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Triple gauge boson parameters.

A note upon the relations between different parameterisations.

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Abstract

There are at least two common parameterisations which one will come across in TGC analysis. The relation between these parameters can be at times confusing since in at least one case there is a different use of the variable known as κ_z, and as a result apparenty mutually incompatible relations appear in different publications.

In this note we attempt to clarify some of these relations.

1 Introduction and executive summary

In this first instance of this note we concern ourselves with two common TGC parameterisations.

- SET1: the formalism which appears in the LEP2 yellow report [1] page 525, Hagiwara et. al. [2] and Gaemers and Gounaris [3]. This uses: $\Delta g_1^z, \Delta\kappa_\gamma, \Delta\kappa_z, \lambda_\gamma, \lambda_z$.

  These are taken to be the correct definition of these parameters.

- SET2: the formalism which appears in Bilenky et. al. [4]. This uses: $\delta_z, x_\gamma, x_z, y_\gamma, y_z$.

  These are taken to be the correct definition of these parameters.

  The problem which arises is that in this paper you will also find reference to $\Delta\kappa_\gamma, \Delta\kappa_z, \lambda_\gamma, \lambda_z$ parameters, however these use a **different** definition of κ_z and are hence incompatible with the ones in SET1.

As a result:

- The relations between SET1 and SET2 given as equations (2) and (3) of [1] are **correct.**
- The relations given as equation (2.7) of [4] are **incompatible** with SET1.

2 Relations between parameters

The best way to see unambiguously the correct relation between these two sets is to look at the expressions for the Lagrangians given as equation (1) page 526 of [1] and equation (2.10) page 26 of [4]. Be warned that the term g_{wwz} has a different meaning in the two cases. In the first it is a fixed quantity $g_{wwz} \equiv \cot\theta_w$, and any anomalous variation is absorbed in the parameter g_1^z. In the second case it is a variable parameter such that $g_{wwz} = \cot\theta_w + \delta_z$ where δ_z is an anomalous part. Therefore for clarity in the following we write either $\cot\theta_w$ or $\cot\theta_w + \delta_z$ explicitly and avoid the use of the ambiguous term g_{wwz}.

By comparing coefficients [1] we see the correct relations:

[1]The direct coefficient comparison yields for the photon terms:

$$
\begin{array}{rcl}
\text{ref [1]} & & \text{ref [4]}
\end{array}
$$

$$
\begin{aligned}
(1+\Delta\kappa_\gamma) &= (1+x_\gamma) && (1)\\
\lambda_\gamma &= y_\gamma && (2)
\end{aligned}
$$

and for the Z terms:

$$
\begin{aligned}
(1+\Delta g_1^z)\cot\theta_w &= \cot\theta_w + \delta_z && (3)\\
(1+\Delta\kappa_z)\cot\theta_w &= \cot\theta_w + \delta_z + x_z && (4)\\
\lambda_z\cot\theta_w &= y_z && (5)
\end{aligned}
$$

$$\begin{array}{rcll} & \text{ref [1]} & \text{ref [4]} & \\ \Delta\kappa_\gamma & = & x_\gamma & (6) \\ \lambda_\gamma & = & y_\gamma & (7) \\ \Delta g_1^{\mathrm{z}} \cot\theta_{\mathrm{w}} & = & \delta_{\mathrm{z}} & (8) \\ \Delta\kappa_{\mathrm{z}} \cot\theta_{\mathrm{w}} & = & \delta_{\mathrm{z}} + x_{\mathrm{z}} & (9) \\ \lambda_{\mathrm{z}} \cot\theta_{\mathrm{w}} & = & y_{\mathrm{z}} & (10) \end{array}$$

where we use the forms $\kappa_\gamma = 1 + \Delta\kappa_\gamma$, $\kappa_{\mathrm{z}} = 1 + \Delta\kappa_{\mathrm{z}}$ and $g_1^{\mathrm{z}} = 1 + \Delta g_1^{\mathrm{z}}$.

These should all be regarded as the correct relations [2] if you wish to abide by the LEP2 report standard. Note that these are identical to those given as equations (2) and (3) of [1].

3 The α parameters

The SU(2)$\times$U(1) motivated approach leads to three parameters [1]. These are $\alpha_{B\phi}$, $\alpha_{W\phi}$ and α_W, and are related to the SET1 and SET2 parameters by:

$$\alpha_{B\phi} = x_\gamma - \delta_{\mathrm{z}} \sin\theta_w \cos\theta_w \tag{11}$$
$$= \Delta\kappa_\gamma - \Delta g_1^{\mathrm{z}} \cos^2\theta_w \tag{12}$$

$$\alpha_{W\phi} = \delta_{\mathrm{z}} \sin\theta_w \cos\theta_w \tag{13}$$
$$= \Delta g_1^{\mathrm{z}} \cos^2\theta_w \tag{14}$$

$$\alpha_W = y_\gamma \tag{15}$$
$$= \lambda_\gamma \tag{16}$$

subject also to the following constraints which arise from gauge invariance:

$$\Delta\kappa_{\mathrm{z}} = -\Delta\kappa_\gamma \tan^2\theta_w + \Delta g_1^{\mathrm{z}} \tag{17}$$
$$\lambda_{\mathrm{z}} = \lambda_\gamma \tag{18}$$

or equivalently

[2] In contrast, the relations given as equation (2.7) page 26 of [4] use a **different** definition of the term κ_{z}. The relation is in fact ${\kappa_{\mathrm{z}}}^{\mathrm{LepII-report}} = (1+\Delta g_1^{\mathrm{z}}){\kappa_{\mathrm{z}}}^{\mathrm{Bilenky}}$. That this is true can be seen by comparing coefficients in the Lagrangians equation (1) of [1] and equation (2.3) of [4].

$$
\begin{aligned}
x_z &= -x_\gamma \tan\theta_w && (19)\\
y_z &= y_\gamma \cot\theta_w && (20)
\end{aligned}
$$

A useful relation which follows is:

$$\Delta\kappa_\gamma = \alpha_{W\phi} + \alpha_{B\phi} \tag{21}$$

4 HISZ model

The HISZ constraints [3] are given in references [6]. These use the two free parameters $\Delta\kappa_\gamma$ and λ_γ, with the constraints:

$$
\begin{aligned}
\Delta g_1^z &= \Delta\kappa_\gamma/(2\cos^2\theta_w) && (22)\\
\Delta\kappa_z &= \Delta\kappa_\gamma(1-\tan^2\theta_w)/2 && (23)\\
&\equiv -\Delta\kappa_\gamma \tan^2\theta_w + \Delta g_1^z &&\\
\lambda_z &= \lambda_\gamma && (24)
\end{aligned}
$$

or

$$
\begin{aligned}
\delta_z &= x_\gamma/(2\sin\theta_w \cos\theta_w) && (25)\\
x_z &= -x_\gamma \tan\theta_w && (26)\\
y_z &= y_\gamma \cot\theta_w && (27)
\end{aligned}
$$

The first two relations are equivalent to demanding $\alpha_{B\phi} = \alpha_{W\phi}$ in the language of the α models. The last relation is the same as α_W. Therefore this model is entirely equivalent [4] to varying $\alpha_{B\phi} = \alpha_{W\phi}$ and α_W independently.

[3] Another small word of warning. The HISZ relations are given as one of the options in the documentation for Gentle v2.0 [5], however due to the confusion which was pointed out in section 1 these relations are incorrect. The authors are now aware of this.

[4] It is noted that the 'orthogonal' combination of $\alpha_{B\phi} = -\alpha_{W\phi}$ corresponds to varying δ_z with all other parameters set to zero.

References

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