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Update of the hadronic contribution to the QED vacuum polarization

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Abstract

We have updated the evaluation of the hadronic contribution to the running of the QED fine structure constant. It is obtained from a dispersion integral over a parametrization of the measured cross section of $e^+e^- \rightarrow$ hadrons. We find this contribution to be 0.0280 ± 0.0007 at s = M_Z^2 corresponding to $\alpha^{-1}(M_Z^2)$ = 128.89 ± 0.09. We have also updated and parametrized the corrections to α as a function of energy and discuss the influence on the luminosity measurements at LEP.

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1 Introduction

Good knowledge of radiative corrections is needed to match the high level of precision obtained experimentally in Z-physics[1].

An important class of electroweak radiative corrections are the boson self energies or vacuum polarization corrections. They are independent of any particular initial or final state. In the case of the photon, the renormalized self-energy function $\Pi_{\gamma\gamma}$ can be absorbed in a running coupling constant $\alpha(s)$:

$$\alpha(s) = \frac{\alpha(0)}{1 + \Pi_{\gamma\gamma}(s)} \qquad\qquad \Delta\alpha(s) = -\Pi_{\gamma\gamma}(s)$$

At Z-energy, the shift of α from the known leptons and the udscb quarks is rather sizeable. From this analysis:

$$\begin{aligned} \Delta\alpha(M_Z^2) = -\Pi_{\gamma\gamma}(M_Z^2) &= 5.94 \pm 0.07\% \\ \text{of which}\quad & \ 2.80 \pm 0.07\% \text{ from udscb quarks} \\ \text{such that } \alpha^{-1}(M_Z^2) &= 128.89 \pm 0.09 \end{aligned}$$

The photon self-energy correction can be calculated using standard Feynman techniques to evaluate the loop diagram corresponding to the photon self energy. For a fermion with charge Q_{f} and a colour factor N_{C} (3 for quarks, 1 for leptons) the result is

$$\Pi_{\gamma\gamma} = \frac{\alpha}{3\pi} N_{\mathrm{C}}\ Q_{\mathrm{f}}^2\ P(s, m^2)$$

where $P(s, m^2)$ is a known analytical expression. The full expression for $P(s, m^2)$ with comments on the implementation in computer code is given in [2]. It is used for leptons but there are problems for quarks: the masses, in particular of the light u and d quarks are not unambiguously defined and QCD corrections are large.

There is a method for obtaining the contribution from quarks to the photon self energy that avoids these problems: the evaluation of the loop diagram can be related to cross-section measurements (symbolized by cutting the loop). The imaginary part of $\Pi_{\gamma\gamma}$ for hadrons is directly related to R_{had}, the QED cross-section of the process $e^+e^- \to$ hadrons normalized to the QED cross-section for muon-pair production:

$$\mathrm{Im}\,\Pi_{\gamma\gamma}(s) = -\frac{\alpha}{3} R_{\mathrm{had}}(s)$$

The real part of $\Pi_{\gamma\gamma}$ can be determined by analyticity via a dispersion integral [3] (Kramers–Kronig rule; the P stands for principal value of the integral):

$$\mathrm{Re}\,\Pi_{\gamma\gamma}(s) = \frac{\alpha\ s}{3\pi}\ \ P\int_{4m_\pi^2}^{\infty} \frac{R_{\mathrm{had}}(s')}{s'(s'-s)} ds'$$

[1] The analysis described in this paper was first presented in [1]. The reader can also find there a more detailed introduction.

The task is now to find a realistic parametrization of $R_{\rm had}(s)$ using experimental data and perform the integration. The integration can be done numerically or analytically if $R_{\rm had}(s)$ is approximated by straight lines. Early evaluations were done as part of radiative corrections to e$^+$e$^-$ cross-sections [4] (the e$^+$e$^-$ point-like QED cross-section is $4\pi\alpha^2(s)/3s$) or in estimates of energy scales for grand unification [5].

One of the authors (HB) started to be involved in this subject in 1981, motivated by the need to update the evaluation of Berends and Komen [4] for accurate measurements of total e$^+$e$^-$ cross-sections at PETRA [6]. The result was parametrized by a simple function suitable for event generators. When applied to s = M_Z^2, the result was $\Delta\alpha_{\rm had} = 2.89 \pm 0.25\%$.

In 1985-86, B.W. Lynn et al. [7] and F. Jegerlehner [8] gave results on the hadronic corrections to $\Delta\alpha$ at s = M_Z^2. Within the framework of a workshop on longitudinal polarization at LEP, an effort was made to update and compare results from Refs. [6], [7] and [8]. The common result, published in 1989 [9] was :

$$\Delta\alpha_{\rm had} = 2.88 \pm 0.09\% \text{ at } \mathrm{M}_Z \text{ assumed to be 92 GeV}$$

There is some arbitrariness involved in extrapolating $R_{\rm had}$ between the measured points and combining systematic errors of different experiments. An important conclusion of the common paper [9] was that the independent programs and parametrization methods gave nearly identical results. Differences in central values obtained from the use of the trapezoidal rule between many data points, partially smoothed functions or broad averages were negligible compared to the experimental errors. The last, simplest method was described in the common paper [9] and is also the basis for this analysis.

We updated $R_{\rm had}(s)$ and recalculated $\Pi_{\gamma\gamma}(s)$ using all currently available data. We chose to represent $R_{\rm had}$ using broad averages in the continuum and to rely, whenever they were available, on published world averages. Our parametrization of $R_{\rm had}$ including resonances for centre-of-mass energies up to 10 GeV is shown as a solid line in Fig. 1. The relative error of the continuum contribution of $R_{\rm had}$ is shown as a band and given in numbers at the bottom of the figure. Measurements of various experiments are shown with their statistical errors. The contributions from the ρ, narrow resonances and different energy regions to $\Delta\alpha$ are summarized in Table 1 and illustrated in Fig. 2.

The changes from the '89 analysis are given in Table 2 and discussed below:

- Above the c.m.s. energy of 12 GeV we have used a parametrization of PETRA, PEP and TRISTAN data made by D. Haidt [10] based on second-order QCD with $\alpha_s = 0.146^{+0.031}_{-0.026}$ at 34 GeV corresponding to a value of $0.124^{+0.021}_{-0.019}$ at M_Z. In calculating $R_{\rm had}$ we made sure that we use the same formulas from the ZFITTER code [11] used by Haidt to calculate $\alpha_s(M_Z)$. This minimizes any theoretical error from QCD. Note that the value for α_s agrees well with measurements at LEP ($0.128 \pm 0.005 \pm 0.002$ from R and 0.123 ± 0.006 from event shape measurements [12]). Our overall 1.7% relative uncertainty for energies above 12 GeV is conservative.

- In the c.m.s. energy region between 7.23 and 10.34 GeV new data have been obtained by the MD-1 detector at the VEPP-4 collider in Novosibirsk [13]. The measurements

range $\sqrt{s}$, GeV	relative error	$\Delta\alpha$ %	comments
ρ	5 %	0.35	Kinoshita param., up to 1.8 GeV
narrow resonances	Γ_e	0.21	P.D.G., undressed width
1.05 – 2.5	15 %	0.24	with ND, DM2
2.5 – 5.0	15 %	0.30	
5 – 7	6 %	0.18	including Crystal Ball
7 – 12	3 %	0.31	Blinov et al., world average '93
> 12	1.7 %	1.21	Haidt's parametrization
		2.80	

Table 1: Contributions to $\Delta\alpha(M_Z^2)$

$\Delta\alpha_{\mathrm{had}}$ =	2.88	$\pm$ 0.09 %	1989 result
	–0.037		Crystal Ball, lower mean and error than MarkI
	–0.005		scaling from M_Z = 92 GeV to 91.1887 GeV
	2.84	$\pm$ 0.07 %	result using Crystal Ball and new M_Z
	–0.021		above 12 GeV (PETRA, PEP, TRISTAN)
	–0.011		other changes, DM2, VEPP–2M, VEPP–4
	–0.008		rad.cor. resonances
	2.80	$\pm$ 0.07 %	

Table 2: Changes in $\Delta\alpha(M_Z^2)$ from 1989 analysis

show no evidence of unknown resonant structure. The same group has also made a compilation of the measurements carried out in this energy region by the PLUTO, DESY-Heidelberg, MARK-I, DASP II, LENA, CUSB, CLEO, Crystal Ball and ARGUS detectors (see Ref. [13] for references and details of compilation). They quote a value for R of 3.579 with an error of 1.8%. However, a possible error correlation between different experiments has not been used in the compilation . We use this mean value for the continuum for energies from 7 to 12 GeV with a conservative uncertainty of 3%.

- New results for the c.m.s. energy region between 5.0 and 7.4 GeV came from the Crystal Ball collaboration at the SLAC storage ring SPEAR [14]. The data show no evidence for a new threshold or resonant structure in this energy range. The average value of R is reported to be $3.44 \pm 0.03 \pm 0.18$. This is significantly smaller than the value of 4.3 ± 0.4 published eight years earlier by the Mark I collaboration [15] for the c.m.s. energy region between 5.0 and 7.8 GeV and used in the previous analysis [9]. Including the Crystal Ball data made the most significant single change in the entire update. The systematic error in the Mark I analysis is quoted as 20% at energies below the J/ψ resonance and decreasing to 10% above 6 GeV. We decided to use an average value of R = 3.48 in this energy range, based on a weighted average of the Crystal Ball and Mark I results, where the uncertainty of the latter was assumed to be 20%.

- Data from the Neutral Detector (ND) at VEPP-2M at energies from 0.8 to 1.4 GeV improved and confirmed the knowledge of the ρ and the onset of the continuum [16]. Measurements at the DM2 detector at the DCI storage ring cover energies from 1.3 to 2.4 GeV with a potential accuracy of 10%. The total hadronic cross-section in this energy range, including the DM2 data, has been the subject of a thesis [17]. Unfortunately, this work has not been published and moreover, does not contain a complete discussion of systematics. Our parametrization for c.m.s energies up to 2.5 GeV has been slightly modified based on the results from the ND and DM2 detectors. We assumed a conservative uncertainty of 15% on our parametrization of the continuum up to 2.4 GeV.

- Resonances were included as in the '89 paper. The contribution is proportional to the undressed electronic width Γ_e. Values and errors are taken from the 1994 version of the review of particle properties [18]. The change in convention of the particle data group in publishing now the dressed width is taken into account (this is discussed in detail in Ref. [19]). The effect on $\Delta\alpha$ of going to undressed width is small, as can be seen from Table 2.

2 Comparison with recent estimates in the Literature

Figure 3 shows a comparison of recent estimates of $\Delta\alpha_{\mathrm{had}}$. The 1989 result is shown at the top of Fig. 3 as it was published, and just below, as modified by the inclusion of Crystal Ball data and scaling to the measured Z-mass. This brings it very close to the value given by Jegerlehner in 1992 [20]. The estimates of Nevzorov et al. [21], Geshkenbein and Morgunov [22] and Martin and Zeppenfeld [23] rely on a theoretical description of hadronic cross-sections (such as quark masses and QCD) except at very low energies. We believe that this introduces some additional theoretical uncertainty and compromises the usefulness of $\Delta\alpha$ in testing electroweak theory and searching for new physics.

The analysis of Swartz [24] is based on a dispersion integral using current data on R_{had} and can therefore be directly compared with our analysis. The result of $\Delta\alpha_{\mathrm{had}} = 2.666 \pm 0.075$ is incompatible with our analysis. The paper incorrectly states that previous analyses [9], [20] used equal weights.

We find excellent agreement with the recent, very detailed analysis of Eidelman and Jegerlehner [19]. They use direct, trapezoidal integration of data and also calculate the corrections to g–2.

3 Parametrization as a function of energy

The simple parametrization of the hadronic contribution to the vacuum polarization [6, 9] is used in many computer programs. It is described by the formula

$$\mathrm{Re}\Pi_h(s) \cong \mathrm{Re}\Pi_h(-t) \cong \mathrm{A} + \mathrm{B}\ln(1 + \mathrm{C}|s|)$$

where s, t are in GeV^2. We have updated the parameters A, B, and C for five different c.m.s. energy regions. This parametrization is better than 0.13 σ in the whole t-channel and exact for M_Z in the s-channel. The values of fitted parameters are given in Table 3. Computer code for the photon self-energy function including this parametrization of the hadronic part is available from the authors.

range $\sqrt{s}$, GeV	A	B	C
0.0 - 2.0	0.0	0.00228770	4.08041425
2.0 - 4.0	0.0	0.00251507	3.09624477
4.0 - 10.0	0.0	0.00279328	2.07463133
10.0 - 91.2	0.00122270	0.00296694	1.0
91.2 - 100000.0	0.00164178	0.00292051	1.0

Table 3: Parametrization of the hadronic vacuum polarization

The update of $\Pi_{\gamma\gamma}$ or, equivalently, $\Delta\alpha$ also has an influence on the precision of the luminosity measurement at LEP. The error of the hadronic contribution to the vacuum polarization given in Ref. [9] has contributed 0.08% and 0.05% [25], respectively, to the precision in calculating the luminosity cross-section for the first and second [26] generations of the luminosity detectors at LEP. The errors are reduced to 0.07% and 0.04% for these two generations of detectors using from this update the error of 4% for the hadronic part of vacuum polarization. It has to be noted that a change in the vacuum polarization contribution implies also a change in the cross-section used for the luminosity determination. As an example we have calculated the cross-section for the ALEPH detector SICAL with the old and new parametrizations of the hadronic contribution to the vacuum polarization. The change in the cross-section is 0.02%. The dominant contributions to $\Delta\alpha$ at the energies relevant in the luminosity determination are from the ρ and the low-energy continuum, as can be seen in Fig. 4.

4 Conclusion

Our updated evaluation of the hadronic contributions to the running of the QED fine structure constant α resulted in $\Delta\alpha_{\mathrm{had}} = 0.0280 \pm 0.0007$ at s = M_Z^2 or $\alpha^{-1}(M_Z^2) = 128.89 \pm 0.09$. The uncertainty is due mainly to the relatively poor knowledge of the hadronic cross-section in e^+e^- collisions at low energies with $\sqrt{s}$ from 1 to 5 GeV (continuum and broader ψ's).

Old measurements have not always been fully analyzed and published (DM2, DELCO at SPEAR). New measurements of $R_{\rm had}$ with a precision of a few percent should be strongly encouraged (Frascati, Novosibirsk, Bejing).

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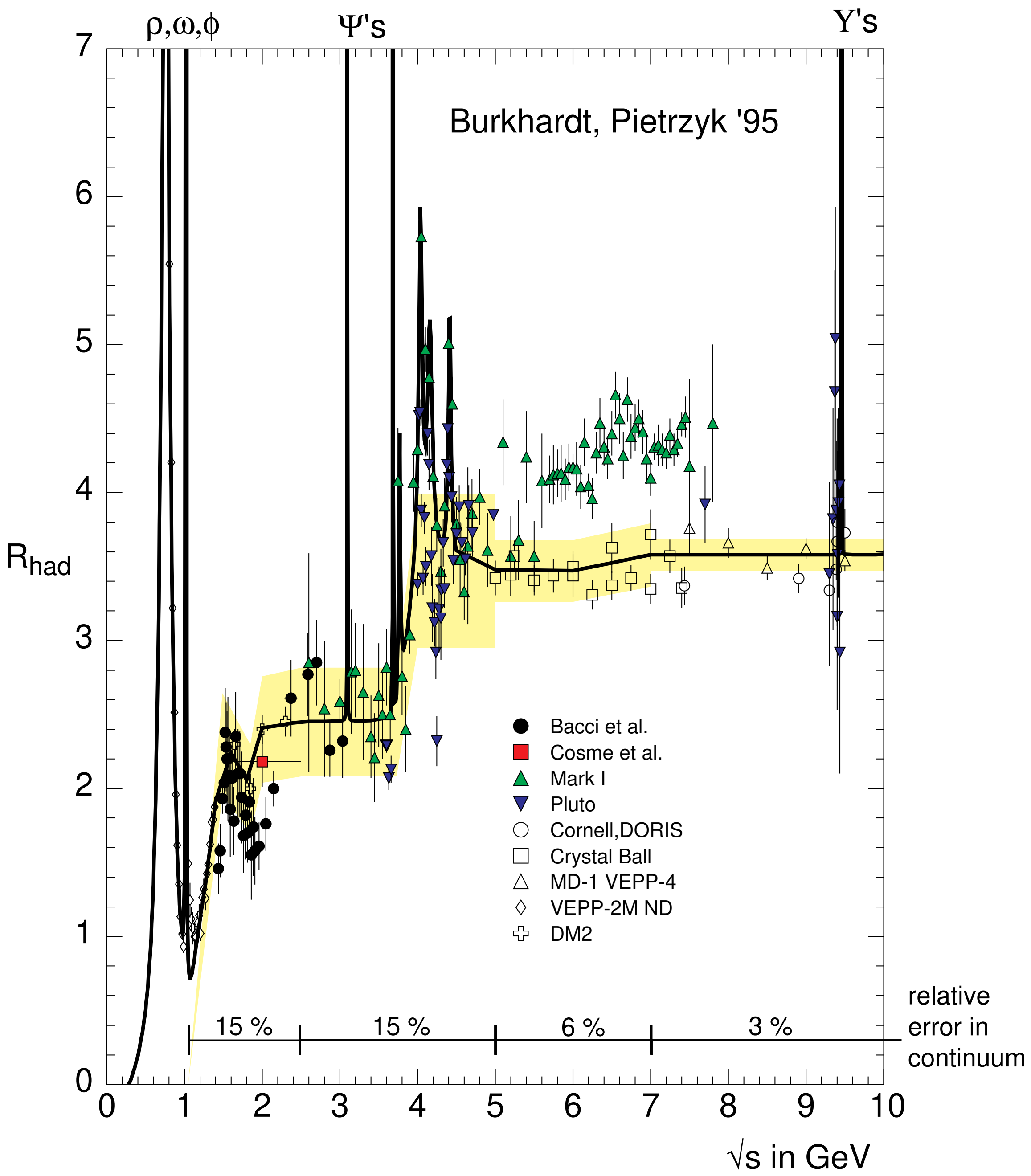

Figure 1: $R_{\rm had}$ including resonances.

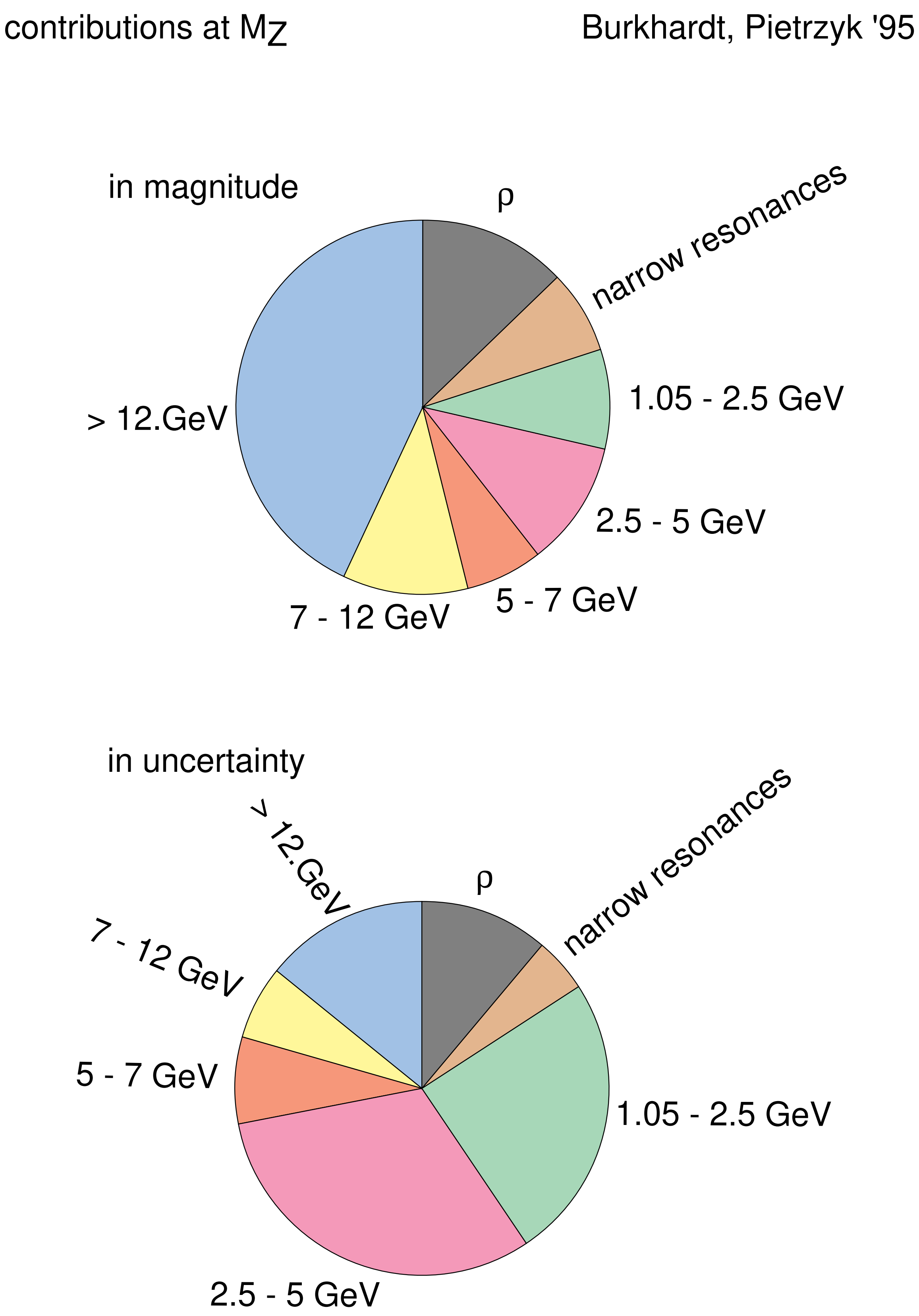

Figure 2: Relative contributions to $\Delta\alpha(M_Z^2)$ in magnitude and uncertainty.

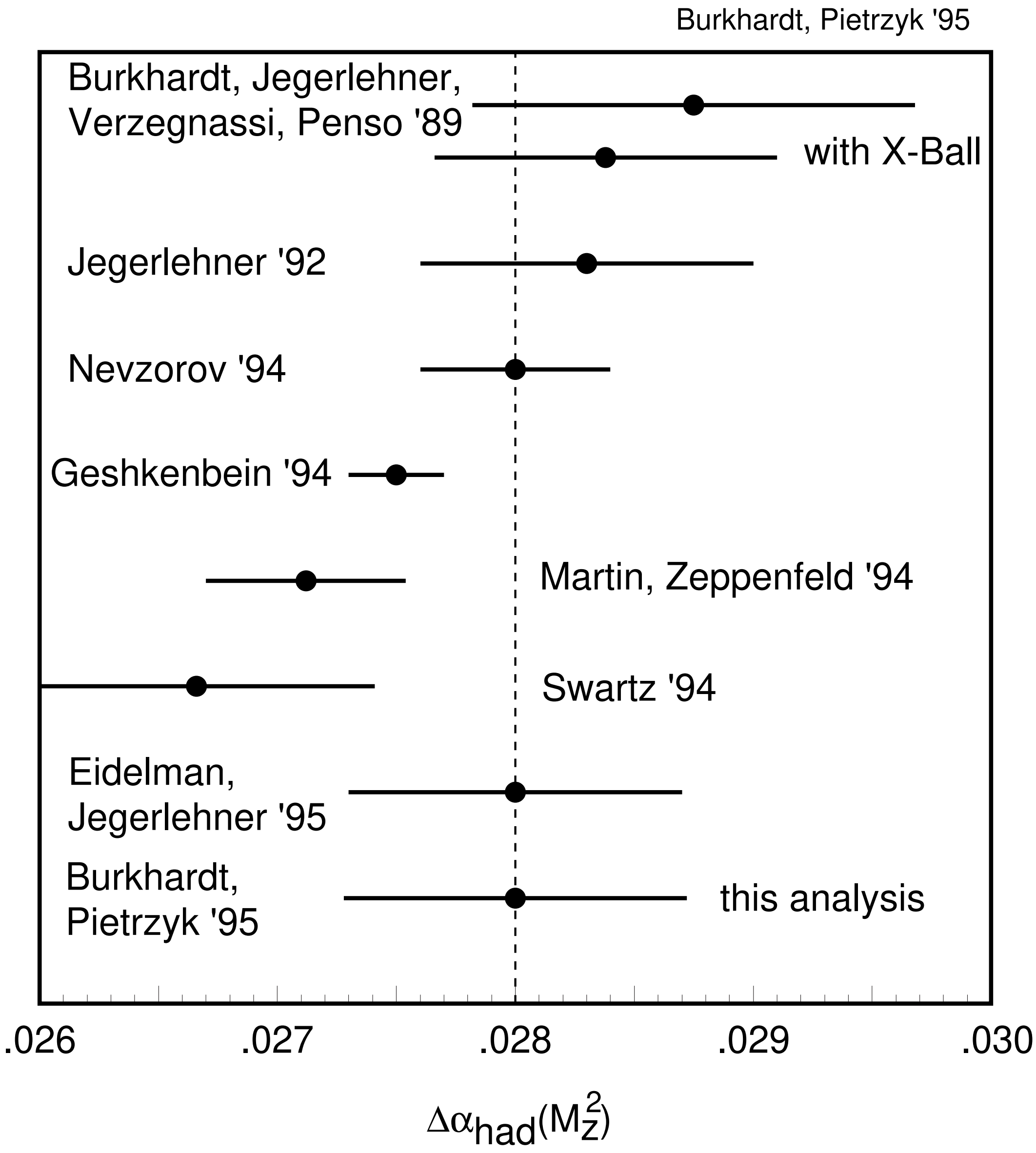

Figure 3: Comparison of recent estimates of $\Delta\alpha_{\rm had}$.

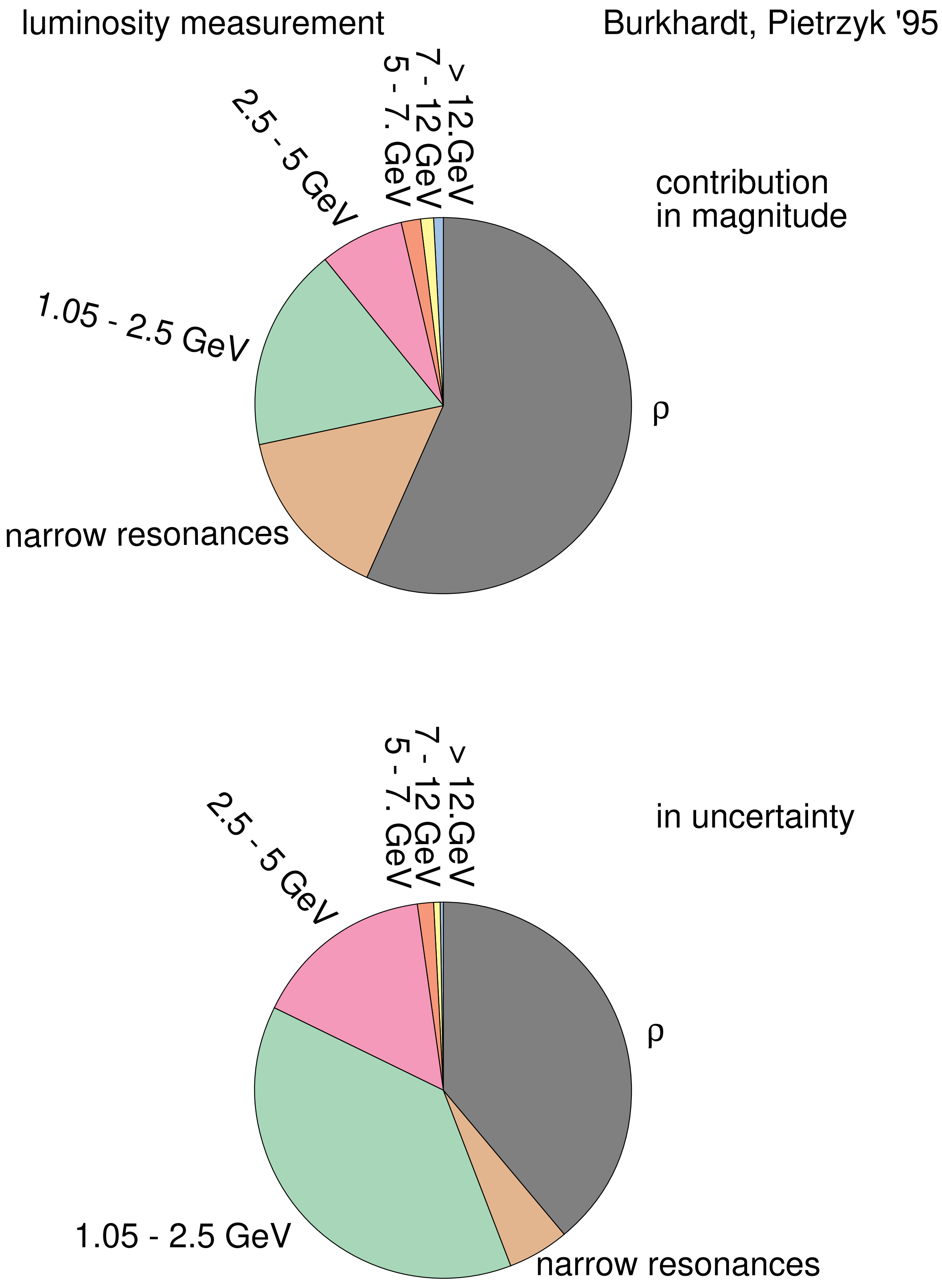

Figure 4: Relative contributions to $\Delta\alpha(t = -1.424\ \mathrm{GeV}^2)$ in magnitude and uncertainty.