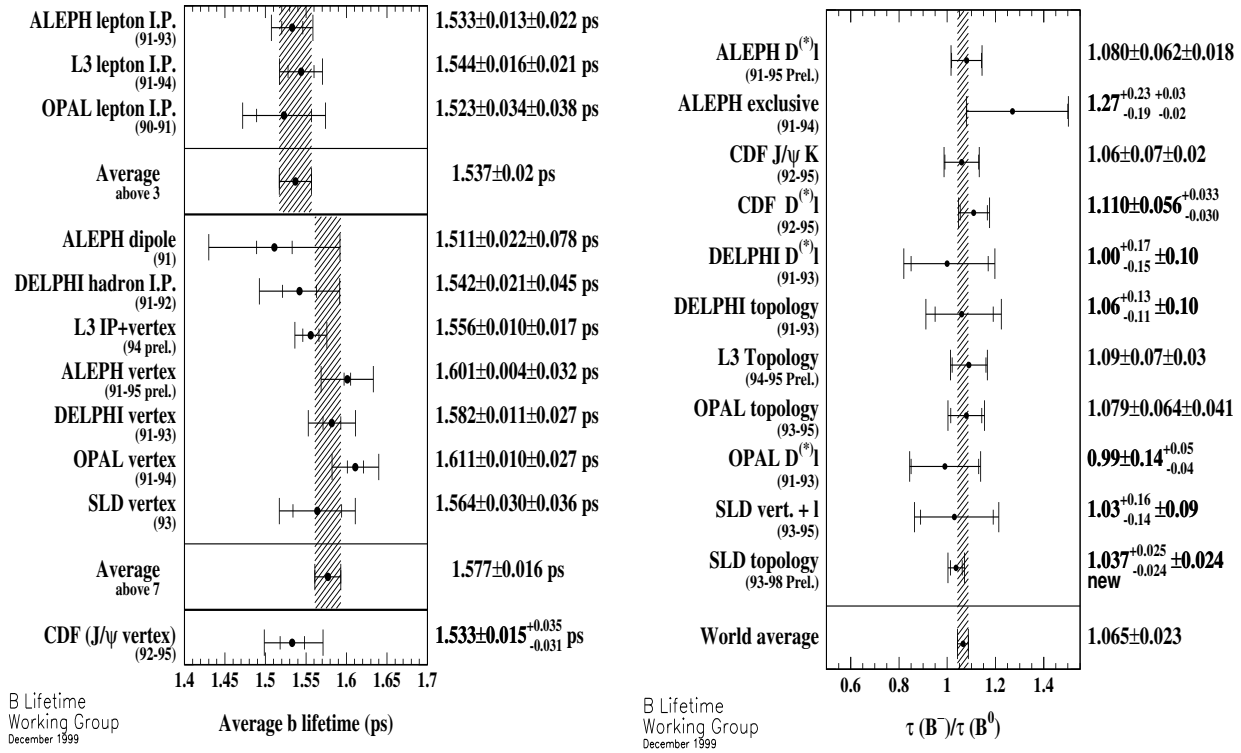


Figure 3: *Left: Measurements of the average b-hadron lifetime. Right: Ratio between the* $\mathrm{B}^-$ *and* $\overline{\mathrm{B}^0_\mathrm{d}}$ *lifetimes. Hatched areas correspond to present averages of the measurements given in the corresponding figures. Internal error bars correspond to statistical uncertainties and full error bars include systematics.*

- measurements at LEP based on the identification of a lepton from a $b$-hadron decay:

$$\tau_b^{sl} \;=\; (1.537 \pm 0.020)\ \mathrm{ps} \tag{28}$$

- measurement at CDF based on the identification of a $\mathrm{J}/\psi$ from a $b$-hadron decay:

$$\tau_b^{\mathrm{J}/\psi} \;=\; (1.533^{+0.038}_{-0.034})\ \mathrm{ps} \tag{29}$$

The reason for this division is that, since the lifetimes of the individual $b$-hadron species are different, then an average lifetime can only be compared for samples in which the composition is the same.

The following sources of correlated systematic uncertainties have been considered when evaluating the averages: $b$- and $c$-quark fragmentation and decay models, $\mathrm{BR}(b \to \ell)$, $\mathrm{BR}(b \to \mathrm{c} \to \overline{\ell})$, $\tau_c$, $\mathrm{BR}(\mathrm{c} \to \overline{\ell})$ and the charged track multiplicity in $b$-hadron decays (see Table 1).

The measurement in set 1 is an estimate of the average $b$-hadron lifetime for a sample of weakly decaying $b$-hadrons produced in Z decays.

$$\tau_b \;=\; f_{\mathrm{B_d}}\tau(\mathrm{B^0_d}) + \mathrm{f_{B^+}}\tau(\mathrm{B^+}) + \mathrm{f_{B_s}}\tau(\mathrm{B^0_s}) + \mathrm{f}_{b\text{-baryon}}\tau(b-\mathrm{baryon}) \tag{30}$$

The measurements of set 2 are based on a sample of hadrons depleted in $b$-baryons and enriched in $\mathrm{B}^+$ with respect to the production fractions, and thus are expected to

correspond to a higher average lifetime. In practice this difference is a second order[7] effect. Assuming that all $b$-hadrons have the same semileptonic partial width (see Section 2), values of the average lifetimes in the inclusive and semileptonic samples can be related using measured values for production rates (Section 4) and lifetimes of the different types of $b$-hadrons, giving:

$$\frac{\tau_b^{sl}}{\tau_b} \;=\; 1.007 \pm 0.005. \tag{31}$$

It can be noticed that, as explained previously, $\tau_b^{sl}$ is expected to be slightly larger than $\tau_b$.

Finally the CDF J/$\psi$ measurement probably corresponds to a different composition in $b$-hadron species compared to any of the previous sets of measurements.

As a consequence, the average $b$-lifetime, $\tau_b$, is obtained by combining the values of $\tau_b^{incl.}$ and $\tau_b^{sl}$, only, and neglecting the small correction given in Equation (31):

$$\tau_b \;=\; (1.564 \pm 0.014)\ \text{ps}. \tag{32}$$

The difference between the values of $\tau_b^{incl.}$ and $\tau_b^{sl}$ amounts to less than two standard deviations and, as the overall $\chi^2$ of all ten corresponding measurements is equal to 8.2, there is no reason to separate the two sets of results.

## 3.6 *b*-hadron lifetime ratios.

Ratios of $b$-hadron lifetimes, relative to the $\mathrm{B^0_d}$ lifetime have also been determined using previous averages of the individual lifetimes and direct measurements of the ratio between $\mathrm{B^+}$ and $\mathrm{B^0_d}$ lifetimes (see Figure 3.5-right).

| Lifetime ratio | Measured value |
|---|---|
| $\tau(\mathrm{B^+})/\tau(\mathrm{B^0_d})$ | $1.065 \pm 0.023$ |
| $\tau(\mathrm{B^0_s})/\tau(\mathrm{B^0_d})$ | $0.937 \pm 0.040$ |
| $\tau(b-\text{baryon})/\tau(\mathrm{B^0_d})$ | $0.773 \pm 0.036$ |

Table 2: *Ratios of b-hadron lifetimes relative to the* $\mathrm{B^0_d}$ *lifetime.*

The original predictions [69] for the meson lifetimes, given in Equation (33), which had encountered theoretical criticism [70], are on the mark. A recent lattice study [71] finds a result quite consistent with the original work which was based on factorization.

$$\frac{\tau(\mathrm{B^+})}{\tau(\mathrm{B^0_d})} \;=\; 1 + 0.05 \times \frac{f_B^2}{(200\ \mathrm{MeV})^2},\ \frac{\tau(\mathrm{B^0_s})}{\tau(\mathrm{B^0_d})} \;=\; 1 \pm \mathcal{O}(1\%),\ \frac{\tau(\Lambda_b^0)}{\tau(\mathrm{B^0_d})} \;=\; 0.90 - 0.95. \tag{33}$$

A large discrepancy is observed for baryons and it remains to be clarified if this problem is related only to the validity of quark models used to determine parameters in case of baryons, as was suggested in original publications [69], or to more basic defects in the whole theoretical picture. Lattice evaluations for $b$-baryons are on the way [72].

[7] When considering that the relative differences between $b$-hadron lifetimes correspond to first order effects.

# 4 $B^0_d$ and $B^0_s$ oscillations and *b*-hadron production fractions.

The four LEP, CDF and SLD Collaborations have published or otherwise released measurements of $\Delta m_d$ [73, 74, 75, 76, 77, 78] and lower limits for $\Delta m_s$ [79, 80, 81, 82, 83]. Combined results as well as estimates for *b*-hadron fractions have been prepared by the B oscillation working group[8].

The estimates of the *b*-fractions, described in Section 4.1, are important inputs needed for the combination of the $B^0_d$ and $B^0_s$ oscillation results. The procedure used to combine $\Delta m_d$ results is explained in Section 4.2, followed by a description of the common systematic uncertainties among the analyses, and the result for the mean value of $\Delta m_d$. Then, combined results on the $B^0_s$ oscillation amplitude, as well as an overall lower limit on $\Delta m_s$ are presented in Section 4.3. More details on these procedures can be found in reference [84].

## 4.1 Measurements of *b*-hadron production rates in *b*-jets.

In this analysis, the production rates (fractions) of the different types of weakly-decaying *b*-hadrons are assumed to be similar in *b*-jets originating from Z decays or being emitted at high transverse momentum in $p-\overline{p}$ collisions at 1.8 TeV. This hypothesis can be justified considering that the last steps of jet hadronization consist in a non-perturbative QCD process occuring at a scale which is of the order of $\Lambda_{\rm QCD}$.

Direct information on production rates is available from measurements of branching ratio products using channels with characteristic signatures. All such direct measurements used in the present analysis are listed in Appendix D.

At CDF and LEP the $\overline{B^0_s}$ production rate has been evaluated using events with a $D^+_s$ accompanied by a lepton of opposite sign in the final state (Table D.1). The rate of these events being given by $f_{B_s} \times BR(B^0_s \to D^-_s \ell^+ \nu_\ell X)$ it is necessary to evaluate $BR(B^0_s \to D^-_s \ell^+ \nu_\ell X)$. This has been done by assuming, from SU(3) symmetry, that the partial semileptonic decay widths into D, $D^*$ and $D^{**}$ final states are the same for all B mesons. A lower value for the $B^0_s$ semileptonic branching fraction, with a $D^-_s$ in the final state, is obtained by assuming that all $\overline{D^{**}_s}$ states decay into a non-strange D meson. The branching fraction corresponding to $B^0_s \to D^{**-}_s \ell^+ \nu_\ell$ is obtained using Equation (1) multiplied by the lifetime ratio $\tau(B^0_s)/\tau(B^0_d)$. This corresponds to a lower limit on $D^-_s$ production because the possibility that $D^{**-}_s$ states decay into $D^-_s \overline{K} K$ or $D^-_s \eta^{(\prime)}$ has been neglected. In addition, assuming that only the measured $D^{(*)}\pi$ final state, in $B^0_d$ or $B^+$ semileptonic decays, corresponds to $D^{(*)}K$ transitions for the $B^0_s$, an upper value can be obtained for the $B^0_s$ decay of interest using the relation:

$$\Gamma(B^0_d \to \overline{D^{(*)0}} \pi^- \ell^+ \nu_\ell) = \frac{4}{3} \Gamma(B^0_s \to \overline{D^{(*)0}} K^- \ell^+ \nu_\ell) \qquad (34)$$

based on isospin symmetry. In practice, lower and upper values are compatible within uncertainties. Using the same assumptions, the CDF Collaboration has measured the

[8]The present members of the B oscillation working group are: V. Andreev, E. Barberio, G. Boix, C. Bourdarios, P. Checchia, O. Hayes, R. Hawkings, M. Jimack, O. Leroy, S. Mele, H-G. Moser, F. Parodi, M . Paulini, P. Privitera, P. Roudeau, O. Schneider, A. Stocchi, T. Usher, C. Weiser, and S. Willocq.

ratio $f_{\rm B_s}/(f_{\rm B^+} + f_{\rm B_d}) = 0.213 \pm 0.068$ from final states with an electron and a charm meson ($\rm D^0$, $\rm D^+$, $\rm D^{*+}$, $\rm D_s^+$) [88][9].

In a similar way, the fraction of $b$-baryons is estimated from the measured production rates of $\Lambda_c\ell$ [91, 92] and $\Xi^{\mp}\ell^{\mp}$ [93, 94] final states (Table D.2) yielding, respectively, $f_{\Lambda_b^0} = (9.8^{+3.7}_{-2.5})\%$ and $f_{\Xi_b^-} = (1.1^{+0.6}_{-0.5})\%$. The value for $\rm BR(\Lambda_b^0 \to \Lambda_c X \ell^- \overline{\nu_\ell})$ has been obtained considering that there could be at maximum one $\Lambda_c$ produced in each decay or, in the other extreme, that all excited baryons, assumed to be produced with a rate similar to $\rm D^{**}$ final states in B mesons, decay into a D meson and a non-charmed baryon. Similar considerations have been applied to $\Xi_b$ semileptonic decays. The semileptonic branching fraction $\rm BR(\Xi_b \to \Xi_c X \ell^- \overline{\nu_\ell})$ has been taken to be equal to $\rm BR(\Lambda_b^0 \to \Lambda_c X \ell^- \overline{\nu_\ell})$ and it has been assumed that, within present uncertainties, $\rm BR(\Xi_c^0 \to \Xi^- X) = BR(\Lambda_c \to \Lambda^0 X)$. The total $b$-baryon production rate is then: $f_{b\text{-baryon}} = (12.0^{+3.9}_{-2.7})\%$. This is then averaged with a direct measurement of $f_{b\text{-baryon}} = (10.2 \pm 2.8)\%$ from the number of protons in $b$-events [95]. As for the $\rm B_s^0$, the CDF Collaboration has measured the production rate of $b$-baryons relative to non-strange B mesons, $f_{b\text{-baryon}}/(f_{\rm B^+} + f_{\rm B_d}) = 0.118 \pm 0.042$, using $\Lambda_c e^-$ final states [88].

The fraction of $\rm B^+$ mesons has also been measured [96] using the charge of a large sample of inclusively reconstructed secondary vertices: $f_{\rm B^+} = (41.4 \pm 1.6)\%$ (Table D.3).

All these measurements have been combined to obtain average production rates for $\rm B_s^0$, $b$-baryons, $\rm B^+$ and $\rm B_d^0$, imposing that:

$$f_{\rm B^+} + f_{\rm B_d} + f_{\rm B_s} + f_{b\text{-baryon}} \;=\; 1 \tag{35}$$

and

$$f_{\rm B^+} = f_{\rm B_d}. \tag{36}$$

This last constraint is expected to be true for $b$-mesons whereas it is known to be wrong for $c$-mesons. In $b$-jets, and also in $c$-jets, similar production rates of mesons, in which the heavy quark is associated to a $\overline{u}$ or $\overline{d}$ antiquark, are expected from isospin invariance of strong interactions. Decays of these states may result in different rates for weakly decaying mesons with a $u$ or a $d$ flavour but, for $\rm B_d^0$ and $\rm B^+$ mesons, no asymmetry is expected. $\rm B^*$ mesons decay electromagnetically leaving the flavour of the light spectator quark in the $b$-meson unchanged. Non-strange $\rm B^{**}$ mesons decaying through strong interactions and having masses away from threshold will not either induce any asymmetry. $\rm B_s^{**0}$ mesons decay also by strong interactions into $\rm B_d^{0(*)}\overline{K^0}$ or $\rm B^{+(*)}K^-$ with equal probabilities. A possible tiny difference between these two rates can be expected for decays of narrow states occurring very close to threshold because of the mass difference between $\rm K^0$ and $\rm K^+$ mesons[10]. The largest part of the observed asymmetry between $\rm D^0$ and $\rm D^+$ production rates in $c$-jets comes from $\rm D^*$ decays which occur, mainly, by strong interactions very close to threshold for $\rm D\pi$ production preventing the transition $\rm D^{*0} \to D^+\pi^-$.

The production rate for $b\overline{s}$ states is expected to be rather similar to the probability ($\gamma_s$) for creating an $s\overline{s}$ pair during the hadronization process ($\gamma_s \sim 12\%$). But, because of their masses, non strange $\rm B^{**}$ mesons are not expected to decay into $\rm B_s^{0(*)}\overline{K}$ whereas strange

[9]CDF has published a second measurement of the same ratio using double semileptonic B decays with $\phi\ell$ and $\rm K^*\ell$ final states as characteristic signatures for $\rm B_s^0$ and non-strange B mesons [89]. At present, this result has not been included in the average for $b$-hadron production fractions because of difficulties in properly correlating certain assumptions in that analysis with the other measurements.

[10]The mass difference between $\rm B_d^0$ and $\rm B^+$ mesons is compatible with zero within $\pm 0.3$ MeV/c$^2$.

$B^{**}$ mesons are decaying mainly into non-strange weakly decaying $b$-mesons. As a result, the fraction of $B_s^0$ in $b$-jets is expected to be given by $f_{B_s} \simeq \gamma_s\ (1 - P(b \to B^{**})) \simeq 10\%$.

Correlated systematics between the different measurements, coming mainly from the poorly measured branching fractions of $D_s^+$ and $\Lambda_c$ charmed hadrons, have been taken into account and results are given in Table 3.

| $b$-hadron fractions | correlation coefficients |
|---|---|
| $f_{B_s} = (9.4 \pm 2.2)\%$ | |
| $f_{b\text{-baryon}} = (10.1 \pm 1.8)\%$ | $\rho(f_{B_s}, f_{b\text{-baryon}}) = -0.31$ |
| $f_{B_d} = f_{B^+} = (40.3 \pm 1.2)\%$ | $\rho(f_{B_d}, f_{B_s}) = -0.68$, $\rho(f_{B_d}, f_{b\text{-baryon}}) = -0.48$ |

Table 3: *Average values of b-hadron production rates, obtained from direct measurements and their correlation coefficients.*

Additional information on the production rates can be obtained from measurements of the time-integrated mixing probability of $b$-hadrons. For an unbiased sample of semileptonic $b$-hadron decays in $b$-jets, with fractions $g_{B_d}$ and $g_{B_s}$ of $B_d^0$ and $B_s^0$ mesons, this mixing probability is equal to:

$$\overline{\chi} = g_{B_s}\ \chi_s + g_{B_d}\ \chi_d \tag{37}$$

where $\chi_d$ is the time-integrated mixing probability for $B_d^0$ mesons (see Section 4.2) and $\chi_s$=1/2 in the corresponding quantity for $B_s^0$ mesons[11]. As mentioned in Section 2 already, the semileptonic width is assumed to be the same for all $b$-hadron species, implying $g_i = f_i\ R_i$, where $R_i = \frac{\tau_i}{\tau_b}$ are the lifetime ratios. This leads to the relation:

$$f_{B_s} \ = \ \frac{1}{R_s}\ \frac{(1+r)\ \overline{\chi} - (1 - f_{b\text{-baryon}}\ R_{b-\text{baryon}})\ \chi_d}{(1+r)\ \chi_s - \chi_d} \tag{38}$$

where $r = R_u/R_d = \tau(B^+)/\tau(B_d^0)$. This is used to extract another determination of $f_{B_s}$ from the $b$-baryon fraction of Table 3, the lifetime ratio averages of Table 2, the world average value of $\chi_d$ from Equation (44) of Section 4.2 and the $\overline{\chi}$ average from the LEPEWWG (see Table 1). This new estimate of $f_{B_s}$, $f_{B_s}^{\text{mixing}} = (10.3 \pm 1.5)\%$, is then combined with the $b$-hadron rates from direct measurements, taking into account correlations[12] and imposing the conditions (35) and (36). The final $b$-hadron fractions are displayed in Table 4.

## 4.2 Combination method for $\Delta m_d$.

The measurements of $\Delta m_d$ are now quite precise and it is important that the correlated systematic uncertainties are correctly handled. Furthermore, many results depend on physics parameters for which different values were used in the original analyses. Before being combined, the measurements of $\Delta m_d$ are therefore adjusted on the basis of a common consistent set of input values.

[11]The assumption $\chi_s = \frac{1}{2}$ can be justified by the existence of limits on $\Delta m_s$ obtained from $D_s$-lepton analyses, which have negligible dependence on $f_{B_s}$.

[12]There is a small statistical correlation between $\chi_d$ and $\overline{\chi}$, arising from the fact that a few $\Delta m_d$ analyses at LEP are based on the same samples of dilepton events as the ones used to extract $\overline{\chi}$, this correlation is ignored, with a negligible effect on the final result.

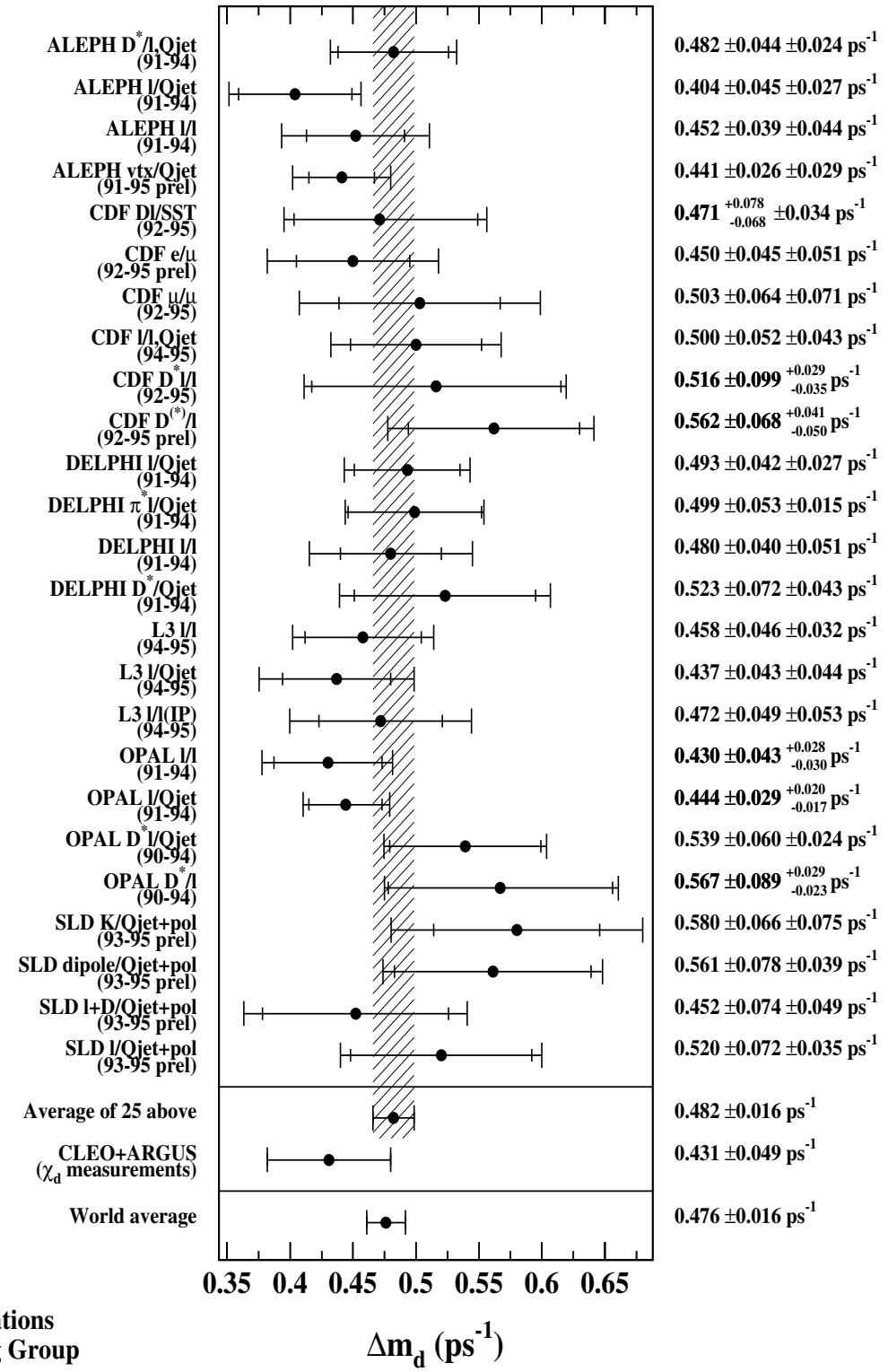


Figure 4: *Individual and combined measurements of $\Delta m_d$ at LEP, CDF and SLD. Note that the individual measurements are as quoted in the original publications, but their average includes the effects of adjustments to a common set of input parameters. The world average, also includes $\chi_d$ measurements [4] performed at the $\Upsilon(4S)$ resonance.*

| $b$-hadron fractions | correlation coefficients |
|---|---|
| $f_{\mathrm{B_s}} = (10.0 \pm 1.2)\%$ | |
| $f_{b\text{-baryon}} = (9.9 \pm 1.7)\%$ | $\rho(f_{\mathrm{B_s}}, f_{b\text{-baryon}}) = -0.03$ |
| $f_{\mathrm{B_d}} = f_{\mathrm{B^+}} = (40.1 \pm 1.0)\%$ | $\rho(f_{\mathrm{B_d}}, f_{\mathrm{B_s}}) = -0.57$, $\rho(f_{\mathrm{B_d}}, f_{b\text{-baryon}}) = -0.81$ |

Table 4: *Average values of b-hadron production rates and their correlation coefficients obtained from direct measurements and including constraints from results on B mixing.*

For each input parameter the $\Delta m_d$ measurement and the corresponding systematic uncertainty are linearly rescaled in accordance with the difference between the originally used parameter value and error, and the new common values. This is done for the systematic uncertainties originating from the $b$-hadron lifetimes, the $b$-hadron fractions and the mixing parameters $\overline{\chi}$ and $\chi_d$. The statistical and systematic uncertainties of each individual measurement are symmetrized. An alternative approach using asymmetric uncertainties (when quoted as such) would produce negligible differences in the combined result.

The combination procedure makes a common fit of $\Delta m_d$ and of the parameters related to the sources of the common systematic uncertainties. It is assumed that $\Delta m_d$ may be expressed as a function $X(Y_1 \ldots Y_{N_{sys}})$ with a weak dependence on the systematic sources $Y_\alpha$. Expanding $X$ then gives:

$$X(Y_1 \ldots Y_{N_{sys}}) \approx X^0 + \sum_{\alpha=1}^{N_{sys}} Y_\alpha \sigma_\alpha^{syst} \tag{39}$$

where the quantities $\sigma_\alpha^{syst}$ are the correlated errors on $\Delta m_d$ from systematic sources $\alpha$ and $Y_\alpha = (z_\alpha - z_\alpha^{fit})/\delta_\alpha$. In this last expression, $z_\alpha$ and $z_\alpha^{fit}$ are the input and fitted values of the systematic parameter $\alpha$ and $\delta_\alpha$ is the variation used to calculate $\sigma_\alpha^{syst}$.

The following $\chi^2$ is then constructed:

$$\chi^2 = \sum_{i=1}^{N} \left( \frac{X_i - \sum_{\alpha=1}^{N_{sys}} Y_\alpha \sigma_{i\alpha}^{syst} - X^0}{\sigma_i^{uncor}} \right)^2 + \sum_{\alpha=1}^{N_{sys}} Y_\alpha^2, \tag{40}$$

where $X_i$ are the measurements of $\Delta m_d$ and $\sigma_i^{uncor}$ the quadratic sum of the statistical and uncorrelated systematic uncertainties on $X_i$. This $\chi^2$ is minimized with respect to the parameters $X^0$ and $Y_\alpha$; the result for $\Delta m_d$ is taken as the value of $X^0$ at the minimum (and the values of $Y_\alpha$ at the minimum are ignored). This method gives the same results as a $\chi^2$ minimization with inversion of a global correlation matrix. As several measurements also have a statistical correlation, the first sum in Equation (40) is then generalized to handle a N×N error matrix describing the statistical and uncorrelated systematic uncertainties.

The following sources of systematic uncertainties, common to analyses from different experiments, have been considered:

- $b$-lifetime measurements (Section 3). Different measurements use the $b$-hadron lifetimes in different ways as input, i.e. some use the actual lifetimes, and others use ratios of lifetimes. As this leads to complicated correlations between the analyses, the following procedure is adopted. For each measurement a single combined value of all $b$-lifetime related systematic uncertainties is computed. This value is

then treated as fully correlated with all other such numbers from the other measurements. Tests show that this procedure gives a negligible bias and results in a conservative evaluation of the uncertainty.

- $b$-quark fragmentation (Section 2). In some analyses the $b$-hadron momentum dependence is treated through the variation of the $\epsilon$ parameter in the Peterson fragmentation function, and in others through the mean scaled $b$-hadron energy $< x_E >$. In such a case the results are treated as fully correlated without explicit dependence on $\epsilon$ or $< x_E >$.

- fraction of cascade decays (Section 2), related mainly to the value of BR($b \to c \to \ell^+ \nu_\ell X$).

- fraction of $\mathrm{B}^+$ mesons remaining in $\mathrm{B}^0_\mathrm{d}$ enriched samples for analyses using $\mathrm{D}^*$ mesons. It depends on decay branching fractions and on the mistag probability.

- production rate of $\mathrm{D}^{**}$ mesons in $b$-hadron semileptonic decays (Section 2).

- $b$-hadron fractions (Section 4.1).

- $\mathrm{D}^0$ lifetime ([4]).

Common systematics, due to purely experiment-dependent factors (i.e. common to different results in a particular experiment), have not been included in the above list, but nonetheless are treated as correlated in the fit.

Following this procedure, a combined value:

$$\Delta m_d^{\mathrm{LEP+CDF+SLD}} \;=\; (0.482 \pm 0.016)\ \mathrm{ps}^{-1} \tag{41}$$

is obtained.

Using the relation[13]:

$$\chi_d = \frac{1}{2}\ \frac{(\Delta m_d\ \tau_{B_d})^2}{(\Delta m_d\ \tau_{B_d})^2 + 1} \tag{42}$$

and the $\mathrm{B}^0_\mathrm{d}$ lifetime of Section 3, the above $\Delta m_d$ result can be converted to:

$$\chi_d^{\mathrm{LEP+CDF+SLD}} \;=\; 0.180 \pm 0.009, \tag{43}$$

Averaging with the time integrated mixing results obtained by ARGUS and CLEO at the Upsilon(4S) ( see Table 1), yields finally:

$$\chi_d^{\mathrm{LEP+CDF+SLD+\Upsilon(4S)}} \;=\; 0.177 \pm 0.008, \tag{44}$$

or equivalently

$$\Delta m_d^{\mathrm{LEP+CDF+SLD+\Upsilon(4S)}} \;=\; (0.476 \pm 0.016)\ \mathrm{ps}^{-1} \;=\; (3.13 \pm 0.11)\ 10^{-4}\ \mathrm{eV}. \tag{45}$$

The individual measurements of $\Delta m_d$ and their combined value are shown in Figure 4.

The determination of $\Delta m_d$ as described above and of the $b$-hadron fractions described in Section 4.1 cannot be performed sequentially: the values of the fractions are needed to perform the $\Delta m_d$ fit, and the best estimates of the fractions can be obtained only once the final $\Delta m_d$ average is known. This circular dependence has been handled by including the calculation of the fractions in the $\Delta m_d$ fitting procedure, in such a way that the final results quoted for $\Delta m_d$ and the $b$-hadron fractions form a consistent set.

[13]Equation (42) assumes that there is no decay width difference in the $\mathrm{B}^0_\mathrm{d} - \overline{\mathrm{B}^0_\mathrm{d}}$ system.

## 4.3 Combination method for $B_s^0$ oscillation amplitudes and derived limits on $\Delta m_s$.

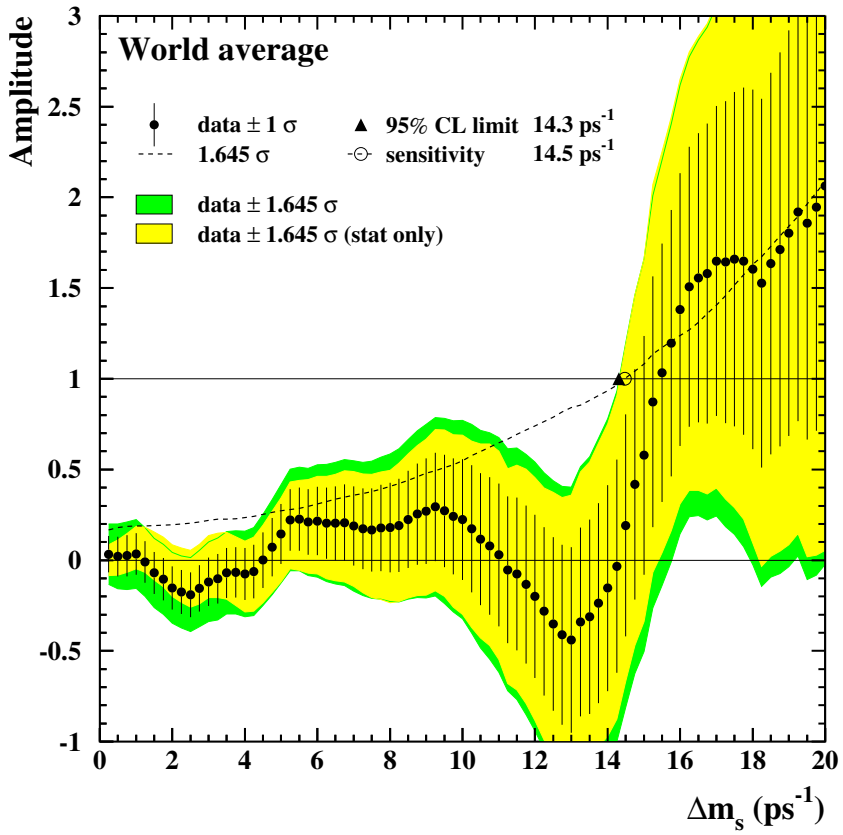


Figure 5: *Combined* $B_s^0$ *oscillation amplitude as a function of* $\Delta m_s$*. A 95% CL lower limit on* $\Delta m_s$ *of 14.3* $\mathrm{ps}^{-1}$ *is derived from this spectrum. Points with error bars are the fitted amplitude values and corresponding uncertainties, including systematics. The dark shaded area is obtained by multiplying these uncertainties by 1.645 such that the integral of the probability distribution, assumed to be Gaussian, is equal to 5% above this range. The clear shaded area is obtained in a similar way but including only statistical uncertainties in the fit; as a consequence, central values of the two regions do not necessarily coincide.*

No experiment has yet directly observed $B_s^0$ oscillations, so the task of a combination procedure is to calculate an overall limit or to quantify the evidence for a signal from the information provided by each measurement. This is done using the amplitude method of reference [97]. At each value of $\Delta m_s$, in the range of interest, an amplitude is measured in each analysis, where the expected value of the amplitude is unity at the true frequency. An overall limit on $\Delta m_s$ is then inferred from the combined amplitude spectrum by excluding regions of $\Delta m_s$ where the amplitude is incompatible with unity.

Studies with Monte Carlo simulation show that the measured amplitude has a Gaussian distribution around its expected value, which is zero for frequencies much lower than the true one, and unity at the true frequency. In addition, the error on the amplitude

depends on the statistical power of the analysis but not on the true value of the oscillation frequency. Therefore, if at a given frequency ($\Delta m_s$) a measured amplitude ($\mu$) with error $\sigma$ is found, this value of the frequency can be excluded at the 95% C.L. if the probability of measuring an amplitude equal or smaller than $\mu$, for a true amplitude of unity, is smaller than 5%, that is $\int_{-\infty}^{\mu} G(A-1,\sigma)dA < 0.05$, or equivalently $\mu + 1.645\sigma < 1$ where $G$ is the Gaussian function.

Before combining the measured amplitudes, the individual central values and systematic uncertainties are adjusted, using the same procedure as for $\Delta m_d$ measurements, to common values of the $b$-hadron fractions (Table 4), $b$-hadron lifetimes (Section 3), and $\Delta m_d$ (Equation (45). The adjustment to a common value of $f_{\mathrm{B_s}}$ is performed first and needs a special treatment for certain analyses, as explained below. Although the final value of $f_{\mathrm{B_s}}$ is calculated under the assumption that $\mathrm{B_s^0}$ mixing is maximal, studies indicate that the effect on the combined $\Delta m_s$ result is negligible.

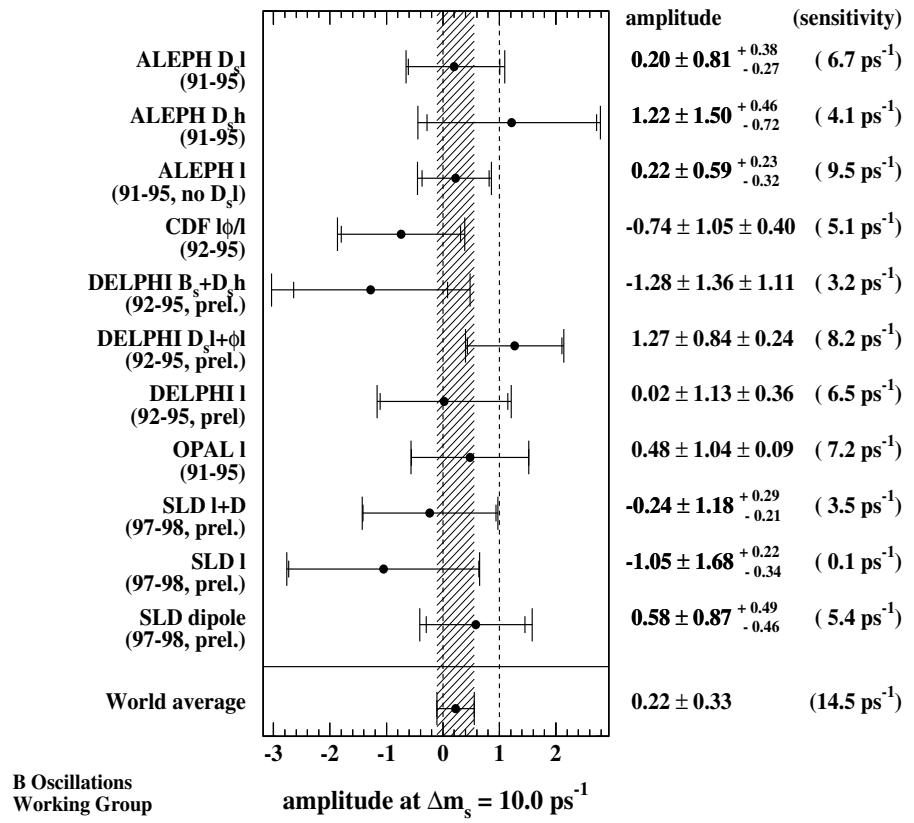


Figure 6: *Measurements of the* $\mathrm{B_s^0}$ *oscillation amplitude. The amplitudes are given at* $\Delta m_s = 10$ ps$^{-1}$*, along with the relevant statistical and systematic errors. The sensitivities are given within parentheses in the column on the right. Note that the individual measurements are as quoted in the original publications, but the average, corresponding to the hatched area, includes the effects of adjustments to a common set of input parameters. Dashed lines, corresponding to amplitude values of 0 and 1 have been drawn.*

The statistical uncertainty on the measured amplitude is expected to be inversely

proportional to the $B_s^0$ purity of the analysed sample [97]. This purity is of the order of $f_{B_s}$ for inclusive analyses. For analyses where a full or partial $B_s^0$ reconstruction is performed, the purity is much less dependent on the assumed value of $f_{B_s}$. Therefore, the statistical uncertainties on the amplitudes measured in inclusive analyses have been multiplied by $f_{B_s}^{used}/f_{B_s}^{new}$, where $f_{B_s}^{used}$ is the $B_s^0$ fraction assumed in the corresponding analysis, and $f_{B_s}^{new}$ is the $B_s^0$ fraction from Table 4. This correction is also applied on the central values of the amplitude measured in inclusive analyses, after having checked that the relative uncertainty on the amplitude is essentially independent of $f_{B_s}$ for large enough values of $\Delta m_s$.

The amplitudes (measured at each value of $\Delta m_s$) are averaged using the same procedure as for the combination of $\Delta m_d$ results. In addition to systematics which are correlated within the individual experiments, the following sources of correlated systematic uncertainties have been taken into account:

- $b$-hadron lifetimes measurements,
- $b$-quark fragmentation,
- direct and cascade semileptonic branching fractions of $b$-hadrons,
- $b$-hadron fractions,
- $\Delta m_d$ measurements,
- $\Delta\Gamma_{B_s^0}$ measurements.

Data using the amplitude method have been provided by ALEPH, DELPHI, OPAL (for their lepton-jet analysis only[14]), CDF and SLD. The combined amplitude spectrum is shown in Figure 5. All values of $\Delta m_s$ below 14.3 ps$^{-1}$ are excluded at the 95% CL. The combined expected sensitivity to $\Delta m_s$ defined as the largest value of $\Delta m_s$ that would have been excluded if the combined amplitudes were zero at all values of $\Delta m_s$, is 14.5 ps$^{-1}$.

In Figure 6 the values of the amplitude at $\Delta m_s = 10$ ps$^{-1}$ from the various analyses, are shown. In addition, the sensitivities are given.

Positive measured amplitudes are found for frequency values near and above the sensitivity limit. Some values deviate from zero by more than twice the total estimated error. The likelihood profile obtained from the amplitude spectrum, using the prescription of [97], shows a minimum at 16.3 ps$^{-1}$. The likelihood value at this minimum is 1.9 units below the asymptotic likelihood value for $\Delta m_s \to \infty$. Because the measurements at different frequencies are correlated, it is not possible to calculate analytically the probability that, in the explored frequency range, a fluctuation as or more unlikely occurs in a sample where the true frequency is far beyond the sensitivity. Therefore, an estimation procedure based on fast Monte Carlo experiments has been developed [98]: the above probability is found to be $(10 \pm 2)\%$, where the quoted uncertainty is due to the assumptions and approximations adopted in the fast Monte Carlo generation.

[14]The OPAL Collaboration has other results on searches for $B_s^0$ oscillations which are not included in this combination (see reference [82] for details).

# 5 Limit on the decay width difference for mass eigenstates in the $B_s^0$-$\overline{B_s^0}$ system.

The CKM picture of weak charge-changing transitions predicts the $B_s^0$ and its charge conjugate to mix. This results in new states $B_s^{heavy}$ and $B_s^{light}$ with masses $m_{B_s^0}^{heavy}$ and $m_{B_s^0}^{light}$ and (probably) different widths $\Gamma_{B_s^0}^{heavy}$ and $\Gamma_{B_s^0}^{light}$.

Neglecting CP violation, the mass eigenstates are also CP eigenstates, the $B_s^{light}$ being CP even and $B_s^{heavy}$ being CP odd. The decay of a $\overline{B_s^0}$ meson via the quark subprocess $b(\overline{s}) \to c\overline{c}s(\overline{s})$ gives rise to predominantly CP-even eigenstates, thus the CP-even eigenstate should have the greater decay rate and hence shorter lifetime. For convenience of notation we therefore substitute $\Gamma_{B_s^0}^{light} \equiv \Gamma_{B_s^0}^{short}$, $\Gamma_{B_s^0}^{heavy} \equiv \Gamma_{B_s^0}^{long}$ and define $\Gamma_{B_s^0} = 1/\tau_{B_s^0} = (\Gamma_{B_s^0}^{long} + \Gamma_{B_s^0}^{short})/2$ and $\Delta\Gamma_{B_s^0} = \Gamma_{B_s^0}^{short} - \Gamma_{B_s^0}^{long}$, in the following.

Theoretical calculations [99] of the ratio $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ at next-to-leading order give:

$$\frac{\Delta\Gamma_{B_s^0}}{\Gamma_{B_s^0}} = \left(\frac{\mathcal{F}_{B_s^0}}{210\ \text{MeV}}\right)^2 [0.006\mathcal{B}_b(m_b) + 0.150\mathcal{B}_s(m_b) - 0.063] \tag{46}$$

where $\mathcal{F}_{B_s^0}$ is the $B_s^0$ decay constant and where $\mathcal{B}_b(m_b)$ and $\mathcal{B}_s(m_b)$ are bag parameters. Using values of $\mathcal{B}_b(m_b) = 0.80 \pm 0.15$, $\mathcal{B}_s(m_b) = 1.19 \pm 0.02 \pm 0.20$ and $\mathcal{F}_{B_s^0} = (245 \pm 30)$ MeV, from recent lattice calculations [100] yields $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0} = 0.16 \pm 0.03 \pm 0.04$, where the uncertainties are from $\mathcal{F}_{B_s^0}$ and $\mathcal{B}_s$ respectively. Care should be taken here as the values for the bag constants are preliminary and are correctly normalised at one loop only so there may be some additional systematic uncertainty to be included.

The width difference and the mass difference are correlated ($\Delta\Gamma_{B_s^0}/\Delta m_s = \frac{3}{2}\pi\frac{m_b^2}{m_t^2}$ to first approximation) thus offering the possibility of measuring $\Delta m_s$ via the lifetime difference rather than the oscillation frequency. This could be particularly important if the oscillation frequency is too fast to be measured with the present experimental proper time resolution. In addition, if $\Delta\Gamma_{B_s^0}$ does turn out to be sizable, the observation of CP violation and the measurement of CKM phases from untagged $B_s^0$ samples can be imagined [101].

The existing experimental constraints on the width difference and the combination of these constraints are reported here[15].

## 5.1 Experimental constraints on $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$.

Experimental information on $\Delta\Gamma_{B_s^0}$ can be extracted by studying the proper time distribution of data samples enriched in $B_s^0$ mesons. An alternative method based on measuring the branching ratio $B_s \to D_s^{(*)+}D_s^{(*)-}$ has also recently been proposed [102]. The available results are summarised in Table 5.

Methods based on double exponential lifetime fits to samples containing a mixture of CP eigenstates have a quadratic sensitivity to $\Delta\Gamma_{B_s^0}$ (inclusive, semileptonic, $D_s^+$-hadron ), whereas methods based on isolating a single CP eigenstate have a linear dependence on $\Delta\Gamma_{B_s^0}$ ($\phi\phi$, J/$\psi\phi$). The latter are therefore, in principle, more sensitive to $\Delta\Gamma_{B_s^0}$ but tend to suffer from reduced statistics.

[15] Members of the $\Delta\Gamma_{B_s^0}$ working group are: P. Coyle, D. Lucchesi, S. Mele, F. Parodi and P. Spagnolo.

| Experiment | Selection | Measurement | $\Delta\Gamma_{\mathrm{B}^0_s}/\Gamma_{\mathrm{B}^0_s}$ |
|---|---|---|---|
| L3 [38] | inclusive B sample | | $< 0.67$ |
| DELPHI [47] | $\overline{\mathrm{B}^0_\mathrm{s}} \to \mathrm{D}_s^+\ell^-\overline{\nu_\ell}X$ | $\tau_{\mathrm{B}^{\mathrm{semi.}}_\mathrm{s}} = (1.42^{+0.14}_{-0.13} \pm 0.03)$ ps | $< 0.46$ |
| OTHERS | $\overline{\mathrm{B}^0_\mathrm{s}} \to \mathrm{D}_s^+\ell^-\overline{\nu_\ell}X$ | $\tau_{\mathrm{B}^{\mathrm{semi.}}_\mathrm{s}} = (1.46 \pm 0.07)$ ps | $< 0.30$ |
| ALEPH [102] | $\mathrm{B}^0_\mathrm{s} \to \phi\phi\mathrm{X}$ | $\mathrm{BR}(\mathrm{B}^{\mathrm{short}}_\mathrm{s} \to \mathrm{D}_\mathrm{s}^{(*)+}\mathrm{D}_\mathrm{s}^{(*)-}) = (23 \pm 7^{+19}_{-\ 9})\%$ | $0.26^{+0.29}_{-0.13}$ |
| ALEPH [102] | $\mathrm{B}^0_\mathrm{s} \to \phi\phi\mathrm{X}$ | $\tau_{\mathrm{B}^{\mathrm{short}}_\mathrm{s}} = (1.27 \pm 0.33 \pm 0.07)$ ps | $0.45^{+0.80}_{-0.49}$ |
| DELPHI [103] | $\overline{\mathrm{B}^0_\mathrm{s}} \to \mathrm{D}_s^+$ hadron | $\tau_{\mathrm{B}^{\mathrm{D_s-had.}}_\mathrm{s}} = (1.53^{+0.16}_{-0.15} \pm 0.07)$ ps | $< 0.69$ |
| CDF [34] | $\mathrm{B}^0_\mathrm{s} \to \mathrm{J}/\psi\phi$ | $\tau_{\mathrm{B}^{\mathrm{J}/\psi\phi}_\mathrm{s}} = (1.34^{+0.23}_{-0.19} \pm 0.05)$ ps | $0.33^{+0.45}_{-0.42}$ |

Table 5: *Experimental constraints on* $\Delta\Gamma_{\mathrm{B}^0_s}/\Gamma_{\mathrm{B}^0_s}$. *The upper limits are quoted at the 95 %C.L.*

In order to obtain an improved limit on $\Delta\Gamma_{\mathrm{B}^0_s}$ the results based on fits to the proper time distributions are used to apply a constraint on the allowed range of $1/\Gamma_{\mathrm{B}^0_s}$. The world average $\mathrm{B}^0_\mathrm{s}$ lifetime is not used as its meaning is not clear if $\Delta\Gamma_{\mathrm{B}^0_s}$ is non-zero. Instead, it is chosen to constrain $1/\Gamma_{\mathrm{B}^0_s}$ to the world average $\tau_{\mathrm{B}^0_\mathrm{d}}$ lifetime ($1/\Gamma_{\mathrm{B}^0_s} \equiv 1/\Gamma_{\mathrm{B}^0_d} = \tau_{\mathrm{B}^0_\mathrm{d}} = (1.562 \pm 0.029)$ ps). This is well motivated theoretically as the total widths of the $\mathrm{B}^0_\mathrm{s}$ and $\mathrm{B}^0_\mathrm{d}$ mesons are expected to be equal within less than one percent [69, 104] and $\Delta\Gamma_{\mathrm{B}^0_d}$ to be negligible.

Further information on the various individual measurements is now given.

- *L3 inclusive* B *sample:* in an unbiased inclusive B sample, all $\mathrm{B}^0_\mathrm{s}$ decay modes are measured, including decays into CP eigenstates. An equal number of $\mathrm{B}^{\mathrm{short}}_\mathrm{s}$ and $\mathrm{B}^{\mathrm{long}}_\mathrm{s}$ mesons are therefore selected and the proper time dependence of the signal is given by:

$$P_{incl}(t) = \frac{1}{2}(\Gamma^{\mathrm{long}}_{\mathrm{B}^0_s} \exp(-\Gamma^{\mathrm{long}}_{\mathrm{B}^0_s} t) + \Gamma^{\mathrm{short}}_{\mathrm{B}^0_s} \exp(-\Gamma^{\mathrm{short}}_{\mathrm{B}^0_s} t)). \tag{47}$$

  If the proper time dependence of this sample is fitted assuming only a single exponential lifetime, then, using the definitions of $\Gamma_{\mathrm{B}^0_s}$ and $\Delta\Gamma_{\mathrm{B}^0_s}$ and assuming that $\Delta\Gamma_{\mathrm{B}^0_s}/\Gamma_{\mathrm{B}^0_s}$ is small, the measured lifetime is given by:

$$\tau_{\mathrm{B}^{\mathrm{incl.}}_\mathrm{s}} = \frac{1}{\Gamma_{\mathrm{B}^0_s}} \frac{1}{1 - \left(\frac{\Delta\Gamma_{\mathrm{B}^0_s}}{2\Gamma_{\mathrm{B}^0_s}}\right)^2}. \tag{48}$$

  L3 effectively incorporates $P_{incl}(t)$ into the proper time fit of an inclusive B sample and applies the constraint $1/\Gamma_{\mathrm{B}^0_s} = (1.49 \pm 0.06)$ ps.

- *DELPHI* $\overline{\mathrm{B}^0_\mathrm{s}} \to \mathrm{D}_s^+\ell^-\overline{\nu_\ell}X$*:* in a semileptonic selection, the ratio of short and long $\mathrm{B}^0_\mathrm{s}$'s selected is proportional to the ratio of the decay widths and the proper time dependence of the signal is:

$$P_{semi}(t) = \frac{\Gamma^{\mathrm{short}}_{\mathrm{B}^0_s}\ \Gamma^{\mathrm{long}}_{\mathrm{B}^0_s}}{(\Gamma^{\mathrm{short}}_{\mathrm{B}^0_s} + \Gamma^{\mathrm{long}}_{\mathrm{B}^0_s})}(\exp(-\Gamma^{\mathrm{short}}_{\mathrm{B}^0_s} t) + \exp(-\Gamma^{\mathrm{long}}_{\mathrm{B}^0_s} t)). \tag{49}$$

If this sample is fitted assuming only a single exponential lifetime for the $B_s^0$, then the measured lifetime is, always in the limit that $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ is small, given by:

$$\tau_{B_s^{\rm semi.}} = \frac{1}{\Gamma_{B_s^0}} \frac{1+\left(\frac{\Delta\Gamma_{B_s^0}}{2\Gamma_{B_s^0}}\right)^2}{1-\left(\frac{\Delta\Gamma_{B_s^0}}{2\Gamma_{B_s^0}}\right)^2}. \tag{50}$$

The single lifetime fit of a semi-leptonic sample is thus more sensitive to the effects of $\Delta\Gamma_{B_s^0}$ than in the fully inclusive case. Information on $\Delta\Gamma_{B_s^0}$ is obtained by scanning the likelihood as a function the two parameters $1/\Gamma_{B_s^0}$ and $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ and applying the $\tau_{B_d^0}$ constraint.

- *OTHERS:* many analyses of the $B_s^0$ semileptonic lifetime have not explicitly considered the possibility of a non-zero $\Delta\Gamma_{B_s^0}$ value. The fact that the single exponential lifetime for this case (Equation (50)) is sensitive to $\Delta\Gamma_{B_s^0}$, allows information on $\Delta\Gamma_s$ to be extracted for these analyses also. The average $B_s^0$ semileptonic lifetime has been recalculated excluding the DELPHI result previously discussed and information on $\Delta\Gamma_{B_s^0}$ is obtained by using Equation (50) and applying the $\tau_{B_d^0}$ constraint. The validity of Equation (50), in the presence of background contributions has been verified using a toy Monte Carlo [105].

- *ALEPH* $B_s^0 \to \phi\phi X$ *(counting method):* only those $B_s^0$ decays which are CP eigenstates can contribute to a width difference between the CP even and CP odd states. An analysis [107] of such decays shows that $B_s^0 \to D_s^{(*)+}D_s^{(*)-}$ is by far the dominant contribution and is almost 100% CP even. Under this assumption, $\Delta\Gamma_{B_s^0} = \Gamma_{B_s^0}^{\rm short}(B_s^{\rm short} \to D_s^{(*)+}D_s^{(*)-})$ and can be related to the branching ratio of $B_s^{\rm short} \to D_s^{(*)+}D_s^{(*)-}$ through:

$${\rm BR}(B_s^{\rm short} \to D_s^{(*)+}D_s^{(*)-}) = \frac{\Gamma_{B_s^0}^{\rm short}(B_s^{\rm short} \to D_s^{(*)+}D_s^{(*)-})}{\Gamma_{B_s^0}^{\rm heavy}} = \frac{\Delta\Gamma_{B_s^0}}{\Gamma_{B_s^0}(1+\frac{\Delta\Gamma_{B_s^0}}{2\Gamma_{B_s^0}})}. \tag{51}$$

ALEPH [102] has measured this branching fraction and this is the only constraint on $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ which does not rely on a measurement of the lifetime of the other CP eigenstate.

- *ALEPH* $B_s^0 \to \phi\phi X$ *(lifetime method):* as the decay $B_s^0 \to D_s^{(*)+}D_s^{(*)-} \to \phi\phi X$, is predominantly CP even, the proper time dependence of the $B_s^0$ component of the sample is therefore just a simple exponential with the appropriate lifetime:

$$P_{single}(t) = \Gamma_{B_s^0}^{\rm short} \exp(-\Gamma_{B_s^0}^{\rm short} t). \tag{52}$$

This lifetime is related to $\Delta\Gamma_{B_s^0}$ and $\Gamma_{B_s^0}$ via the expression:

$$\frac{\Delta\Gamma_{B_s^0}}{\Gamma_{B_s^0}} = 2\left(\frac{1}{\Gamma_{B_s^0}\ \tau_{B_s^{\rm short}}} - 1\right). \tag{53}$$

ALEPH [102] has measured the lifetime of $\phi\phi X$ events and information on $\Delta\Gamma_s/\Gamma_s$ has been obtained with the assumption $1/\Gamma_{B_s^0} \equiv \tau_{B_d^0}$.

- *DELPHI inclusive* $D_s^+$*:* for a fully inclusive $D_s^+$ selection, the sample is expected to have an increased CP-even content, as the $B_s^0 \to D_s^{(*)+} D_s^{(*)-}$ contribution is enhanced. If $f_{DsDs}$ is the fraction of $D_s^{(*)+} D_s^{(*)-}$ in the sample, then the proper time dependence is expected to be:

$$P(t)_{D_s-had.} = f_{DsDs} P_{single}(t) + (1 - f_{DsDs}) P_{semi}(t). \tag{54}$$

  For the DELPHI analysis a value $f_{DsDs} = (22 \pm 7)\%$ is estimated from simulation. Scanning the likelihood as a function $1/\Gamma_{B_s^0}$ and $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ and applying the $\tau_{B_d^0}$ constraint yields an upper limit on $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$.

- *CDF* $B_s^0 \to J/\psi\phi$*:* the final state $B_s^0 \to J/\psi\phi$ is thought to be predominantly CP even (i.e. measures mainly $\tau_{B_s^{short}}$) [107]. An update [106] of the CDF measurement of the polarization in $B_s^0 \to J/\psi\phi$ decays measures the fraction of CP even in the final state to be $f_{short} = (79 \pm 19)\%$ and supports this expectation. For this case the proper time dependence of the $B_s^0$ component of the sample is:

$$P_{J/\psi\phi}(t) = f_{short} P_{single}(\Gamma_{B_s^0}^{short}, t) + (1 - f_{short}) P_{single}(\Gamma_{B_s^0}^{long}, t). \tag{55}$$

  CDF measures the lifetime of $J/\psi\phi$ events and information on $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ is obtained after applying the $1/\Gamma_{B_s^0} \equiv \tau_{B_d^0}$ constraint and including the experimental uncertainty on $f_{short}$.

## 5.2 Combined limit on $\Delta\Gamma_{B_s^0}$.

In order to combine the analyses summarised in Table 5, the result of each analysis has been converted to a two-dimensional log-likelihood in the $(1/\Gamma_{B_s^0}, \Delta\Gamma_{B_s^0}/\Gamma_{B_s^0})$ plane. This log-likelihood has either been provided by each experiment or reconstructed from the measured lifetimes using the expected dependence of this quantity on $1/\Gamma_{B_s^0}$ and $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$. The latter procedure was necessary for the OTHERS and CDF entries of Table 5. The L3 analysis is not included in the average as the two-dimensional likelihood was not provided and as it could not be reconstructed from the available information. The $\tau_{B_d^0}$ constraint is not applied on $1/\Gamma_{B_s^0}$ at this stage. Systematic uncertainties are included in the individual log-likelihood distributions.

The log-likelihood distributions have been summed and the variation of the global negative log-likelihood function has been measured with respect to its minimum ($\Delta\mathcal{L}$). The 68%, 95% and 99% C.L. contours of the combined negative log-likelihood are shown in Figure 7a. The corresponding limit on $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ is:

$$\begin{aligned} \Delta\Gamma_{B_s^0}/\Gamma_{B_s^0} &= 0.24^{+0.16}_{-0.12} \\ \Delta\Gamma_{B_s^0}/\Gamma_{B_s^0} &< 0.53 \text{ at the } 95\% \text{ C.L.} \end{aligned} \tag{56}$$

An improved limit on $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ can be obtained by applying the $\tau_{B_d^0} = (1.562 \pm 0.029)$ ps constraint. When expressed as a probability density, this constraint is:

$$f_C(1/\Gamma_{B_s^0}) = \frac{1}{\sqrt{2\pi}\sigma_{\tau_{B_d^0}}} \exp\left( -\frac{(1/\Gamma_{B_s^0} - \tau_{B_d^0})^2}{2\sigma^2_{\tau_{B_d^0}}} \right). \tag{57}$$

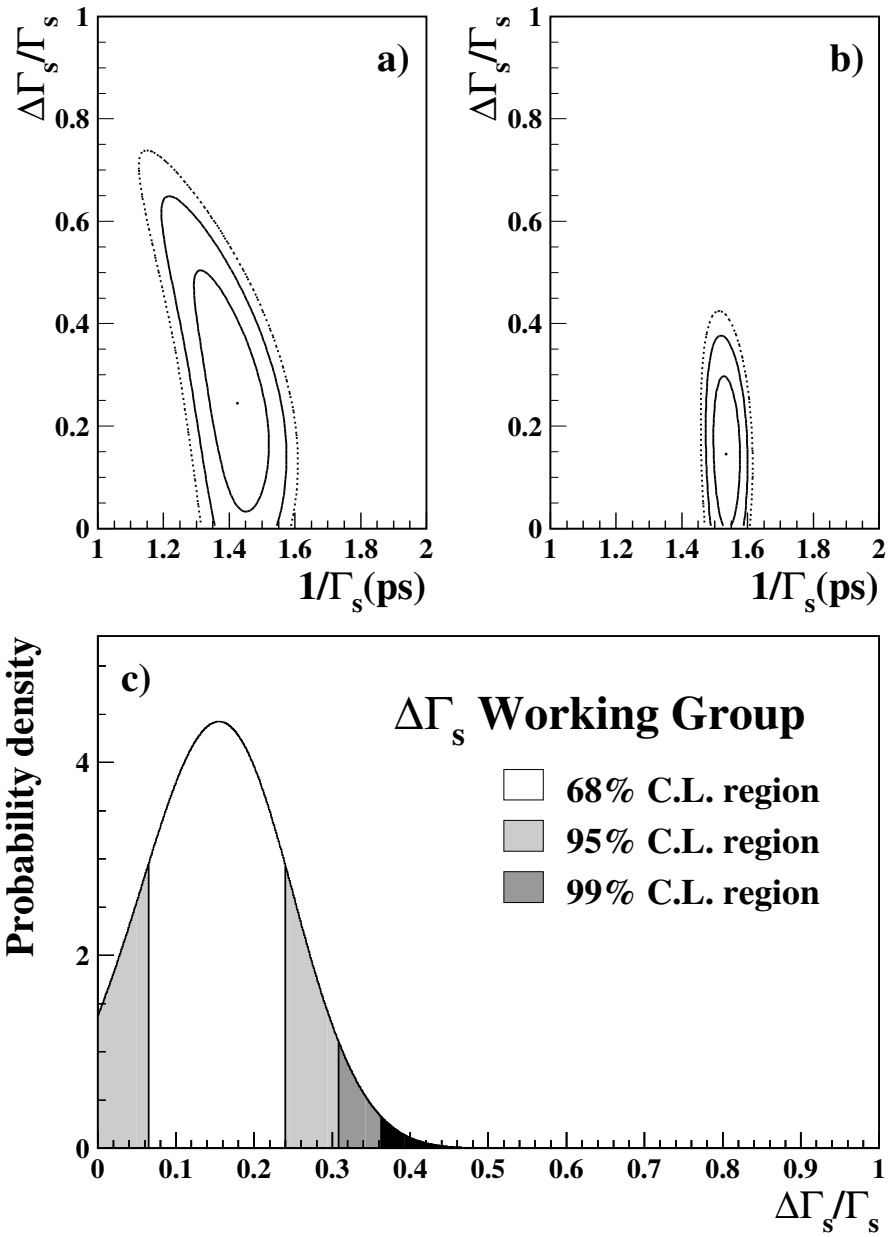


Figure 7: *a) 68%, 95% and 99% C.L. contours of the negative log-likelihood distribution in the plane* $1/\Gamma_{B_s^0}$-$\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$. *b) Same as a) but with the constraint* $1/\Gamma_{B_s^0} \equiv \tau_{B_d^0}$ *supposed to be exact. c) Probability density distribution for* $\Delta\Gamma_{B_s^0}/\Gamma_{B_s^0}$ *after applying the constraint; the three shaded regions show the limits at the 68%, 95% and 99% C.L. respectively.*

The value of the uncertainty, $\sigma_{\tau_{\mathrm{B}^0_\mathrm{d}}}$ includes, eventually, the theoretical uncertainty on the equality $1/\Gamma_{\mathrm{B}^0_\mathrm{s}} = \tau_{\mathrm{B}^0_\mathrm{d}}$, expected to be of the order of one % or below. Using Bayes theorem, the probability density for $\Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}}$, with the constraint applied, is obtained by convoluting $f_C(1/\Gamma_{\mathrm{B}^0_\mathrm{s}})$ with the 2-D probability density for $1/\Gamma_{\mathrm{B}^0_\mathrm{s}}$ and $\Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}}$ ($\mathcal{P}(\tau_{\mathrm{B}^0_\mathrm{s}}, \Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}})$), and normalizing the result to unity:

$$\mathcal{P}(\Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}}) = \frac{\int \mathcal{P}(1/\Gamma_{\mathrm{B}^0_\mathrm{s}}, \Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}}) f_C(1/\Gamma_{\mathrm{B}^0_\mathrm{s}}) d(1/\Gamma_{\mathrm{B}^0_\mathrm{s}})}{\int \mathcal{P}(1/\Gamma_{\mathrm{B}^0_\mathrm{s}}, \Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}}) f_C(1/\Gamma_{\mathrm{B}^0_\mathrm{s}}) d(1/\Gamma_{\mathrm{B}^0_\mathrm{s}}) d\Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}}} \tag{58}$$

where $\mathcal{P}(1/\Gamma_{\mathrm{B}^0_\mathrm{s}}, \Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}})$ is proportional to $\exp(-\Delta\mathcal{L})$.

The two-dimensional log-likelihood obtained, after including the constraint, supposed to be exact, is shown in Figure 7b. The resulting probability density distribution for $\Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}}$ is shown in Figure 7c. The corresponding limit on $\Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}}$ is:

$$\begin{aligned} \Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}} &= 0.16^{+0.08}_{-0.09} \\ \Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}} &< 0.31 \text{ at the } 95\% \text{ C.L.} \end{aligned} \tag{59}$$

If an additional 2% uncertainty, assumed to be Gaussian distributed, is incorporated to account for the theory assumption $\tau_{\mathrm{B}^0_\mathrm{d}} = 1/\Gamma_{\mathrm{B}^0_\mathrm{s}}$, the effect on the result is small:

$$\begin{aligned} \Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}} &= 0.17^{+0.09}_{-0.10} \\ \Delta\Gamma_{\mathrm{B}^0_\mathrm{s}}/\Gamma_{\mathrm{B}^0_\mathrm{s}} &< 0.32 \text{ at the } 95\% \text{ C.L.} \end{aligned} \tag{60}$$

Given the current status of the theoretical estimates of the bag parameters, it is considered premature to quote a corresponding constraint on the range of $\Delta m_s$ values.

# 6 Average of LEP $|\mathrm{V}_{cb}|$ measurements.

Within the framework of the Standard Model of electroweak interactions, the elements of the Cabibbo-Kobayashi-Maskawa mixing matrix are free parameters, constrained only by the requirement that the matrix be unitary. Relevant branching fractions have to be determined experimentally and inputs from theory are needed to obtain the values of the matrix elements. Operator Product Expansion (OPE) and Heavy Quark Effective Theory (HQET) provide means to determine $|\mathrm{V}_{cb}|$ with relatively small theoretical uncertainties, by studying the decay rates of, respectively, inclusive and exclusive semileptonic $b$-decays.

There are two methods to measure $|\mathrm{V}_{cb}|$: the inclusive method, which uses the semileptonic decay width of $b$-decays; and the exclusive method, where $|\mathrm{V}_{cb}|$ is extracted by studying the exclusive $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu}_\ell$ decay process. The $\mathrm{B} \to \mathrm{D}\ell^-\overline{\nu_\ell}$ channel has not been averaged to date.

In this note, both methods are used to determine values for $|\mathrm{V}_{cb}|$, which are then combined to produce a single average[16]. The semileptonic $b$-decay width, determined by the LEP heavy flavour electroweak fit to ALEPH, DELPHI, L3 and OPAL data, is used to determine $\mathrm{BR}(b \to \ell^-\overline{\nu_\ell}\mathrm{X})$ needed to extract $|\mathrm{V}_{cb}|$ from the inclusive method. Results

[16]Members of the $|\mathrm{V}_{cb}|$ working group are: D. Abbaneo, E. Barberio, A. Bellerive, S. Blyth, M. Calvi, P. Gagnon, R. Hawkings, M. Margoni, S. Mele, F. Muheim, D. Rousseau and F. Simonetto.

from ALEPH [108][17], DELPHI [109][18] and OPAL [110] are used to perform a LEP $|V_{cb}|$ average in the $\overline{B^0_d} \to D^{*+}\ell^-\overline{\nu}_\ell$ decay channel. These measurements are combined using a method similar to that used by the B oscillations working group.

Theoretical input parameters needed to extract $|V_{cb}|$ from actual measurements are detailed in Appendix E. In both analyses, the validity of the quark/hadron duality hypothesis has been assumed.

## 6.1 Inclusive $|V_{cb}|$ determination.

In the inclusive method, the partial width for semileptonic B mesons decays to charmed mesons is related to $|V_{cb}|$ using the following expression (Appendix E):

$$\begin{aligned} |V_{cb}| &= 0.0411 \sqrt{\frac{1.55}{0.105}\Gamma(b \to \ell^-\overline{\nu_\ell}X_c)} \left(1 - 0.024\left(\frac{\mu_\pi^2 - 0.5}{0.2}\right)\right) \times \\ &\quad (1 \pm 0.030(pert.) \pm 0.020(m_b) \pm 0.024(1/m_b^3)). \end{aligned} \tag{61}$$

where $X_c$ represents any state containing a charmed hadron. The rest of the expression, within parentheses, represents the correction to the muon decay formalism, depending on the $b$- and $c$-quark masses and on the strong coupling constant.

Experimentally, the semileptonic width of a $b$-hadron is determined from its semileptonic branching ratio and lifetime. In Z decays, a mixture of $\overline{B^0_d}$, $B^-$, $\overline{B^0_s}$ and $b$-baryons is produced, such that the inclusive semileptonic branching ratio measured at LEP is an average over the different produced hadrons:

$$\begin{aligned} \mathrm{BR}(b \to \ell^-\overline{\nu_\ell}X_c) &= f_{B_d}\frac{\Gamma(\overline{B^0_d} \to \ell^-\overline{\nu_\ell}X_c)}{\Gamma(\overline{B^0_d})} + f_{B^+}\frac{\Gamma(B^- \to \ell^-\overline{\nu_\ell}X_c)}{\Gamma(B^-)} \\ &\quad + f_{B_s}\frac{\Gamma(\overline{B^0_s} \to \ell^-\overline{\nu_\ell}X_c)}{\Gamma(\overline{B^0_s})} + f_{b\text{-baryon}}\frac{\Gamma(b-\text{baryon} \to \ell^-\overline{\nu_\ell}X_c)}{\Gamma(b-\text{baryon})} \\ &\simeq \Gamma(b \to \ell^-\overline{\nu_\ell}X_c)(f_{B_d}\tau_{B^0} + f_{B^+}\tau_{B^-} + f_{B_s}\tau_{B_s} + f_{b\text{-baryon}}\tau_{b-\text{baryon}}) \\ &= \Gamma(b \to \ell^-\overline{\nu_\ell}X_c)\tau_b \end{aligned} \tag{62}$$

where $\tau_b$ is the average $b$-hadron lifetime. Therefore the semileptonic width can be obtained using the inclusive semileptonic branching ratio and the average $b$-hadron lifetime. The two last equalities in Equation (62) assume that all $b$-hadrons have the same semileptonic width. This hypothesis may be incorrect for $b$-baryons. Taking into account the present precision of LEP measurements of $b$-baryon semileptonic branching ratios and lifetimes, the maximal possible correction to Equation (62) is about 1.5% (see Section 2).

The average LEP value for $\mathrm{BR}(b \to \ell^-\overline{\nu_\ell}X) = (10.58 \pm 0.07(\text{stat.}) \pm 0.17(\text{syst.}))\%$ is taken from the global fit which combines the heavy flavour measurements performed at the Z (see Section 2). The $\mathrm{BR}(b \to \ell^-\overline{\nu_\ell}X_u)$ contribution is subtracted from $\mathrm{BR}(b \to \ell^-\overline{\nu_\ell}X)$, using the LEP average value given in Section 7. For the average $b$-hadron lifetime, the world average value of $\tau_b$ is used, as obtained in Equation (32).

[17] Updated to use the parametrisation of [111], the Ligeti model [19, 112] for $\overline{B^0_d} \to D^{**+}\ell^-\overline{\nu}_\ell$ decays and various updated inputs.

[18] Updated to use the Ligeti model [19] for $\overline{B^0_d} \to D^{**+}\ell^-\overline{\nu}_\ell$ decays.

### 6.1.1 Sources of systematic errors.

The systematic errors assumed at present in the determination of the inclusive semileptonic decay width can be grouped into the following categories:

- *errors related to the efficiency and purity of the b-tagging algorithm.*

  As a lifetime $b$-tag is involved, effects due to the uncertainties in the sample composition in terms of different heavy hadrons, and uncertainties in the heavy hadron lifetimes, are considered.

- *input parameters influencing the signal and background normalisation.*

  The values of the production fractions: $f_{\mathrm{B_d}}$, $f_{\mathrm{B^+}}$, $f_{\mathrm{B_s}}$, $f_{b\text{-baryon}}$ were taken from Section 4.1. The branching ratios $\mathrm{BR}(b \to \overline{c} \to \ell)$, $\mathrm{BR}(b \to \tau \to \ell)$, $\mathrm{BR}(b \to \mathrm{J}/\psi \to \ell^+\ell^-)$ and the rates of gluon splitting $\mathrm{P}(\mathrm{g} \to \mathrm{c\overline{c}})$ and $\mathrm{P}(\mathrm{g} \to \mathrm{b\overline{b}})$ have been fixed to the values of [2]. $\mathrm{R}_b$, $\mathrm{R}_\mathrm{c}$, $\mathrm{BR}(b \to c \to \overline{\ell})$ and $\mathrm{BR}(c \to \overline{\ell})$ are parameters of the LEPEWWG fit. Values for all these quantities are listed in Table 1.

- *the average fraction of the beam energy carried by the weakly decaying b-hadron.*

  Different models have been considered for the shape of the fragmentation function and the free parameter of the model has been determined from the data.

- $\Lambda_b$ *polarization* effects on the lepton spectra have been included.

- *semileptonic decay models.* The average LEP value for $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X})$ is taken from the global LEPEWWG fit which combines the heavy flavour measurements performed at the Z [2]. but removing the forward-backward asymmetry measurements (see Section 2).

- *detector specific items.*

  They include: lepton efficiencies, misidentification probabilities, detector resolution effects, jet reconstruction, etc.

The errors listed in the last item are all uncorrelated among the different experiments. The others have been split into their uncorrelated and correlated parts. The error on the average $b$-hadron lifetime is assumed to be uncorrelated with the error on the semileptonic branching ratio. The propagation of these errors to the error on $|\mathrm{V}_{cb}|$ is done assuming that they are Gaussian in the branching ratio and the lifetime, respectively.

### 6.1.2 Inclusive $|\mathrm{V}_{cb}|$ average.

Using the expression given in Equation (61), the following value is obtained:

$$|\mathrm{V}_{cb}|^{incl.} = (40.76 \pm 0.41(\mathrm{exp.}) \pm 2.04(\mathrm{theo.})) \times 10^{-3} \tag{63}$$

where the first error is experimental and the second is from theory. The experimental contributions due to the semileptonic branching ratio and the lifetime are $\pm 0.37 \times 10^{-3}$ and $\pm 0.18 \times 10^{-3}$, respectively.

## 6.2 Exclusive $|V_{cb}|$ determination.

In the exclusive method, the value of $|V_{cb}|$ is extracted by studying the decay rate for the process $\overline{B^0_d} \to D^{*+}\ell^-\overline{\nu}_\ell$ as a function of the recoil kinematics of the $D^{*+}$ meson. The decay rate is parameterized as a function of the variable $w$, defined as the product of the four-velocities of the $D^{*+}$ and the $\overline{B^0_d}$ mesons. This variable is related to the square of the four-momentum transfer from the $\overline{B^0_d}$ to the $\ell^-\overline{\nu}_\ell$ system, $q^2$, by:

$$w = \frac{m^2_{D^{*+}} + m^2_{B^0_d} - q^2}{2 m_{B^0_d} m_{D^{*+}}}, \tag{64}$$

and its values range from 1.0, when the $D^{*+}$ is produced at rest in the $\overline{B^0_d}$ rest frame, to about 1.50. Using HQET, the differential partial width for this decay is given by:

$$\frac{d\Gamma}{dw} = \mathcal{K}(w)\mathcal{F}^2_{D^*}(w)\,|V_{cb}|^2 \tag{65}$$

where $\mathcal{K}(w)$ is a known phase space term and $\mathcal{F}_{D^*}(w)$ is the hadronic form factor for the decay. Although the shape of this form factor is not known, its magnitude at zero recoil, $w = 1$, can be estimated using HQET. In the heavy quark limit ($m_b \to \infty$), $\mathcal{F}(w)$ coincides with the Isgur-Wise function [113, 114] which is normalised to unity at the point of zero recoil. Corrections to $\mathcal{F}_{D^*}(1)$ have been calculated to take into account the effects of finite quark masses and QCD corrections [115]. Calculations of this correction yield $\mathcal{F}_{D^*}(1) = 0.88 \pm 0.05$ (Appendix E). Since the phase space factor $\mathcal{K}(w)$ tends to zero as $w \to 1$, the decay rate vanishes at $w = 1$ and the accuracy of the extrapolation relies on achieving a reasonably constant reconstruction efficiency in the region close to $w = 1$. The unknown function $\mathcal{F}_{D^*}(w)$ is approximated with an expansion around $w = 1$ [111]:

$$\mathcal{F}_{D^*}(w) = \mathcal{F}_{D^*}(1) \times \left[1 - 8\rho^2 z + (53\rho^2 - 15)z^2 - (231\rho^2 - 91)z^3\right], \tag{66}$$

where $\rho^2$ is the slope parameter at zero recoil and $z = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$. Theoretical predictions restrict values of $\rho^2$ to be in the range: $-0.14 < \rho^2 < 1.54$. An alternative parametrization, obtained earlier, can be found in [112].

## 6.3 Sources of systematic uncertainties.

In this section, the systematic uncertainties assumed at present in the determination of $|V_{cb}|$ using the semileptonic decay $\overline{B^0_d} \to D^{*+}\ell^-\overline{\nu}_\ell$ are listed. They are grouped into the following categories:

- *normalisation*: $\overline{B^0_d}$ meson production rate, $D^{(*)}$ branching fraction to the tagged final states (including topological BR), $\overline{B^0_d}$ lifetime (this is needed to obtain the $\overline{B^0_d} \to D^{*+}\ell^-\overline{\nu_\ell}$ decay partial width);

- *input parameters which influence the reconstruction efficiency* (e.g. $b \to B$ fragmentation function, value of the $\overline{B^0_d}$ lifetime);

- *background from physical processes*, comprising $\overline{B^0_d} \to D^{*+}\tau^-\overline{\nu}_\tau$ $\overline{B^0_d} \to D^{*+}X_{\overline{c}}$ (followed by the semileptonic decay $\tau/X_{\overline{c}} \to \ell^-\overline{\nu_\ell}X$), and, particularly, the intermediate production of excited charm mesons which subsequently decay to a $D^{*+}$;

- *detector specific items*: selection efficiency (lepton identification, tracking, vertexing), non physics background (combinatorial, hadron mis-identification), resolution, fitting, etc. This last set is treated as uncorrelated among experiments, and therefore will be ignored in the following discussion.

### 6.3.1 Normalisation.

The $\overline{\mathrm{B^0_d}}$ production rate at LEP is given by the product:

$$\frac{\Gamma(\mathrm{Z} \to b\overline{b})}{\Gamma(\mathrm{Z} \to \mathrm{hadrons})}\ \mathrm{BR}(b \to \overline{\mathrm{B^0_d}}) \ = \ \mathrm{R}_b\ f_{\mathrm{B_d}} \tag{67}$$

where $\mathrm{R}_b$ is determined in dedicated analyses [2] (Table 1) and $f_{\mathrm{B_d}}$ is taken from Section 4.1.

The values for the charm meson decay branching ratios are taken from the PDG [4]. Care has been taken in properly accounting for correlations among some of them (e.g. $\mathrm{D^0} \to \mathrm{K^-}\pi^+$ with $\mathrm{D^0} \to \mathrm{K}n\pi$, etc.). Analyses based on the inclusive reconstruction of pions from $\mathrm{D^{*+}}$ cascade decays may be affected by the knowledge of the topological $\mathrm{D^0}$ branching ratios which are taken from MARKIII measurements [116].

The $\overline{\mathrm{B^0_d}}$ lifetime determined in Section 3.2 has been used.

Knowledge of the $\overline{\mathrm{B^0_d}}$ fragmentation function is necessary in order to compute the fraction of $\overline{\mathrm{B^0_d}}$ which were not reconstructed because they did not have enough energy to be detected. In the LEP environment, $\overline{\mathrm{B^0_d}}$ hadrons carry on average a large fraction, $< x_E >$, of the beam energy (Table 1) and only a small fraction of them are outside the selection acceptance.

### 6.3.2 Physics background.

Charged $\mathrm{D^*}$ mesons can be accompanied by a lepton of opposite charge in two decay mechanisms of $b$-hadrons:

- $\overline{\mathrm{B}} \to \mathrm{D^{*+}}\tau^-\overline{\nu_\tau}$ with $\tau \to \ell^-\overline{\nu_\ell}\nu_\tau$,
- $\overline{\mathrm{B}} \to \mathrm{D^{*+}}\overline{\mathrm{D}}\mathrm{X}$ with $\overline{\mathrm{D}} \to \ell^-\overline{\nu_\ell}\mathrm{Y}$.

Their respective branching fractions have been evaluated in Section 2.

The production rates of the different $\mathrm{D^{**}}$ states and the variation of their corresponding form factors, as a function of $w$, has also to be considered. Published results are based on the old Isgur-Wise model [20], which predicts a sizeable rate of background near the end-point spectrum. As a consequence, the error on the overall amount of $\mathrm{D^{**}}$ in this model has a large effect on $|\mathrm{V}_{cb}|$ while having a negligible contribution on the slope parameter $\rho^2$. However, HQET predicts that, in the infinite charm mass limit, the rate near $w = 1$ is suppressed by a further factor $(w^2 - 1)$ when compared with the signal [18, 117]. In this case, the rate uncertainty would have a large effect on the slope with only a small influence on $|\mathrm{V}_{cb}|$ [109]. However, models in this extreme case fail to predict the $\mathrm{R^{**}}$ ratio, between the production rates of the two narrow states, of Equation (19).

A treatment which accounts for $\mathcal{O}(1/m_c)$ corrections is proposed in [19]. Several possible approximations of the form factors are provided, depending on five different expansion schemes and on three different input parameters. To be conservative, at the

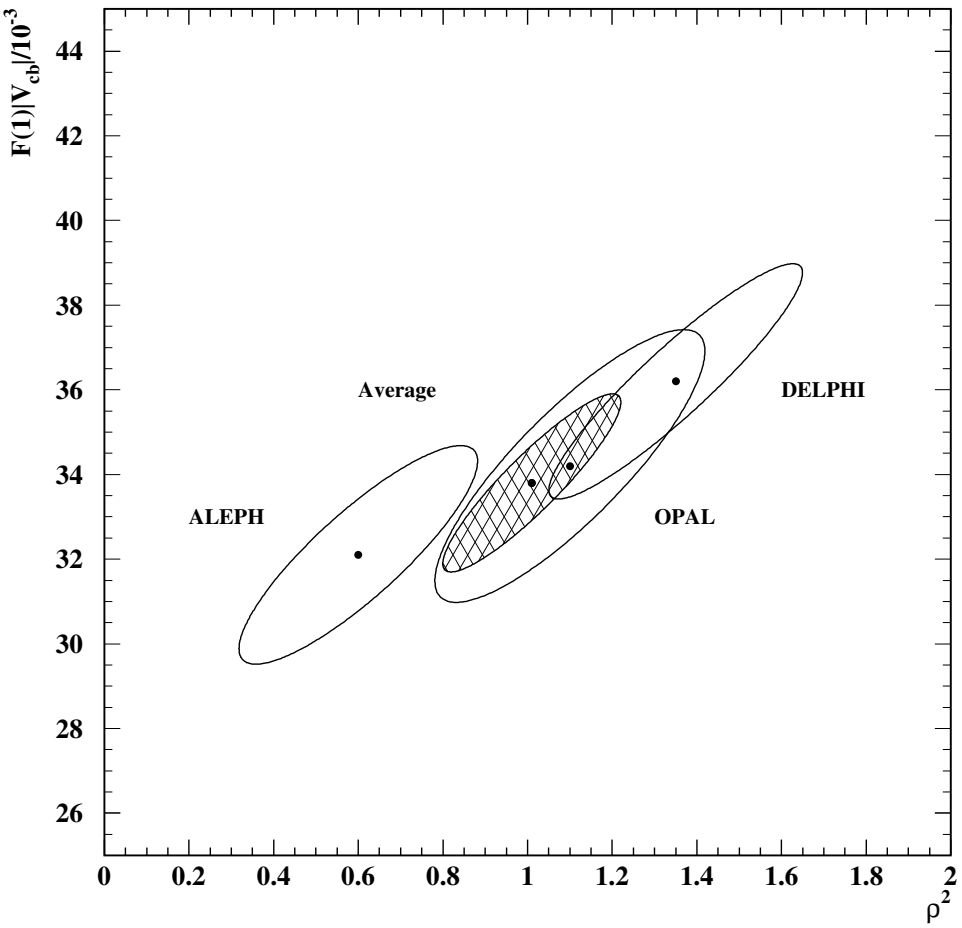


Figure 8: *Corrected measurements and LEP average using the exclusive method. The error ellipses, centred on the different measurements, correspond to one standard deviation and include systematics. The hatched ellipsis corresponds to the average of the three measurements.*

present stage of the analysis, each proposed scheme was tested in turn and the input parameters were varied over their full range. The quoted central value corresponds to the arithmetic average of the values obtained with the two extreme models. The systematic error due to the modelling of the $D^{**}$ background was computed as half the difference between the two extreme results.

### 6.3.3 Corrections applied to the measurements.

Since the three LEP measurements have been performed using different methods and inputs, they must be brought on the same footings before being averaged. ALEPH and DELPHI measurements have been updated to use the Caprini-Lellouch-Neubert [111] extrapolating method and Ligeti [19] $\overline{B_d^0} \to D^{**+}\ell^-\overline{\nu}_\ell$ model, while OPAL uses the Caprini-Neubert extrapolating method and JETSET $\overline{B_d^0} \to D^{**+}\ell^-\overline{\nu}_\ell$ model, corrections to OPAL measurement have been estimated using the ALEPH analysis which is similar.

| experiment | $\mathcal{F}_{D^*}(1)\lvert V_{\rm cb}\rvert$ ($\times 10^3$) | $\rho^2$ |
|---|---|---|
| ALEPH [108] | $32.1 \pm 2.1 \pm 1.5$ | $0.60 \pm 0.25 \pm 0.13$ |
| DELPHI [109] | $36.2 \pm 1.4 \pm 2.4$ | $1.35 \pm 0.13 \pm 0.27$ |
| OPAL [110] | $34.2 \pm 1.9 \pm 2.6$ | $1.10 \pm 0.24 \pm 0.21$ |

Table 6: *Experimental results corrected for common inputs.*

Corrections for the input parameters, listed in Table 7, have been calculated as for the $\Delta m_d$ measurement (see Section 4.2). The central value of each analysis is adjusted according to the difference between the used and desired parameter values and to the associated systematic error. The systematic error itself is then scaled to reflect the desired uncertainty on the input parameter. Table 6 lists the corrected results for the three experiments.

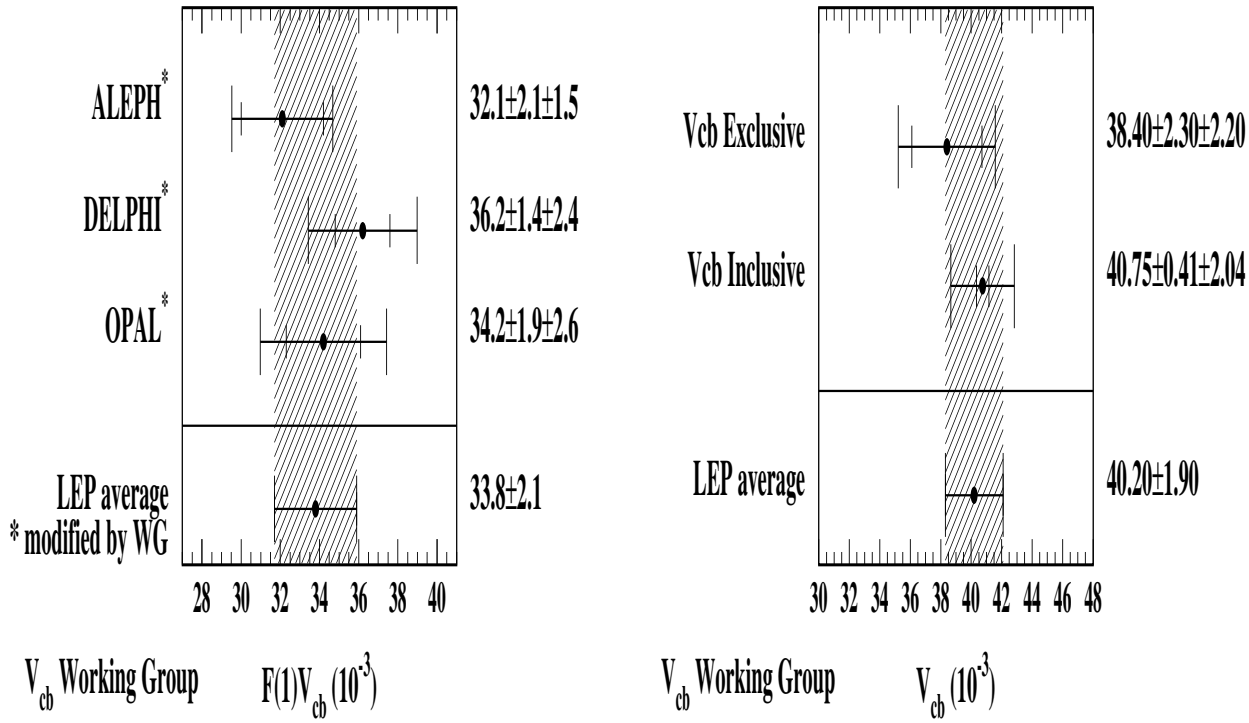


Figure 9: *Left: Corrected $\mathcal{F}_{D^*}(1)\lvert V_{cb}\rvert$ values and LEP average using the exclusive method. The values shown in the plot have been adjusted by the working group and are those used to perform the average. The original values can be found in the experimental papers. Right: $\lvert V_{cb}\rvert$ LEP average.*

### 6.3.4 Exclusive $\lvert V_{cb}\rvert$ average.

The combination method for $\mathcal{F}_{D^*}(1)\lvert V_{cb}\rvert$ and $\rho^2$ is the same as the method used for $\Delta m_d$ (see Section 4.2) generalized to the combination of two or more correlated parameters.

| Source | $\sigma(\mathcal{F}_{D^*}(1)\lvert V_{\rm cb}\rvert)/\mathcal{F}_{D^*}(1)\lvert V_{\rm cb}\rvert$ | $\sigma(\rho^2)$ |
|---|---|---|
| **BR's** | | |
| $\mathrm{BR}(\mathrm{D}\to \mathrm{K}n\pi)$ | 1.0 | 0.01 |
| $\mathrm{BR}(\mathrm{D}^{*+}\to \mathrm{D}^0\pi^+)$ | 1.0 | - |
| $\Gamma_{\mathrm{b\overline{b}}}/\Gamma_{\rm had}$ | 0.19 | - |
| $f_{\mathrm{B_d}}$ | 1.2 | - |
| **Background** | | |
| $\mathrm{B}^-\to \mathrm{D}^{*+}\pi^-\ell^-\overline{\nu_\ell}$ | 4.5 | 0.16 |
| $\overline{\mathrm{B}}\to \mathrm{D}^{*+}\mathrm{X}_{\overline{\mathrm{c}}}(\to \ell^-\mathrm{X})$ | 0.20 | - |
| $\overline{\mathrm{B}}\to \mathrm{D}^{*+}\tau^-\overline{\nu_\tau}$ | 0.17 | - |
| **Detector** | 2.1 | 0.10 |
| **Other inputs** | | |
| Fragmentation | 0.9 | - |
| $\overline{\mathrm{B^0_d}}$ lifetime | 1.2 | - |
| **Total syst.** | 5.5 | 0.19 |
| **Statistical** | 2.7 | 0.09 |

Table 7: *Dominant systematic uncertainties on $\mathcal{F}_{D^*}(1)|V_{cb}|$ and $\rho^2$ expressed, respectively, as relative and absolute values.*

The LEP average (see Figure 9-right) gives:

$$\mathcal{F}_{D^*}(1)|V_{cb}| = (33.8 \pm 0.9(\text{stat.}) \pm 1.9(\text{syst.})) \times 10^{-3} \quad (68)$$

$$\rho^2 = 1.01 \pm 0.09 \pm 0.19 \quad (69)$$

The parameters used are listed in Table 1. The uncertainty on the variation with $w$ of the $\overline{\mathrm{B^0_d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu}_\ell$ form factors is taken to be fully correlated between experiments. The dominant systematic uncertainties on $\mathcal{F}_{D^*}(1)|V_{cb}|$ are listed in Table 7. The largest comes from the $\overline{\mathrm{B^0_d}} \to \mathrm{D}^{**+}\ell^-\overline{\nu}_\ell$ contribution (mainly the shape of the form factors). The confidence level of the fit is 15%. The error ellipses of the corrected measurements and of the LEP average are shown on Figure 8.

The theoretical estimate, $\mathcal{F}_{D^*}(1)$=$0.88 \pm 0.05$ (see Appendix E), is used to determine:

$$|\mathrm{V}_{cb}|^{excl.} = (38.4 \pm 1.0(\text{stat.}) \pm 2.1(\text{syst.}) \pm 2.2(\text{theo.})) \times 10^{-3}. \quad (70)$$

## 6.4 Overall $|\mathrm{V}_{cb}|$ average.

The combined $|\mathrm{V}_{cb}|$ average (see Figure 9-right) can be extracted taking into account correlations between the inclusive and exclusive methods. The most important source of correlations comes from theoretical uncertainties in the evaluation of $\mu_\pi$, the average momentum of the $b$-quark inside the $b$-hadron (Appendix E). In the determination of experimental systematic uncertainties, theoretical uncertainties in the modelling of $b \to \ell$ decays and the exact amount of $b \to \mathrm{D}^{**}$ decays are taken as fully correlated. If these correlations were neglected, the central value for the $|\mathrm{V}_{cb}|$ average would change by 0.3%. Uncertainties from lepton identification and background also contribute but to a much lesser extent. All other sources amount to negligible contributions to the correlated error.

The various contributions to the uncertainty are listed in Table 8. The combined value is:

$$|\mathrm{V}_{cb}| = (40.2 \pm 1.9) \times 10^{-3} \tag{71}$$

where, within the total error of 1.9, 1.5 comes from uncorrelated sources and 1.2 from correlated sources.

| source | correlated | | uncorrelated | |
|---|---|---|---|---|
| | $\|\mathrm{V}_{cb}\|$ incl. | $\|\mathrm{V}_{cb}\|$ excl. | $\|\mathrm{V}_{cb}\|$ incl. | $\|\mathrm{V}_{cb}\|$ excl. |
| theory | 2.4% | 2.7% | 4.4% | 5.0% |
| exp. syst. | 0.6% | 4.6% | 0.7% | 3.0% |
| stat. | | | 0.4% | 2.7% |
| total | 2.5% | 5.3% | 4.5% | 6.4% |

Table 8: *Contributions to correlated and uncorrelated errors on* $|\mathrm{V}_{cb}|^{incl.}$ *and* $|\mathrm{V}_{cb}|^{excl.}$ *measurements, expressed as relative errors.*

# 7 Average of LEP $|\mathrm{V}_{ub}|$ measurements.

The first LEP combined determinations of $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)(\ell^- = e^- \text{ or } \mu^-)$ and the derivation of $|\mathrm{V}_{ub}|$ have been obtained by the combination of the results reported by the ALEPH [118], DELPHI [119] and L3 [120] Collaborations[19]. The three analyses rely on different techniques to measure the inclusive yield of $b \to u$ transitions in semileptonic $b$-hadron decays. The experimental details can be found in the original publications referred above. All three experiments reported evidence for the $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$ transition and measured its rate $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$. DELPHI fits directly $|\mathrm{V}_{ub}| \,/\, |\mathrm{V}_{cb}|$ from the fraction of candidate $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$ decays in the selected data sample. For this averaging, the corresponding value of $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$ has been derived by using the value of $|\mathrm{V}_{cb}|$ obtained in Section 6.4 and the relationship between $|\mathrm{V}_{ub}|$ and $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$ given below.

In order to average these results, the sources of systematic uncertainties have been divided into two categories. The first one defines uncorrelated systematics. These account for experimental systematics, such as lepton identification, $b$-tagging, vertexing efficiency and energy resolution and uncorrelated systematics from signal modelling and background description. The second category of systematic uncertainties consists of those deriving from simulation of signal and background that have been assumed to be fully correlated for the different analyses. The contributions from the statistical, experimental, uncorrelated and correlated systematic uncertainties are summarised in Table 9.

The sources of correlated systematics belong to both the description of background $b \to c$ and to the modelling of signal $b \to u$ transitions as summarised in Table 10. The differences in the analysis techniques adopted by the three experiments are reflected by differences in the size of systematic uncertainties estimated from each common source. An important common systematic is due to the charm topological branching ratios and

[19] Members of the $|\mathrm{V}_{ub}|$ working group are: D. Abbaneo, M. Battaglia, P. Henrard, E. Piotto, Ph. Rosnet, C. Schwick and S. Mele.

| Experiment | BR | (stat.) | (exp.) | (uncorrelated) | (correlated) |
|---|---|---|---|---|---|
| ALEPH [118] | 1.73 | $\pm$ 0.48 | $\pm$ 0.29 | $\pm$ 0.29 ($^{\pm 0.29}_{\pm 0.04}$ $^{b\to c}_{b\to u}$) | $\pm$ 0.47 ($^{\pm 0.43}_{\pm 0.19}$ $^{b\to c}_{b\to u}$) |
| DELPHI [119] | 1.57 | $\pm$ 0.35 | $\pm$ 0.37 | $\pm$ 0.17 ($^{\pm 0.12}_{\pm 0.12}$ $^{b\to c}_{b\to u}$) | $\pm$ 0.42 ($^{\pm 0.34}_{\pm 0.20}$ $^{b\to c}_{b\to u}$) |
| L3 [120] | 3.30 | $\pm$ 1.00 | $\pm$ 0.80 | $\pm$ 0.68 ($^{\pm 0.68}_{-}$ $^{b\to c}_{b\to u}$) | $\pm$ 1.40 ($^{\pm 1.29}_{\pm 0.54}$ $^{b\to c}_{b\to u}$) |

Table 9: *The results for $10^3\times$BR$(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$ from the LEP experiments with the sources of the statistical, experimental, model uncorrelated and model correlated uncertainties.*

to the rate of D $\to$ $\mathrm{K}^0$ decays. ALEPH and L3 are sensitive to the uncertainties in the $b$ fragmentation function due to the use of kinematical variables for discriminating $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$ from $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_c$ decays. The DELPHI result depends on the assumed composition in $b$-hadron species due to the use of kaon anti-tagging to reject $b \to c$, thus rejecting also $\mathrm{B}_\mathrm{s}^0$ and $\Lambda_b^0$ decays.

| Source | ALEPH | DELPHI | L3 |
|---|---|---|---|
| $b$ species | 0.01 | 0.12 | - |
| $b$ fragmentation | 0.22 | 0.03 | 0.32 |
| $b \to \ell$ model | 0.11 | 0.08 | 1.24 |
| $c \to \ell$ model | 0.14 | 0.13 | 0.12 |
| D topological BR's | 0.31 | 0.06 | - |
| BR(D$\to \mathrm{K}^0$) | 0.08 | 0.19 | - |
| $\mathrm{D}^{**}$, $\mathrm{D}^{(*)}\pi$ production | 0.04 | 0.19 | - |
| $b \to u$ exclusive model | 0.05 | 0.18 | 0.20 |
| $b \to u$ inclusive model | 0.18 | 0.08 | 0.25 |
| $\Lambda_b$ decay model | 0.04 | - | 0.44 |

Table 10: *Correlated sources of systematic uncertainties (in units of $10^{-3}$) entering in the measurement of* BR$(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$.

The systematic uncertainties, on the $b \to u$ signal, have been grouped into *inclusive model* and *exclusive model* classes which are assumed to be fully correlated. The first corresponds to the uncertainty in the modelling of the kinematics of the $b$-quark in the heavy hadron. It has been estimated by the spread of the results obtained with the ACCMM [121], the Dikeman-Shifman-Uraltsev [122] and the parton [123] models in the ALEPH and DELPHI analyses. In the case of the L3 analysis, the uncertainties in the single pion and in the lepton energy spectra were evaluated from the discrepancies between the model of Ref [124] and the ISGW [20] model respectively to the JETSET 7.4 [125] prediction have been used. The *exclusive model* uncertainty corresponds to the modelling of the hadronic final state in the $b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$ decay. These uncertainties have been estimated by replacing the parton shower fragmentation model in JETSET [125] with the fully exclusive ISGW2 [21] model by ALEPH and DELPHI and by propagating a 100% uncertainty on the B $\to \pi \ell^- \overline{\nu_\ell}$ rate by L3. ALEPH and L3 have also taken into account the uncertainty from the modelling of the charmless semi-leptonic decay of $b$-baryons. This has not been considered by DELPHI as they remove decays containing

identified protons and kaons, thus suppressing the contribution of $b$-baryons as mentioned above. In addition to these sources, ALEPH has estimated a $b \to u$ uncertainty from the energy cut-off value $\Lambda$ for the hybrid model adopted [126], while DELPHI allowed for the $b$-quark pole mass $m_b$ and the expectation value of the kinetic energy operator $< p_b^2 >$ uncertainties. They have been assumed to be uncorrelated.

The three measurements of $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$ have been averaged using the Best Linear Unbiased Estimate (B.L.U.E.) technique [127]. Using the inputs from Table 9 and Table 10, the LEP average value for $\mathrm{BR}(b \to \ell^- \nu \mathrm{X}_u)$ was found to be:

$$\begin{aligned} \mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u) &= (1.67 \pm 0.36(\mathrm{stat.} + \mathrm{exp.}) \pm 0.37(b \to c) \pm 0.20(b \to u)) \times 10^{-3} \\ &= (1.67 \pm 0.55) \times 10^{-3} \end{aligned} \tag{72}$$

with a confidence level for the combination of 0.70 (see Figure 10 in which the separated and combined values are presented).

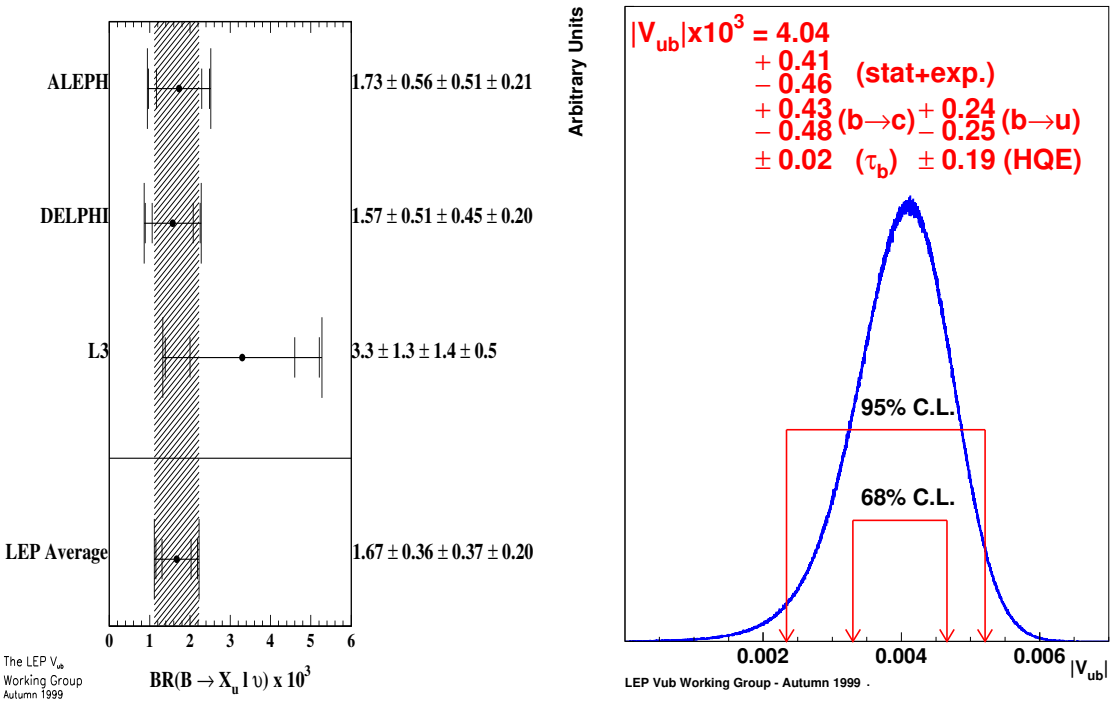


Figure 10: *Left: the determinations of* $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$ *by ALEPH, DELPHI and L3 and the resulting LEP average. Right: the probability density function for* $|\mathrm{V}_{ub}|$ *corresponding to the LEP average value of* $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u)$ *with the value of the median and two confidence intervals indicated. Unphysical negative entries have been discarded and the probability density function renormalised accordingly.*

The value of the $|\mathrm{V}_{ub}|$ element has been extracted using the following relationship derived in the context of Heavy Quark Expansion [128, 129]:

$$\begin{aligned} |\mathrm{V}_{ub}| &= 0.00445 \sqrt{\frac{\mathrm{BR}(b \to \mathrm{X}_u \ell^- \overline{\nu_\ell})}{0.002} \frac{1.55\mathrm{ps}}{\tau_b}} \times \\ &\quad (1 \pm 0.010(\mathrm{pert}) \pm 0.030(1/m_b^3) \pm 0.035(m_b)) \end{aligned} \tag{73}$$

by assuming $m_b = (4.58 \pm 0.06)$ GeV/c$^2$ (Appendix E).

From the LEP average of BR($b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$) obtained above, the $b$-lifetime value, $\tau_b$, obtained in Section 3 and the quoted $|\mathrm{V}_{ub}|$ uncertainty coming from HQE ($\pm 4.7$ % relative), a probability density function for $|\mathrm{V}_{ub}|$ has been calculated. The resulting distribution is shown in Figure 10, where all errors are convoluted together assuming that they are Gaussian on BR($b \to \ell^- \overline{\nu_\ell} \mathrm{X}_u$), with the exception of the HQE error assumed to be Gaussian on $|\mathrm{V}_{ub}|$. The part of this function in the negative, unphysical region is negligible, corresponding to only 0.14%. This has been discarded from the following analysis and the probability density function renormalised accordingly. The median of this function has been chosen as the best estimate of the $|\mathrm{V}_{ub}|$ value and the $\pm 34.135\%$ and $\pm 47.725\%$ of the integral of the probability density function around this value have been used to define the $1\sigma$ and $2\sigma$ confidence regions, denoted hereafter as the 68% and 95% confidence level, obtaining:

$$|\mathrm{V}_{ub}| = (4.04^{+0.62}_{-0.74}) \times 10^{-3} \text{ at the 68\% C.L.} \tag{74}$$

and

$$|\mathrm{V}_{ub}| = (4.04^{+1.17}_{-1.71}) \times 10^{-3} \text{ at the 95\% C.L.} \tag{75}$$

All the uncertainties have been included in these estimates. The application of the above procedure for each error source separately yields the detailed result for the 68% confidence level:

$$\begin{aligned} |\mathrm{V}_{ub}| \;=\; & (4.04^{+0.41}_{-0.46}\ (\text{stat.} + \text{det.})^{+0.43}_{-0.48}\ (b \to c \text{ syst.})^{+0.24}_{-0.25}\ (b \to u \text{ syst.}) \\ & \pm 0.02\ (\tau_b) \pm 0.19\ (\text{HQE})) \times 10^{-3}. \end{aligned} \tag{76}$$

# 8 Summary of all results.

This paper provides precise combined results (see Table 11), from measurements submitted to the 1999 Summer Conferences, on parameters which govern production and decay properties of $b$-hadrons emitted in high energy $b$-quark jets.

At the Z pole, the polarization of $b$-baryons is found to be significantly different from zero but it is reduced as compared to the initial $b$-quark polarization of $-0.94$. Production of $\Sigma_b^{(*)}$ baryons has been invoked to explain this difference.

The accuracy on $\overline{\mathrm{B}^0_\mathrm{d}}$ and $\mathrm{B}^-$ lifetimes is better than 2%. The $\mathrm{B}^-$ lifetime is significantly larger than the $\overline{\mathrm{B}^0_\mathrm{d}}$ lifetime, in agreement with original expectations based on OPE and parton-hadron duality [69]. But the clear difference between measured and expected lifetimes of $b$-baryons remains to be explained. It may point to a failure of parton-hadron duality in inclusive B decays or to a failure of quark models used to evaluate the mean value of operators contributing in $b$-baryon decays.

The accuracy on $\overline{\mathrm{B}^0_\mathrm{d}}$ and $\mathrm{B}^-$ production rates in $b$-quark jets has improved by a factor two as compared to published values [4]. In conjunction with the better determination of the $\overline{\mathrm{B}^0_\mathrm{d}}$ lifetime, this measurement has given a more precise determination of $|\mathrm{V}_{cb}|$ from $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+} \ell^- \overline{\nu_\ell}$ decays. The $\overline{\mathrm{B}^0_\mathrm{s}}$ fraction in jets is close to 10% with an uncertainty which has been reduced by 30%. Its determination is an important input in searches for $\mathrm{B}^0_\mathrm{s}$-$\overline{\mathrm{B}^0_\mathrm{s}}$ oscillations. The accuracy on the $b$-baryon rate has improved by about a factor two because of new measurements using spectator baryons.

| $b$-hadron lifetimes | World averages, Section 3 |
|---|---|
| $\tau(\mathrm{B_d^0})$ | $(1.562 \pm 0.029)$ ps |
| $\tau(\mathrm{B^+})$ | $(1.656 \pm 0.025)$ ps |
| $\tau(\mathrm{B_s^0})$ | $(1.464 \pm 0.057)$ ps |
| $\tau(\Lambda_b^0)$ | $(1.229^{+0.081}_{-0.079})$ ps |
| $\tau(\Xi_b)$ | $(1.39^{+0.34}_{-0.28})$ ps |
| $\tau(b-\mathrm{baryon})$ | $(1.208^{+0.051}_{-0.050})$ ps |
| $\tau_b^{incl.}$ | $(1.564 \pm 0.014)$ ps |
| $\frac{\tau(\mathrm{B^-})}{\tau(\mathrm{B_d^0})}$ | $1.065 \pm 0.023$ |
| **$b$-hadron production rates** | World averages, Section 4 |
| $f_{\mathrm{B_s}}$ | $(10.0 \pm 1.2)\%$ |
| $f_{b\text{-baryon}}$ | $(9.9 \pm 1.7)\%$ |
| $f_{\mathrm{B_d}} = f_{\mathrm{B^+}}$ | $(40.1 \pm 1.0)\%$ |
| $\rho(f_{\mathrm{B_s}}, f_{b\text{-baryon}})$ | $-0.03$ |
| $\rho(f_{\mathrm{B_d}}, f_{\mathrm{B_s}})$ | $-0.57$ |
| $\rho(f_{\mathrm{B_s}}, f_{b\text{-baryon}})$ | $-0.81$ |
| **$\mathrm{B^0} - \overline{\mathrm{B^0}}$ oscillations** | World averages, Section 4 |
| $\Delta m_d$ | $(0.476 \pm 0.016)$ ps$^{-1}$ |
| $\chi_d$ | $0.177 \pm 0.008$ |
| $\Delta m_s$ | $> 14.3$ ps$^{-1}$at the 95% C.L. |
| **Limit on $\Delta\Gamma_{\mathrm{B_s^0}}$** | World averages, Section 5 |
| $\Delta\Gamma_{\mathrm{B_s^0}}/\Gamma_{\mathrm{B_s^0}}$ | $< 0.31$ at the 95% C.L. |
| **Measurement of $\lvert V_{cb}\rvert$** | LEP averages, Section 6 |
| $\mathrm{BR}(b \to \ell X)$ | $(10.58 \pm 0.07 \pm 0.17)\%$ |
| $\mathcal{F}_{D^*}(1)\lvert V_{cb}\rvert$ | $(33.8 \pm 0.9(\mathrm{stat.}) \pm 1.9(\mathrm{syst.})) \times 10^{-3}$ |
| $\rho^2$ | $1.01 \pm 0.09 \pm 0.19$ |
| $\lvert V_{cb}\rvert^{incl.}$ | $(40.76 \pm 0.41(\mathrm{exp.}) \pm 2.04(\mathrm{theo.})) \times 10^{-3}$ |
| $\lvert V_{cb}\rvert^{excl.}$ | $(38.4 \pm 1.0(\mathrm{stat.}) \pm 2.1(\mathrm{syst.}) \pm 2.2(\mathrm{theo.})) \times 10^{-3}$ |
| $\lvert V_{cb}\rvert$ | $(40.2 \pm 1.9) \times 10^{-3}$ |
| **Measurement of $\lvert V_{ub}\rvert$** | LEP averages, Section 7 |
| $\lvert V_{ub}\rvert$ | $(4.04^{+0.62}_{-0.74}) \times 10^{-3}$ |
| **$\Lambda_b^0$ polarization in Z decays** | LEP averages, Section 2 |
| $\mathcal{P}(\Lambda_b^0)$ | $-0.45^{+0.17}_{-0.15} \pm 0.08$ |
| **$\mathrm{D^{**}}$ properties in semileptonic B decays** | World averages, Section 2 |
| $\mathrm{BR}(\overline{\mathrm{B_d^0}} \to \mathrm{D^{**+}}\ell^-\overline{\nu_\ell})$ | $(3.04 \pm 0.44)\%$ |
| $\mathrm{BR}(\overline{\mathrm{B_d^0}} \to \mathrm{D^{**+}}\ell^-\overline{\nu_\ell}) \times \mathrm{BR}(\mathrm{D^{**+}} \to \mathrm{D^*X})$ | $(1.75 \pm 0.21 \pm 0.08)\%$ |
| $\frac{\mathrm{BR}(\overline{\mathrm{B_d^0}} \to \mathrm{D^{*+}}\ell^-\overline{\nu_\ell}) + \mathrm{BR}(\overline{\mathrm{B_d^0}} \to \mathrm{D^{*+}}X\ell^-\overline{\nu_\ell})}{\mathrm{BR}(\overline{\mathrm{B_d^0}} \to \ell^-\overline{\nu_\ell}X)}$ | $0.49 \pm 0.03$ |
| $\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D_1}\ell^-\overline{\nu_\ell})$ | $(0.6 \pm 0.1)\%$ |
| $\mathrm{BR}(\overline{\mathrm{B}} \to \mathrm{D_2^*}\ell^-\overline{\nu_\ell})$ | $< 0.4\%$ at the 95% CL. |

Table 11: *Summary of the results obtained on b-hadron production rates and decay properties using data available by mid 1999. More details on systematic uncertainties can be found in the corresponding sections.*

The mass difference between mass eigenstates of the $B^0_d$-$\overline{B^0_d}$ system is measured with a relative precision better than 4% whereas the corresponding quantity for the $B^0_s$-$\overline{B^0_s}$ system is still unmeasured, in spite of an impressive progress in the sensitivity reached by the experiments.

A first combined result on the decay widths difference between mass eigenstates of the $B^0_s$-$\overline{B^0_s}$ system has been obtained.

Because of the very accurate measurements obtained on the inclusive $b$-lifetime and semileptonic branching fraction, the measurement accuracy of $|V_{cb}|$ from inclusive semileptonic decays is entirely limited by theoretical uncertainties. An experimental control of these uncertainties is thus needed. The measurement of $|V_{cb}|$ from exclusive $\overline{B^0_d} \to D^{*+} X \ell^- \overline{\nu_\ell}$ decays is limited by uncertainties related to $D^{**}$ production and decay mechanisms. Present experimental results on production characteristics of these states, in $b$-hadron semileptonic decays, have been detailed in Section 2 of the present report.

The reduced correlation between theoretical uncertainties attached to the determination of $|V_{cb}|$ from inclusive and exclusive measurements has been used in evaluating a global average.

The new measurement of $|V_{ub}|$ from LEP experiments has reached a similar accuracy as previous determinations from experiments running at the $\Upsilon(4S)$ but with a better sensitivity over a larger fraction of the phase space as compared with measurements using the end-point lepton energy region.

New measurements from LEP and SLD are still expected in the year 2000 and will improve present determinations of $b$-hadron production and decay properties providing already stringent constraints on the shape of the CKM unitarity triangle before the era of B-factories.

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# A Production rates of the $D_1$ and $D_2^*$ mesons in semileptonic *b*-decays.

| Experiment | Channel | Value $\times 10^3$ | Ref. |
|---|---|---|---|
| ALEPH | $BR(b \to \overline{B})BR(\overline{B} \to D_1^+ \ell^- \overline{\nu_\ell})BR(D_1^+ \to D^{*0}\pi^+)$ | $2.06^{+0.55\ +0.29}_{-0.51\ -0.40}$ | [12] |
| ALEPH | $BR(b \to \overline{B})BR(\overline{B} \to D_1^0 \ell^- \overline{\nu_\ell})BR(D_1^0 \to D^{*+}\pi^-)$ | $1.68^{+0.40\ +0.28}_{-0.36\ -0.29}$ | [12] |
| ALEPH | $BR(b \to \overline{B})BR(\overline{B} \to D_1^0 \ell^- \overline{\nu_\ell})BR(D_1^0 \to D^{*+}\pi^-)$ | $3.62^{+1.78}_{-1.48} \pm 0.77$ | [12] |
| CLEO | $BR(B^- \to D_1^0 \ell^- \overline{\nu_\ell})BR(D_1^0 \to D^{*+}\pi^-)$ | $3.73 \pm 0.85 \pm 0.52 \pm 0.24$ | [22] |

Table 12: *Measured values of the* $D_1$ *production rate in b-semileptonic decays. The third systematic uncertainty, quoted in the CLEO analysis, comes from the variation of the detection efficiency when changing the parameters of the form factors, corresponding to the signal, in the ISGW2 model [21].*

| Experiment | Channel | Value $\times 10^4$ | Ref. |
|---|---|---|---|
| ALEPH | $BR(b \to \overline{B})BR(\overline{B} \to D_2^{*+} \ell^- \overline{\nu_\ell})BR(D_2^{*+} \to D^0\pi^+)$ | $3.1^{+2.4\ +0.4}_{-2.2\ -0.6}$ | [12] |
| ALEPH | $BR(b \to \overline{B})BR(\overline{B} \to D_2^{*0} \ell^- \overline{\nu_\ell})BR(D_2^{*0} \to D^{*+}\pi^-)$ | $5.1^{+3.0\ +0.8}_{-2.6\ -0.9}$ | [12] |
| ALEPH | $BR(b \to \overline{B})BR(\overline{B} \to D_2^{*0} \ell^- \overline{\nu_\ell})BR(D_2^{*0} \to D^+\pi^-)$ | $3.8^{+2.4\ +0.8}_{-1.9\ -0.8}$ | [12] |
| CLEO | $BR(B^- \to D_2^{*0} \ell^- \overline{\nu_\ell})BR(D_2^{*0} \to D^{*+}\pi^-)$ | $5.9 \pm 6.6 \pm 1.0 \pm 0.4$ | [22] |

Table 13: *Measured values of the* $D_2^*$ *production rate in b-semileptonic decays. In the original publication from ALEPH, as these values differ from zero only by one or two standard deviations, only upper limits were quoted. The third systematic uncertainty, quoted in the CLEO analysis, comes from the variation of the detection efficiency when changing the parameters of the form factors, corresponding to the signal, in the ISGW2 model [21].*

# B $\Lambda_b^0$ polarization measurements.

| Experiment | Value | Ref. |
|---|---|---|
| ALEPH | $-0.23^{+0.24\ +0.08}_{-0.20\ -0.07}$ | [27] |
| DELPHI | $-0.49^{+0.32\ +0.17}_{-0.30\ -0.17}$ | [28] |
| OPAL | $-0.56^{+0.20\ +0.09}_{-0.13\ -0.09}$ | [29] |

Table 14: *Measured values of the* $\Lambda_b^0$ *polarization in* Z *decays.*

# C Measurements used in the evaluation of *b*-hadrons lifetimes.

In the Tables below, new measurements available since Winter 1999 are labelled with "(n)" and preliminary results with "(p)".

| Experiment | Method | Data set | $\tau_{B^0}$ (ps) | Reference |
|---|---|---|---|---|
| ALEPH (n, p) | $D^{(*)}\ell$ | 91–95 | $1.524 \pm 0.053^{+0.035}_{-0.032}$ | [31] |
| ALEPH | Excl. rec. | 91–94 | $1.25^{+0.15}_{-0.13} \pm 0.05$ | [32] |
| ALEPH | Partial rec. $\pi^+\pi^-$ | 91–94 | $1.49^{+0.17+0.08}_{-0.15-0.06}$ | [32] |
| CDF | $D^{(*)}\ell$ | 92–95 | $1.474 \pm 0.039^{+0.52}_{-0.51}$ | [33] |
| CDF | Excl. $(J/\psi K)$ | 92–95 | $1.58 \pm 0.09 \pm 0.02$ | [34] |
| DELPHI | $D^{(*)}\ell$ | 91–93 | $1.61^{+0.14}_{-0.13} \pm 0.08$ | [35] |
| DELPHI | Charge sec. vtx. | 91–93 | $1.63 \pm 0.14 \pm 0.13$ | [36] |
| DELPHI | Inclusive $D^*\ell$ | 91–93 | $1.532 \pm 0.041 \pm 0.040$ | [37] |
| L3 | Charge sec. vtx. | 94–95 | $1.52 \pm 0.06 \pm 0.04$ | [38] |
| L3 | Inclusive $D^*\ell$ | 94 | $1.74 \pm 0.12 \pm 0.04$ | [39] |
| OPAL | $D^{(*)}\ell$ | 91–93 | $1.53 \pm 0.12 \pm 0.08$ | [40] |
| OPAL (n) | Charge sec. vtx. | 93–95 | $1.523 \pm 0.057 \pm 0.053$ | [41] |
| SLD | Charge sec. vtx.$\ell$ | 93–95 | $1.56^{+0.14}_{-0.13} \pm 0.10$ | [42] |
| SLD (n) | Charge sec. vtx. | 93–98 | $1.585 \pm 0.021 \pm 0.043$ | [43] |
| Average | | | $1.553 \pm 0.029$ | |

Table 15: *Measurements of the* $B^0_d$ *lifetime.*

| Experiment | Method | Data set | $\tau_{B^+}$(ps) | Reference |
|---|---|---|---|---|
| ALEPH (n, p) | $D^{(*)}\ell$ | 91-95 | $1.646 \pm 0.056^{+0.036}_{-0.034}$ | [31] |
| ALEPH | Excl. rec. | 91-94 | $1.58^{+0.21+0.04}_{-0.18-0.03}$ | [32] |
| CDF | $D^{(*)}\ell$ | 92-95 | $1.637 \pm 0.058^{+0.45}_{-0.43}$ | [33] |
| CDF | Excl. $(J/\psi K)$ | 92-95 | $1.68 \pm 0.07 \pm 0.02$ | [34] |
| DELPHI | $D^{(*)}\ell$ | 91-93 | $1.61 \pm 0.16 \pm 0.12$ | [35][a] |
| DELPHI | Charge sec. vtx. | 91-93 | $1.72 \pm 0.08 \pm 0.06$ | [36][a] |
| L3 | Charge sec. vtx. | 94-95 | $1.66 \pm 0.06 \pm 0.03$ | [38] |
| OPAL | $D^{(*)}\ell$ | 91-93 | $1.52 \pm 0.14 \pm 0.09$ | [40] |
| OPAL (n) | Charge sec. vtx. | 93-95 | $1.643 \pm 0.037 \pm 0.025$ | [41] |
| SLD | Charge sec. vtx. $\ell$ | 93-95 | $1.61^{+0.13}_{-0.12} \pm 0.07$ | [42] |
| SLD (n) | Charge sec. vtx. | 93-98 | $1.623 \pm 0.020 \pm 0.034$ | [43] |
| Average | | | $1.639 \pm 0.025$ | |

Table 16: *Measurements of the* $B^+$ *lifetime.*
*a) The combined DELPHI result quoted in [36] is (1.70 ± 0.09) ps.*

| Experiment | Method | Data set | $\tau_{B_s}$ (ps) | Reference |
|---|---|---|---|---|
| ALEPH | $D_s\ell$ | 91-95 | $1.54^{+0.14}_{-0.13}\pm 0.04$ | [44] |
| ALEPH | $D_s h$ | 91-95 | $1.47\pm 0.14 \pm 0.08$ | [45] |
| CDF | $D_s\ell$ | 92-96 | $1.36\pm 0.09^{+0.06}_{-0.05}$ | [46] |
| CDF | Excl. $J/\psi\phi$ | 92-95 | $1.34^{+0.23}_{-0.19}\pm 0.05$ | [34] |
| DELPHI (n) | $D_s\ell$ | 91-95 | $1.42^{+0.14}_{-0.13} \pm 0.03$ | [47] |
| DELPHI (n) | $D_s h$ | 91-95 | $1.49^{+0.16\ +0.07}_{-0.15\ -0.08}$ | [47] |
| DELPHI | $D_s$ inclus. | 91-94 | $1.60\pm 0.26^{+0.13}_{-0.15}$ | [49] |
| OPAL | $D_s\ell$ | 90-95 | $1.50^{+0.16}_{-0.15}\pm 0.04$ | [50] |
| OPAL | $D_s$ inclus. | 90-95 | $1.72^{+0.20+0.18}_{-0.19-0.17}$ | [51] |
| Average | | | $1.464 \pm 0.057$ | |

Table 17: *Measurements of the* $B_s^0$ *lifetime.*

| Experiment | Method | Data set | $\tau_{\Lambda_b}$ (ps) | Reference |
|---|---|---|---|---|
| ALEPH | $\Lambda\ell$ | 91-95 | $1.20^{+0.08}_{-0.08}\pm 0.06$ | [52] |
| ALEPH | $\Lambda_c\ell$ | 91-95 | $1.18^{+0.13}_{-0.12}\pm 0.03$ | [52] |
| ALEPH | $\Lambda\ell^-\ell^+$ | 91-95 | $1.30^{+0.26}_{-0.21}\pm 0.04$ | [52] |
| ALEPH | $\Xi\ell$ | 90-95 | $1.35^{+0.37+0.15}_{-0.28-0.17}$ | [53] |
| CDF | $\Lambda_c\ell$ | 91-95 | $1.32\pm 0.15\pm 0.06$ | [54] |
| DELPHI | $\Lambda\ell\pi$ vtx | 91-94 | $1.16\pm 0.20 \pm 0.08$ | [55][a] |
| DELPHI | $\Lambda\mu$ i.p. | 91-94 | $1.10^{+0.19}_{-0.17} \pm 0.09$ | [56][a] |
| DELPHI | $\Lambda_c\ell$ | 91-94 | $1.11^{+0.19}_{-0.18} \pm 0.05$ | [55][a] |
| DELPHI | $p\ell$ | 91-94 | $1.19\pm 0.14\pm 0.07$ | [55][a] |
| DELPHI | $\Xi\ell$ | 91-93 | $1.5\ ^{+0.7}_{-0.4}\pm 0.3$ | [57] |
| OPAL | $\Lambda\ell$ i.p. | 90-94 | $1.21^{+0.15}_{-0.13}\pm 0.10$ | [58][b] |
| OPAL | $\Lambda\ell$ vtx. | 90-94 | $1.15\pm 0.12\pm 0.06$ | [58][b] |
| OPAL | $\Lambda_c\ell\&\Lambda\ell^-\ell^+$ | 90-95 | $1.29^{+0.24}_{-0.22}\pm 0.06$ | [59] |
| Average | | | $1.208 \pm 0.051$ | |

Table 18: *Measurements of the b-baryon lifetime.*
*a) The combined DELPHI result quoted in [55] is (1.14*$\pm$*0.08 $\pm$ 0.04) ps.*
*b) The combined OPAL result quoted in [58] is (1.16 $\pm$ 0.11 $\pm$ 0.06) ps.*
*The DELPHI and ALEPH* $\Xi\ell$ *results are not included in the quoted average since the selected data samples contain mostly* $\Xi_b$ *while the data samples in the other measurements contain mostly* $\Lambda_b$*. The combination of these two measurements gives :*
$\tau_{\Xi_b} = (1.39^{+0.34}_{-0.28})$ *ps.*
*Considering only the measurements obtained with* $\Lambda_c\ell$ *and* $\Lambda\ell^-\ell^+$ *correlations the average is :* $\tau_{\Lambda_b} = (1.229^{+0.081}_{-0.079})$ *ps.*
*Excluding the measurements obtained with* $\Lambda_c\ell$ *and* $\Lambda\ell^-\ell^+$ *correlations the average is :* $\tau_{b-\mathrm{baryon}} = (1.17^{+0.066}_{-0.054})$ *ps.*

| Experiment | Method | Data set | $\tau_B$ (ps) | Reference |
|---|---|---|---|---|
| ALEPH | Lepton i.p. (3D) | 91-93 | $1.533 \pm 0.013 \pm 0.022$ | [60] |
| L3 | Lepton i.p. (2D) | 91-94 | $1.544 \pm 0.016 \pm 0.021$ | [61][b] |
| OPAL | Lepton i.p. (2D) | 90-91 | $1.523 \pm 0.034 \pm 0.038$ | [62] |
| Average set 1 | | | $1.537 \pm 0.020$ | |
| ALEPH | Dipole | 91 | $1.511 \pm 0.022 \pm 0.078$ | [63] |
| ALEPH | Sec. vert. | 91-95 | $1.601 \pm 0.004 \pm 0.032$ | [64] |
| DELPHI | All track i.p.(2D) | 91-92 | $1.542 \pm 0.021 \pm 0.045$ | [65][a] |
| DELPHI | Sec. vert. | 91-93 | $1.582 \pm 0.011 \pm 0.027$ | [66][a] |
| L3 | Sec. vert. + i.p. | 91-94 | $1.556 \pm 0.010 \pm 0.017$ | [61][b] |
| OPAL | Sec. vert. | 91-94 | $1.611 \pm 0.010 \pm 0.027$ | [67] |
| SLD | Sec. vert. | 93 | $1.564 \pm 0.030 \pm 0.036$ | [68] |
| Average set 2 | | | $1.577 \pm 0.016$ | |
| Average sets 1-2 | | | $1.564 \pm 0.014$ | |
| CDF | J/$\psi$ vert. | 92-95 | $1.533 \pm 0.015\ ^{+0.035}_{-0.031}$ | [34] |

Table 19: *Measurements of the average b-hadron lifetime.*
*a) The combined DELPHI result quoted in [66] is (1.575 ± 0.010 ± 0.026) ps.*
*b) The combined L3 result quoted in [61] is (1.549 ± 0.009 ± 0.015) ps.*

| Experiment | Method | Data set | Ratio $\tau_+/\tau_0$ | Reference |
|---|---|---|---|---|
| ALEPH (n) | $D^{(*)}\ell$ | 91-95 | $1.080 \pm 0.062 \pm 0.018$ | [31] |
| ALEPH | Excl. rec. | 91-94 | $1.27^{+0.23+0.03}_{-0.19-0.02}$ | [32] |
| CDF | $D^{(*)}\ell$ | 92-95 | $1.110 \pm 0.056^{+0.033}_{-0.030}$ | [33] |
| CDF | Excl. (J/$\psi K$) | 92-95 | $1.06 \pm 0.07 \pm 0.02$ | [34] |
| DELPHI | $D^{(*)}\ell$ | 91-93 | $1.00^{+0.17}_{-0.15} \pm 0.10$ | [35] |
| DELPHI | Charge sec. vtx. | 91-93 | $1.06^{+0.13}_{-0.11} \pm 0.10$ | [36] |
| L3 | Charge sec. vtx. | 94-95 | $1.09 \pm 0.07 \pm 0.03$ | [38] |
| OPAL | $D^{(*)}\ell$ | 91-93 | $0.99 \pm 0.14^{+0.05}_{-0.04}$ | [40] |
| OPAL (n) | Charge sec. vtx. | 93-95 | $1.079 \pm 0.064 \pm 0.041$ | [41] |
| SLD | Charge sec. vtx. $\ell$ | 93-95 | $1.03^{+0.16}_{-0.14} \pm 0.09$ | [42] |
| SLD (n) | Charge sec. vtx. | 93-98 | $1.037^{+0.025}_{-0.024} \pm 0.024$ | [43] |
| Average | | | $1.065 \pm 0.023$ | |

Table 20: *Measurements of the ratio* $\tau_{B^+}/\tau_{B^0_d}$.

# D Measurements of *b*-hadron production rates.

## D.1 $B_s^0$ production rate.

| Quantity | Value | Ref. |
|---|---|---|
| $f_{B_s}$ BR($B_s^0 \to D_s^- \ell^+ \nu_\ell X$) BR($D_s^- \to \phi\pi^-$) | $(2.87 \pm 0.31 \pm {}^{0.25}_{0.41}) \times 10^{-4}$ | [85] |
| $f_{B_s}$ BR($B_s^0 \to D_s^- \ell^+ \nu_\ell X$) BR($D_s^- \to \phi\pi^-$) | $(4.2 \pm 1.9) \times 10^{-4}$ | [86] |
| $f_{B_s}$ BR($B_s^0 \to D_s^- \ell^+ \nu_\ell X$) BR($D_s^- \to \phi\pi^-$) | $(3.9 \pm 1.1 \pm 0.8) \times 10^{-4}$ | [87] |
| Average of the three measts. | $(3.00 \pm {}^{0.38}_{0.41}) \times 10^{-4}$ | |
| $f_{B_s}/(f_{B^+} + f_{B_d})$ BR($D_s^- \to \phi\pi^-$) | $(7.7 \pm 1.5) \times 10^{-3}$ | [88] |

Table 21: *Inputs used in the calculation of the* $B_s^0$ *production rate .*

All published results have been multiplied by the branching fraction for the decay $D_s^- \to \phi\pi^-$ to be independent of the assumed central value and uncertainty of this quantity; quoted uncertainties have been reevaluated accordingly.

The ALEPH measurement [90] of $f_{B_s}$ BR($B_s^0 \to D_s^- X$) BR($D_s^- \to \phi\pi^-$) $= (3.1 \pm 0.7 \pm 0.6) \times 10^{-3}$ has not been used to obtain an additional measurement on $f_{B_s}$ because of the model dependence attached to the evaluation of BR($B_s^0 \to D_s^- X$). Instead, the value of $f_{B_s}$, quoted in Table 4, can be used to extract, from this measurement, the inclusive branching fraction for $D_s^-$ production in $B_s^0$ decays:

$$\text{BR}(B_s^0 \to D_s^- X)\ \text{BR}(D_s^- \to \phi\pi^-) = (3.1 \pm 0.7 \pm 0.7) \times 10^{-2} \tag{77}$$

and, using the value for BR($D_s^- \to \phi\pi^-$) given in Table 1:

$$\text{BR}(B_s^0 \to D_s^- X) = 0.86 \pm 0.19 \pm 0.29. \tag{78}$$

## D.2 *b*-baryon production rate.

| Quantity | Value | Ref. |
|---|---|---|
| BR($b \to \Lambda_b^0$) BR($\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu_\ell} X$) BR($\Lambda_c \to pK^-\pi^+$) | $(3.78 \pm 0.31 \pm 0.23) \quad (10^{-4})$ | [91] |
| BR($b \to \Lambda_b^0$) BR($\Lambda_b^0 \to \Lambda_c^+ \ell^- \overline{\nu_\ell} X$) BR($\Lambda_c \to pK^-\pi^+$) | $(5.19 \pm 1.14 \pm {}^{1.19}_{0.66}) \quad (10^{-4})$ | [92] |
| Average of the two measts. | $(3.90 \pm 0.38)\ 10^{-4}$ | |
| $f_{b\text{-baryon}}/(f_{B^+} + f_{B_d})$ BR($\Lambda_c \to pK^-\pi^+$) | $(5.9 \pm 1.4) \times 10^{-3}$ | [88] |
| BR($b \to \Xi_b$) BR($\Xi_b^- \to \Xi^- \ell^- \overline{\nu_\ell} X$) | $(\ 5.4 \pm 1.1 \pm 0.8) \times 10^{-4}$ | [93] |
| BR($b \to \Xi_b$) BR($\Xi_b^- \to \Xi^- \ell^- \overline{\nu_\ell} X$) | $(\ 5.9 \pm 2.1 \pm 1.0) \times 10^{-4}$ | [94] |
| Average of the two measts. | $(5.53 \pm 1.17)\ 10^{-4}$ | |
| $f_{b\text{-baryon}}$ | $0.102 \pm 0.007 \pm 0.027$ | [95] |

Table 22: *Inputs used in the calculation of the b-baryon production rate .*

All published results have been multiplied by the branching fraction for the decay $\Lambda_c \to pK^-\pi^+$ to be independent of the assumed central value and uncertainty of this quantity; quoted uncertainties have been reevaluated accordingly.

## D.3 $B^+$ production rate.

| Quantity | Value | Ref. |
|---|---|---|
| $f_{B^+}$ | $0.414 \pm 0.016$ | [96] |

Table 23: *Direct measurement of the* $B^+$ *production rate .*

This value is obtained from the measurement of the production rate of charged weakly decaying $b$-hadrons. A small correction has been applied to account for $\Xi_b^-$ production, as given in Table D.2.

# E Theoretical uncertainties relevant to the measurements of $|V_{ub}|$ and $|V_{cb}|$.

At beginning of June 1999 a workshop entitled "Informal Workshop on the Derivation of $|V_{cb}|$ and $|V_{ub}|$: Experimental Status and Theory Uncertainties"[20] was held at CERN.

As, at that time, there was not a general consensus on the values for theoretical errors to be used to evaluate $|V_{cb}|$ and $|V_{ub}|$ [117], the main aim of this workshop has been to scrutinize the uncertainties attached to the different parameters and to define a common set of values to be adopted for the derivation of $|V_{ub}|$ and $|V_{cb}|$. The various contributing systematic uncertainties have been added in quadrature. This procedure is justified when several independent sources of uncertainties contribute, independently of their exact distributions, unless a single source clearly dominates. A linear sum can be justified if uncertainties are fully and positively correlated, which is not really the case of the presently considered quantities. In addition, theoretical errors correspond usually to a confidence level larger than 68% which is assumed when these uncertainties are taken as standard deviations of Gaussian distributions.

It is clear that the present procedure is not entirely satisfactory; it is simply pragmatic in the absence, at present, of a direct experimental control or of other evaluations with different theoretical techniques, of the main sources of systematic theoretical uncertainties. More details can be found in [130].

## E.1 Measurement of $|V_{ub}|$ using the decay $b \to \ell^- \overline{\nu_\ell} X_u$.

Based on studies developed independently by two groups [128, 129], a relative theoretical uncertainty of 5% has been evaluated for the extraction of $|V_{ub}|$ from the measured inclusive charmless semileptonic rate $\mathrm{BR}(b \to \ell^- \overline{\nu_\ell} X_u)$. The central value of the two analyses also agree[21] and the following relationship has been adopted in the extraction of $|V_{ub}|$:

$$\begin{aligned} |V_{ub}| = \quad & 0.00445 \left(\frac{\mathrm{BR}(b\to\ell^-\overline{\nu_\ell}X_u)}{0.002}\right)^{1/2} \left(\frac{1.55\ \mathrm{ps}}{\tau_B}\right)^{1/2} \\ & \times \left(1 \pm 0.010_{pert.} \pm 0.030_{1/m_b^3} \pm 0.035_{m_b}\right) \end{aligned} \tag{79}$$

There have been extensive discussions concerning the uncertainty to be attributed to the $b$-quark mass value $m_b$. The analyses reported in [131, 132] independently indicate a range of $\pm 0.060$ GeV/c$^2$ or less to be assigned to $m_b$. However during the workshop this narrow mass range raises concerns from a third independent analysis described in [133]. As a result of this ongoing discussion, finally, the value $m_b(1\ \mathrm{GeV}) = (4.58 \pm 0.060)\mathrm{GeV/c^2}$, where the error is meant to define the 68% confidence level, has been adopted for the derivations of $|V_{ub}|$ and $|V_{cb}|$[22].

[20] Participants at this workshop were: D. Abbaneo, P. Ball, E. Barberio, M. Battaglia, M. Beneke, I.I. Bigi, G. Buchalla, M. Calvi, O. Cooke, F. Defazio, L. di Ciaccio, R. Fleischer, P. Gagnon, P. Henrard, A. Hoang, L. Lellouch, J. Lu, S. Mele, E. Piotto, P. Rosnet, P. Roudeau, D. Rousseau, Ch. Schwick and F. Simonetto.

[21] In practice the initial central value quoted in [128] has been corrected and the value obtained in [129] is $|V_{ub}| = 0.00443 \left(\frac{\mathrm{BR}(b\to\ell\overline{\nu_\ell}X_u)}{0.002}\right)^{1/2} \left(\frac{1.55\ \mathrm{ps}}{\tau_b}\right)^{1/2} \times (1 \pm 0.020_{pert.} \pm 0.030_{m_b})$.

[22] As explained in Section E.3, in the inclusive determination of $|V_{cb}|$, all theoretical uncertainties have been multiplied by two, in a rather arbitrary way.

$m_b(\mu)$ has been defined in a way which leads to a linear dependence on $\mu$:

$$\frac{dm_b(\mu)}{d\mu} \;=\; -\,c_m\,\frac{\alpha_s(\mu)}{\pi} \;+\; .... \tag{80}$$

The number $c_m$ specifies the concrete definition within this general class; reasonable choices are $c_m \;=\; \frac{4}{3}$ or $\frac{16}{9}$.

There has been a claim [134] that a larger fractional uncertainty of $^{+13}_{-10}\%$, as well as a 7.5% lower central value for $|\mathrm{V}_{ub}|$ has to be used. From the arguments which have been developed at the workshop, it has been concluded that results of the analyses [128, 129] have been performed on well defined theoretical grounds and that they provide a reliable estimate of the theoretical uncertainties.

Recent measurements performed at LEP, based on the inclusive selection of $b \to \ell^-\overline{\nu_\ell}\mathrm{X}_u$ decays, are sensitive to a large fraction of the mass distribution of the hadronic system and to the full range of the lepton energy spectrum. This mass distribution $M_X$ has been studied by theorists [135] for the $b \to \ell^-\overline{\nu_\ell}\mathrm{X}_u$ transition. The conclusion was[23] that the additional theoretical uncertainty on $|\mathrm{V}_{ub}|$ from $m_b$ and $\mu_\pi^2$ is less than 10% if the experimental inclusive technique is sensitive up to at least $M_X \simeq 1.5$ GeV/c$^2$.

It has thus been concluded that theoretical errors attached to the determination of $|\mathrm{V}_{ub}|$ using the present experimental technique are under control.

## E.2 Measurement of $|\mathrm{V}_{cb}|$ using the decay $\overline{\mathrm{B}^0_\mathrm{d}} \to \mathrm{D}^{*+}\ell^-\overline{\nu_\ell}$.

It has been proposed to use the parametrization given in [136] to account for the dependence in $w$ and to extract the value at $w = 1$ of the differential decay rate. The quantity $\mathcal{F}_{D^*}(1)\,|\mathrm{V}_{cb}|$ is then obtained.

In order to measure $|\mathrm{V}_{cb}|$, the value at $w = 1$ of the form factor is taken from theory by evaluating different corrections [137] which have to be applied to the naive expectation of $\mathcal{F}_{D^*}(1) = 1$:

$$|\mathcal{F}_{D^*}(1)|^2 \;=\; \xi_A(\mu) - \Delta^A_{1/m^2} - \Delta^A_{1/m^3} - \sum_{0<\epsilon_i<\mu} |\mathcal{F}_i|^2 \tag{81}$$

The central value of $\mathcal{F}_{D^*}(1)$ has been recently lowered by 2% by evaluating higher order perturbative corrections [138].

Central value and uncertainty for the $b$ quark mass are the same as used in Section E.1.

The charm quark mass is obtained from the mass difference $m_b - m_c$, inferred from the measured values of the spin averaged beauty and charm mesons masses:

$$\begin{array}{rcl} m_b \;-\; m_c \;= & \langle M_B\rangle - \langle M_C\rangle \;+\; \mu_\pi^2\left(\frac{1}{2m_c} - \frac{1}{2m_b}\right) \;+\; \mathcal{O}\left(1/m^2_{c,b}\right) \\ = & (\;3.50 \;+\; 0.040\frac{\mu_\pi^2 - 0.5}{0.1} \;+\; \Delta M_2\;)\;(\mathrm{GeV/c^2}) \end{array} \tag{82}$$

The main difference between the theoretical analyses given in [137] and [139] comes from the evaluation of the uncertainties on non-perturbative corrections. A different approach can be found in [140].

[23] In practice experimental analyses take into account the expected mass distribution of the hadronic system and the variation of the detection efficiencies versus this mass, to evaluate the real importance of this systematic uncertainty.

The detailed balance of uncertainties given in [137], revised by L. Lellouch and P. Roudeau at this workshop for consistency with the values adopted for the different parameters, is the following:

$$\mathcal{F}_{D^*}(1) \;=\; 0.880 - 0.024\frac{\mu_\pi^2 - 0.5\ \mathrm{GeV}^2}{0.1\ \mathrm{GeV}^2} \pm 0.035_{excit.} \pm 0.010_{pert.} \pm 0.025_{1/m^3} \qquad (83)$$

In the following, are reviewed the hypotheses or results on which the contributions of the different terms are based.

### E.2.1 $\mu_\pi^2$.

The value $\mu_\pi^2 = (0.5 \pm 0.1)\ \mathrm{GeV}^2$ has been used and it gives $\pm 0.024$ uncertainty on $\mathcal{F}_{D^*}(1)$.

From QCD sum rules, the following value has been obtained [141]:

$$\mu_\pi^2 = (0.5 \pm 0.1)\ \mathrm{GeV}^2 \qquad (84)$$

and a model-independent lower bound has been established [142]:

$$\mu_\pi^2 > \mu_G^2 \simeq \frac{4}{3}(M_{B^*}^2 - M_B^2) \approx 0.4\ \mathrm{GeV}^2 \qquad (85)$$

### E.2.2 High mass excitations.

In QCD sum rules, used in [137, 139], the effect expected from high mass hadronic states has been introduced in a rather arbitrary way. In [137], it has been assumed that their effect, on the correction terms which behave as $m_Q^{-2}$, can vary between 0 and 100% of $\Delta^A_{1/m^2}$. Such a variation corresponds to $\pm 0.035$ on $\mathcal{F}_{D^*}(1)$.

### E.2.3 Higher order non-perturbative corrections.

The value of $\pm 0.025$ uncertainty related to the contribution from terms of order at least $m_Q^{-3}$ is considered as being optimistic from the authors of [137].

### E.2.4 Adopted value.

Combining the uncertainties quoted in (83) gives:

$$\mathcal{F}_{D^*}(1) \;=\; 0.88 \;\pm 0.05. \qquad (86)$$

### E.2.5 Expected improvements in the control of theoretical errors.

The uncertainty attached to "excited states" is expected to be decreased if better measurements of $D^{**}$ production rates and hadronic mass distributions for these states are obtained[24].

The uncertainty related to $\mu_\pi^2$ is expected to decrease by measuring moments of the lepton momentum in the B rest frame.

[24]By $D^{**}$ all decay modes which are not $\overline{B} \to D\ell^-\overline{\nu_\ell}$ and $\overline{B} \to D^*\ell^-\overline{\nu_\ell}$ have been implied.

### E.2.6 Form factors for D** production near to $w = 1$.

It has been realized that the $w$-dependence of form factors used to describe D** production in $b$-hadron semileptonic decays is of importance to evaluate the systematic uncertainty attached to $|V_{cb}|$ related to the fraction of D* mesons coming from D** decays. It has been proposed to use the model described in [19] which predicts a rate for the ratio of $2^+$ over $1^+$ narrow states more in agreement with corresponding experimental results. Additional information, on D** production which may bring constraints on this type of models is welcome.

## E.3 Measurement of $|V_{cb}|$ using the inclusive semileptonic decay $b \to \ell^- X$ rate.

The expression relating $|V_{cb}|$ to the inclusive semileptonic branching fraction can be found in [137]:

$$\begin{aligned} |V_{cb}| = \quad & 0.0411 \left(\frac{BR(b\to\ell^-\overline{\nu_\ell}X_c)}{0.105}\right)^{1/2} \left(\frac{1.55\ ps}{\tau_b}\right)^{1/2} \\ & \times \left(1 - 0.012\frac{\mu_\pi^2 - 0.5 GeV^2}{0.1 GeV^2}\right) \\ & \times \left(1 \pm 0.015_{pert.} \pm 0.010_{m_b} \pm 0.012_{1/m_Q^3}\right) \end{aligned} \tag{87}$$

The central value of reference [137] has been lowered by 2% to account for a different choice of the $b$-quark mass. A very similar result on the central value and corresponding uncertainties can be found in [129].

### E.3.1 Uncertainties related to quark masses.

It has been assumed that $m_b(\mu)$ can be determined with an uncertainty of $\pm 60$ MeV/c$^2$, as discussed in Section E.1, and that the difference between the $b$- and $c$-quark masses is known with an uncertainty of $\pm 40$ MeV/c$^2$, related to the error on $\mu_\pi^2$. These variations induce $\pm 0.010$ and $\pm 0.012$ uncertainties on $|V_{cb}|$, respectively.

### E.3.2 Adopted value.

Adding in quadrature the quoted uncertainties in Equation (87) leads to a $\pm 2.5\%$ relative uncertainty on $|V_{cb}|$. Since these errors have not been cross-checked by other theoretical approaches or experimental measurements, **we decided to inflate the total error by an arbitrary factor of two**, leading to a theoretical uncertainty of $\pm 5\%$ (see reference [130] for a more detailed discussion). This implies, in practice, that uncertainties of $\pm 120$ MeV/c$^2$, $\pm 80$ MeV/c$^2$ and $\pm 0.2$ GeV$^2$ have been attached to the respective determinations of $m_b$, $m_b - m_c$ and $\mu_\pi^2$.

## E.4 Common sources of theoretical errors for the two determinations of $|V_{cb}|$.

Theoretical uncertainties attached to the exclusive and inclusive measurements of $|V_{cb}|$, from these two methods, are largely uncorrelated. When evaluating the average, only uncertainties related to $\mu_\pi^2$ and $m_b$ have been considered as fully correlated.

## E.5 Sources of theoretical errors entering into the measurement of the ratio $\frac{|V_{ub}|}{|V_{cb}|}$.

In the evaluation of the ratio $\frac{|V_{ub}|}{|V_{cb}|}$, several common experimental and model systematics have a reduced effect.

For the ratio $\frac{|V_{ub}|_{incl}}{|V_{cb}|_{incl}}$, uncertainties originating from the determination of $m_b$ have to be considered as fully correlated. Leading effects of order $\mathcal{O}(1/m^3)$ are expected to be uncorrelated between $|V_{ub}|$ and $|V_{cb}|$ determinations [130]. This is also expected for perturbative uncertainties.

With the conventions used in the previous sections, the relative errors on the ratio $\frac{|V_{ub}|_{incl}}{|V_{cb}|_{incl}}$ are:

$$\pm 0.015(m_b) \ \pm 0.032(pert.) \ \pm 0.024(\mu_\pi^2) \pm 0.038(1/m^3) \ = \ \pm 0.06 \qquad (88)$$

For the ratio $\frac{|V_{ub}|_{incl}}{|V_{cb}|_{excl}}$, all uncertainties can be considered as uncorrelated giving:

$$\pm 0.035(m_b) \ \pm 0.014(pert.) \ \pm 0.024(\mu_\pi^2) \pm 0.039(1/m^3) \pm 0.035(excit.) \ = \ \pm 0.07 \qquad (89)$$

In practice, the ratio $\frac{|V_{ub}|}{|V_{cb}|}$ will be obtained using the average of the two measurements of $|V_{cb}|$ and taking into account common systematics with $|V_{ub}|$.

## E.6 Conclusions and summary.

The conclusions of the workshop which are given in this Appendix have left aside several subtleties concerning the exact meaning of the parameters entering into the different expressions for $|V_{ub}|\,(incl.)$, $|V_{cb}|\,(incl.)$ and $|V_{cb}|\,(excl.)$. The interested reader has therefore to consult the documents quoted in the references for more information.

Two groups (at least) have obtained consistent results on central values and uncertainties for $|V_{ub}|\,(incl.)$ and $|V_{cb}|\,(incl.)$.

Uncertainties on $|V_{cb}|\,(incl.)$ have been enlarged, in a rather arbitrary way, by a factor two to have some margin because of possible additional contributions (reliability of $1/m_c$ expansion, need for other techniques to evaluate $m_b$,...).

Dedicated experimental studies on $D^{**}$ production and on the distributions of moments of the lepton momentum in the B rest frame are needed to evaluate with better accuracy and greater confidence the most important sources of systematic errors contributing in the theoretical expressions.

As said in the introduction, this proposal is a first conservative attempt, and it is hoped that in the near future quoted values for the uncertainties can be reduced and/or corrected for possible mistakes.

The following central values and theoretical uncertainties have been used in the measurements of $|V_{ub}|$ and $|V_{cb}|$ obtained from combined LEP analyses done in Summer 1999:

- inclusive measurement of $|V_{ub}|$:

$$\begin{aligned} |V_{ub}| = \quad & 0.00445 \left(\frac{\mathrm{BR}(b\to \ell^- \overline{\nu_\ell} X_u)}{0.002}\right)^{1/2} \left(\frac{1.55\ \mathrm{ps}}{\tau_b}\right)^{1/2} \\ & \times \left(1 \pm 0.010_{pert.} \pm 0.030_{1/m_b^3} \pm 0.035_{m_b}\right) \end{aligned} \qquad (90)$$

- inclusive measurement of $|\mathrm{V}_{cb}|$:

$$\begin{aligned} |\mathrm{V}_{cb}| = \quad & 0.0411 \left(\frac{\mathrm{BR}(b\to\ell^-\overline{\nu_\ell}\mathrm{X}_c)}{0.105}\right)^{1/2} \left(\frac{1.55\ \mathrm{ps}}{\tau_b}\right)^{1/2} \\ & \times \left(1 - 0.024\frac{\mu_\pi^2 - 0.5\mathrm{GeV}^2}{0.2\mathrm{GeV}^2}\right) \\ & \times \left(1 \pm 0.030_{pert.} \pm 0.020_{m_b} \pm 0.024_{1/m_Q^3}\right) \end{aligned} \tag{91}$$

- exclusive measurement of $|\mathrm{V}_{cb}|$:

$$\mathcal{F}_{D^*}(1) \ = \ 0.880 - 0.024\frac{\mu_\pi^2 - 0.5\ \mathrm{GeV}^2}{0.1\ \mathrm{GeV}^2} \pm 0.035_{excit.} \pm 0.010_{pert.} \pm 0.025_{1/m^3} \tag{92}$$

All uncertainties have been added in quadrature.

The correlated uncertainty between $|\mathrm{V}_{cb}|\,(incl.)$ and $|\mathrm{V}_{cb}|\,(excl.)$ measurements stem from the effect of $\mu_\pi^2$.

The correlated uncertainty between $|\mathrm{V}_{ub}|\,(incl.)$ and $|\mathrm{V}_{cb}|\,(incl.)$ measurements is limited to the effect of $m_b$. Hence no correlated uncertainty between $|\mathrm{V}_{ub}|\,(incl.)$ and $|\mathrm{V}_{cb}|\,(excl.)$ measurements is assumed.