



# *Low Energy QCD and ChPT Tests in the NA48/2 Experiment*

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on Behalf of the NA48/2 Collaboration

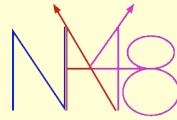
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New Trends In High-Energy Physics  
Yalta - Crimea (Ukraine), September 15<sup>th</sup> - 22<sup>nd</sup> 2007



# Outline

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- > NA48/2 Experimental Setup
- > Low Energy QCD:
  - >  $K_{e4}$ : Form Factors and  $\pi\pi$  Scattering Length
  - >  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ : "Cusp" Effect and  $\pi\pi$  Scattering Length
- > ChPT Tests:
  - >  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ : First Evidence of Interference Term
  - >  $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$ : First Observation



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48

# NA48/2

# *Experimental Setup*



# History



**NA48 (1997-2000): Direct CP-Violation in neutral K**

$$> \text{Re}(\varepsilon'/\varepsilon) = (14.7 \pm 2.2) \cdot 10^{-4}$$

**NA48/1 (2002): Rare  $K_S$  decays**

$$> \text{BR}(K_S \rightarrow \pi^0 e^+ e^-) = (5.8^{+2.8}_{-2.3} \pm 0.8) \cdot 10^{-9}$$

$$> \text{BR}(K_S \rightarrow \pi^0 \mu^+ \mu^-) = (2.8^{+1.5}_{-1.2} \pm 0.2) \cdot 10^{-9}$$

**NA48/2 (2003-2004): Direct CP-Violation in charged K**

$$> A_g(K^\pm \rightarrow \pi^\pm \pi^+ \pi^-) = (-1.5 \pm 2.1) \cdot 10^{-4}$$

$$> A_g(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = (1.8 \pm 1.8) \cdot 10^{-4}$$

**...and many other results on kaon and hyperon decays**

1997	$\varepsilon'/\varepsilon$ run	$K_L + K_S$
1998	$\varepsilon'/\varepsilon$ run	$K_L + K_S$
1999	$\varepsilon'/\varepsilon$ run	$K_S$ Hi. Int.
2000	$K_L$ only	$K_S$ High Intensity <i>NO Spectrometer</i>
2001	$\varepsilon'/\varepsilon$ run	$K_S$ High Int.
2002	$K_S$ High Intensity	
2003	$K^\pm$ High Intensity	
2004	$K^\pm$ High Intensity	



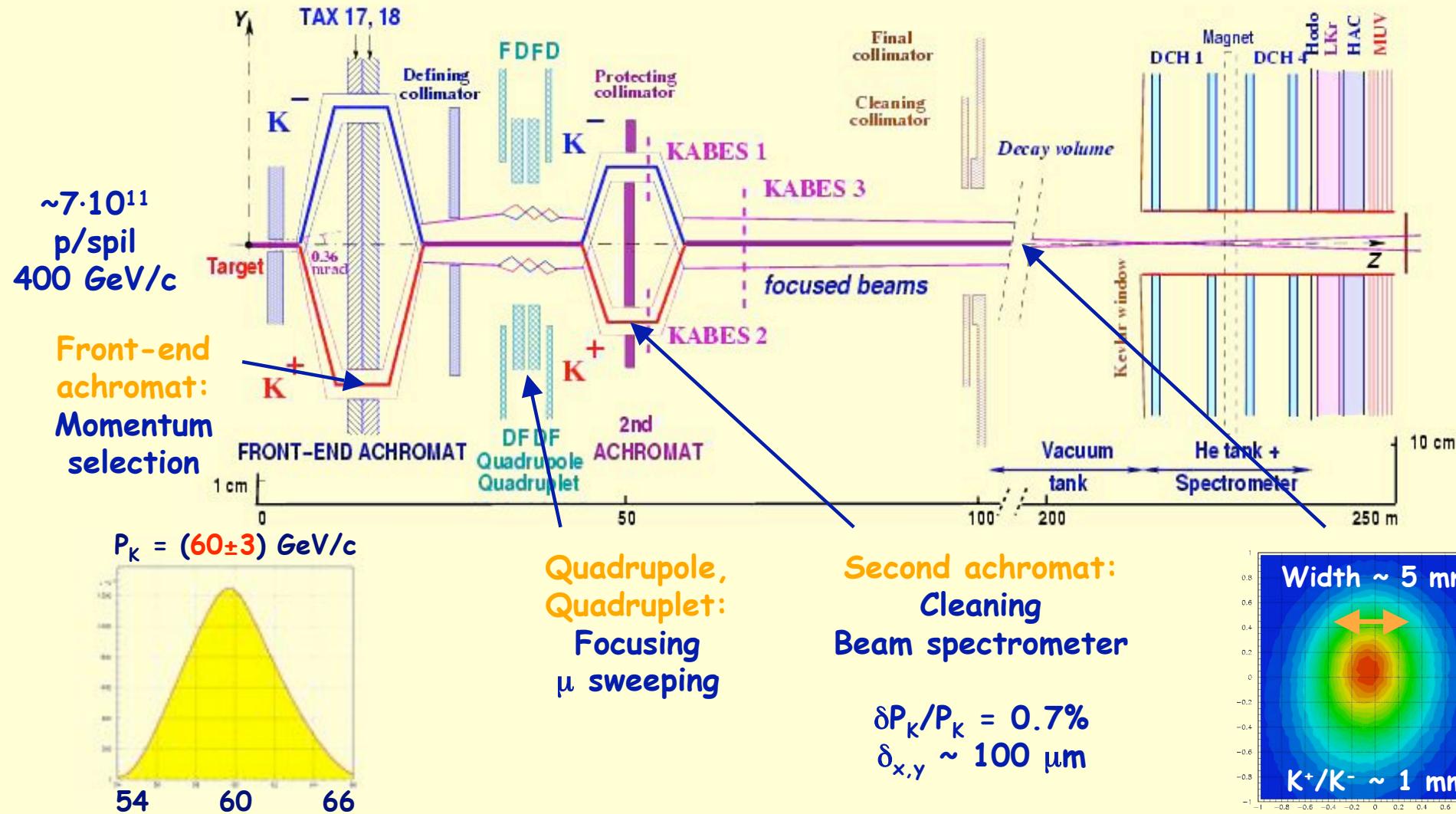
# Simultaneous Beam



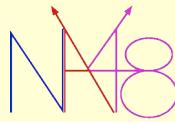
2-3M K/spill ( $\pi/K \sim 10$ )  
 $\pi$  decay products stay in pipe  
Flux ratio:  $K^+/K^- \sim 1.8$

Simultaneous  $K^+$  and  $K^-$  beams:  
large charge symmetrization  
of experimental conditions

Beams coincide within  $\sim 1\text{mm}$   
all along 114m decay  
volume



# Detector



## Magnetic spectrometer (4 DCHs):

- > 4 view / DCH  $\rightarrow$  high efficiency
- >  $\sigma_p/P = 1.0\% + 0.044\% \cdot P$  [GeV/c]

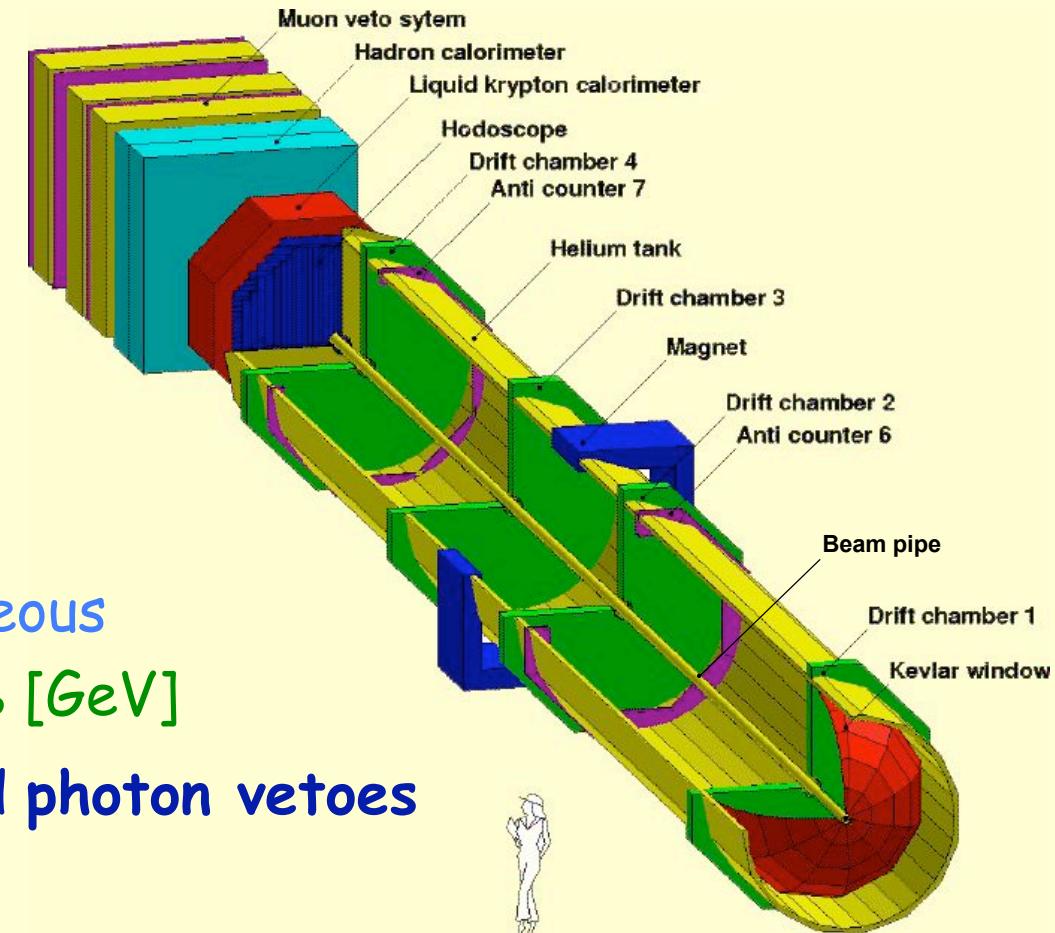
## Hodoscope:

- > Fast trigger
- >  $\sigma_t = 150\text{ps}$

## Electromagnetic calorimeter (LKr):

- > High granularity, quasi-homogeneous
- >  $\sigma_E/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\%$  [GeV]

## Hadron calorimeter, muon veto and photon vetoes

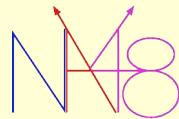


## Trigger:

- > Fast hardware trigger (L1): hodoscope & DCHs multiplicity
- > Level 2 trigger (L2): on-line processing of DCHs & LKr information



# Data Taking



## Run periods:

- > 2003: ~ 50 days
- > 2004: ~ 60 days

## Total statistics in 2 years:

- >  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ : ~  $4 \cdot 10^9$
- >  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ : ~  $1 \cdot 10^8$

-> >200 TB of data recorded



A view of the NA48/2 beam line

Rare  $K^\pm$  decays can be measured down to  $BR \sim 10^{-9}$

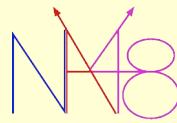


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# *Low Energy QCD: $K_{e4}$ Decay*



# Formalism



The  $K^\pm \rightarrow \pi^+\pi^-e^\pm\nu$  ( $K_{e4}$ ) dynamic is fully described by 5 variables (Cabibbo-Maksymovicz):

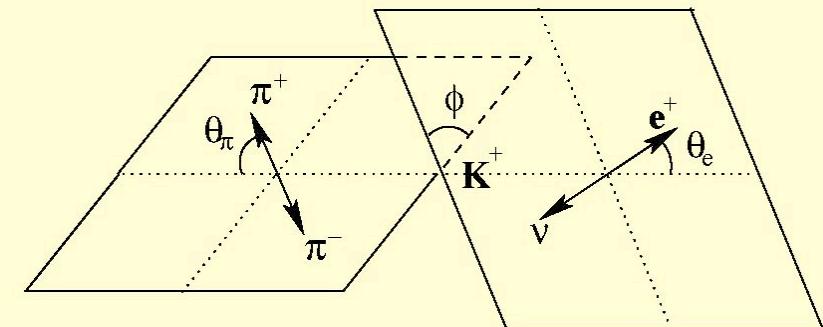
$$M_{\pi\pi}^2, M_{e\nu}^2, \cos\theta_\pi, \cos\theta_e \text{ and } \phi$$

The transition amplitude can be written using 2 axial and 1 vector Form Factors that can be developed in a partial wave expansion:

$$F = F_s \cdot e^{i\delta_s} + F_p \cdot e^{i\delta_p} \cdot \cos\theta_\pi + \text{terms}_{d\text{-wave}}$$

$$G = G_p \cdot e^{i\delta_p} + \text{terms}_{d\text{-wave}}$$

$$H = H_p \cdot e^{i\delta_p} + \text{terms}_{d\text{-wave}}$$



The Form Factors can be expanded as a function of  $q^2 = (M_{\pi\pi}^2/4m_\pi^2 - 1)$  and  $M_{e\nu}^2$ :

$$\begin{aligned} F_s &= f_s + f_s' \cdot q^2 + f_s'' \cdot q^4 + \\ &\quad + f_e' \cdot (M_{e\nu}^2/4m_\pi^2) + \dots \end{aligned}$$

$$F_p = f_p + f_p' \cdot q^2 + \dots$$

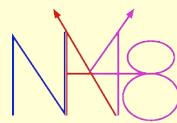
$$G_p = g_p + g_p' \cdot q^2 + \dots$$

$$H_p = h_p + h_p' \cdot q^2 + \dots$$

**$F_s, F_p, G_p, H_p$  and  $\delta = \delta_s - \delta_p$  used as fit parameters**

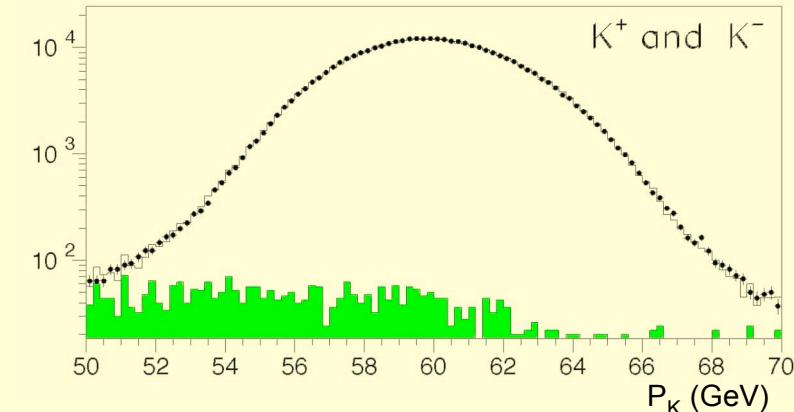


# Selection and Background



## Signal selection:

- > 3 charged tracks
- > 2 opposite sign  $\pi$ s
- > 1 e: LKr & DCH info E/P
- > 1  $\nu$ : some missing energy &  $P_T$

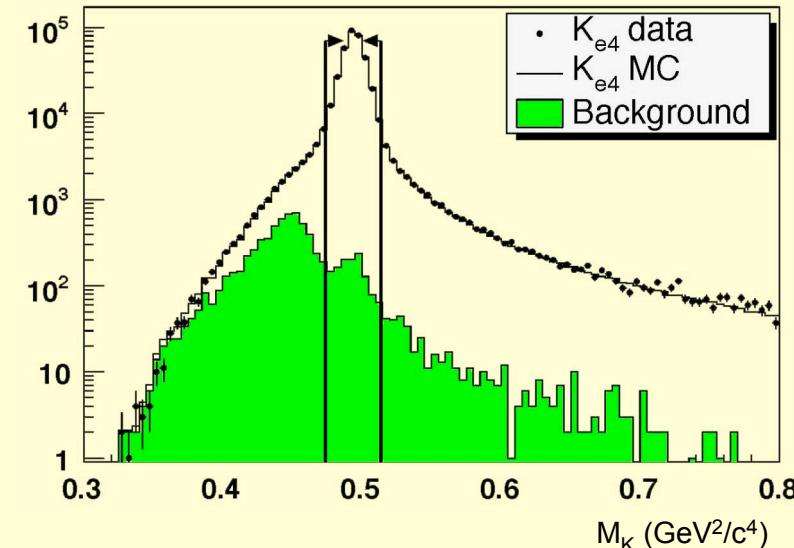


## Main background sources:

- >  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  decay with  $\pi \rightarrow e\nu$  (dominant) or
- >  $\pi$  mis-ID as e
- >  $K^\pm \rightarrow \pi^\pm \pi^0 (\pi^0)$  decay with  $\pi^0 \rightarrow e^+ e^- \gamma$  and e
- > mis-ID as  $\pi + \gamma$ s undetected
- > Background is studied using the electron "wrong sign" events (assuming  $\Delta Q = \Delta S$  and total charge  $\pm 1$ ) and cross checked with MC

2003: 677500 events

Total background can be kept @ ~0.5% level





# Fitting Procedure



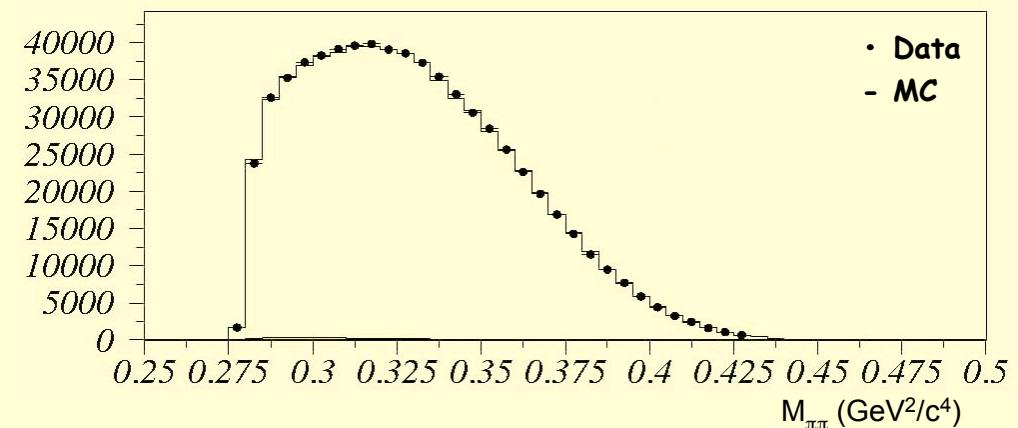
Using iso-populated bins in the 5-D space of the C.M. variables one defines a grid of:

$$10(M_{\pi\pi}) \times 5(M_{e\nu}) \times 5(\cos\theta_e) \times 5(\cos\theta_\pi) \times 12(\phi) = 15000 \text{ boxes}$$

The set of Form Factor values is used to minimize a log-likelihood estimator well suited for small numbers of data event/bin and taking into account the statistics of the simulation (simulated and expected events/bin)

Assuming constant Form Factors over single boxes, ten independent fits in  $M_{\pi\pi}$  bins have been performed to get model independent results

		Data	MC
$K^+$	Events	435654	10.0 M
	Events/bin	29	667
$K^-$	Events	241856	5.6 M
	Events/bin	16	373

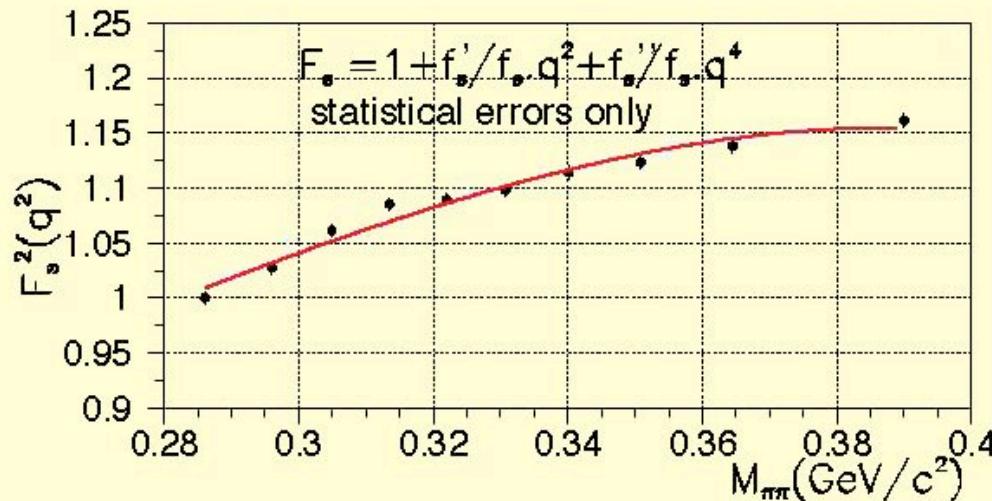




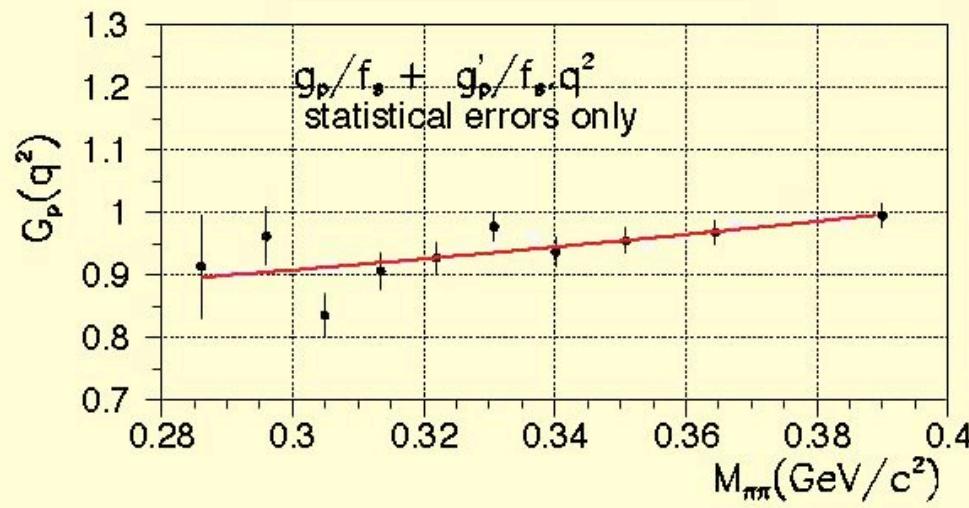
# Results (I)

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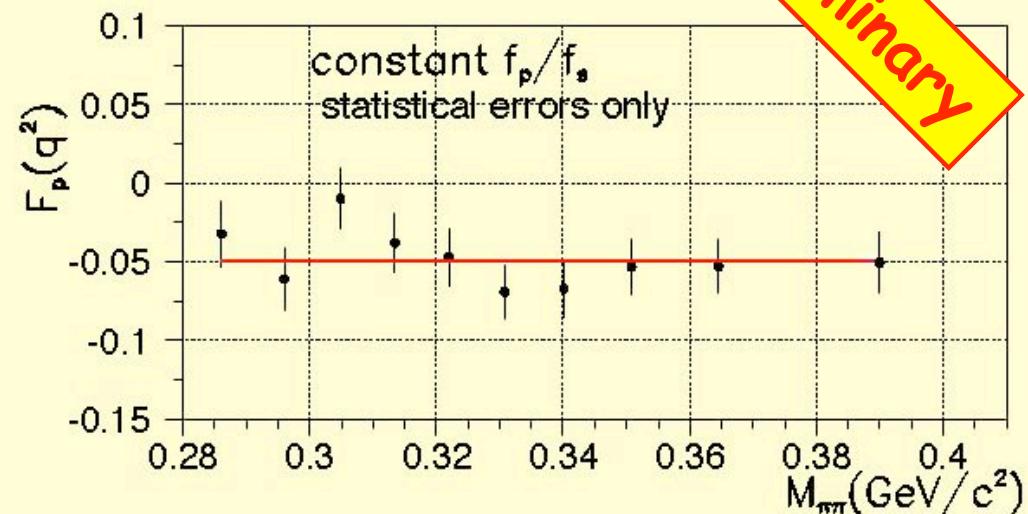
Preliminary



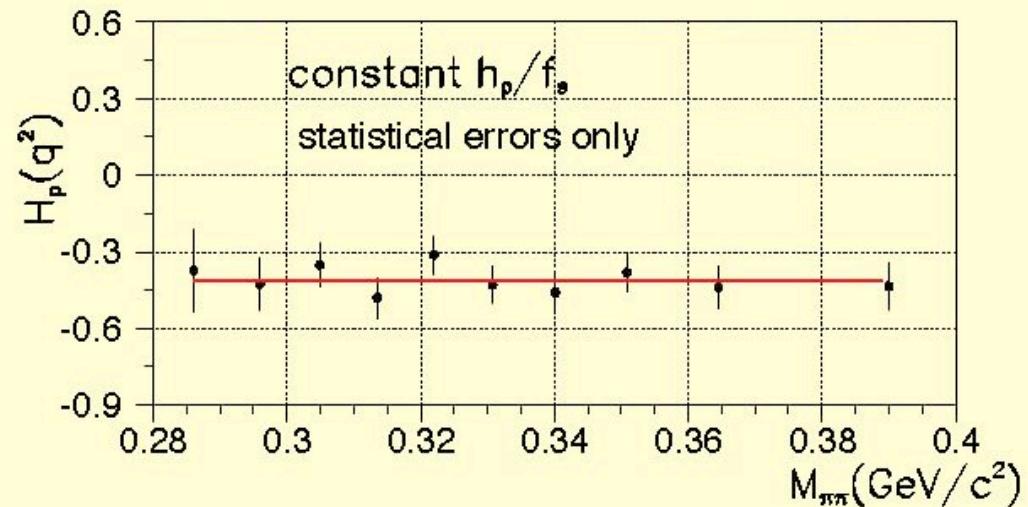
Quadratic in  $q^2$



Linear in  $q^2$



First measurement of  $F_p \neq 0$



No linear term ( $h_p'$ )



# Results (II)



value ± stat ± syst

$$\begin{aligned} f_s'/f_s &= 0.165 \pm 0.011 \pm 0.006 \\ f_s''/f_s &= -0.092 \pm 0.011 \pm 0.007 \\ f_e'/f_s &= 0.081 \pm 0.011 \pm 0.008 \\ f_p/f_s &= -0.048 \pm 0.004 \pm 0.004 \\ g_p/f_s &= 0.873 \pm 0.013 \pm 0.012 \\ g_p'/f_s &= 0.081 \pm 0.022 \pm 0.014 \\ h_p/f_s &= -0.411 \pm 0.019 \pm 0.007 \end{aligned}$$

## Systematics checks:

- > Acceptance
- > Background
- > Particles ID
- > Radiative corrections

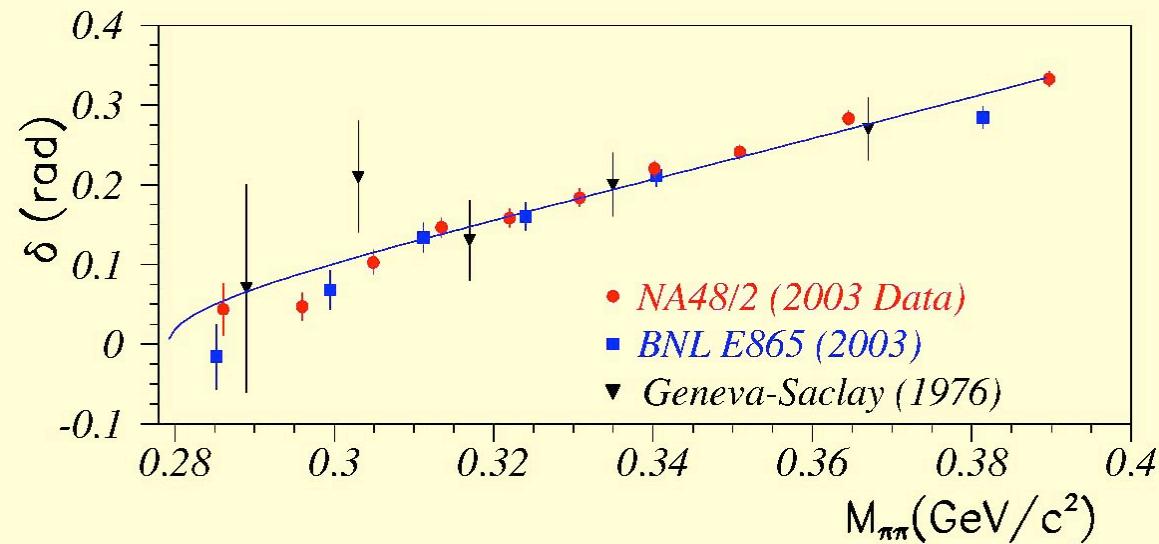
- > All the Form Factors are measured relatively to  $f_s$  (no BR measurement)
- > Because of different beam geometries for  $K^+$  and  $K^-$ , the event samples are fitted separately and the results combined according to their statistical precision ( $K^+/K^- \sim 1.8$ )
- > First evidence of  $f_p \neq 0$  and  $f_e' \neq 0$
- > Neglected  $M_{ev}$  dependence of the normalization
- > The Form Factors are measured at level of <5% of relative precision while the slopes at ~15% relative precision (factor 2÷3 improvement wrt. previous measurements)



# $\delta$ Dependence

Preliminary  
NLO

- > The extraction of the  $\pi\pi$  scattering lengths from the  $\delta = \delta_s - \delta_p$  phase shift needs external theoretical and experimental data inputs
- > The Roy equations provide this relation between  $\delta$  and  $(a_0, a_2)$  near threshold, extrapolating from the  $M_{\pi\pi} > 0.8$  GeV/c<sup>2</sup> region. The precision of these data defines the width of the Universal Band in the  $(a_0, a_2)$  plane
- > The fit of the experimental points using the Roy equations in the Universal Band allows to extract the  $a_0$  and  $a_2$  values

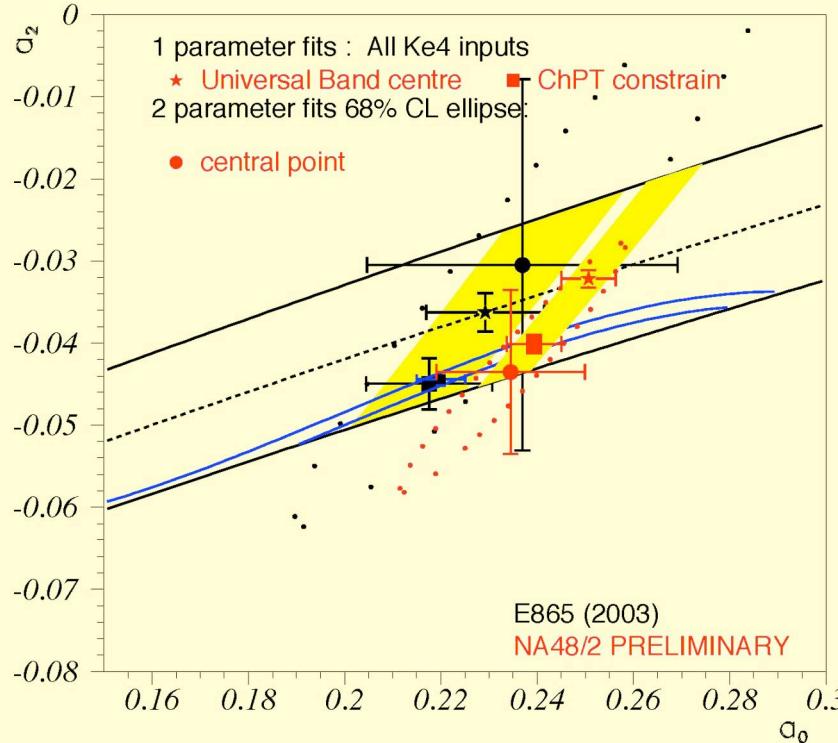




# ( $a_0, a_2$ ) Plane (I)

Preliminary

Minimizing the  $\chi^2$  in the 2-D fit it's possible to identify the favoured solution (and the corresponding ellipse)



Single parameter fit ( $a_2$  constrained to the center line of the UB):

$$a_0 \cdot m_{\pi^+} = 0.256 \pm 0.006_{\text{stat}} \pm 0.005_{\text{syst}}$$
$$(a_2 \cdot m_{\pi^+} = -0.031 \pm 0.001_{\text{stat}} \pm 0.001_{\text{syst}})$$

Two parameters fit:

$$a_0 \cdot m_{\pi^+} = 0.233 \pm 0.016_{\text{stat}} \pm 0.012_{\text{syst}}$$
$$a_2 \cdot m_{\pi^+} = -0.047 \pm 0.011_{\text{stat}} \pm 0.008_{\text{syst}}$$

- > The E865 and NA48/2 results agreement is marginal (mainly due to the last  $\delta$  point in E865)
- > The correlation between  $a_0$  and  $a_2$  is ~96% (similar for both experiment)

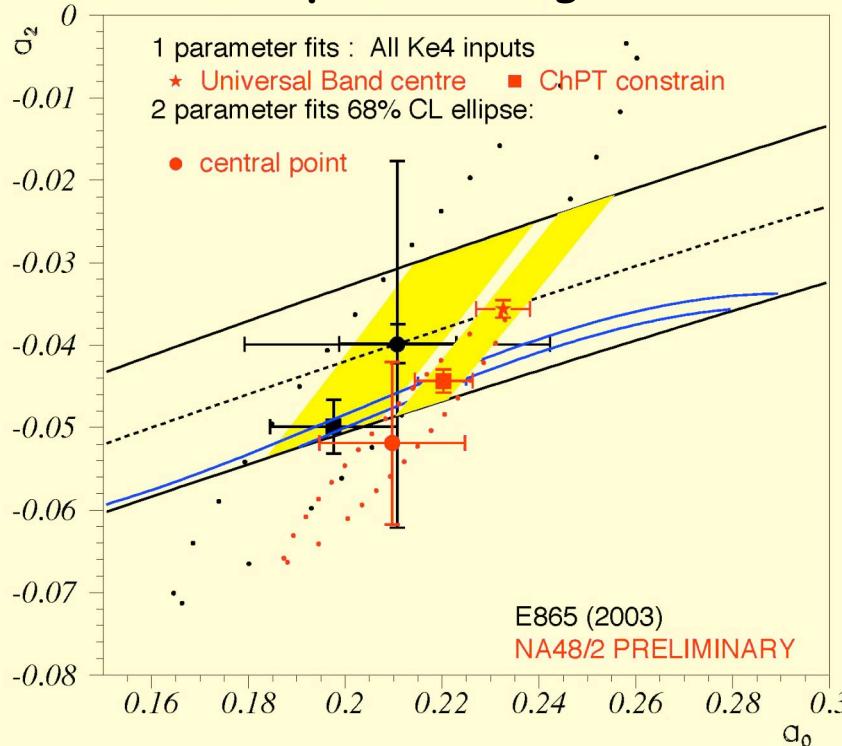


# $(a_0, a_2)$ Plane (II)

Preliminary

Following recent developments one can correct the measured  $K_{e4}$  phases for isospin symmetry breaking effect before extracting  $a_0$  (the correction is  $\sim 10 \div 12$  mrad (negative))

## With isospin breaking correction

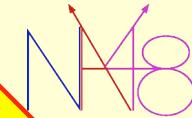


- > Using preliminary isospin corrections, both  $a_0$  and  $a_2$  values decrease (by  $\sim 0.02$  and  $\sim 0.004$  respectively) with statistical and systematic errors unchanged
- > The new values would then be in very good agreement with the preferred ChPT prediction ( $a_0 = 0.220$ ,  $a_2 = -0.0444$ ) and the most recent lattice calculations ( $a_2 = -0.04330 \pm 0.00042$ )

Both bands shift left and down in the  $(a_0, a_2)$  plane



# $K_{e4}$ "Neutral"



Preliminary

## Signal selection:

- > 1 e in the DCHs
- > 4  $\gamma$ s in the LKr
- >  $\pi^0$  mass constraints
- > 1  $\nu$ : missing  $P_T$

## Main background sources:

- >  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  with a mis-ID  $\pi^\pm$
- >  $K^\pm \rightarrow \pi^0 e^\pm \nu \gamma$  with 1 accidental  $\gamma$  and the 2  $\gamma$ s faking a  $\pi^0$

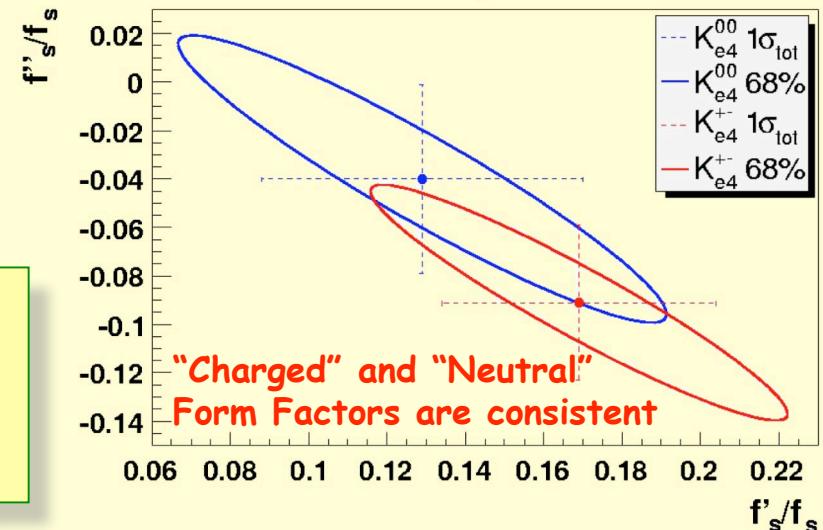
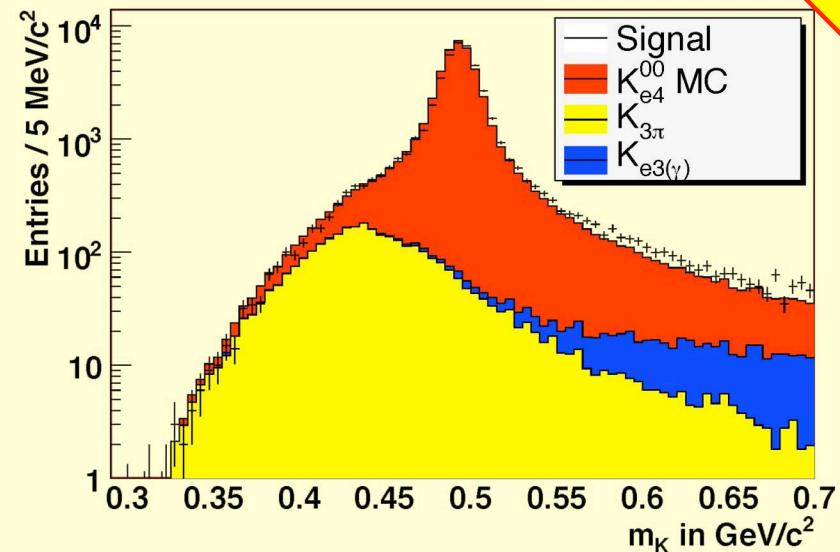
2003+2004: ~38000 events with  
~2÷3% of residual background

$$BR = (2.587 \pm 0.026_{\text{stat}} \pm 0.019_{\text{syst}} \pm 0.029_{\text{norm}}) \cdot 10^{-5}$$

$$f'_s/f_s = 0.129 \pm 0.036_{\text{stat}} \pm 0.020_{\text{syst}}$$

$$f''_s/f_s = -0.040 \pm 0.034_{\text{stat}} \pm 0.020_{\text{syst}}$$

- > BR 10 times better than current PDG value (KEK-E470 based on 216 signal events)
- >  $f'_e$  consistent with 0 within the present statistics





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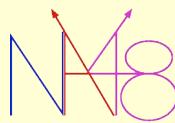
# *Low Energy QCD:*

## $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ Decay

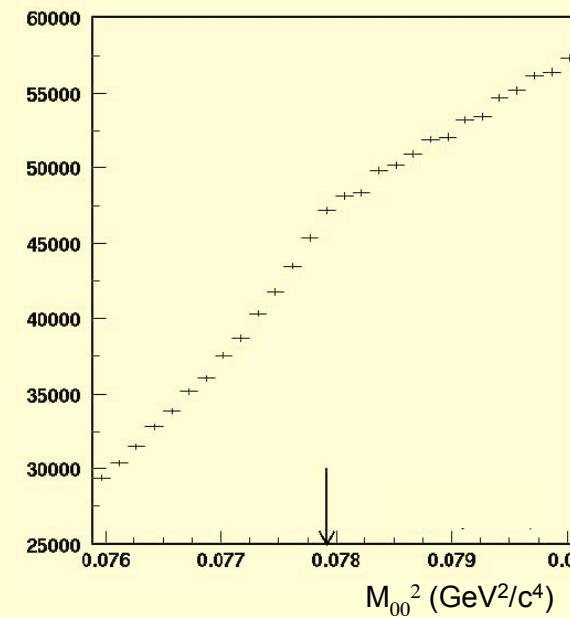
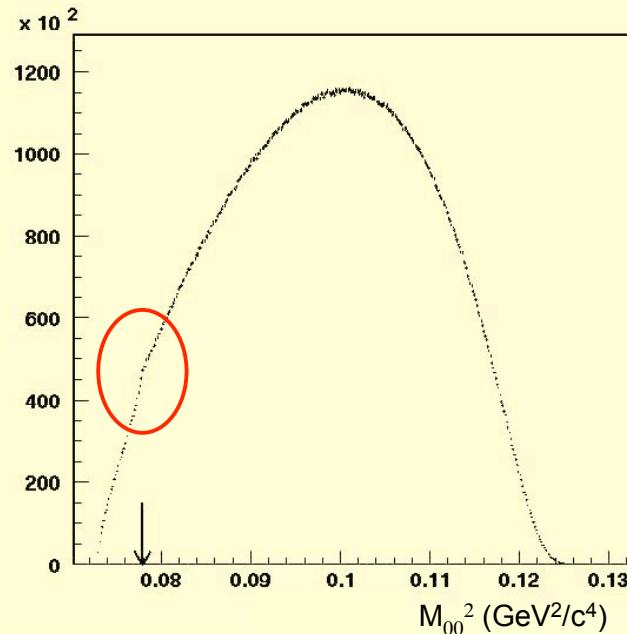




# A "Cusp"



- From  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  decay (partial sample of 2003 data) we observed an anomaly in the  $M_{00}^2$  invariant mass distribution in the region around  $M_{00}^2 = (2m_{\pi^+})^2 = 0.07792 \text{ (GeV}^2/\text{c}^4)$



- This anomaly has been interpreted as a final state charge exchange scattering process of  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^- (\pi^+ \pi^- \rightarrow \pi^0 \pi^0)$
- The parameter  $a_0 - a_2$  (difference between the S-wave  $\pi\pi$  scattering lengths in the isospin  $I=0$  and  $I=2$  states) can be precisely measured using this sudden anomaly ("cusp")



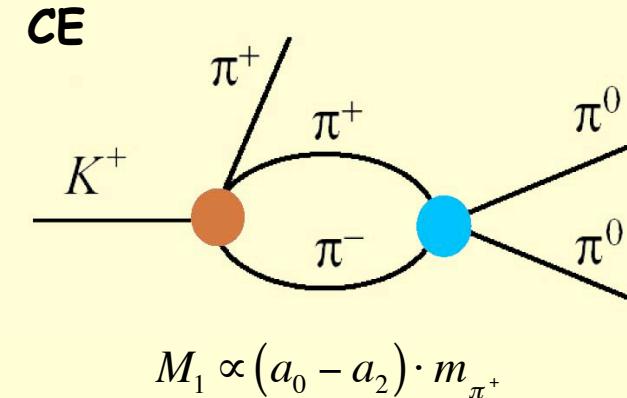
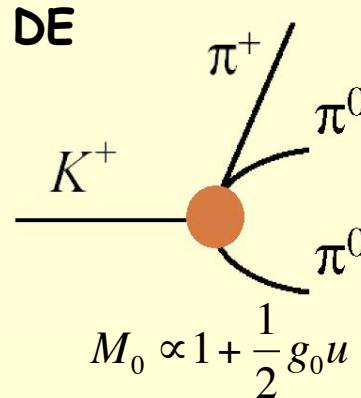
# Interpretation (I)



Re-scattering model: two amplitudes contribute to  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$

$$M(K^\pm \rightarrow \pi^\pm \pi^0 \pi^0) = M_0 + M_1$$

- >  $M_0$ : Direct emission
- >  $M_1$ : Charge exchange in final state of  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$  ( $\pi^+ \pi^- \rightarrow \pi^0 \pi^0$ )



The singularity in the invariant mass spectrum at  $\pi^+ \pi^-$  threshold is mainly caused by the destructive interference of  $M_0$  and  $M_1$

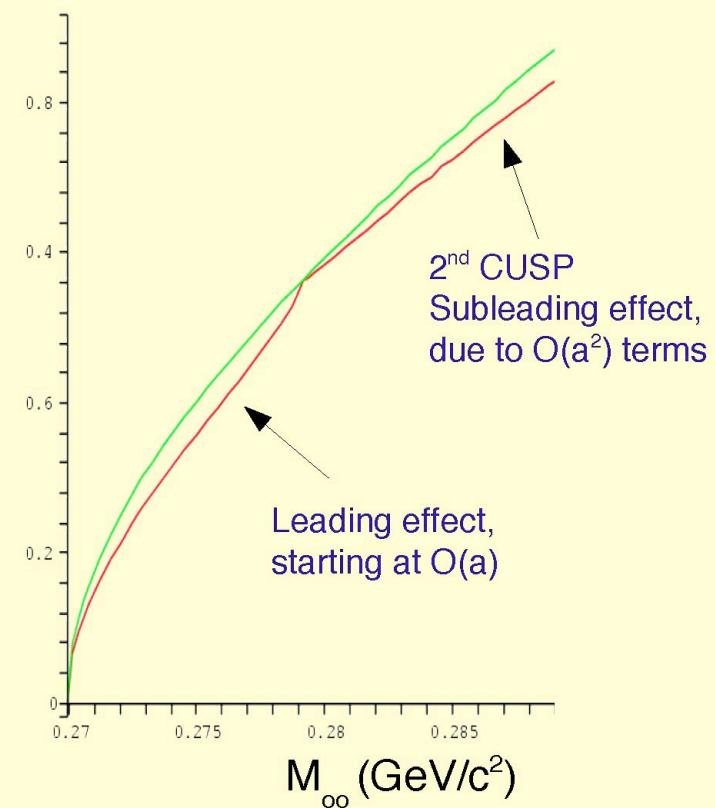
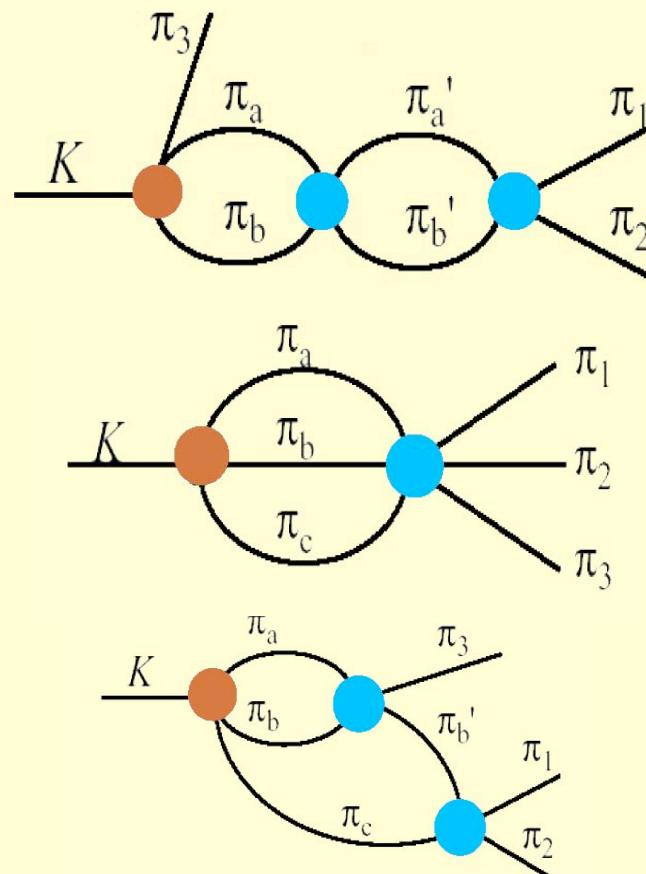
The effect is present below the threshold and not above it  
(re-scattering model at one-loop (N. Cabibbo: PRL 93 (2004) 121801))



# Interpretation (II)



More complete formulation of the model including all re-scattering processes at one-loop and two-loop level (N. Cabibbo and G. Isidori: JHEP 0503 (2005) 21) has been used to extract NA48/2 results





# Results (I)



Systematic checks: acceptance determination, trigger efficiency and fitting interval

$$g_0 = 0.645 \pm 0.004_{\text{stat}} \pm 0.009_{\text{syst}}$$

$$h' = -0.047 \pm 0.012_{\text{stat}} \pm 0.011_{\text{syst}}$$

$$(a_0 - a_2) \cdot m_{\pi^+} = 0.268 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{ext}}$$

$$a_2 \cdot m_{\pi^+} = -0.041 \pm 0.022_{\text{stat}} \pm 0.014_{\text{syst}}$$

Predictions in ChPT (PLB 488 (2000) 261):

- >  $(a_0 - a_2) \cdot m_{\pi^+} = 0.265 \pm 0.004$
- >  $a_2 \cdot m_{\pi^+} = -0.0444 \pm 0.0010$

Fit imposing ChPT constraint between  $a_0$  and  $a_2$  (PRL 86 (2001) 5008)

$$a_0 \cdot m_{\pi^+} = 0.220 \pm 0.006_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.011_{\text{ext}}$$

$$(a_0 - a_2) \cdot m_{\pi^+} = 0.264 \pm 0.006_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{ext}}$$

Published results (2003): PLB 633 (2006) 173-182



# Results (II)

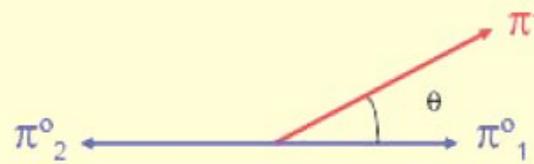


Preliminary

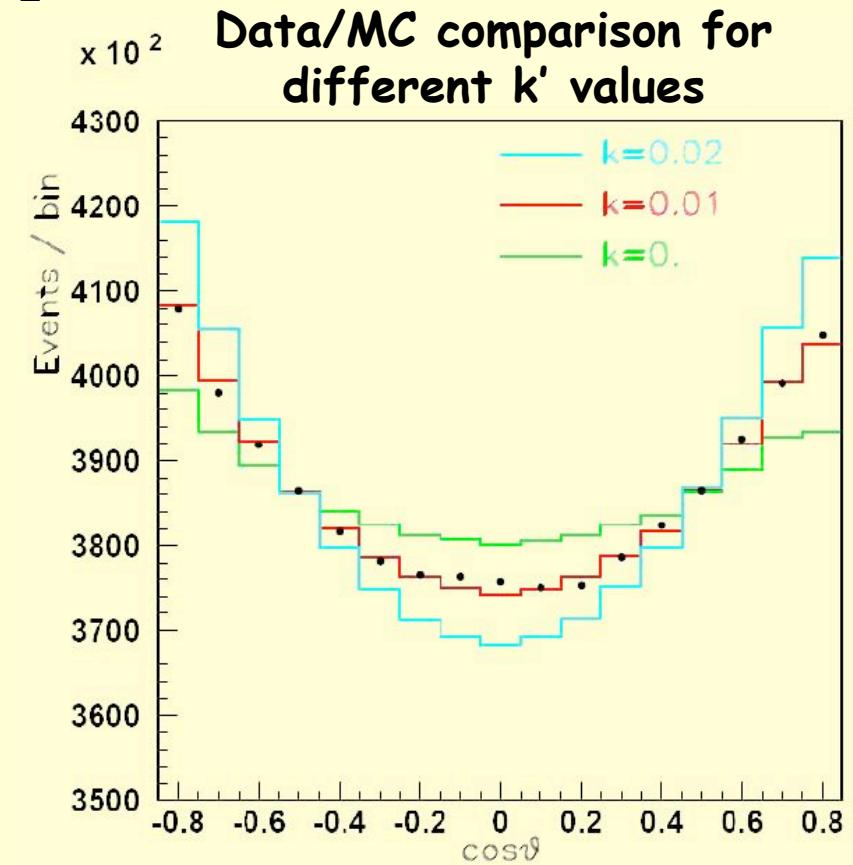
Fitting Dalitz plot above cusp finds evidence for  $k' > 0$  term

$$M_{+00}^2 = 1 + \frac{1}{2}g_0 u + \frac{1}{2}h' u^2 + \frac{1}{2}k' v^2 + \dots$$

Change of Dalitz variables, from  $(s_3, s_2-s_1)$  to  $(s_3, \cos\theta)$ . Define  $\theta$  as angle between  $\pi^\pm$  and  $\pi^0$  in  $\pi^0\pi^0$  rest frame:



$g_0$  and  $h'$  change (2% and 25% respectively) but no change in  $(a_0 - a_2) \cdot m_{\pi^+}$  and  $a_2 \cdot m_{\pi^+}$



$$k' = 0.0097 \pm 0.0003_{\text{stat}} \pm 0.0008_{\text{syst}}$$



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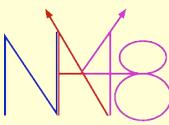
# *ChPT Tests:*

## *$K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ Decay*



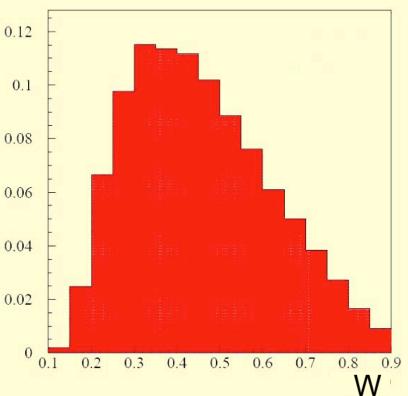
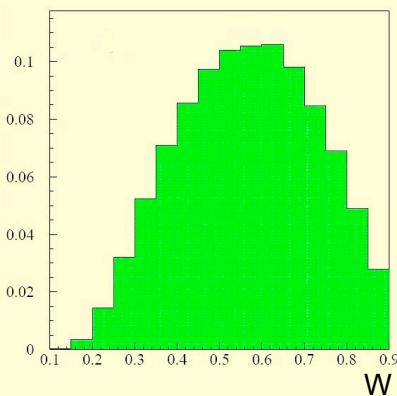
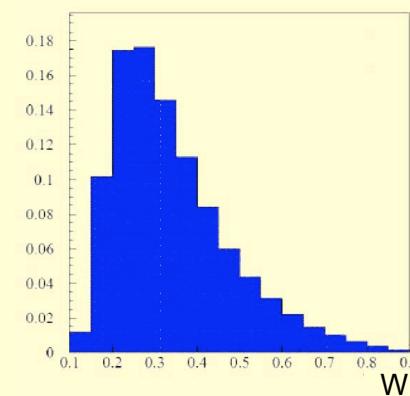
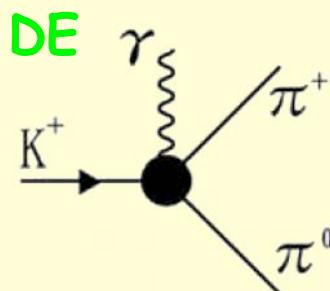
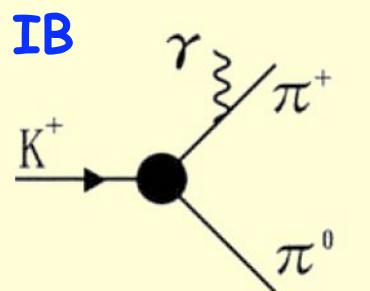


# Introduction



$$\frac{d\Gamma^\pm}{dW} \approx \left( \frac{d\Gamma^\pm}{dW} \right)_{IB} \left[ 1 + 2 \left( \frac{m_\pi}{m_K} \right)^2 \cdot W^2 \cdot |E| \cos((\delta_l - \delta_0) \pm \phi) + \left( \frac{m_\pi}{m_K} \right)^4 \cdot W^4 \cdot (|E|^2 + |M|^2) \right]$$

IB                                    INT                            DE



$$W^2 = \frac{(P_K^* \cdot P_\gamma^*)(P_\pi^* \cdot P_\gamma^*)}{(m_K m_\pi)^2}$$

PDG (55 MeV <  $T_\pi^*$  < 90 MeV)

IB:  $(2.75 \pm 0.15) \cdot 10^{-4}$

DE:  $(4.4 \pm 0.8) \cdot 10^{-6}$

INT: not yet measured

All previous measurements have been performed setting INT to zero

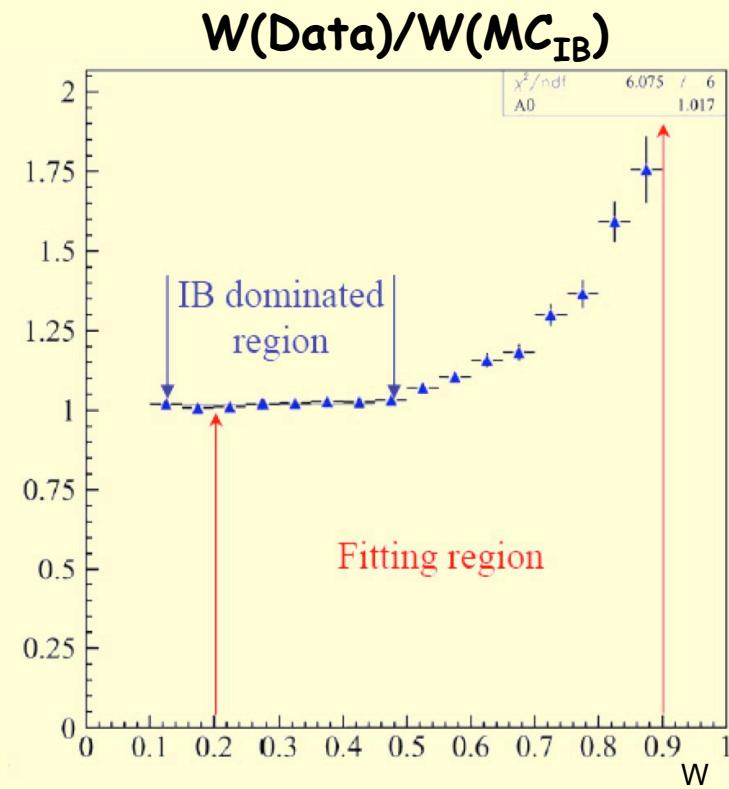
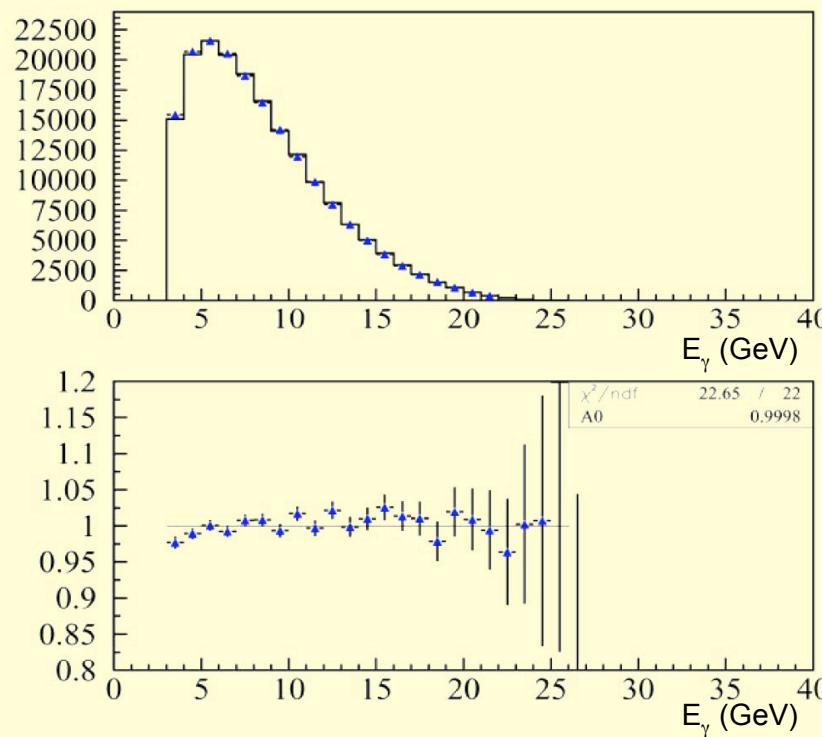


# Data/MC Comparison



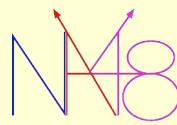
In the 2003 data sample ( $\sim 30\%$  of the whole statistics)  $\sim 220 \cdot 10^3$   $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$  have been selected:

- > After trigger efficiency correction good agreement between Data and MC for  $E_\gamma$ , in particular for  $E_\gamma > 5$  GeV (used for final result)
- > The ratio  $W(\text{Data})/W(\text{MC}_{\text{IB}})$  is in good agreement for IB dominated region and clearly shows DE



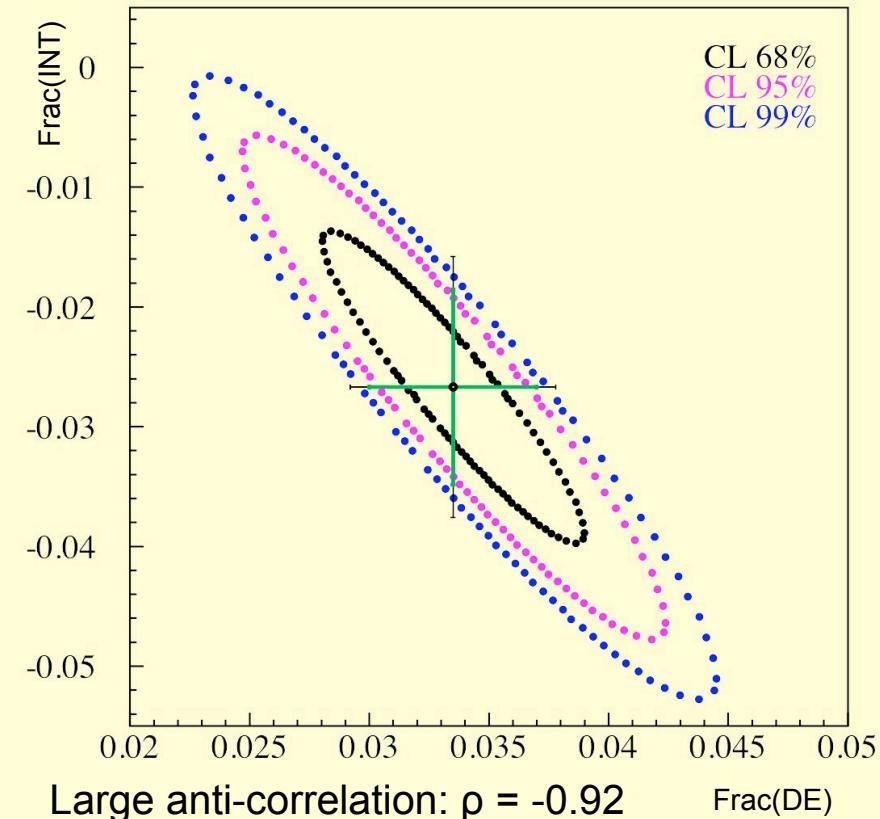


# Results



Use extended Maximum Likelihood for  $0.2 < W < 0.9$  to fit in the region  $0 \text{ MeV} < T_{\pi}^* < 80 \text{ MeV}$  (based on  $124 \cdot 10^3$  events)

-> First evidence of Interference between Inner Bremsstrahlung and Direct Emission amplitudes



$$\text{Frac(DE)} = (3.35 \pm 0.35_{\text{stat}} \pm 0.25_{\text{syst}}) \%$$

$$\text{Frac(INT)} = (-2.67 \pm 0.81_{\text{stat}} \pm 0.73_{\text{syst}}) \%$$



NH8

*ChPT Tests:*

$K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$  Decay



# BR Measurement

Preliminary

Never observed before. Naïve estimation of the BR:

$$\text{BR}(\pi^\pm e^+ e^- \gamma) = \text{BR}(\pi^\pm \gamma\gamma) \cdot 2\alpha \sim 1.6 \cdot 10^{-8}$$

Theoretical expectation (ChPT, Gabbiani99):

$$\text{BR}(\pi^\pm e^+ e^- \gamma) = (0.9 \div 1.6) \cdot 10^{-8}$$

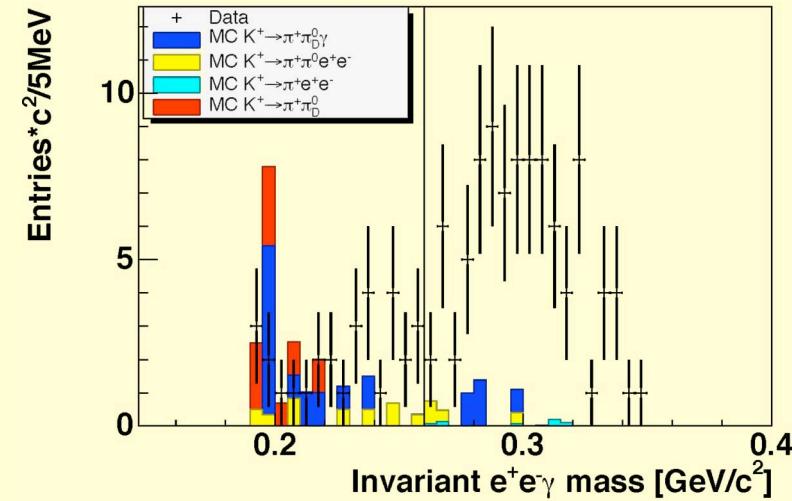
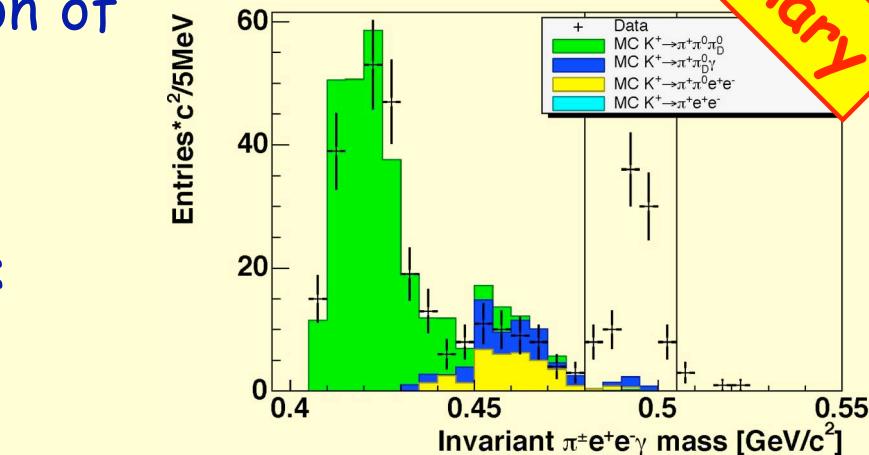
depending on  $\hat{c}$

Event sample (2003+2004):

- > 92 candidates events
- >  $1 \pm 1$  accidental background
- >  $5.1 \pm 1.7$  physical background

Normalization channel:

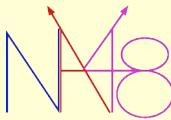
$$K^\pm \rightarrow \pi^\pm \pi^0_D: 14 \cdot 10^{-6} \text{ events}$$



$$\text{BR}(K^\pm \rightarrow \pi^\pm e^+ e^- \gamma) = (1.27 \pm 0.14_{\text{stat}} \pm 0.05_{\text{syst}}) \cdot 10^{-8}$$



# Summary (I)



- > NA48/2 has improved measurements of the  $K_{e4}$  Form Factors in the "charged" and "neutral" modes (5–30% relative statistical precision)
- > Using a conservative theoretical approach, preliminary values of  $a_0$  and  $a_2$  are obtained (2-D fit):

$$a_0 \cdot m_{\pi^+} = 0.233 \pm 0.016_{\text{stat}} \pm 0.012_{\text{syst}}$$

$$a_2 \cdot m_{\pi^+} = -0.047 \pm 0.011_{\text{stat}} \pm 0.008_{\text{syst}}$$

- > More elaborated theoretical corrections would bring down those values in close agreement with ChPT predictions
- > A new "cusp" structure in  $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$  was observed ( $\pi\pi$  final state charge exchange process of  $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ ) which provides a new method for the extraction of the  $\pi\pi$  scattering lengths:

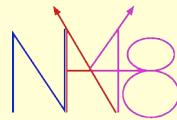
$$(a_0 - a_2) \cdot m_{\pi^+} = 0.268 \pm 0.010_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.013_{\text{theor}}$$

- > Parameter  $a_2$  directly measured for the first time even though with low accuracy:

$$a_2 \cdot m_{\pi^+} = -0.041 \pm 0.022_{\text{stat}} \pm 0.014_{\text{syst}}$$



# Summary (II)



- > The first measurement of Direct Emission and Interference terms in  $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$  has been performed in the region  $0 \text{ MeV} < T_{\pi}^* < 80 \text{ MeV}$ :

$$\text{Frac(DE)} = (3.35 \pm 0.35_{\text{stat}} \pm 0.25_{\text{syst}}) \%$$

$$\text{Frac(INT)} = (-2.67 \pm 0.81_{\text{stat}} \pm 0.73_{\text{syst}}) \%$$

- > A first evidence of a negative Interference has been found and therefore a non negligible contribution of electric term to Direct Emission amplitude

- > The  $K^\pm \rightarrow \pi^\pm e^+ e^- \gamma$  decay has been observed for the first time:

$$\text{BR}(K^\pm \rightarrow \pi^\pm e^+ e^- \gamma) = (1.27 \pm 0.14_{\text{stat}} \pm 0.05_{\text{syst}}) \cdot 10^{-8}$$