Precision tests of QCD from experimental studies of $K^{\pm}$ decays with the NA48/2 experiment at CERN

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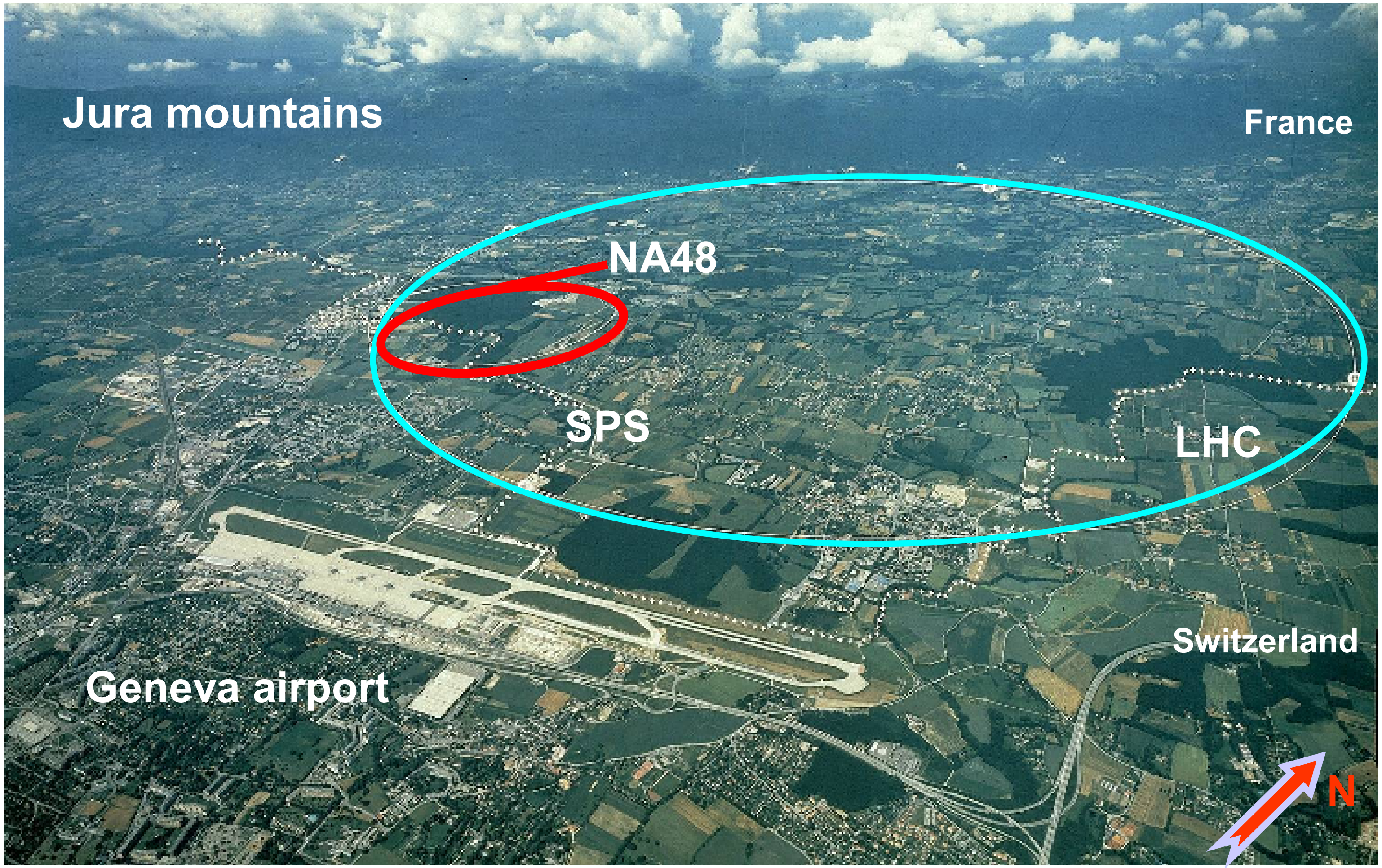
Outline

- **Beam line, detector** : $K^{\pm}$ beams, experimental setup, performances
- **QCD measurements** from kaons decays
- **$K_{3\pi}$ decays ($K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \pi^{0}$) :** the "cusp effect"
  Dalitz plot parameters and $\pi\pi$ scattering lengths a_0, a_2
- **K_{e4} decays ($K^{\pm} \rightarrow e^{\pm} \nu \pi^{+} \pi^{-}$) :**
  Form Factors, $\pi\pi$ phase shift δ and a_0, a_2
- **Comparison with theory (χPT)**
- **Summary**

NA48/2 : CERN experiment dedicated to Kaon physics

The NA48/2 collaboration: ~100 physicists from 15 Institutes

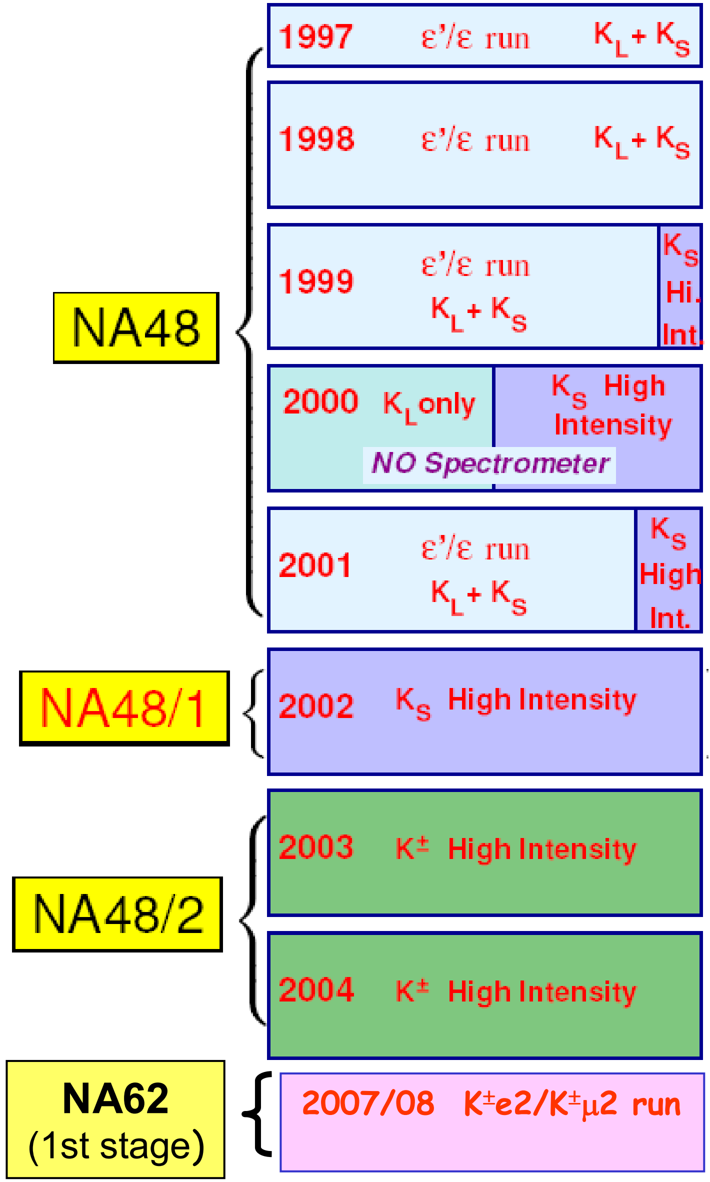
Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien



NA48 over a decade

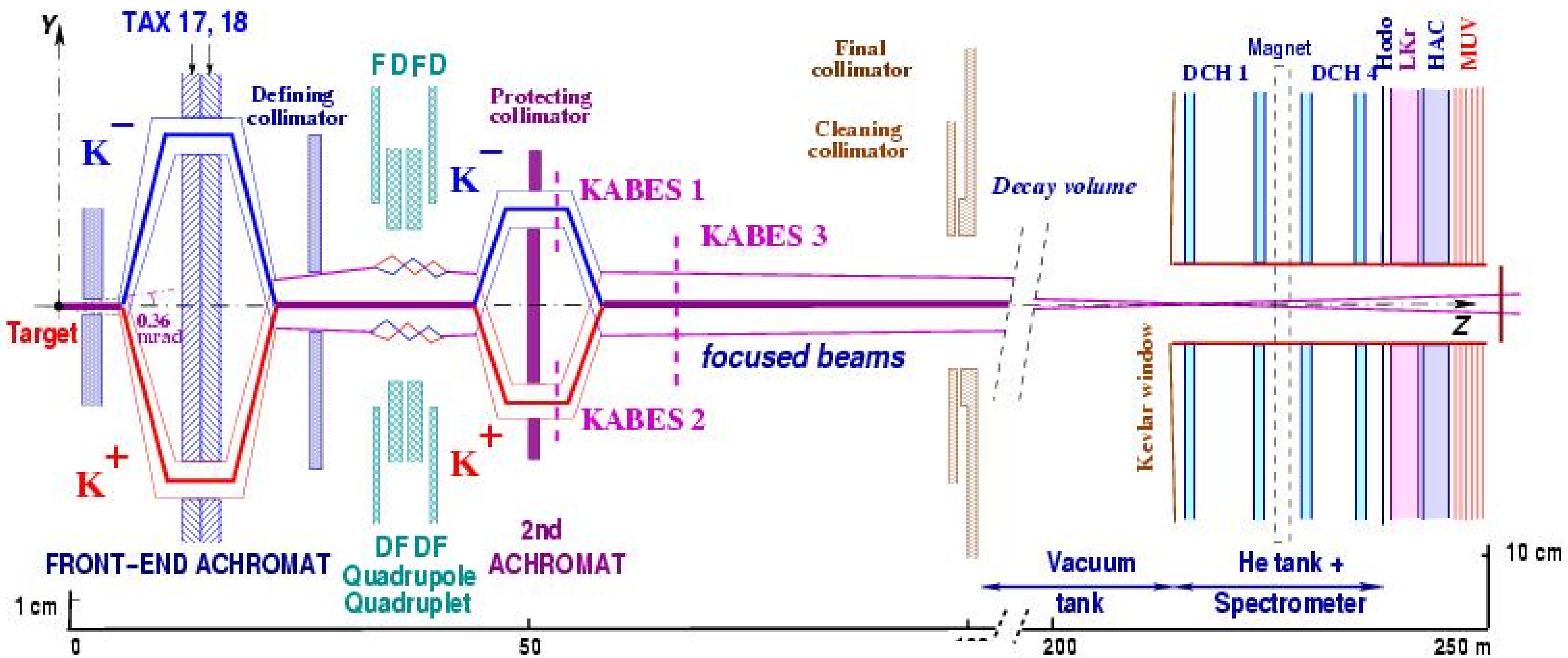
NA48 designed for measurements
of CPV quantities in kaon decays

- **NA48 1997-2001** : ε'/ε measurement
- **NA48/1 2002 : K_s** High intensity
- **NA48/2 2003-2004** : search for direct CP violating charge asymmetries A_g in $K^{\pm}\to3\pi$ decays + many rare decays
- **NA62** (1[st] stage) 2007-2008: study of **$K^{\pm}_{e2}/K^{\pm}_{\mu2}$ ratio**
- NA62 2011-2012: search for $K^{+}\to\pi^{+}\nu\bar{\nu}$

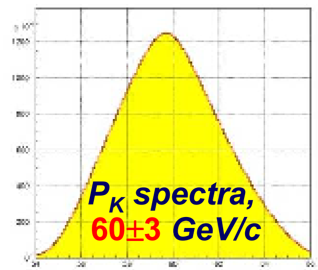


NA48/2 at CERN-SPS

2003 run: ~ 50 days + 2004 run: ~ 60 days

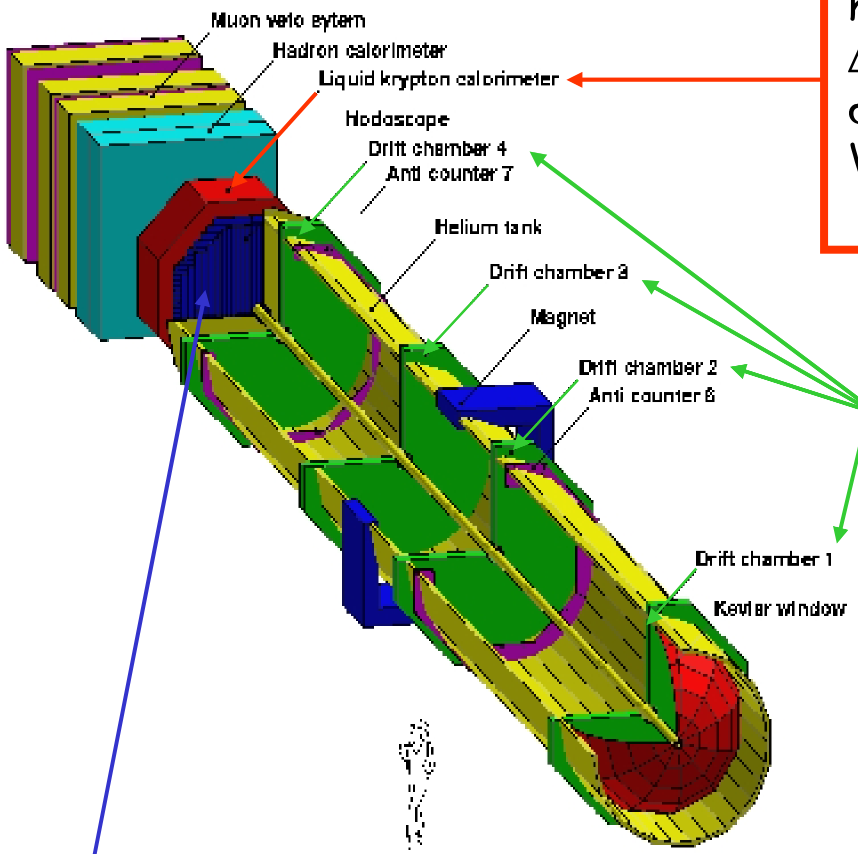


Simultaneous K^+ and K^- beams: large charge symmetrization of experimental conditions



Beams coincide within ~1mm all along the 114m decay volume flux ratio K^+/K^- ~1.8

The NA48/2 experiment: detector and performances



LKr electromagnetic calorimeter :
homogeneous and high granularity
$\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV)
$\sigma_x = \sigma_y \sim 1.5$ mm for E=10 GeV
Very good resolution for neutrals ($\pi^0 \rightarrow \gamma\gamma$)
e.g. $\sigma\,(M\pi\pi^0\pi^0) = 1.3$ MeV/c^2

Magnetic spectrometer :

- 4 DCH's + dipole magnet
- $\Delta p/p = (1.0 \oplus 0.044\ p)\%$ (p in GeV/c)
- Very good resolution for charged invariant masses: $\sigma\,(M3\pi^{\pm}) = 1.7$ MeV/c^2

E/p ratio used for e /π discrimination

Hodoscope for charged fast trigger $\sigma_t = 150$ ps

The NA48/2 data taking in 2003-2004 : (50 + 60) days

~10^{10} triggers and > 200 TB of data recorded

Total statistics in 2 years:

$K^{\pm} \to \pi^{+} \pi^{-} \pi^{\pm}$: ~ 4. 10^{9} evts

$K^{\pm} \to \pi^{0} \pi^{0} \pi^{\pm}$: ~1. 10^{8} evts

$K^{\pm} \to \pi^{+} \pi^{-} e^{\pm} \nu$: ~1.1 10^{6} evts

$K^{\pm} \to \pi^{0} \pi^{0} e^{\pm} \nu$: ~3.1 10^{4} evts

and more… in particular radiative decays

Published:

Ag_{+}(2003+2004) Phys.Lett.B, 634(2006)474

Dalitz plot param. Phys.Lett.B, 649(2007)349

Ag_{0}(2003+2004) Phys.Lett.B, 638(2006) 22

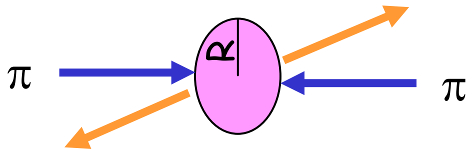
common method EPJC 52 (2007) 875

$K_{3\pi}$ **cusp 2003** Phys. Lett. B, 633(2006) 173

K_{e4} 2003 EPJC 54 (2008) 411

Results from full statistics 2003+2004 on $K_{3\pi}$ cusp and K_{e4} are presented

$\pi\pi$ scattering lengths: why interesting?



angular wave number: $k = \frac{\sqrt{2mE}}{\hbar}$

- Low energy kR <<1 ⇨ S-wave dominates the total cross section
- isospin I = 0, 2 allowed by Bose statistics
- scattering matrix ⇨ $S|\pi\pi\rangle = \exp(2i\delta)|\pi\pi\rangle$

  2 phases: $\delta_{0,2} = a_{0,2}\, k$ related to scattering lengths a_0, a_2

  scattering lengths a_0, a_2 are essential parameters of χPT

→ *spontaneous symmetry breaking from $\langle q\,\bar{q}\rangle$ condensate ?*

QCD tests from Kaon decays

low energy regime of hadron physics below 1 GeV

- described by effective field theory and isospin symmetry limit
- spontaneous symmetry breaking (χPT) is treated as a perturbation (Low Energy Constants $\bar{l}_i$)
- lattice QCD also promising to compute LECs

M_π and form factor F_π functions of m_q, LECs and quark condensate $\langle q\,\bar{q}\rangle$

$$M_\pi^2 = M^2\left(1 - \frac{M^2}{32\pi^2F^2}\bar{\ell}_3 + O(p^4)\right)$$

$$M^2 \equiv -\frac{m_u + m_d}{F^2}\langle 0|\bar{q}q|0\rangle$$

Gell-Mann, Oakes, Renner (68)

$$F_\pi = F\left(1 + \frac{M^2}{16\pi^2F^2}\bar{\ell}_4 + O(p^4)\right)$$

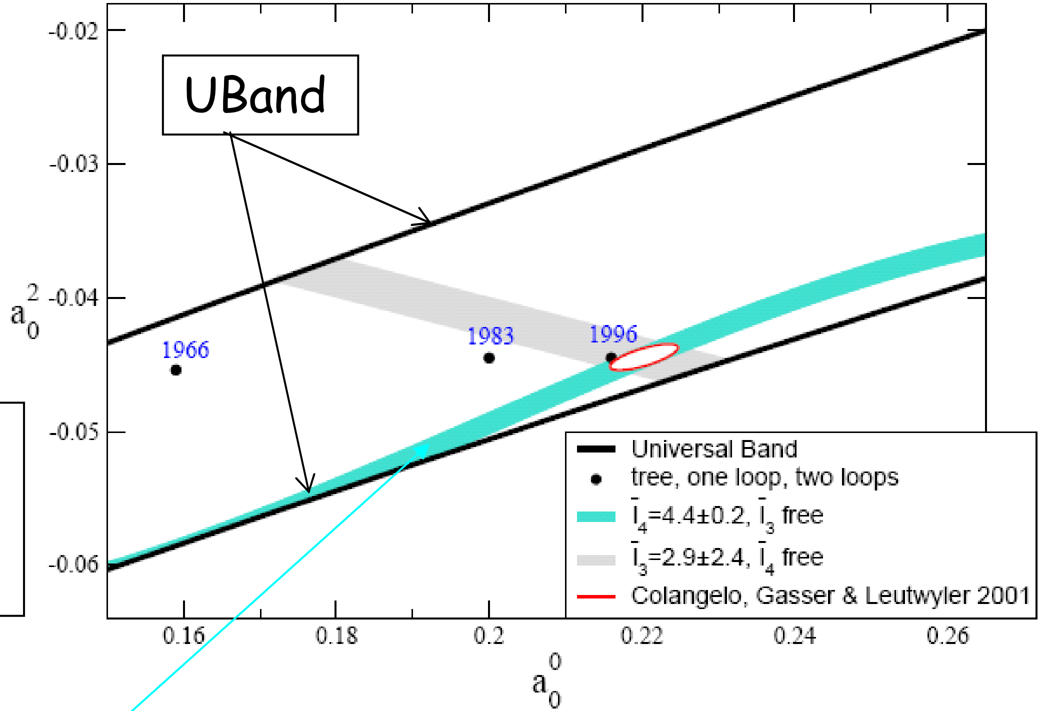
calculating ππ scattering lengths

ππ scattering amplitudes with ≠ Isospin related through Roy equations where a_0 and a_2 are essential parameters
Roy equations numericaly solved by ACGL(2001) and DFGS (2002)

Universal Band :

$a_2 = (-0.0849 + 0.232\, a_0 - 0.0865\, a_0^2)\pm 0.0088$

ACGL= Bern Group : B. Ananthanarayan, G. Colangelo, J. Gasser, H. Leutwyler DFGS = Orsay Group : S. Descotes-Genon, N. Fuchs, L. Girlanda, J. Stern



χPT constrains a_0 and a_2 further in
χPT band:
$a_2 = (-0.0444 + 0.236\,(a_0 - 0.22) - 0.61\,(a_0 - 0.22)^2 - 9.9\,(a_0 - 0.22)^3) \pm 0.0008$

precise predictions from χPT (CGL 2001)

$a_0 = 0.220 \pm 0.005$

$a_2 = -0.0444 \pm 0.0008$

$\Rightarrow (a_0 - a_2) = 0.265 \pm 0.004$

measuring $\pi\pi$ scattering lengths

several approaches have been developed

- $K^{\pm} \rightarrow \pi^{+} \pi^{-} e^{\pm} \nu$ decays though semi-rare BR = $(4.09 \pm 0.09)\ 10^{-5}$ give

  very clean environment (no other hadron)
- a_0 and a_2 measured with angular distributions
- used since 1960's but limited by statistics

  Geneva-Saclay CERN/PS S118 experiment: 30 000 K^{+} (1977)

  BNL E865 experiment: 400 000 K^{+} (2003)

  CERN/SPS NA48/2: 1 130 000 $K^{\pm}$ (2009)

Pionium atoms : DIRAC CERN/PS $\pi\pi$ life time measurement

Sensitivity to $(a_0-a_2)^2$, PLB 619 (2005) ~40% data analyzed

- $K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$ decays have copious BR = $(1.757 \pm 0.024)\ 10^{-2}$
- cusp in $M\pi_0\pi_0$ sensitive to (a_0-a_2) and a_2

  CERN/SPS NA48/2: 16×10^{6} (2006) and 60×10^{6} (2009)

Cusp effect in $K^{\pm} \to \pi^0 \pi^0 \pi^{\pm}$: first observation

matrix element of $K^{\pm} \to \pi^0 \pi^0 \pi^{\pm}$ decay described as an expansion with Dalitz plot variables u and v

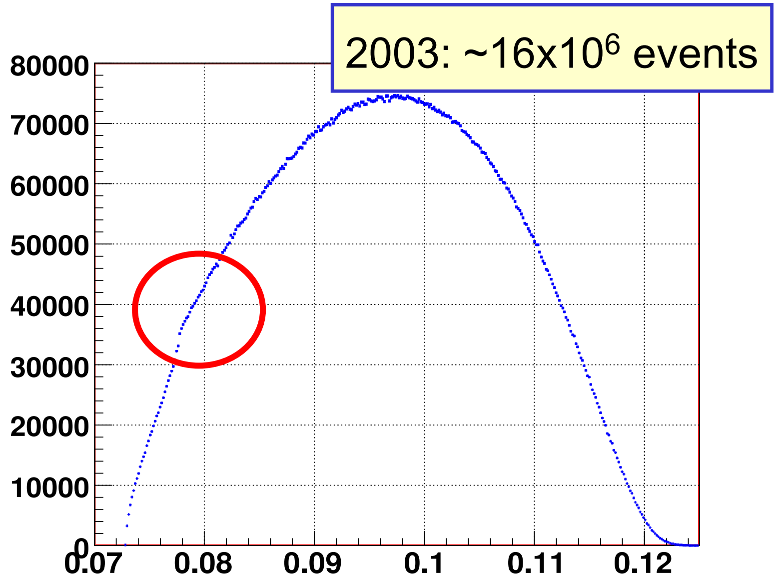
$u = (s_3 - s_0)/m_\pi^2 \qquad v = (s_2 - s_1)/m_\pi^2$

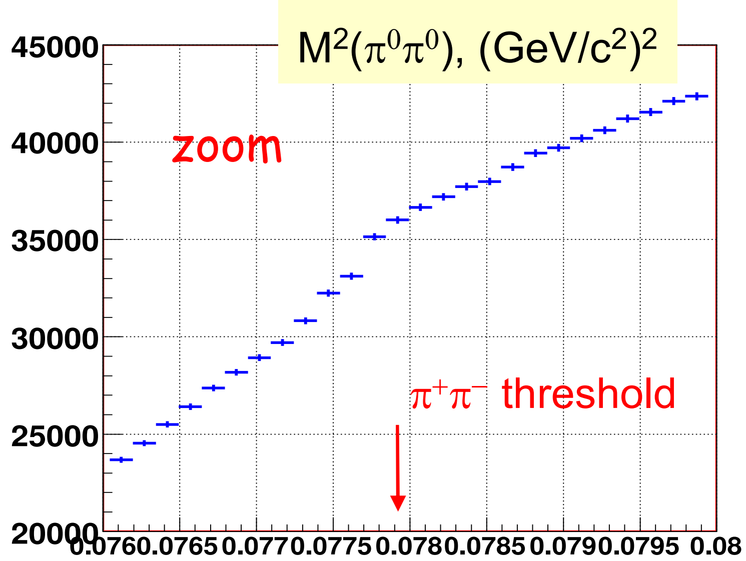
$s_0 = (s_1 + s_2 + s_3)/3 \qquad s_i = (P_K - P_{\pi i})^2 = M_{jk}^{\ 2}$

$|M_0|^2$ (PDG) $\sim 1 + gu + hu^2 + kv^2$ (PDG) or re-written as

$M_0 = A_0(1 + g_0 u/2 + h'_0 u^2/2 + k'_0 v^2/2)$

with $g_0 \approx g$, $h'_0 \approx h - g^2/4$, $k'_0 \approx k$





First observation of a cusp structure was made with 16×10^6 events collected in 2003, PLB 633 (2006), [arXiv:hep-ex/0511056] , very good mass resolution proved to be decisive

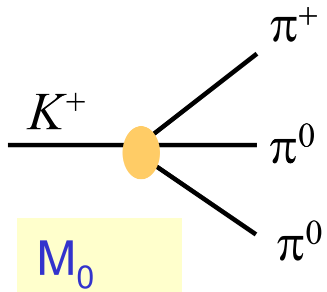
statistics increased with 44×10^6 more events from 2004 $\Rightarrow$ final analysis 60×10^6 events

Cusp interpreted as re-scattering effect of $K^{\pm}\to\pi^{\pm}\pi^{+}\pi^{-}$

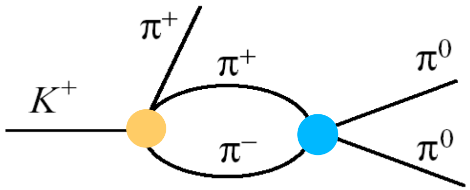
The cusp structure in the vicinity of $\pi^{+}\pi^{-}$ threshold is the signature of $\pi\pi$ re-scattering in the $K^{\pm}\to \pi^{+}\pi^{-}\pi^{\pm}$ decay

$$M_1 = -\frac{2}{3}(a_0 - a_2)\, m_{\pi^+} M_+ \sqrt{1-\left(M_{\pi^0\pi^0}/2\,m_{\pi^+}\right)^2}$$

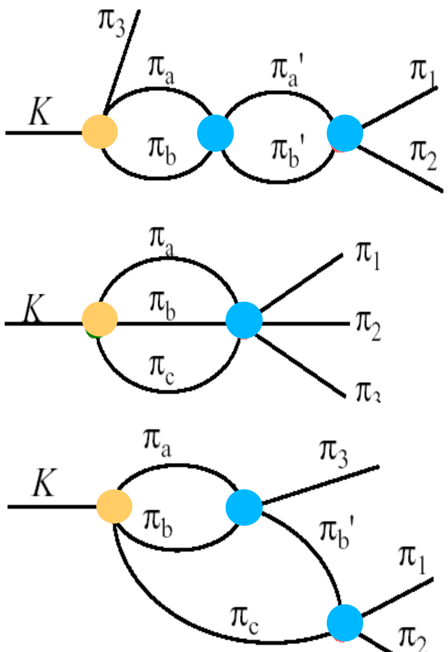
M_1 real below threshold, imaginary above



M_0

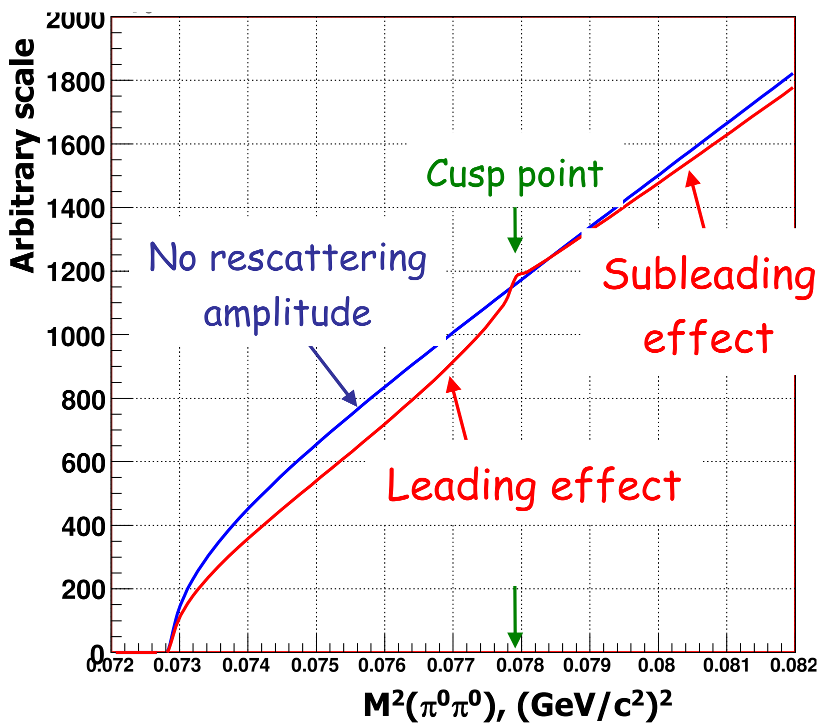


M_1



and more ..

Distortion due to loop effects



Two different approaches to extract scattering lengths

Cabibbo-Isidori (CI)

Cabibbo PRL93(2004) , CI JHEP 0503(2005)

- $M = M_0 + M_1$

above threshold $|M|^2 = |M_0|^2 + |M_1|^2$
below threshold $|M|^2 = |M_0|^2 + |M_1|^2 + 2 M_0 M_1$

- Two-loop effects included
- Radiative corrections not included,

Bern-Bonn (BB)

CGKR PLB 638 (2006) , and recently BFGKR NPB 806(2009)

- effective field theory approach based on non-relativistic Lagrangian
- two-loop formulation, different from CI, introduces different (larger) correlations between scattering lengths and Dalitz plot parameters
- electromagnetic effects included in the amplitudes (can be switched off for comparisons)

very close and fruitful collaboration between theorists and experimentalists to implement latest developments

Both formulations (CI and BB) are used to extract the physics parameters

$(g_0, h'_0, a_0-a_2, a_2)$

Cusp fitting procedure

Fit the M^2_{00} distribution using the detector response matrix Rij obtained from GEANT-based Monte-Carlo simulation and 4 physics parameters to minimize χ^2

$$\chi^2(g_0, h_0', a_0 - a_2, a_2, x) = \sum_{j=1}^{N_{bin}} \frac{(N_j - xM_j)^2}{N_j + x^2 M_j}$$

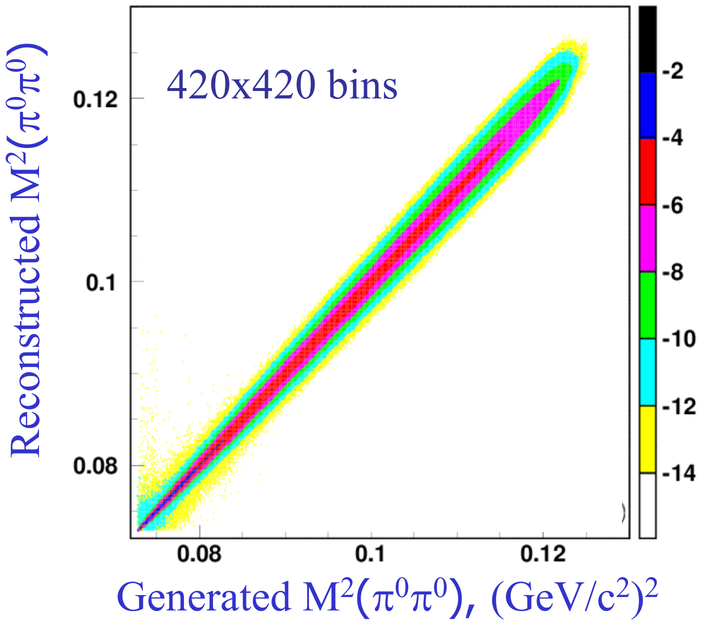
N_j data events/bin

M_j simulated events/bin = $\Sigma\, R_{ij} G_i$

x M_j expected events/bin

x = Data/MC normalization

LOG (R_{ij})



a_0, a_2 experimental precision: stat + syst

4p fit

$\Delta(a_0 - a_2) = \pm\ 0.005 \pm 0.002$ (CI,BB)

$\Delta a_2 = \pm\ 0.009 \pm 0.006$ (CI)

$\Delta a_2 = \pm\ 0.013 \pm 0.009$(BB)

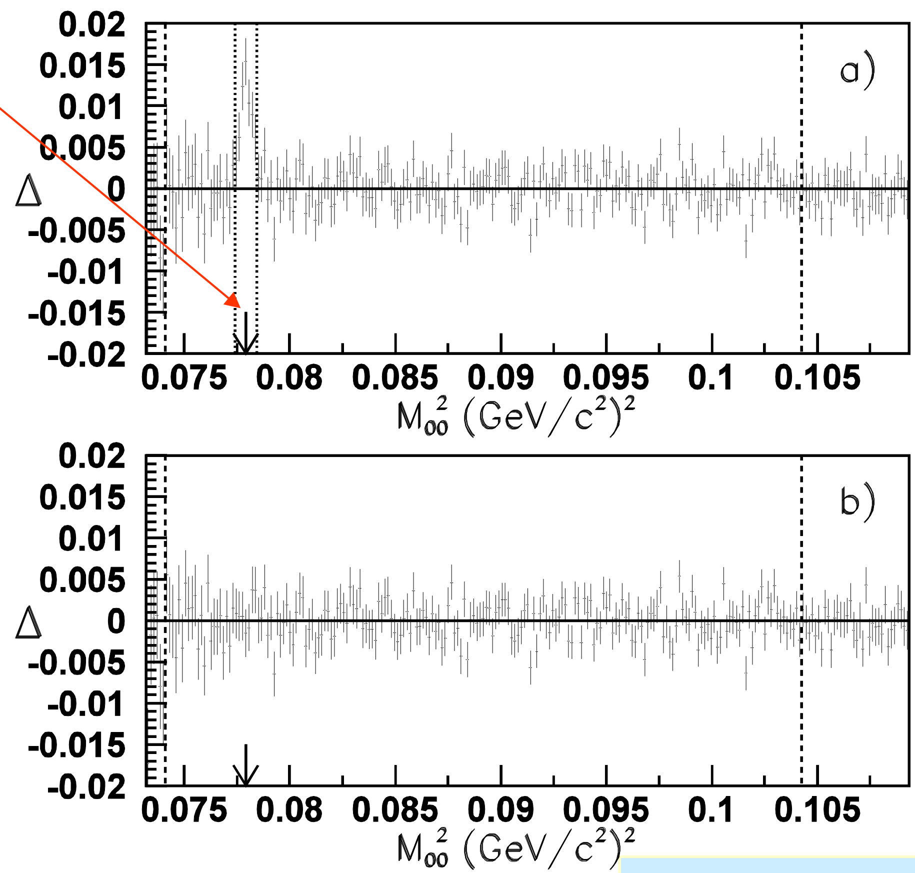
3p fit using $a_2 = f(a_0)$

$\Delta(a_0 - a_2) = \pm\ 0.002 \quad \pm\ 0.001$

Cusp : fit quality

- 7 bins around $2m_{\pi+}$ threshold are excluded from the fit to avoid the "perturbation" of pionium
- Pionium model without radiative correction Silagadze JETP Lett. 60(1994)689 predicts ~60%
- Pionium formation taken into account in the fit and including unbound $\pi\,\pi$ states as computed by Gevorkian et al. hep-ph/0612129

Δ = (Data – fit) / Data



$M^2(\pi^0\pi^0)$ $(Gev/c^2)^2$

Cusp: scattering lengths results

Preliminary (2003+2004)

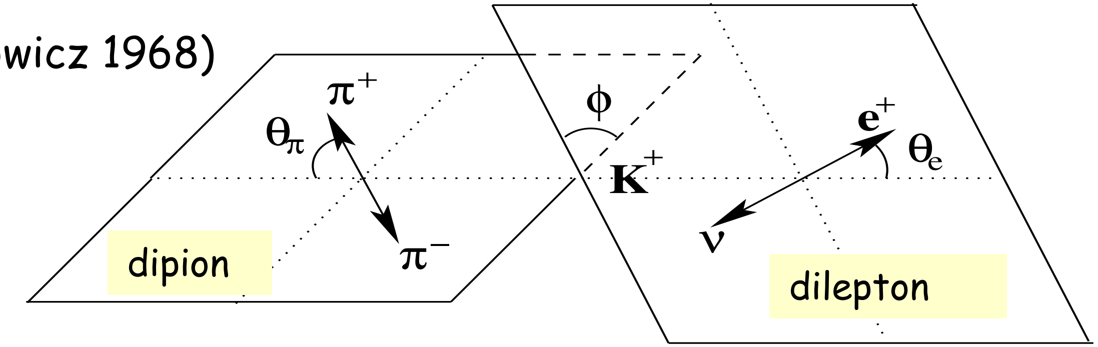
a_2 free	correlations between a_2 and other parameters are larger in BB model than in CI model
CI model	$a_0 - a_2 = 0.248 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.001_{ext} \pm 0.009_{th}$ $a_2 = -0.009 \pm 0.009_{stat} \pm 0.007_{syst} \pm 0.001_{ext} \pm 0.015_{th}$
BB model	$a_0 - a_2 = 0.257 \pm 0.005_{stat} \pm 0.002_{syst} \pm 0.001_{ext} \pm 0.009_{th}$ $a_2 = -0.024 \pm 0.013_{stat} \pm 0.009_{syst} \pm 0.002_{ext} \pm 0.015_{th}$

Using ChPT constraint	Note : ext is mainly due to $R=(A_+/A_0)^2 = 3.175 \pm 0.050$
CI model	$a_0 - a_2 = 0.265 \pm 0.002_{stat} \pm 0.001_{syst} \pm 0.002_{ext} \pm 0.005_{th}$ ($a_0 = 0.2203$ $a_2 = -0.0443$)
BB model	$a_0 - a_2 = 0.263 \pm 0.002_{stat} \pm 0.001_{syst} \pm 0.002_{ext} \pm 0.005_{th}$ ($a_0 = 0.2186$ $a_2 = -0.0447$)

K_{e4} decays : formalism

kinematic variables (Cabibbo-Maksymowicz 1968)

$S_\pi = M^2_{\pi\pi}$, $S = M^2_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



s, p waves expansion (Pais-Treiman 1968)
+ T-invariance give reach to δ_l^I

$$\delta_0^0 \equiv \delta_s \text{ and } \delta_1^1 \equiv \delta_p$$

F, G = 2 Axial Form Factors

$F = F_s e^{i\delta s} + F_p e^{i\delta p} \cos\theta_\pi$

$G = G_p e^{i\delta g}$

H = 1 Vector Form Factor

$H = H_p e^{i\delta h}$

F, G, H are complex and dimensionless

Fit the 5D distributions with 4 Form factors and one phase shift , assuming identical phases for the p-wave Form Factors F_p, G_p, H_p:

The fit parameters are :

F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

(F_s , F_p , $G_{p,}$, H_p are real)

Ke4 decays: event selection and background

Signal ($\pi^+\pi^- e^\pm \nu$) topology :

- 3 charged tracks and a good vertex
- two opposite sign pions,
- 1 electron (E/p ~ 1),
- some missing energy and p_T (ν)
- reconstruct PK (missing ν hypothesis)

Background main sources :

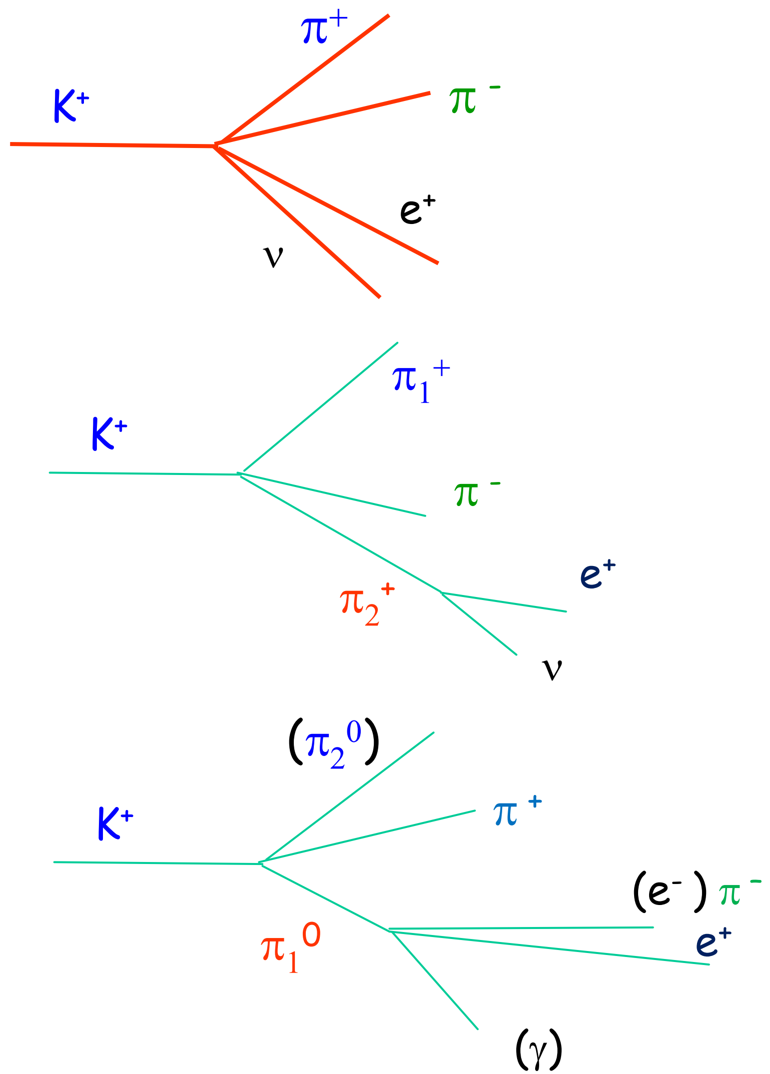
$K^+ \rightarrow \pi^-$ π_2^+ π_1^+ (dominant)

↳ e^- ν or mis-ident e^-

$K^+ \rightarrow \pi^0$ (π^0) π^+

↳ $(e^+e^-\gamma)$

↳ mis-ident π^+ and γ (s) undetected



Ke4 decays: background rejection

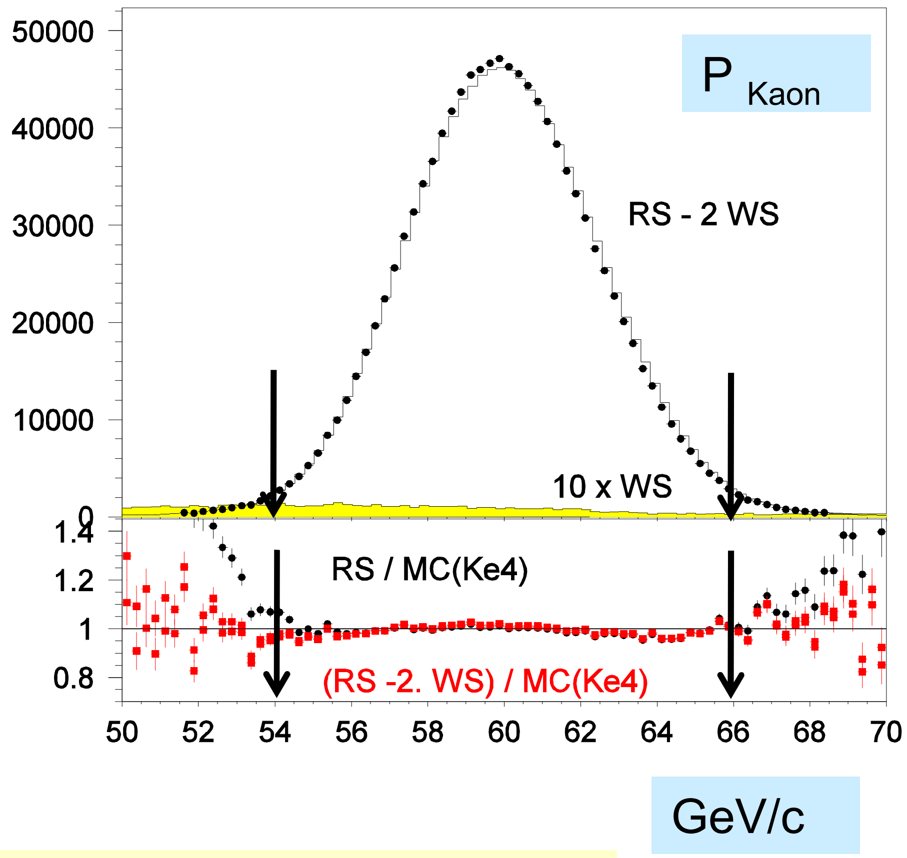
Control sample from data (assuming $\Delta S=\Delta Q$ valid)
$K^{\pm} \rightarrow \pi^{\pm}\ \pi^{\pm}\ e^{\mp}\ \nu$ "Wrong Sign" events

- total charge (±1) as "Right Sign" events
- electron charge opposite to total charge
- same sign pions

Rate (RS/WS) events:

2 if coming from $K_{3\pi}$ (dominant)

1 if coming from $K_{2\pi(\pi 0)}$



Total background level can be kept at ~ 2 x 0.3 % relative level estimated from WS events rate and checked from MC simulation

Ke4 decays : fitting procedure

Total (2003+2004) 1.13×10^6 Ke4 decays

Using iso-populated bins in the 5D space of kinematic variables $M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ one defines a grid of

10x5x5x5x12=15000 bins with "flexible" size

In each $M_{\pi\pi}$ "slice" a set of 4 parameters is fitted over 1500 bins to minimize the difference between the data and MC populations

The normalisation F_s^2 is obtained in each bin/slice by the ratio $x_{slice} = \Sigma_{j\ in\ slice} Nj / \Sigma_{j\ in\ slice} MCj$

Data sample	MC sample
K^+ sample (726 400 events) 48 events/box	K^+ MC (17.4×10^6 events) 1160 events/box
K^- sample (404 400 events) 27 events/box	K^- MC (9.7×10^6 events) 650 events/box

K^+ and K^- fitted separately in 10 independent $M_{\pi\pi}$ bins & combined on statistical basis

model independent analysis since no assumption is made on the variation of δ and FF versus $M_{\pi\pi}$

Ke4 Form Factors : fit results

Preliminary (2003+2004)

Following Amoros Bijnens (1999) FF are expanded versus q^2 ($q^2 = S_\pi/4m_\pi^2 - 1$) and $S_e/4m_\pi^2$ (isospin symmetry assumed)

$$F_s^2 = f_s^2(1 + f_s'/f_s\, q^2 + f_s''/f_s\, q^4 + f_e'/f_s\, S_e/4m_\pi^2)^2$$

Correlation	f_s''/f_s	f_e'/f_s
f_s'/f_s	-0.95	0.08
f_s''/f_s		0.02

$$G_p/f_s = g_p/f_s + g_p'/f_s\, q^2$$

Correlation -0.91

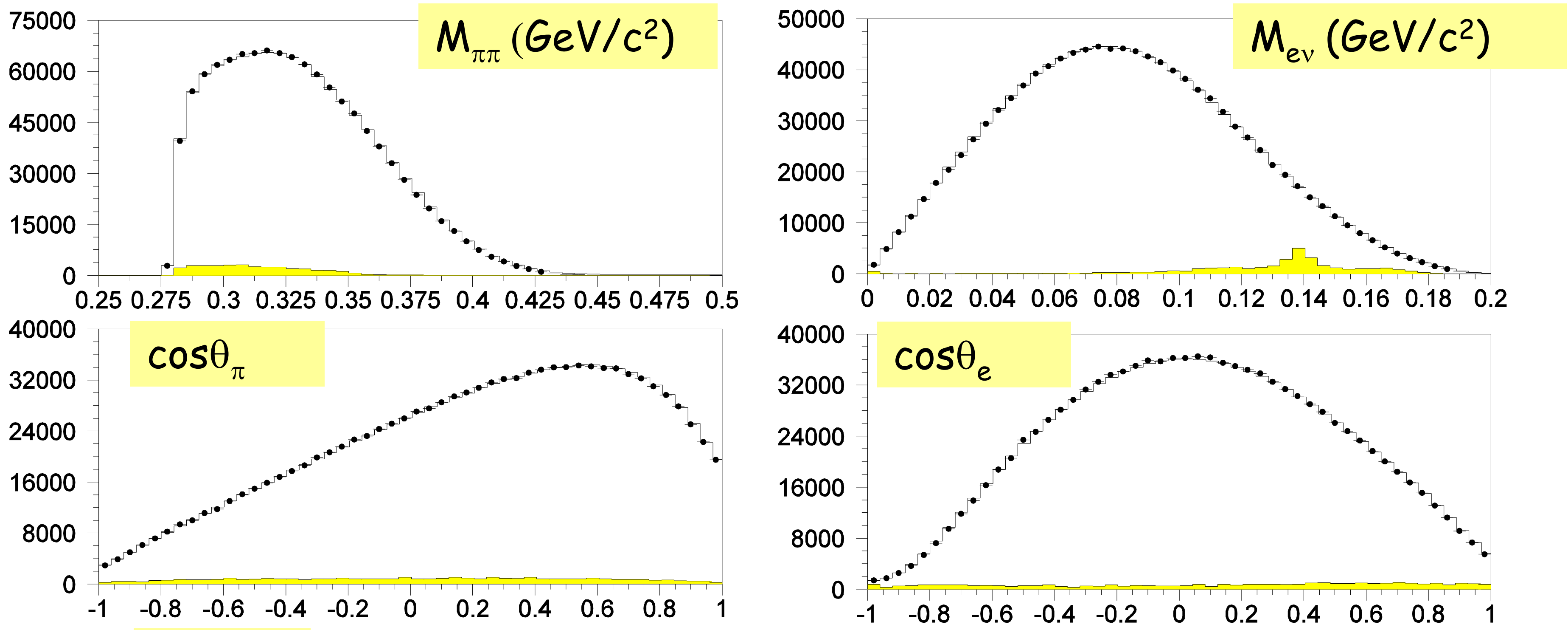
New !

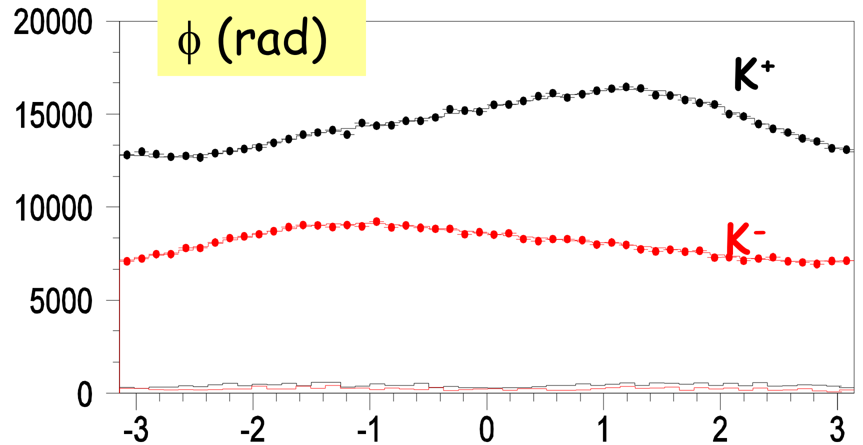
	value ± stat ± syst
f_s'/f_s	$0.152 \pm 0.007 \pm 0.005$
f_s''/f_s	$-0.073 \pm 0.007 \pm 0.006$
f_e'/f_s	$0.068 \pm 0.006 \pm 0.007$
f_p/f_s constant	$-0.048 \pm 0.003 \pm 0.004$
g_p/f_s	$0.868 \pm 0.010 \pm 0.010$
g_p'/f_s	$0.089 \pm 0.017 \pm 0.013$
h_p/f_s constant	$-0.398 \pm 0.015 \pm 0.008$

systematics

- mostly from background + acceptance control
- systematic error ≤ statistical error

Ke4 decays : Data/MC comparison after fit





(data marker)	= Data
histogram	= Simulation after fit
Yellow hist	= WS Background (x 10 to be visible)

CP symmetry :
(K$^+$) ϕ distribution is opposite of (K$^-$) ϕ distribution

Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)

$\pi\pi$ phase shifts predicted from data above 0.8 GeV using **Roy equations** with parameters **a_0 and a_2**

Numerical solutions calculated in the **Isospin symmetry limit (UB)**, need several corrections

factorization of electromagnetic and mass effects :

$\underbrace{\text{Gamow factor x PHOTOS generator}}$ x $\underbrace{\text{Isospin corrections}}$

Radiative effects included in the simulation,

Gamow factor : Coulomb force between charged particles

PHOTOS generator: real photon(s) are emitted and tracked in the simulation

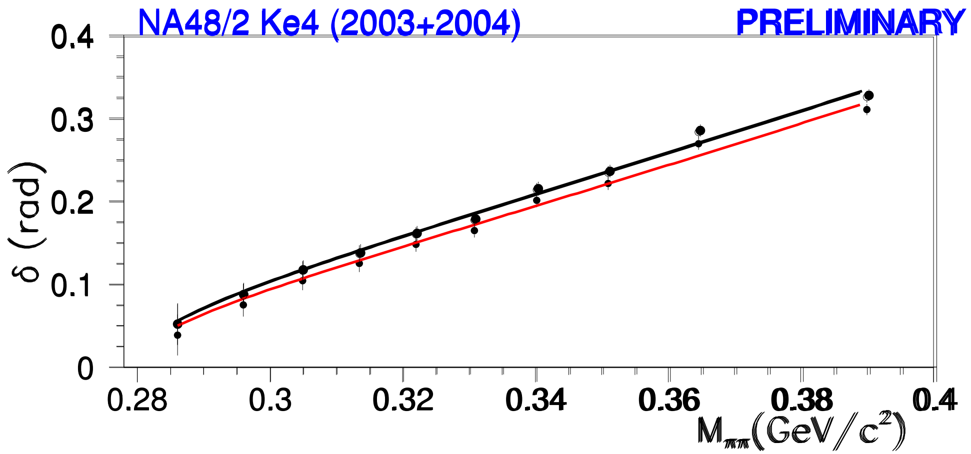
(-> effect on event selection + possible bias on reconstructed quantities)

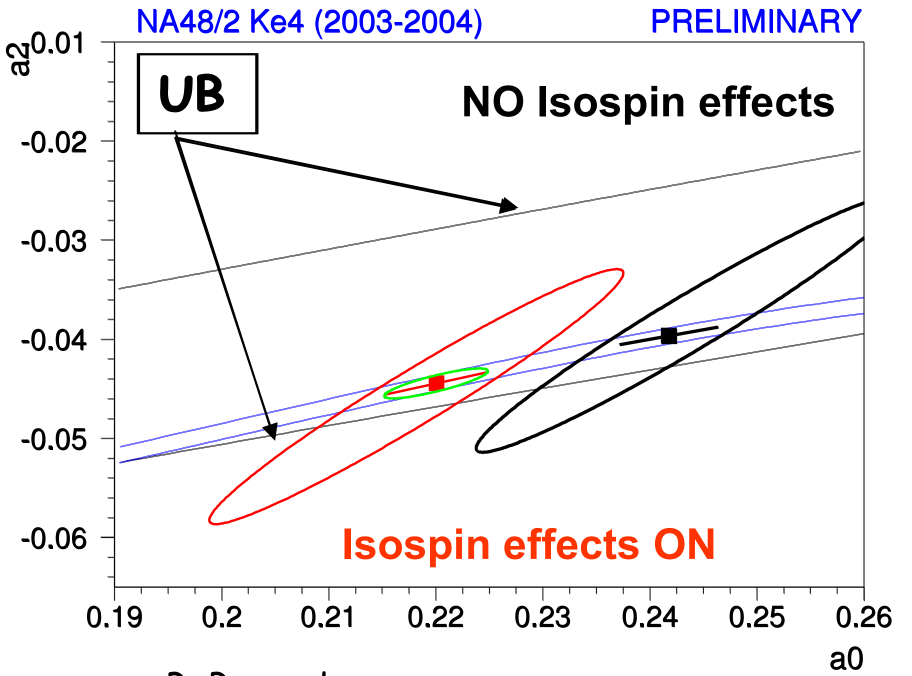
Mass effects:

- computed as a correction to $\delta_{\pi\pi}$
- larger than experimental error !

CGR EPJ C59 (2009) 777,
DK preliminary June 2008 in progress

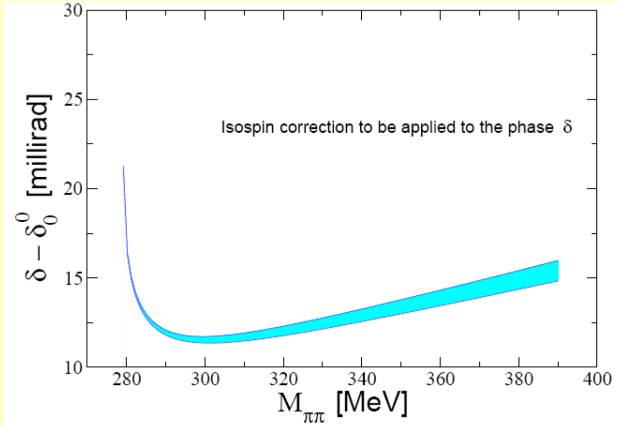
Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)





Stat error is 7-8 mrad

Isospin correction is 10-15 mrad



This induces a large **change** on (a_0,a_2) values

from a 2p fit	from a 1p fit
Δa_0 = -0.025, Δa_2 = -0.007	Δa_0 = -0.022
$\sigma(a_0)$: ± 0.013 ± 0.005	$\sigma(a_0)$: ± 0.005 ± 0.002
$\sigma(a_2)$: ± 0.0084 ± 0.0034	stat syst
stat syst	

Ellipses are 68% CL contours in 2p fits

Comparison with theoretical predictions

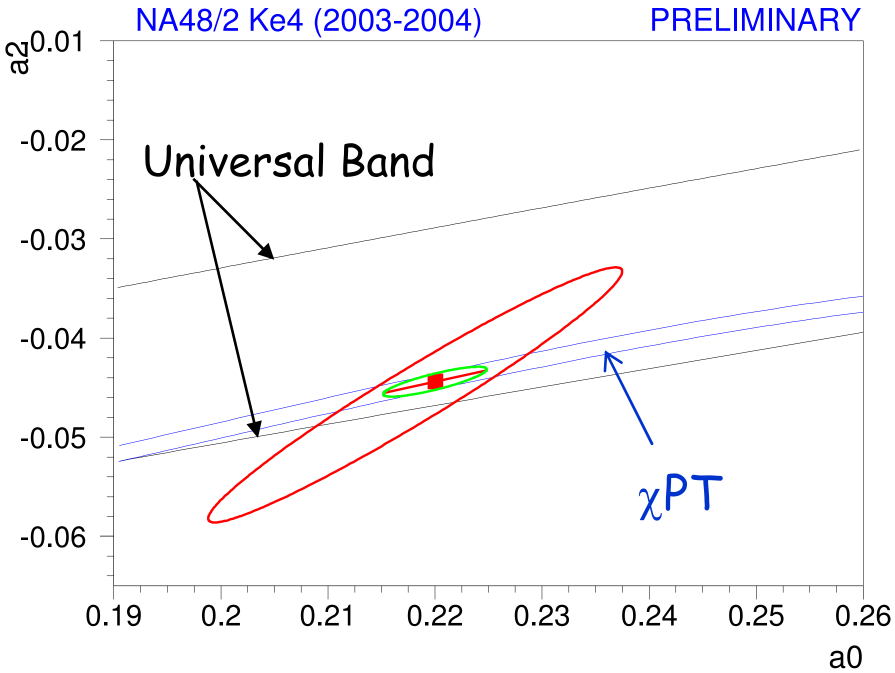
Preliminary (2003+2004)

THEORY prediction

Using more inputs from χPT and low energy constants, the prediction is better constrained (CGL NPB603(2001),PRL86(2001)):

$$a_0 = 0.220 \pm 0.005$$

$$a_2 = -0.0444 \pm 0.0008$$



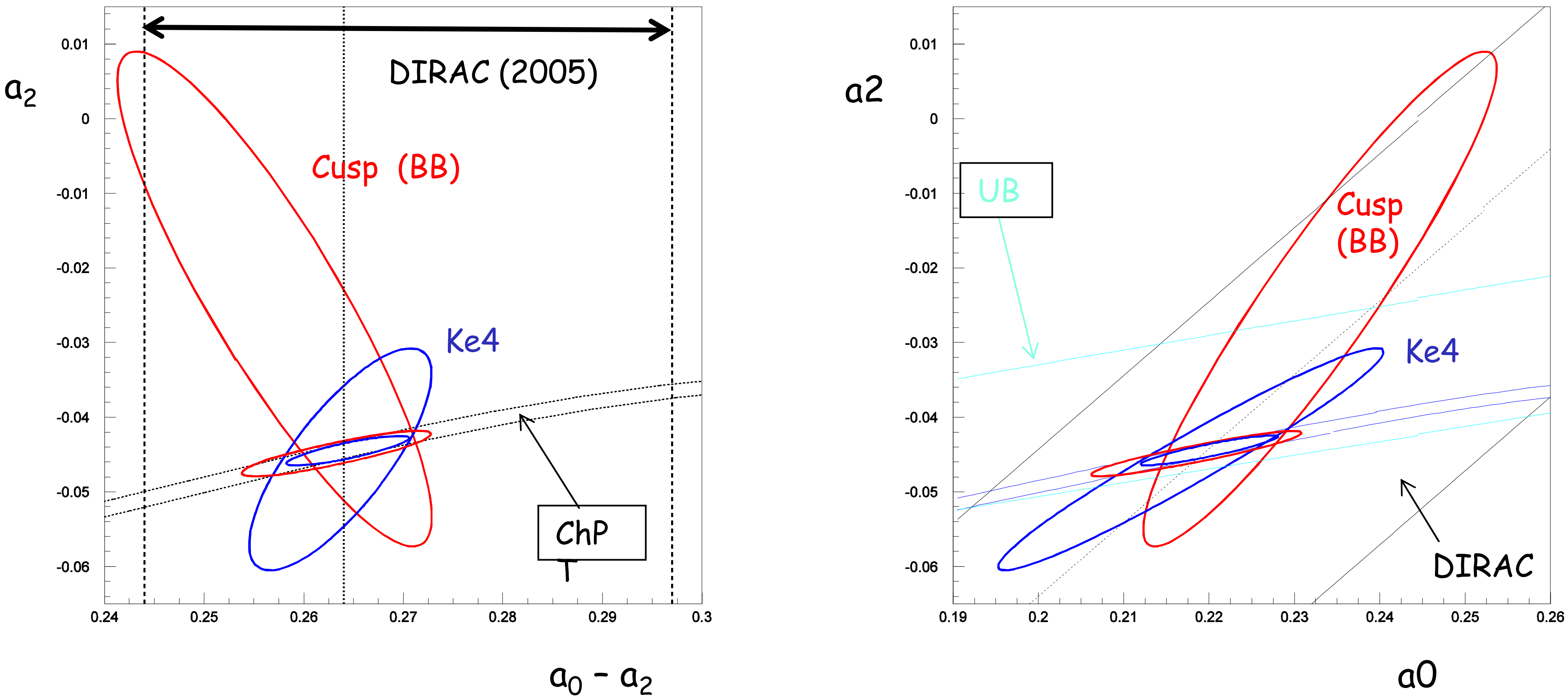
Experimental measurement

a_0 χPT 1p fit	0.2206 $\pm$ 0.0049 stat $\pm$ 0.0018 syst $\pm$ 0.0064 theo *
a_0 free a_2 free 2p fit	0.2220 $\pm$ 0.0128 stat $\pm$ 0.0050 syst $\pm$ 0.0037 theo* -0.0432 $\pm$ 0.0086 stat $\pm$ 0.0034 syst $\pm$ 0.0028 theo* Correlation 96.7%

* error evaluated from control of the isospin corrections & inputs to Roy equation numerical solutions (CGR EPJ C59 (2009)777)

Cusp and Ke4 : scattering lengths results

Preliminary (2003+2004)



Two statistically independent measurements by NA48/2

Large overlap in the (a_0,a_2) plane favoring the χPT prediction region

Impressive agreement with most precise χPT predictions

Comparison with other experimental measurements

Preliminary (2003+2004)

Cusp : (a_0-a_2) χPT fit with 2 models, BB is the most complete in terms of radiative corrections

K_{e4} : apply isospin corrections to published phase points of all experiments and perform a_0 χPT fit

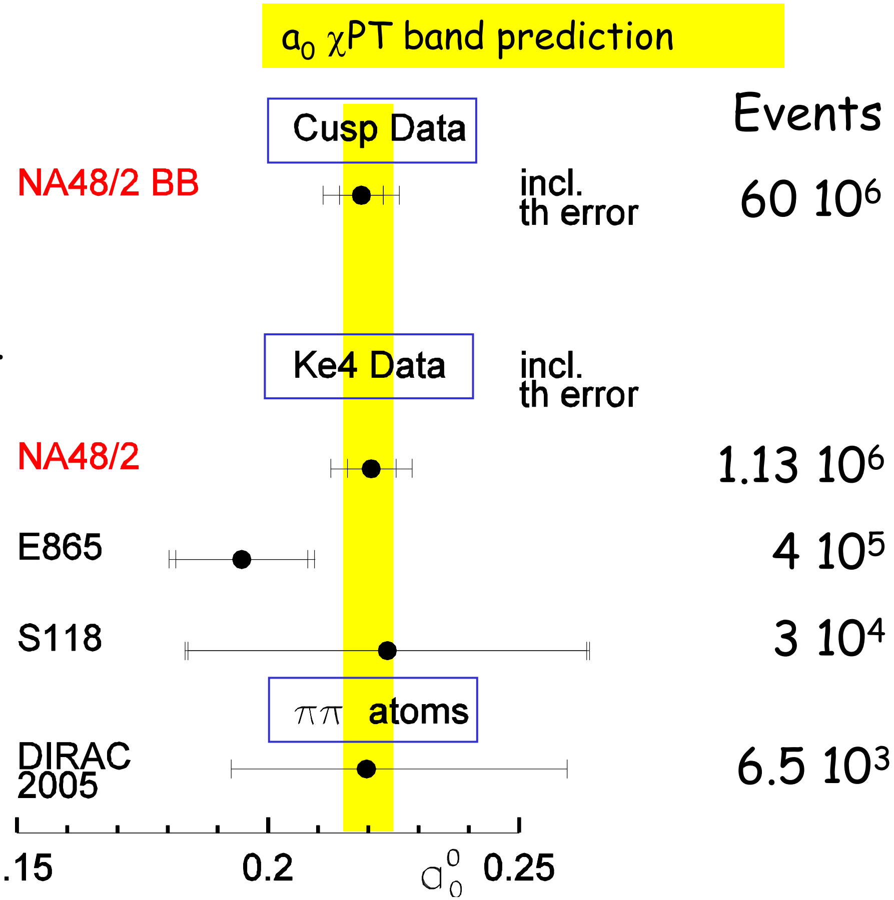
Note: E865 result dominated by highest energy data point, otherwise compatible

$\pi\pi$ atoms: $|a_0-a_2|$ errors from PLB619 (2005), use χPT constraint

(40% Data analyzed)

Yellow band is χPT prediction

$a_0 = 0.220 \pm 0.005$



NA48/2 experiment and most precise theory prediction now at the same level of accuracy!

Summary

$\sim 60\times10^6$ $K_{3\pi}$ and $\sim 1.13\times10^6$ K_{e4} events (2003+2004) analyzed by NA48/2

- Ke4 Form Factors and $K_{3\pi}$ Dalitz parameters measured with improved precision
- using recent theoretical calculations the scattering lengths are extracted

$\Rightarrow$ experimental consistency and impressive agreement with χPT

Theory $a_0 = 0.220 \pm 0.005$, $a_2 = -0.0444 \pm 0.0008$, $a_0 - a_2 = 0.264 \pm 0.004$

cusp (BB model) $a_0 - a_2 = 0.263 \pm 0.002_{stat} \pm 0.001_{syst} \pm 0.002_{ext} \pm 0.005_{th}$

K_{e4} $a_0 = 0.2206 \pm 0.0049_{stat} \pm 0.0018_{syst} \pm 0.0064_{th}$

Remarkable agreement between data and theory ($\pm$ 0.005) demonstrates that the pion mass is dominated (>90%) by the quark condensate (spontaneous breakdown of χPT)