CHIPS '99

Glauber model
and
centrality estimators
in
nucleus-nucleus collisions

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Glauber model

[ref : "High Energy Collision Theory", in "Lectures in Theoretical Physics", Vol. I, Interscience Publishers, New York, 1959, pp. 315-414.]

- Nucleons inside a nucleus are independent.
- The nuclear ground state wave function factorizes:

$$|\psi_A(\vec{q}_1, \ldots, \vec{q}_A)|^2 \;=\; \prod_{i=1}^{A} \rho_A^{(i)}(\vec{q}_i)$$

$\rho_i(\vec{q}_i) =$ normalized, single-particle density for the i-th nucleon.

- The range r of interaction between colliding nucleons is much smaller than the nuclear radius.
- Protons and neutrons are indistinguishable particles.

Other simplifications

- Spin degrees of freedom are neglected.
- Optical limit (nucleon size $\ll$ nucleus size)
- Small angle, very high energy scattering.
- Nucleus-nucleus collision = superposition of N-N collisions

Nucleon-Nucleon Collisions

Probability of a collision between two nucleons at impact parameter b :

$$\tau(\vec{b}) = \sigma_{_0}\, t(\vec{b}) \ .$$

- $t(\vec{b})$ is the thickness function, normalized :

$$\int \mathrm{d}^2 b \; t(\vec{b}) = 1$$

- N-N inelastic cross-section without diffractive component :

$$\sigma_{_0} = 30 \text{ mb} \ .$$

- Unpolarized nucleons $\Rightarrow t(\vec{b}) = t(b)$

- A frequent approximation is :

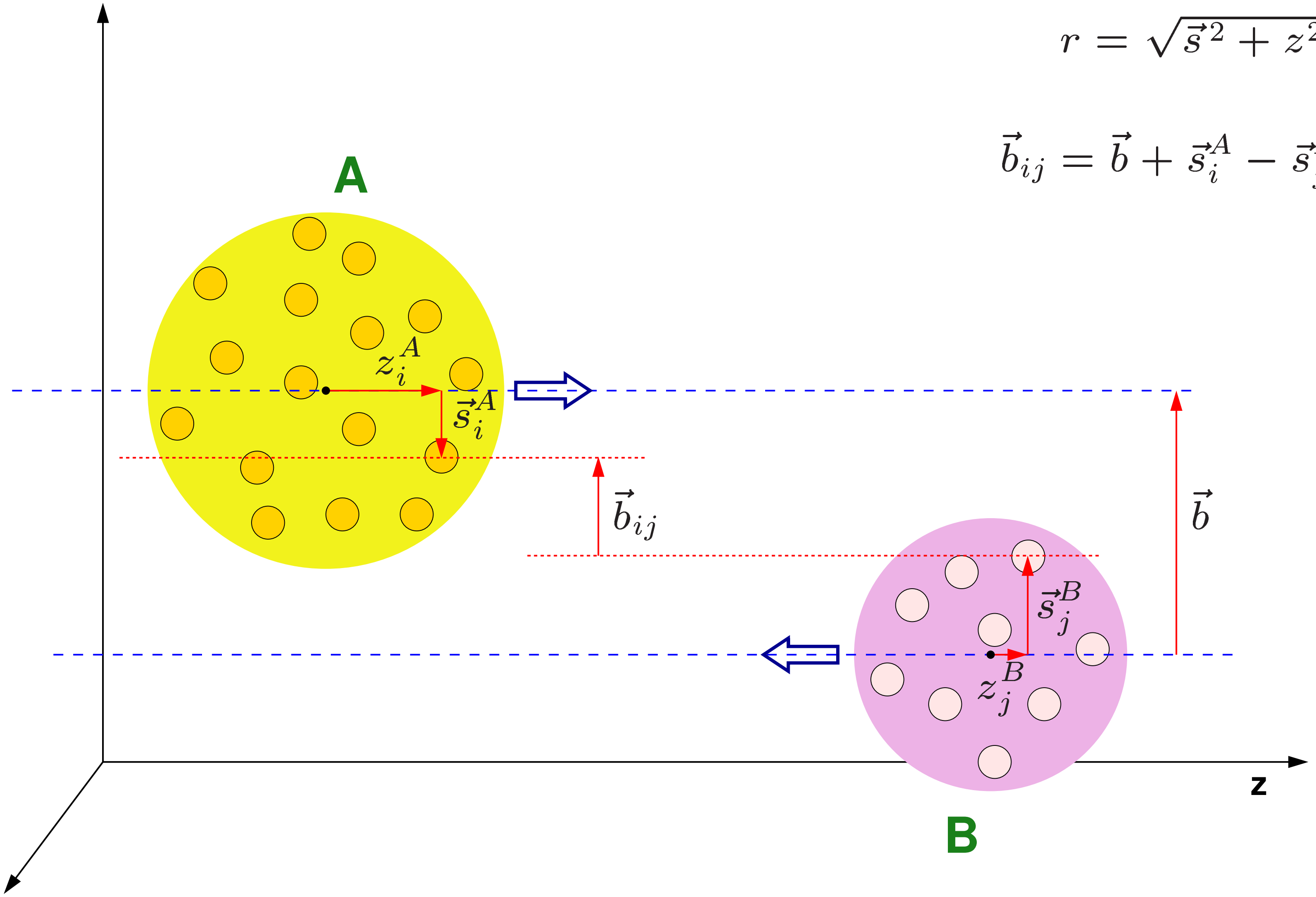
$$t(\vec{b}) = \delta^{(2)}(\vec{b}) \, .$$

Definitions of variables in a nucleus A-nucleus B collision

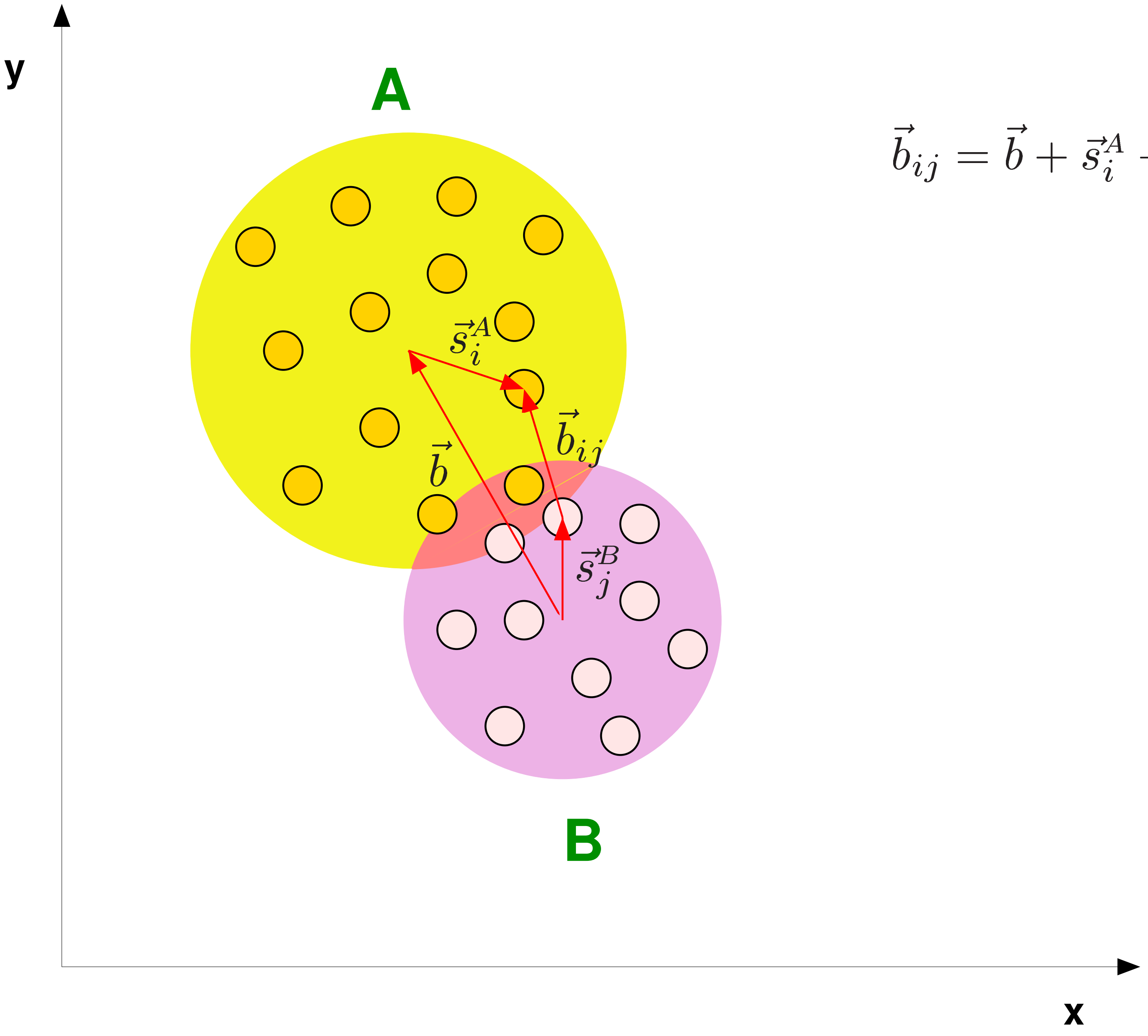
$$\rho(r) = \rho(\vec{s}, z)$$

$$r = \sqrt{\vec{s}^{\,2} + z^2}$$

$$\vec{b}_{ij} = \vec{b} + \vec{s}_i^{\,A} - \vec{s}_j^{\,B}$$



Definitions of variables in a nucleus A-nucleus B collision



$$\vec{b}_{ij} = \vec{b} + \vec{s}_i^A - \vec{s}_j^B$$

Nucleus-Nucleus Collisions

A nuclear configuration is defined by the set of coordinates of its constituent nucleons. For instance, for nucleus A :

$$\{(\vec{s}_1^{\,A}, z_1^A), ..., (\vec{s}_i^{\,A}, z_i^A), ..., (\vec{s}_A^{\,A}, z_A^A)\} \Longrightarrow \{(\vec{s}_i^{\,A}, z_i^A)\}$$

The probability of finding a nucleon in the volume element $\mathrm{d}^2 s \, \mathrm{d}z$ at the position $(\vec{s}, z)$ is :

$$\rho_A(\vec{s}_A, z_A)\, \mathrm{d}^2 s_A \, \mathrm{d}z_A \;\; ;$$

normalization : $\int \rho_A(\vec{s}_A, z_A)\, \mathrm{d}^2 s_A \, \mathrm{d}z_A = 1$.

Forward scattering :

- the transverse coordinates $\{\vec{s}_i^{\,A}\}$ do not change after a collision
- the longitudinal coordinates $\{z_i^A\}$ are not relevant !

In this approximation, the nuclear configuration is determined simply by the transverse coordinates : $\{\vec{s}_i^{\,A}\}$.

The nuclear thickness function :

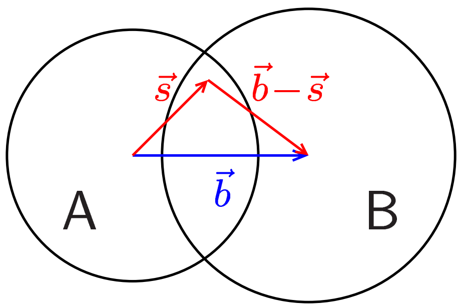
$$T_A(\vec{s}^A) = \int_{-\infty}^{\infty} \mathrm{d}z^A \, \rho(\vec{s}^A, z^A)$$

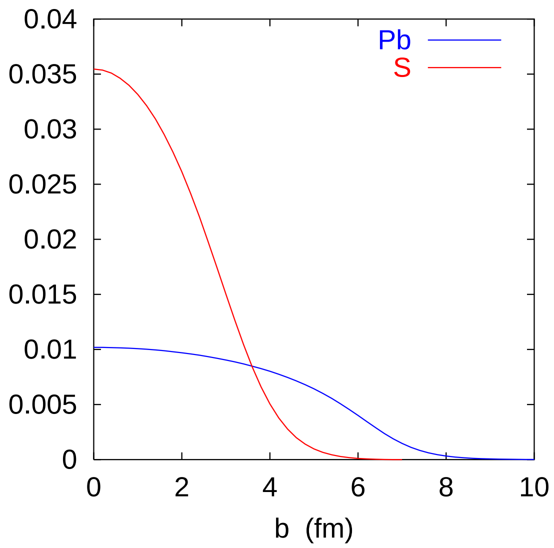
is the probability density of finding a nucleon in nucleus A at transverse coordinate $\vec{s}^A$. It is normalized :

$$\int \mathrm{d}^2 s^A \, T_A(s^A) = 1$$

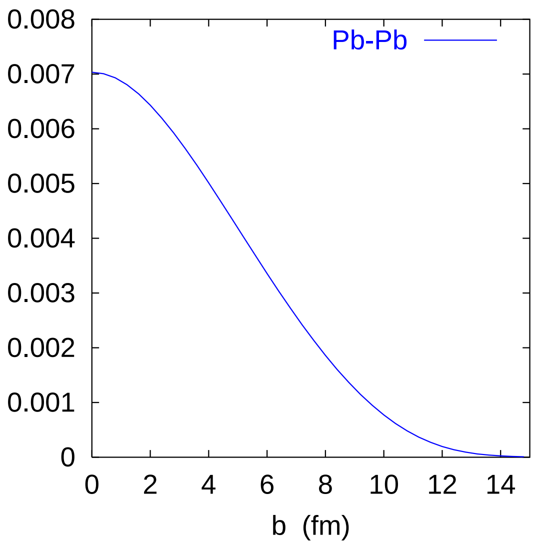
The nuclear overlap function is defined as :

$$T_{AB}(\vec{b}) = \int \mathrm{d}^2 s \, T_A(\vec{s}) \, T_B(\vec{b} - \vec{s}) \ .$$





$T_A(b)$



$T_{AB}(b)$ for Pb-Pb

Calculation of the number of Collisions

Probability of the occurrence of a N-N inelastic collision when the nuclei A and B collide with impact parameter $\vec{b}$:

$$
\begin{aligned}
P_1(\vec{b}) \; &= \; \int \mathrm{d}^2 s_A \mathrm{d} z_A \rho_A(\vec{s}_A, z_A) \; \mathrm{d}^2 s_B \mathrm{d} z_B \rho_B(\vec{s}_B, z_B) \times \\
& \quad \times \tau(\vec{b} + \vec{s}_A - \vec{s}_B) = \\
&= \; \int \mathrm{d}^2 s_A \; \mathrm{d}^2 s_B \; T_A(\vec{s}_A) \; T_B(\vec{s}_B) \; \sigma_0 \, t(\vec{b} + \vec{s}_A - \vec{s}_B) = \\
&= \; \sigma_0 \int \mathrm{d}^2 s T_A(\vec{s}) \; T_B(\vec{b} - \vec{s}) \; = \; \sigma_0 \; T_{AB}(\vec{b})
\end{aligned}
$$

Probability of the occurrence of ν inelastic collisions :

$$
P_\nu(b) = \begin{pmatrix} AB \\ \nu \end{pmatrix} \left[\sigma_0 \; T_{AB}(b)\right]^\nu \; \left[1 - \sigma_0 \; T_{AB}(b)\right]^{AB-\nu}
$$

Probability of a nucleus-nucleus inelastic interaction at impact parameter $\vec{b}$:

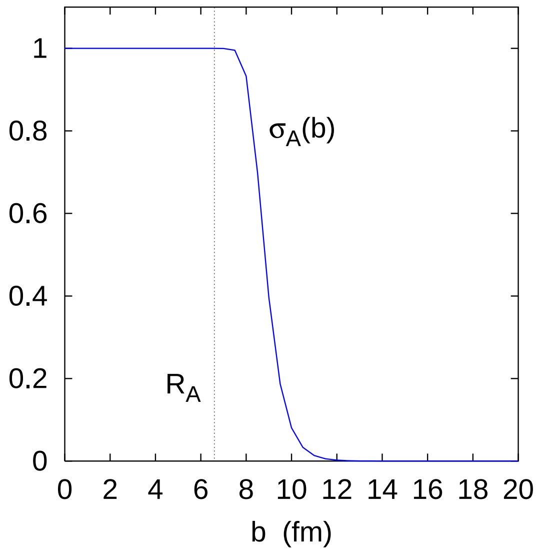
$$
\sigma_{AB}(b) \equiv \frac{\mathrm{d}\sigma_{AB}}{\mathrm{d}b^2} = \sum_{\nu=1}^{AB} P_\nu(b) = 1 - \left[1 - \sigma_0 \; T_{AB}(b)\right]^{AB}
$$

Average number of inelastic N-N collisions (fixed $\vec{b}$) :

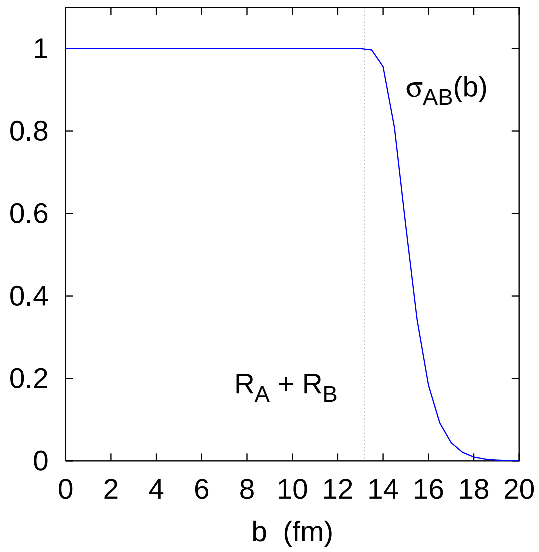
$$\langle \nu(b) \rangle = \sum_{k=1}^{AB} k P_k(b) = \sigma_0 \; AB \; T_{AB}(b)$$

Average number of inelastic N-N collisions, under the additional condition of the occurrence of an inelastic A-B collision (conditional probability) :

$$N_c(b) = \frac{\sum_{k=1}^{AB} k P_k(b)}{\sum_{k=1}^{AB} P_k(b)} = \frac{\sigma_0 \; AB \; T_{AB}(b)}{\sigma_{AB}(b)} \simeq \sigma_0 \; AB \; T_{AB}(b)$$



$\sigma_A(b)$ for p-Pb



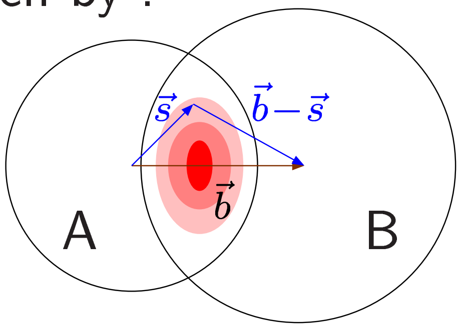
$\sigma_{AB}(b)$ for Pb-Pb

Average over the impact parameter :

$$\langle N \rangle = \frac{\int \mathrm{d}^2 b\, N_c(b)\, \sigma_{AB}(b)}{\int \mathrm{d}^2 b\, \sigma_{AB}(b)} = $$
$$= \frac{\sigma_0\, AB}{\sigma_{AB}} \int \mathrm{d}^2 b\, T_{AB}(b) = \frac{\sigma_0\, AB}{\sigma_{AB}}$$

The local collision density $n_c(\vec{b}, \vec{s})$ is given by :

$$n_c(\vec{b}, \vec{s}) = \frac{\sigma_0\, AB}{\sigma_{AB}(b)}\, T_A(\vec{s})\, T_B(\vec{b} - \vec{s})$$



In p-A collision : B = 1, $T_B(\vec{b}) = \delta^{(2)}(\vec{b}) \Longrightarrow T_{AB}(\vec{b}) = T_A(\vec{b})$

$$\frac{\mathrm{d}\sigma_{pA}}{\mathrm{d}b^2} = 1 - \left[1 - \sigma_0\, T_A(b)\right]^A$$
$$\langle \nu(b) \rangle = \sigma_0\, A\, T_A(b)$$

Wounded nucleons

A nucleon is wounded if it underwent at least one inelastic interaction during a nuclear collision.

[ref : A. Białas et al., Nucl. Phys. B **111** (1976) 461-176]

Computation of the number of wounded nucleons

Probability of collision of nucleon 1 of nucleus B (fixed $\{s_j^B\}$) with anyone of the nucleons of A (fixed $\{s_i^A\}$), at impact parameter $\vec{b}$:

$$p_A\left(s_1^B; b; \{s_i^A\}\right) = 1 - \prod_{i=1}^{A}\left[1 - \tau(\vec{b} + \vec{s}_i^A - \vec{s}_1^B)\right] \equiv p_A(s_1^B) \ .$$

Probability that the nucleons $s_1^B, \ldots, s_{w_B}^B$ collide and $s_{w_B+1}^B, \ldots, s_B^B$ do not :

$$p_A(s_1^B) \ldots p_A(s_{w_B}^B)\ [1 - p_A(s_{w_B+1}^B)] \ldots [1 - p_A(s_B^B)]$$

Probability of having exactly w_B wounded nucleons in B :

$$P_{AB}^{w_B}\left(b; \{s_i^A\}\right) = \begin{pmatrix} B \\ w_B \end{pmatrix} \int d^2 s_1^B \ldots d^2 s_B^B\ T_B(s_1^B) \ldots T_B(s_B^B)\ \times$$

$$\times\ p_A(s_1^B) \ldots p_A(s_{w_B}^B)\ [1 - p_A(s_{w_B+1}^B)] \ldots [1 - p_A(s_B^B))] =$$

$$= \begin{pmatrix} B \\ w_B \end{pmatrix} \left[\bar{p}_A\left(b; \{s_i^A\}\right)\right]^{w_B} \left[1 - \bar{p}_A\left(b; \{s_i^A\}\right)\right]^{B - w_B}$$

where $$\bar{p}_A\left(b; \{s_i^A\}\right) \equiv \int d^2 s^B\ T_B(s^B) p_A\left(s^B; b; \{s_i^A\}\right) \ .$$

Finally, we integrate over the configurations of A :

$$P_{AB}^{w_B}(b) = \int \mathrm{d}^2 s_1^A \ldots \mathrm{d}^2 s_A^A \; T_A(s_1^A) \ldots T_A(s_A^A) \; P_{AB}^{w_B}\left(b; \{s_i^A\}\right) \; .$$

For $A = 1$ (proton-nucleus) : $P_{pB}^{w_B} = \int \mathrm{d}^2 s \; t(s) P_{pB}^{w_B}(b; s)$.

If $A > 1$, $P_{AB}^{w_B}(b)$ can be evaluated only for very small nuclei
$\Longrightarrow$ we need an approximation ! (optical limit) .

Average number of wounded nucleons in an A-B collision :

$$N_{AB}^{w} \equiv \langle w_{AB} \rangle = \langle w_A \rangle + \langle w_B \rangle$$

Average number of wounded nucleons in nucleus B

$$\langle w_B(b) \rangle = \frac{\sum_{w_B=1}^{B} w_B \; P_{AB}^{w_B}(b)}{\sum_{w_B=1}^{B} \; P_{AB}^{w_B}(b)}$$

The normalization is the probability of interaction between nucleus A and nucleus B, at impact parameter $\vec{b}$:

$$\sigma_{AB}(b) = \sum_{w_B=1}^{B} \; P_{AB}^{w_B}(b) = 1 - P_{AB}^{w_B=0}(b) =$$

$$= 1 - \int \mathrm{d}^2 s_1^A \ldots \mathrm{d}^2 s_A^A \; T_A(s_1^A) \ldots T_A(s_A^A) \left[1 - \bar{p}_A(b; s_i^A)\right]^B$$

In the optical limit (large A,B) :

[ref : A. Białas et al., Nucl. Phys **B 111** (1976) 461-476; C. Pajares and A.V. Ramallo, Phys. Rev. **D 31** (1985) 2800-2812; W. Czyż and L.C. Maximon, Annals of Physics **52** (1969) 59-121]

$$\sigma_{AB}^{opt} = 1 - \left\{1 - \sigma_0 T_{AB}(b)\right\}^{AB}$$

The number of wounded nucleons in B is :

$$\langle w_B(b)\rangle = \frac{1}{\sigma_{AB}(b)}\, B \int \prod_{k=1}^{A} \mathrm{d}^2 s_k^A\, T_A(s_k^A)\, \bar{p}_A(b; s_i^A) =$$

$$\left[\text{using} \qquad \tau(\vec{b} + \vec{s}_i^A - \vec{s}^B) = \sigma_0\, \delta^{(2)}(\vec{b} + \vec{s}_i^A - \vec{s}^B)\right]$$

$$= \frac{B}{\sigma_{AB}(b)} \int \mathrm{d}^2 s^B\, T_B(\vec{s}^B) \left\{1 - \left[1 - \sigma_0\, T_A(\vec{b} - \vec{s}^B)\right]^A\right\} =$$

$$= \frac{B}{\sigma_{AB}(b)} \int \mathrm{d}^2 s^B\, T_B(\vec{s}^B) \sigma_A(\vec{b} - \vec{s})$$

The total number of wounded nucleons in an A-B collision at impact parameter $\vec{b}$ is :

$$N_{AB}^{w}(b) = \frac{\int \mathrm{d}^2 s \left[A\, T_A(\vec{b} - \vec{s})\, \sigma_B(\vec{s}) + B\, T_B(\vec{s})\, \sigma_A(\vec{b} - \vec{s})\right]}{\sigma_{AB}(b)}$$

Average over impact parameter :

$$\langle w_B \rangle = \frac{\int \mathrm{d}^2 b \, \langle w_B(b) \rangle \, \sigma_{AB}(b)}{\int \mathrm{d}^2 b \, \sigma_{AB}(b)} = \frac{B \, \sigma_A}{\sigma_{AB}}$$

where $\sigma_A = \int \mathrm{d}^2 b \{1 - [1 - \sigma_0 \, T_A(b)]^A\}$ is the total proton-nucleus A cross section.

From

$$N^w_{AB}(b) = \int \mathrm{d}^2 s \, \frac{\mathrm{d}^2 N^w_{AB}}{\mathrm{d}s^2}(b)$$

we see that the local wounded nucleon density is given by :

$$\begin{aligned} n^w_{AB}(\vec{b}, \vec{s}) &= n^w_{AB}(b, s, \frac{\vec{b} \cdot \vec{s}}{bs}) \equiv \frac{\mathrm{d}^2 N^w_{AB}}{\mathrm{d}s^2}(b) = \\ &= \frac{1}{\sigma_{AB}(b)} \left\{ A T_A(\vec{s}) \left[1 - \left(1 - \sigma_0 \, T_B(\vec{b} - \vec{s}) \right)^B \right] + \right. \\ &\qquad \left. + B \, T_B(\vec{b} - \vec{s}) \left[1 - \left(1 - \sigma_0 \, T_A(\vec{s}) \right)^A \right] \right\} = \\ &= \frac{A \, T_A(\vec{s}) \, \sigma_B(\vec{b} - \vec{s}) + B \, T_B(\vec{b} - \vec{s}) \, \sigma_A(\vec{s})}{\sigma_{AB}(b)} \end{aligned}$$

$E_T \longleftrightarrow b$ correlation

$E_T = \varepsilon N_h$

$N_h \propto \ ?$ $\Leftarrow$ input from some model

$N_h = \alpha N_w$ consistent with exp. data

$N_h = \beta N_c$ unrealistic !

$N_h = \alpha N_w + \beta N_c$??? *small contributions ($\beta \ll \alpha$) from collisions can not be excluded at SPS energy*

In the wounded nucleon model :

$$E_T = (\varepsilon\alpha)N_w = qN_w$$

Fluctuations

Two sources of fluctuations :

- The number of participants varies, for a given b, around the mean value.
- The transverse energy per wounded nucleon has a dispersion around q.

It is reasonable to assume that $E_T(b)$ is distributed according to a gaussian law, around its mean value $\langle qN_w(b)\rangle$, with a dispersion $(\Delta N_w)^2 = aN_w(b)$:

$$\mathcal{P}(E_T, b) = \frac{1}{\sqrt{2\pi\, a\, q^2\, N_w(b)}} \exp\left\{-\frac{\left[E_T - q\, N_w(b)\right]^2}{2\, a\, q^2\, N_w(b)}\right\} ,$$

normalized according to :

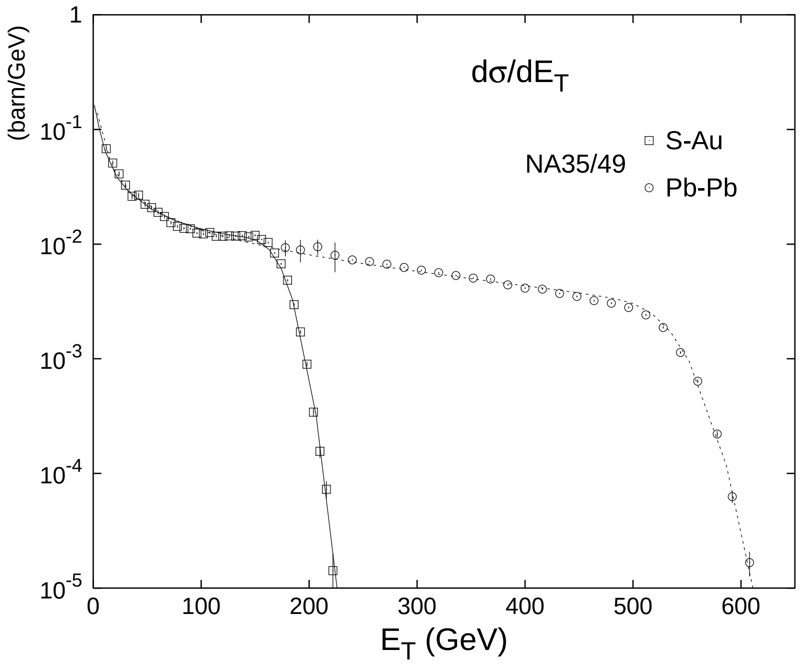
$$\int \mathrm{d}E_T\, \mathcal{P}(E_T, b) = 1$$

Minimum bias E_T distribution

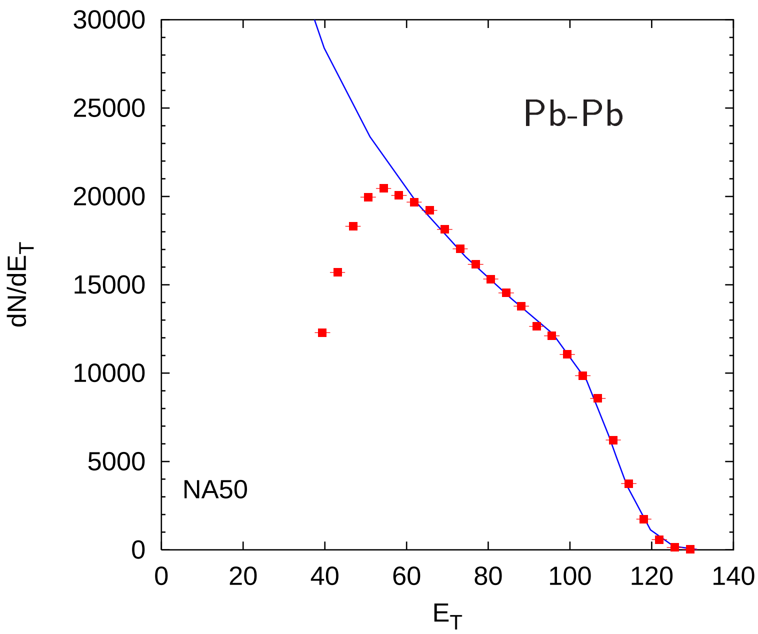
$$\frac{\mathrm{d}\sigma_{mb}}{\mathrm{d}E_T} = \int \mathrm{d}^2 b\, \mathcal{P}(E_T, b)\, \sigma_{AB}(b)$$

Examples :

$q = 1.5$ GeV (S-Au),
$q = 1.4$ GeV (Pb-Pb);
$a = 1$



$q = 0.285$ GeV ;
$a = 1.5$



Drell-Yan pair production

Hard process : it takes place, with constant probability σ_0^h/σ_0 (if no energy loss...) at each N-N collision.

$$\begin{aligned}
\frac{\mathrm{d}\sigma_{AB}^h}{\mathrm{d}E_T} &= \frac{\sigma_0^h}{\sigma_0} \int \mathrm{d}^2b\, \mathcal{P}(E_T, b)\, N_c(b)\sigma_{AB}(b) = \\
&= \frac{\sigma_0^h}{\sigma_0} \int \mathrm{d}^2b\, \mathcal{P}(E_T, b)\, \sigma_0\, AB\, T_{AB}(b) \\
\sigma_{AB}^h &= \sigma_0^h \int \mathrm{d}^2b\, T_{AB}(b)\, AB = \sigma_0^h\, AB\ .
\end{aligned}$$