Results from NA48/2 on $\pi\,\pi$ scattering length measurements in Ke4 and K3π decays

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Quark Confinement and the Hadron Spectrum

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On behalf of the NA48/2 collaboration:

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Wien

outline

- Introduction
- The NA48/2 experiment
- Ke4 decays ($K^\pm \to \pi^+\pi^- e^\pm \nu$) :

  Form Factors and $\pi\pi$ scattering lengths
- K3π decays ($K^\pm \to \pi^0\pi^0\pi^\pm$) : the "cusp" effect

  Dalitz plot parameters and $\pi\pi$ scattering lengths
- Ke4 and Cusp results: a comparison
- Summary

Why/How measure $\pi\pi$ scattering lengths?

Refer to Gerhard Eckert's talk today for the theoretical aspects !

Experimentally: 3 kinds of measurements have been developed

Ke4 decay mode $K^{\pm} \rightarrow \pi^{+} \pi^{-} e^{\pm} \nu$ (BR = $(4.09 \pm 0.09)\, 10^{-5}$)

- Very clean environment (no other hadron)
- Known for long but limited statistics (small branching fraction)

Geneva-Saclay CERN/PS experiment: 30 000 K^{+} (1977)

E865 BNL experiment: 400 000 K^{+} (2003)

NA48/2 CERN/SPS: 1 150 000 $K^{\pm}$ (2008)

Pionium atoms : DIRAC CERN/PS $\pi\pi$ life time measurement

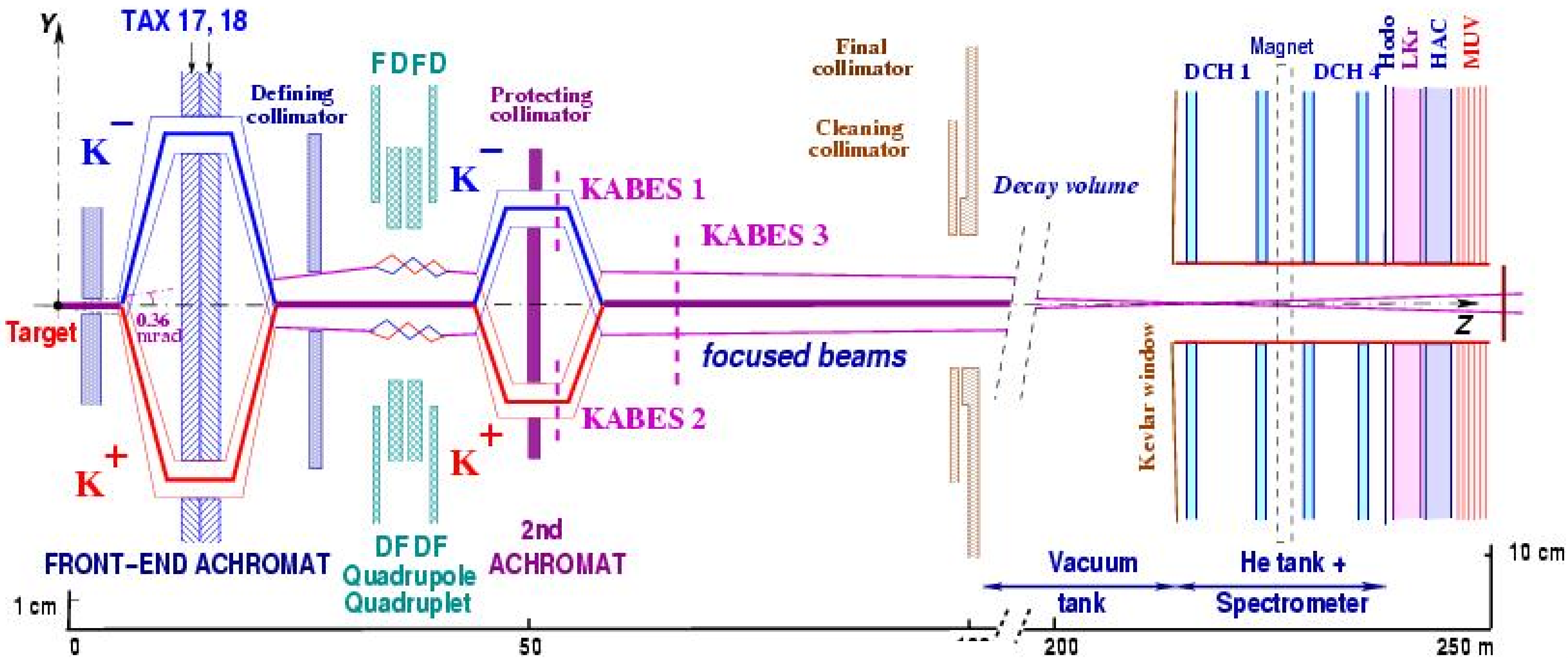
K3π modes (cusp): $K^{\pm} \rightarrow \pi^{0}\pi^{0}\pi^{\pm}$ (BR = $(1.757 \pm 0.024)\, 10^{-2}$)

NA48/2 CERN/SPS (16 M 2006): 60M (2008)

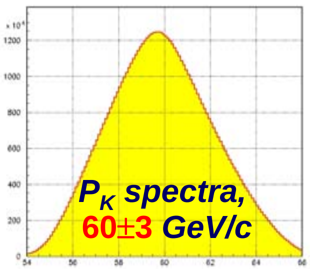
$K_{L} \rightarrow \pi^{0}\pi^{0}\pi^{0}$ (BR = $(19.56 \pm 0.14)\, 10^{-2}$)

KTeV (68M) and NA48 (100M): work in progress…

The NA48/2 experiment at the CERN-SPS : primarily designed for CP violating charge asymmetries studies in K3π decays



Simultaneous K^+ and K^- beams: large charge symmetrization of experimental conditions



Beams coincide within ~1mm all along the 114m decay volume flux ratio K^+/K^- ~1.8

The NA48/2 experiment: detector and performances

Magnetic spectrometer :

4 high-resolution DCH's + dipole magnet

$\Delta p/p$ = (1.0 $\oplus$ 0.044 p)% **(p in GeV/c)**

Very good resolution for charged invariant masses: σ (**M3$\pi^{\pm}$**) = **1.7 MeV/c^2**

Hodoscope for charged fast trigger

σt = 150 ps

LKr electromagnetic calorimeter :

quasi-homogenous and high granularity

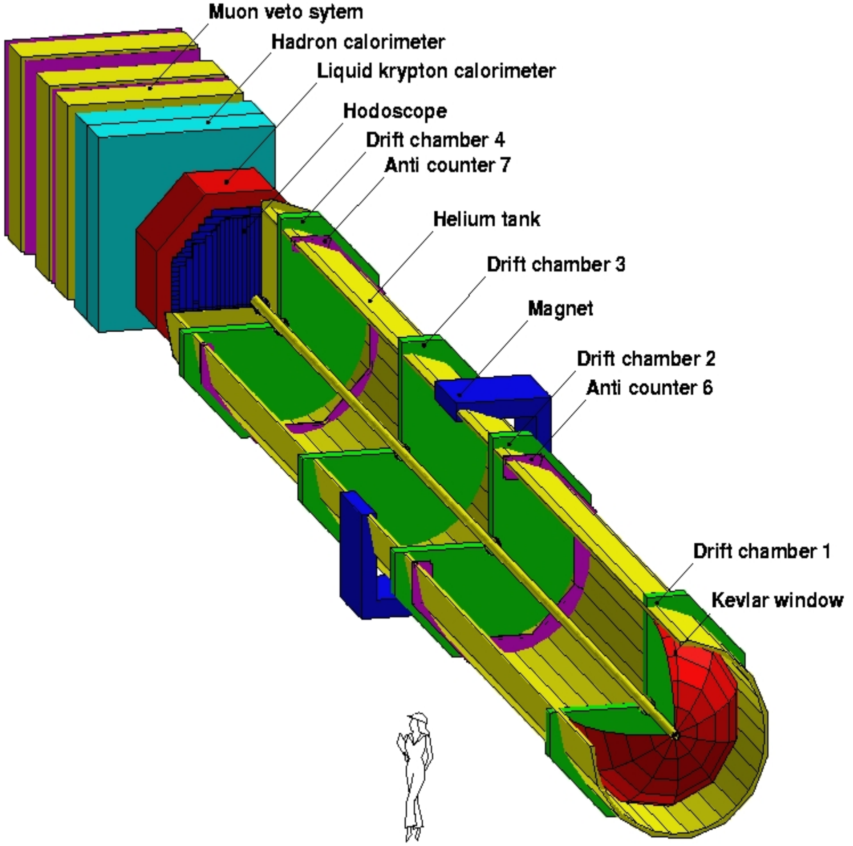
$\Delta E/E = (3.2/\sqrt{E} \oplus 9.0/E \oplus 0.42)\%$ (E in GeV)

$\sigma x = \sigma y \sim 1.5$ mm for E=10 GeV

Very good resolution for neutrals (π^0)

σ (M$\pi\pi^0\pi^0$) = 1.4 MeV/c^2

E/p ratio used for e/π discrimination



The NA48/2 data taking > 200 TB of data recorded

2003 run: ~ 50 days + 2004 run: ~ 60 days

Total statistics in 2 years:

$K^{\pm} \to \pi^{+} \pi^{-} \pi^{\pm}$: ~ 4. 10^{9} evts

$K^{\pm} \to \pi^{0} \pi^{0} \pi^{\pm}$: ~1. 10^{8} evts

$K^{\pm} \to \pi^{+} \pi^{-} e^{\pm} \nu$: ~1.1 10^{6} evts

$K^{\pm} \to \pi^{0} \pi^{0} e^{\pm} \nu$: ~3.1 10^{4} evts

and many more rare decays, in particular radiative decays (cf talk by E.Marinova)

Published:

Ag(2003+2004) Phys.Lett.B, 634(2006)474

Ag$_0$(2003+2004) Phys.Lett.B, 638(2006) 22

+ EPJC 52 (2007) 875

Cusp 2003 Phys.Lett.B, 633(2006) 173

Ke4 2003 EPJC 54 (2008) 411

Preliminary results from the full sample will be presented here

Ke4 decays: event selection and background rejection

Signal ($\pi^+\pi^- e^{\pm}\nu$) topology :

- 3 charged tracks and a good vertex
- two opposite sign pions,
- 1 electron (LKr info E/p ~ 1),
- some missing energy and p_T (ν)
- reconstruct PK (missing ν hypothesis)

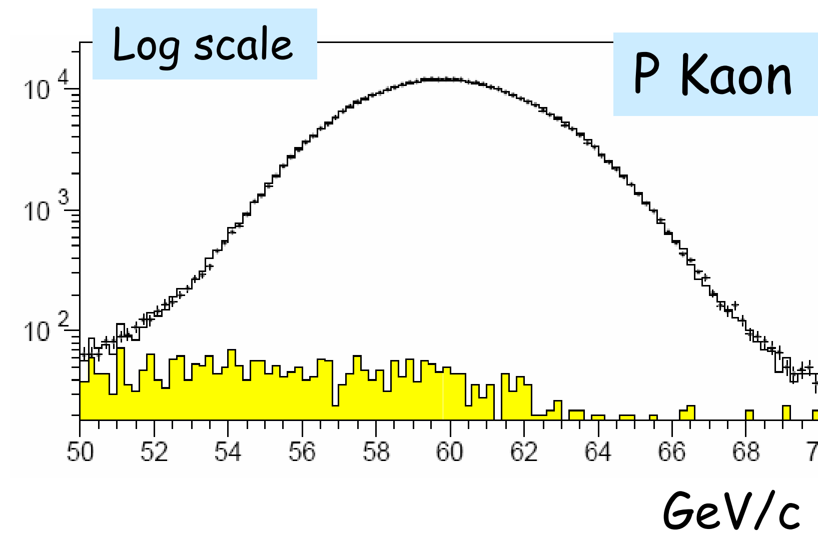
Control sample from data assuming $\Delta S=\Delta Q$:
$K^{\pm} \rightarrow \pi^{\pm}\pi^{\pm}e^{\mp}\nu$ "Wrong Sign" events

- total charge (±1) as Right Sign events
- electron charge opposite to total charge

Rate (RS/WS) events:
2 if coming from $K3\pi$
1 if coming from $K2\pi(\pi^0)$

Background main sources :

$K^{\pm} \rightarrow \pi^+\pi^-\pi^{\pm}$ (dominant)
↳$e\nu$ or misidentified as e
$K^{\pm} \rightarrow \pi^0(\pi^0)\pi^{\pm}$
↳($e^+e^-\gamma$) + 1e misidentified as π
and γ (s) undetected



Total background level can be kept at ~ 0.5 % relative level estimated from WS events rate and checked from MC simulation

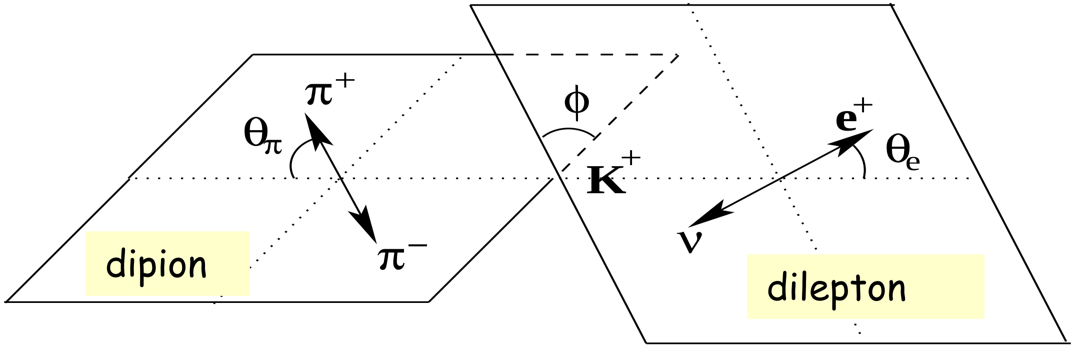
Ke4 decays : formalism

Five kinematic variables

(Cabibbo-Maksymowicz 1965)

S_π ($M^2_{\pi\pi}$), S_e ($M^2_{e\nu}$),

$\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



Partial Wave expansion of the amplitude

(Pais-Treiman 1968)

F, G = 2 Axial Form Factors

$F = F_s\, e^{i\delta s} + F_p\, e^{i\delta p} \cos\theta_\pi$ + d-wave term…

$G = G_p\, e^{i\delta g}$ + d-wave term…

H = 1 Vector Form Factor

$H = H_p\, e^{i\delta h}$ + d-wave term…

Later, use a **Taylor expansion** in powers of q^2, $Se/4m\pi^2$ (Amoros-Bijnens 1999)

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e \left(S_e/4m_\pi^2\right) + ..$$

$$F_p = f_p + f_p' q^2 + ..$$

$$G_p = g_p + g_p' q^2 + ..$$

$$H_p = h_p + h_p' q^2 + ..$$

($q^2 = (S\pi / 4m\pi^2 - 1)$)

The fit parameters are : F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

Ke4 decays : fitting procedure

Total (2003+2004) 1.15 Million Ke4 decays

Using iso-populated boxes in the 5-dimension space of the C.M. variables, ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ) one defines a grid of

10x5x5x5x12=15000 boxes.

The set of fit parameters is used to minimize T^2, a log-likelihood estimator well suited for small numbers of data events/box Nj and taking into account the statistics of the simulation = Mj simulated events/box and Rj expected events/box.

$$T^2 = 2\sum_{j=1}^{Nbox} \left\{ N_j Log\left[\frac{N_j}{R_j}\left(1-\frac{1}{Mj+1}\right)\right] + (N_j + M_j + 1) Log\left[\frac{1+\frac{R_j}{M_j}}{1+\frac{N_j}{M_j+1}}\right] \right\}$$

K^+ sample (739 500 events) 49 events/box	K^+ MC (17.7 Million events) 1180 events/box
K^- sample (411 000 events) 27 events/box	K^- MC (9.8 Million events) 650 events/box
Data sample	MC sample

Ke4 decays : fit strategy

- **step 1**: K^+ and K^- samples fitted separately in 10 independent $M\pi\pi$ bins, results combined according to their statistical error.

No assumption is made on the variation of the phase δ (and FF) from one $M_{\pi\pi}$ bin to the next. This allows a model independent analysis.

- **step 2** Without the overall normalization from branching fraction, only relative form factors (F_p, G_p, H_p)/F_s are measured.
F_s^2 is obtained from the relative bin to bin overall normalization Data/MC after fit.
Relative Form Factor are corrected by the measured F_s to get the **absolute form factors** (to a constant term f_s=$F_s(q^2=0)$)

- **step 3**: the 10 values of each Form Factor and δ phase are used to extract more information.

Taylor expansion used to describe the FF variations, under the isospin symmetry limit

Scattering lengths extracted from phase variation

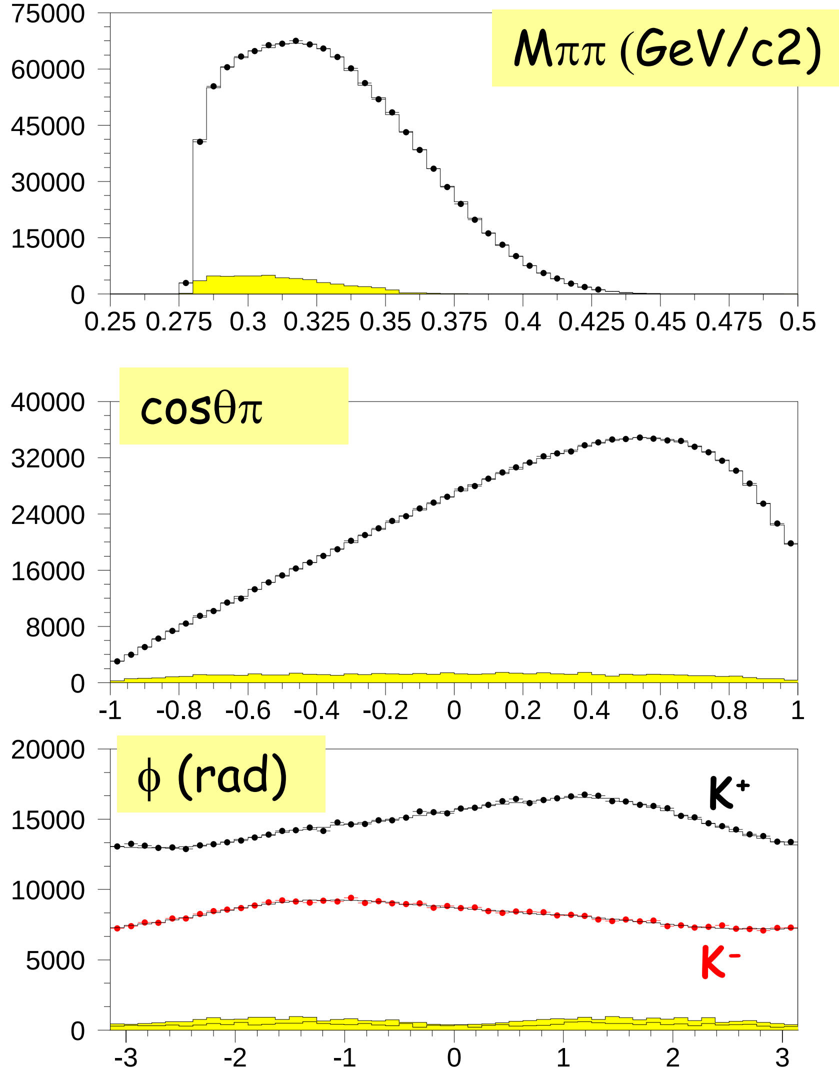
(work in progress by the Bern-Bonn group and others)

Preliminary (2003+2004)

	new value	new stat	2003 syst
f'_s/f_s	0.158	± 0.007	± 0.006
f''_s/f_s	-0.078	± 0.007	± 0.007
f'_e/f_s	0.067	± 0.006	± 0.009
f_p/f_s constant	-0.049	± 0.003	± 0.004
g_p/f_s	0.869	± 0.010	± 0.012
g'_p/f_s	0.087	± 0.017	± 0.015
h_p/f_s constant	-0.402	±0.014	± 0.008

Results in agreement with published 2003 data analysis, systematics taken from EPJC54 (2008), being revisited for the whole data sample

Ke4 decays : Data/MC comparison after fit



= Data

histogram = Simulation after fit

Yellow hist = WS Background (x 10 to be visible)

CP symmetry : (K^+) ϕ distribution is opposite of (K^-) ϕ distribution

Ke4 decays : δ phase and S-wave scattering lengths (a_0 ,a_2)

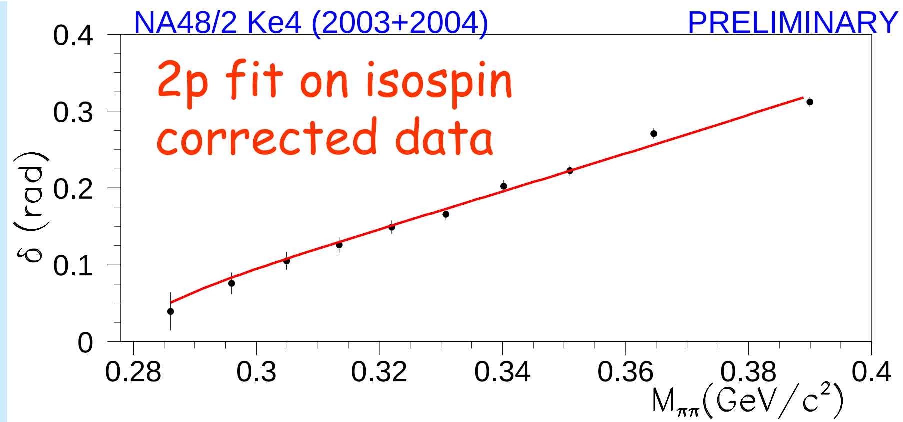
To extract information from the $\delta = (\delta_0^0 - \delta_1^1)$ variation, some external inputs are necessary:

- theoretical work : numerical solutions of Roy equations (ACGL Phys. Rep.353 (2001), DFGS EPJ C24 (2002)) relate δ and 2 subtraction constants (a_0, a_2)
- Experimental data (I=2 $\pi\pi$ data @Higher energy ≥ 800 MeV)

  the Universal Band represents the allowed predicted values within a wide range ($\Delta a_2 = \pm 0.0088$) reflecting the experimental data precision

  a 2-parameter fit can be performed with (a_0,a_2) as free parameters
- the ChPT constrain (CGL NPB603(2001)) gives a precise prediction:

  $a_0 = 0.220 \pm 0.005$, $a_2 = -0.0444 \pm 0.0008$



Experimental precision: stat + syst

2p fit using Roy equ.:

$\Delta a_0 = \pm 0.013 \quad \pm 0.007$

$\Delta a_2 = \pm 0.0084 \pm 0.0041$

1p fit using $a_2=f(a_0)$

$\Delta a_0 = \pm 0.005 \quad \pm 0.002$

Isospin symetry breaking effects can/should now be considered
("Bern" arxiv:hep-ph/0710.3048 by JG et al, work in progress by other groups)

Ke4 decays : isospin corrections to δ

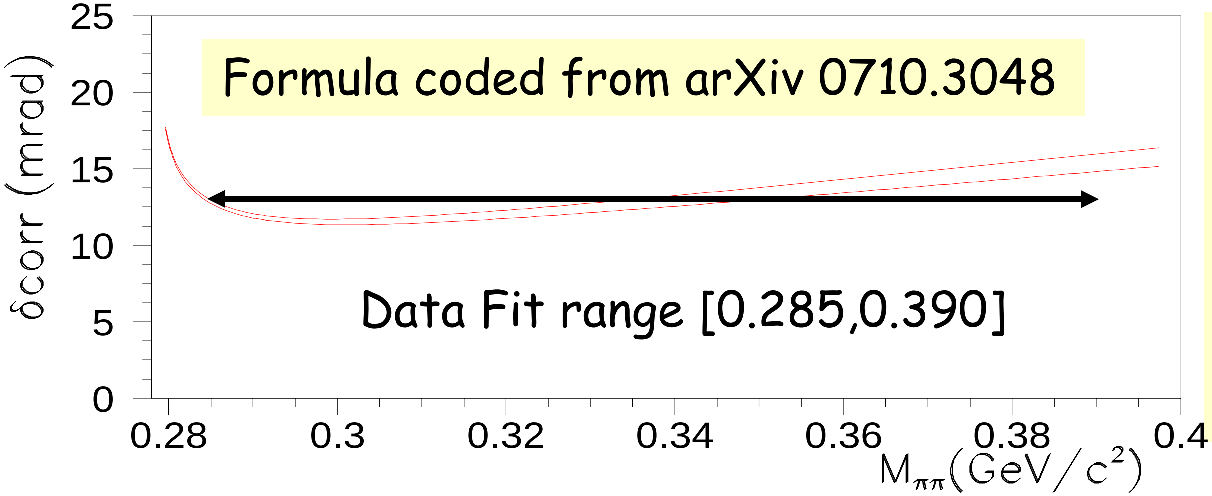
"Bern" corrections well adapted to the analysis which applies radiative and masses Isospin symetry breaking effects in a multiplicative way :
Gamow factor x PHOTOS generator x Isospin corrections (mass effects)

simulation

Correction to data

$$\delta_0^0 \to \delta = \frac{1}{32\pi F_0^2}\left\{(4\Delta_\pi + s_\pi)\sigma + (s_\pi - M_{\pi^0}^2)\left(1+\frac{3}{2R}\right)\sigma_0\right\}$$

Includes one-loop calculation for rescattering



δcorr =11 to 15 mrad over the fitted $M\pi\pi$ range

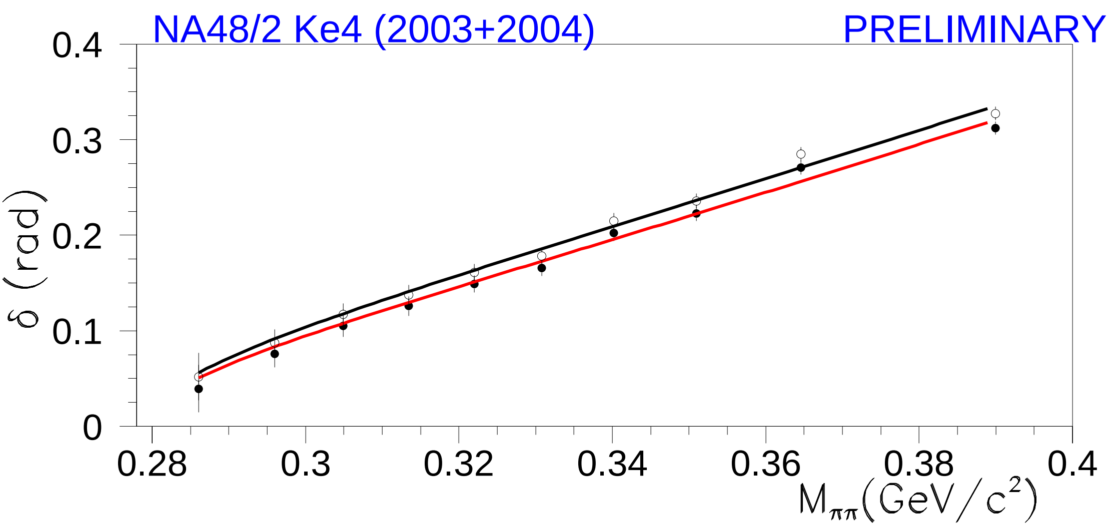
Uncertainty R ± ΔR = 37 ± 4 (R=(ms-mud)/(md-mu))

ΔR translates into $\Delta\delta$corr = at most ± 0.5 mrad @ 0.390 GeV/c^2, marginal

Ke4 decays : isospin corrections to δ

All Data K+ and K- combined
= Isospin corr OFF
= Isospin corr ON

Correction (~10mrad) is larger than the statistical error on each point above 0.3 GeV/c2 (7-8 mrad)



Preliminary (2003+2004)

2p fit	Isospin corr OFF	Isospin corr ON
a_0	0.244 ± 0.013	0.218 ± 0.013
a_2	-0.0385 ± 0.0084	-0.0457 ± 0.0084

Large effect ~2σ on a_0 and a_2, cannot be neglected !

Ke4 decays : scattering lengths (1 151 100 decays)

Preliminary (2003+2004)

value ± stat.

	2003 EPJC 54(2008)	2003 + 2004 preliminary
a_0 ChPT 1p fit $a_2=f(a_0)$	0.223 ± 0.006 -0.0437 ± 0.0015	**0.220 ± 0.005** **-0.0444 ±0.0011**
2p fit a_0 free a_2 free	0.209 ± 0.016 -0.0529±0.0105	**0.218 ± 0.013** **-0.0457±0.0084**

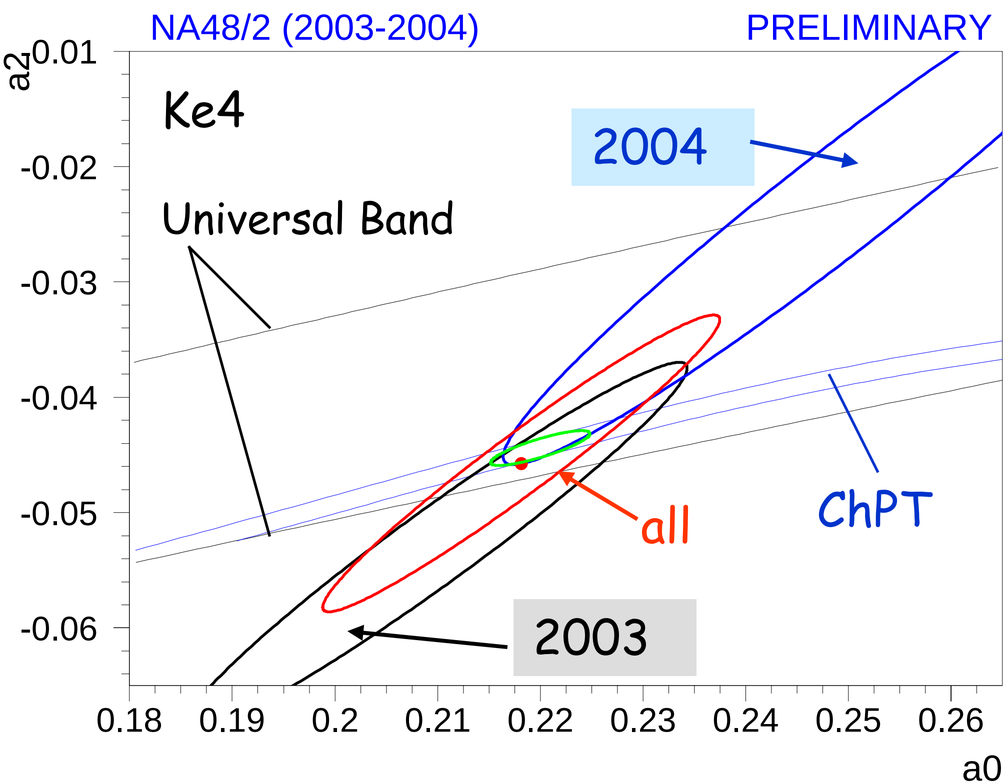
Scattering length a_0 now measured with ~2% relative precision (ChPT 1p fit),

Improved precision for a_0 (~6%) and a_2 (~18%) in a 2-free parameter fit

Systematic uncertainty quoted as for 2003 (conservative, will be revisited) but already ~0.5 x (σ stat) for a_0,a_2

Ke4 charged decays : $\pi\ \pi$ scattering lengths

Isospin corrections ON (Bern prescription)



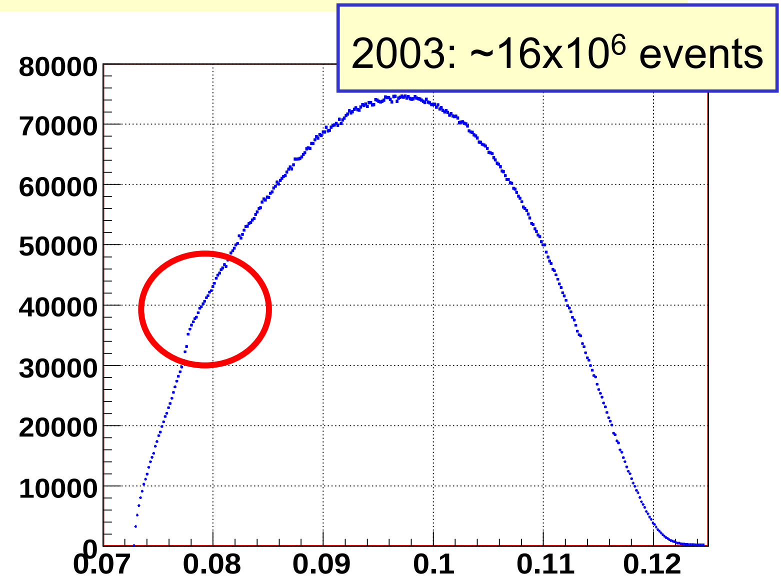
Preliminary (2003+2004)

2p fit	
a_0 free	0.218 ± 0.013 stat ± 0.007 syst
a_2 free	-0.0457 ± 0.0084 stat ± 0.0041 syst
a_0 ChPT 1p fit	0.220 ± 0.005 stat ± 0.002 syst

Precise ChPT predictions : a_2 = -0.0444 ± 0.0008 and a_0 = 0.220 ± 0.005

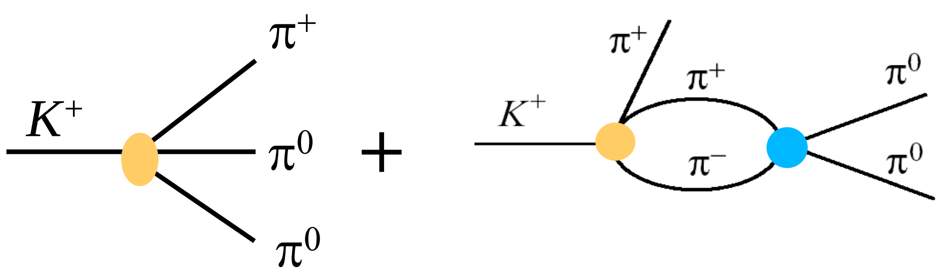
Cusp effect : first observation and interpretation (Cabibbo PRL93(2004))

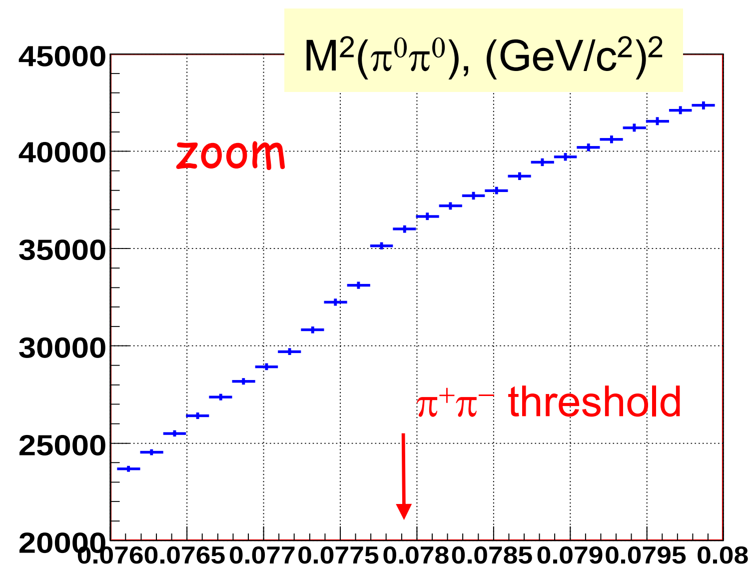
In $K^{\pm} \rightarrow \pi^0 \pi^0 \pi^{\pm}$ decay, the matrix element is usually described as a polynomial expansion using the Dalitz Plot variables u and v



First observation of a cusp structure was made with 16 M events collected in 2003 thanks to the very good mass resolution.

The structure at $\pi^+ \pi^-$ threshold was interpreted as due to the $\pi\pi$ rescattering in the $K^{\pm} \rightarrow \pi^+ \pi^- \pi^{\pm}$ final state





increased statistics with 44 M more data from 2004

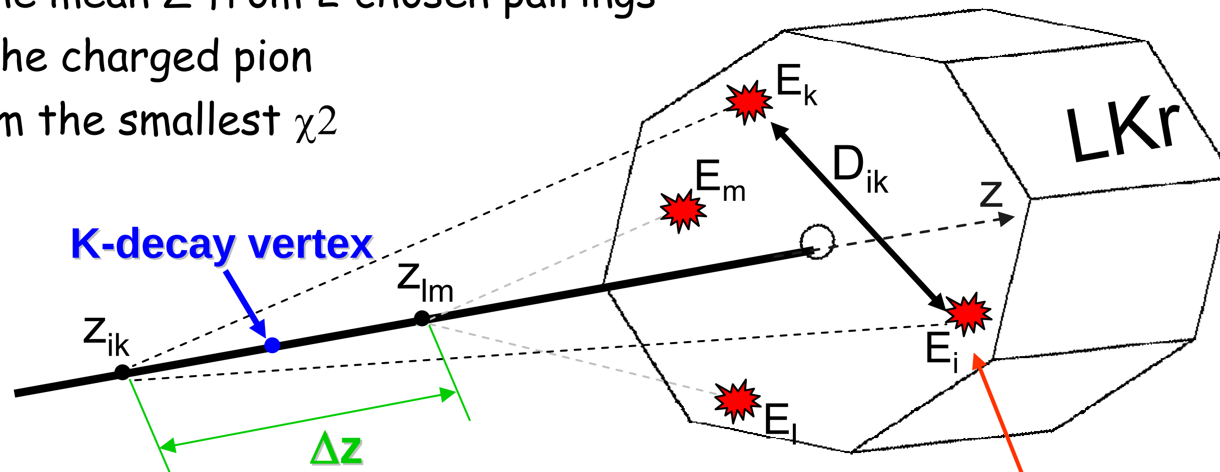
CUSP; $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$ event selection

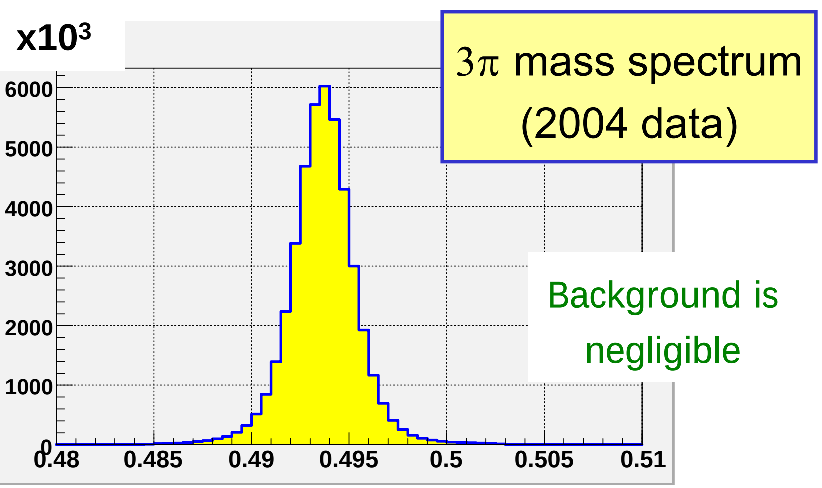
- Each photon pair (i,k) defines a decay vertex under the assumption of $\pi^0 \rightarrow \gamma\gamma$ decay

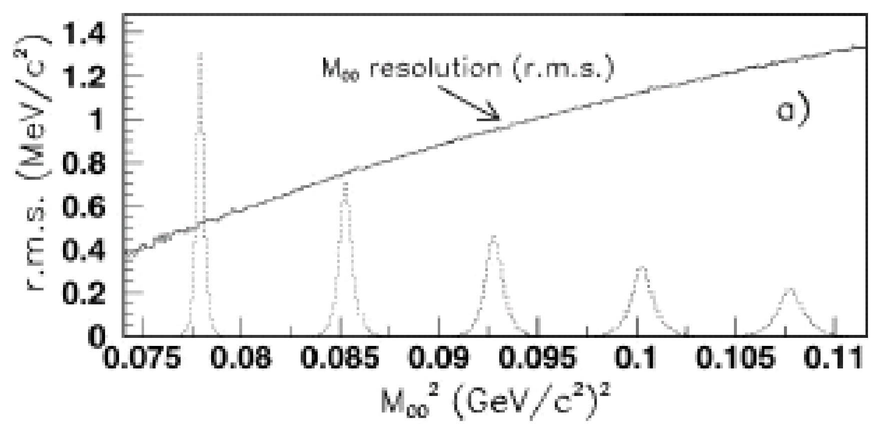
$$Z_{ik}^2 \cong E_i E_k D_{ik}^2 / m_{\pi^0}^2$$

- Neutral vertex defined as the mean Z from 2 chosen pairings
- Reconstruct K mass adding the charged pion
- Chose the best 2 γ pairs from the smallest $\chi 2$

$$\chi^2 = \left(\frac{\Delta Z}{\sigma_Z}\right)^2 + \left(\frac{\Delta m_K}{\sigma_{m_K}}\right)^2$$







Cusp: Cabibbo-Isidori approach (CI JHEP 0503(2005)) $M = M_0 + M_1$

Without re-scattering, M_0 parameterization (as PDG)

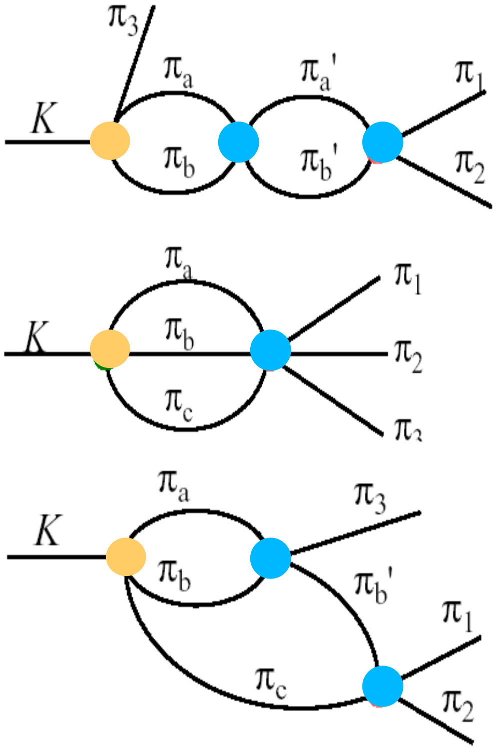
$$M_0 = A_0(1+g_0 u/2+h'_0 u^2/2+k'_0 v^2/2)$$

M1 describes loop diagrams

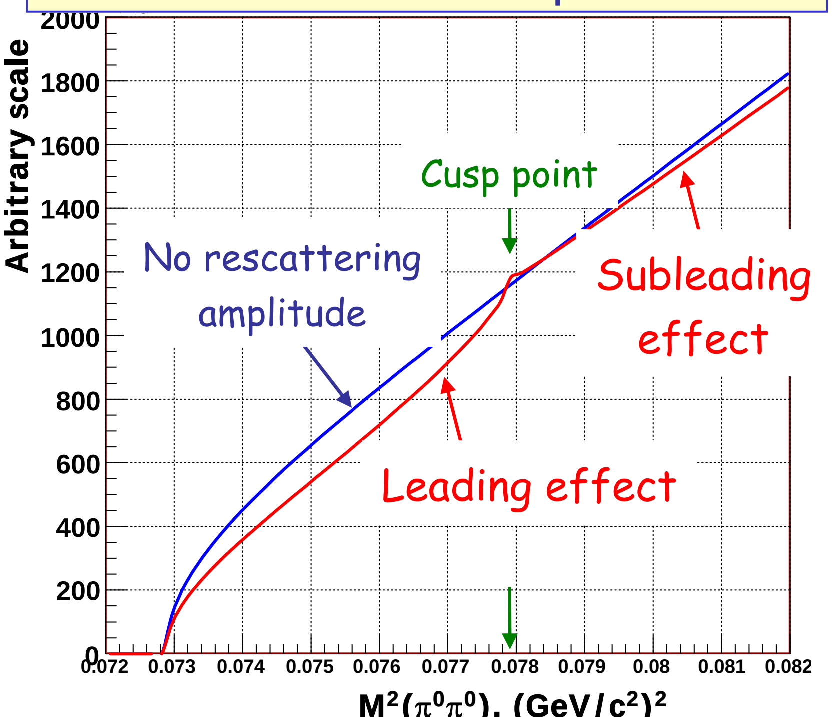
$$M_1 = -\frac{2}{3}(a_0 - a_2) m_{\pi^+} M_+ \sqrt{1 - M_{00}^2 / 2m_{\pi^+}}$$

above threshold $|M|^2 = |M_0|^2 + |M_1|^2$

below threshold $|M|^2 = |M_0|^2 + |M_1|^2 + 2 M_0 M_1$



Distortion due to loop effects



Cusp: Colangelo, Gasser, Kubis, Rusetsky (CGKR PLB638 (2006))

- effective field theory approach based on non-relativistic Lagrangian
- electromagnetic effects included in the amplitudes
- two-loop formulation different from CI introduces different correlations between parameters
- now available as arXiv:0807.0515 (Bissegger, Fuhrer, Gasser, Kubis, Rusetsky)

In both approaches, a very close and fruitful collaboration between theory and experiment is still going to implement latest developments

Cusp fitting procedure : basics

Fit the M^2_{00} projection using the detector response matrix Rij obtained from a GEANT-based Monte-Carlo simulation and 4 physics parameters to minimize χ^2

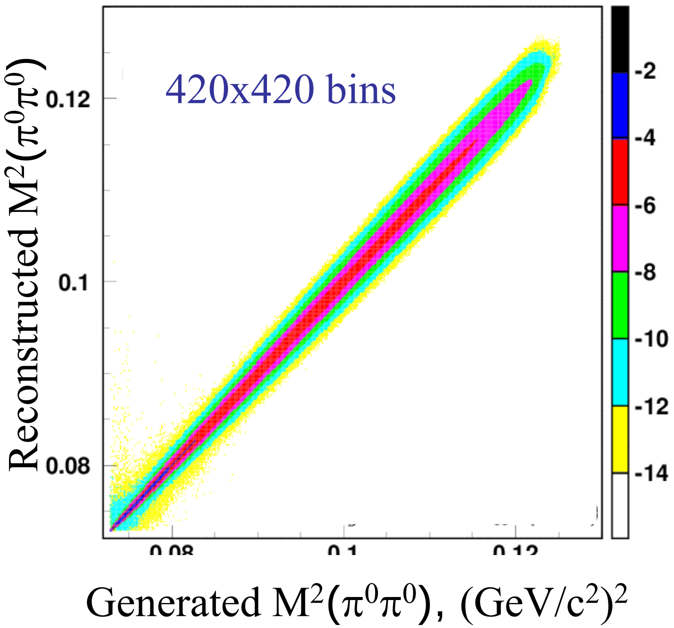
$$\chi^2(g,h,a_0-a_2,a_2,x)=\sum_{j=1}^{N_{bin}}\frac{\left(N_j - xM_j\right)^2}{N_j+x^2M_j}$$

Nj data events/bin

Mj simulated events/bin = $\Sigma\, R_{ij}G_i$

x Mj expected events/bin

x = Data/MC normalization



a_0,a_2 experimental precision: stat + syst

4p fit

$\Delta(a_0-a_2)$ = ± 0.005 ± 0.002

Δa_2 = ± 0.009 ± 0.006 (CI)

Δa_2 = ± 0.015 ± 0.010 (CGKR)

3p fit using $a_2=f(a_0)$

$\Delta(a_0-a_2)$ = ± 0.003 ± 0.002

Cusp fitting procedure: more details

Both formulations (CI and GCKR) are used to extract the physics parameters ($g, h, a_0 - a_2, a_2$)

the constant k'_0 (v-dependent term) is fixed to the value recently measured by a 2d fit of the $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$ Dalitz plot

$k'_0 = 0.0095 \pm 0.0002$ stat ± 0.0005 syst

In the CI fit, the M_+ amplitude is fixed using the recently measured slopes parameters of the $K^{\pm} \rightarrow 3\pi^{\pm}$ amplitude (NA48/2 PLB649(2007))

$g_+ = -0.21117(15)$; $h'_+ = 0.00671(26)$;
$k'_+ = -0.00477(8)$

In the CGKR fit the M_+ amplitude is obtained from a simultaneous fit of the $K^{\pm} \rightarrow 3\pi^{\pm}$ and $K^{\pm} \rightarrow \pi^{\pm}\pi^0\pi^0$ Dalitz plots within the same code :

$g_+ = -0.1837$; $h'_+ = 0.00043$;
$k'_+ = -0.0059$

$$M_0 = A_0(1 + g_0\, u/2 + h'_0\, u^2/2 + k'_0\, v^2/2)$$

$$M_+ = A_+(1 + g_+\, u/2 + h'_+\, u^2/2 + k'_+\, v^2/2)$$

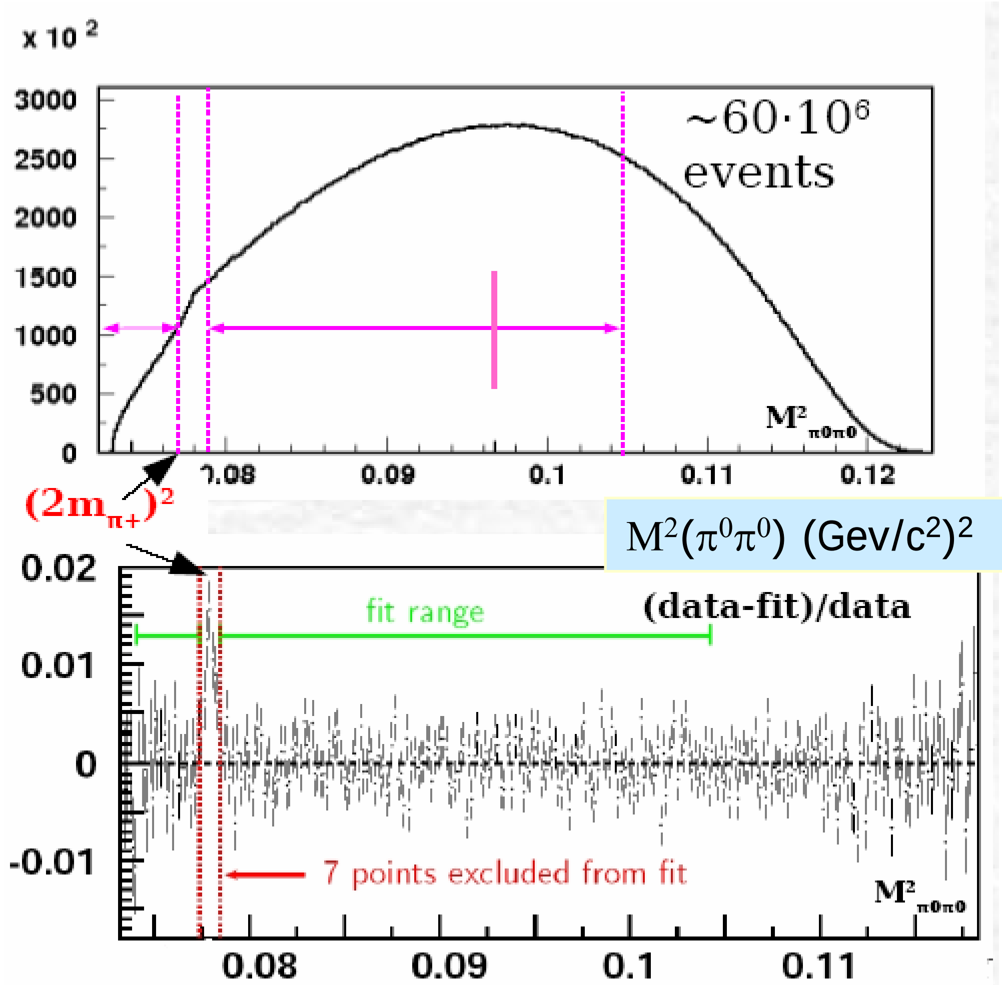
$$\mathbf{M_1} = -\frac{2}{3}(a_0 - a_2)\mathbf{m}_{\pi^+}\mathbf{M}_+ \sqrt{1 - \mathbf{M}_{00}^2 / 2\mathbf{m}_{\pi^+}}$$

Note that the fit results have a large sensitivity to the amplitude ratio $R = A_+/A_0$, fixed from the branching fractions and amplitude shapes: $R = 1.975 \pm 0.015$

Cusp : fit quality

-7 bins around 2mπ+ threshold are excluded from the fit to avoid the "perturbation" of the event excess at the cusp position

-Fit range extended from 176 bins to 226 bins to reduce the total experimental error (good compromise between statistical and systematic error contributions)



Cusp : Pionium atom ($A_{2\pi}$) signature

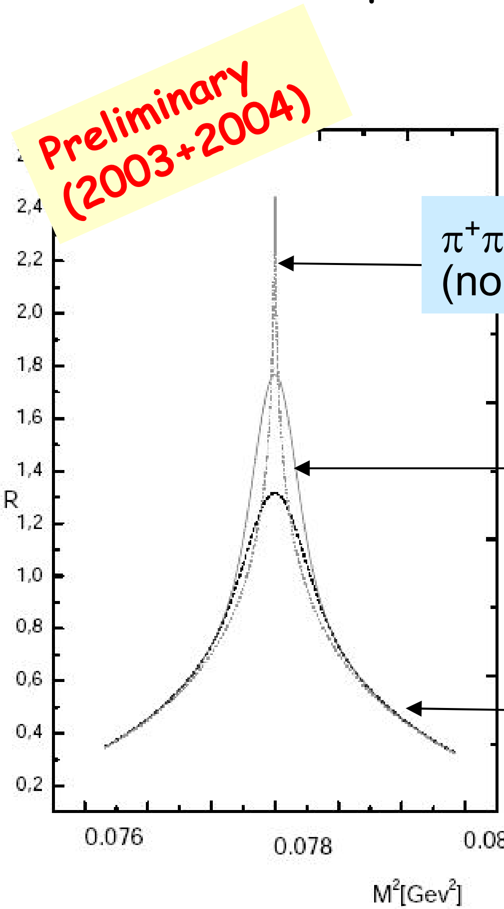
Event excess around the $M(\pi^+\pi^-)$ threshold can be explained as:

-pionium production decaying to $\pi^0\pi^0$
(Silagadze, JETP Lett.60 (1994))

$R = \Gamma(K^\pm \rightarrow \pi^\pm A_{2\pi}) / \Gamma(K^\pm \rightarrow \pi^\pm \pi^+\pi^-)$
$= (1.8 \pm 0.3)\times10^{-5}$
while the prediction is $R = 0.8\times10^{-5}$

-Additional unbound state with resonant structure
(Gevorkian et al, hep-ph/0612129)

$N / \Gamma(K^\pm \rightarrow \pi^\pm\pi^0\pi^0)$: $= (5.6\pm1.0)\times10^{-5}$



$\pi^+\pi^-$ resonant structure (no experimental resolution)

$\pi^+\pi^-$ atoms + resonant structure with experimental resolution

$\pi^+\pi^-$ resonant structure with experimental resolution

Cusp: scattering lengths results

Preliminary (2003+2004)

Using ChPT constraint	Note : ext is mainly due to $R=A_+/A_0=1.975\pm0.015$ th(CI) ~5% probably pessimistic (under evaluation)
CI model	$a_0-a_2 = 0.268 \pm 0.003stat \pm 0.002syst \pm 0.001ext \pm 0.013th$
CGKR model	$a_0-a_2 = 0.266 \pm 0.003stat \pm 0.002syst \pm 0.001ext$

a_2 free	Note : correlations between a_2 and other parameters are larger in CGKR model
CI model	$a_0-a_2 = 0.266 \pm 0.005stat \pm 0.002syst \pm 0.001ext \pm 0.013th$ $a_2 = -0.039 \pm 0.009stat \pm 0.006syst \pm 0.002ext$
CGKR model	$a_0-a_2 = 0.273 \pm 0.005stat \pm 0.002syst \pm 0.001ext$ $a_2 = -0.065 \pm 0.015stat \pm 0.010syst \pm 0.002ext$

Cusp: Dalitz plot slopes result

Reminder: $M(K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}) = M_0 + M_1$

$M_0 \sim (1+g_0 u/2+h'_0 u^2/2+k'_0 v^2/2)$ but …

$|M_0|^2$ (PDG) $\sim (1+gu+hu^2+kv^2)$ so $g_0 \approx g$, $h'_0 \approx h - g^2/4$, $k'_0 \approx k$

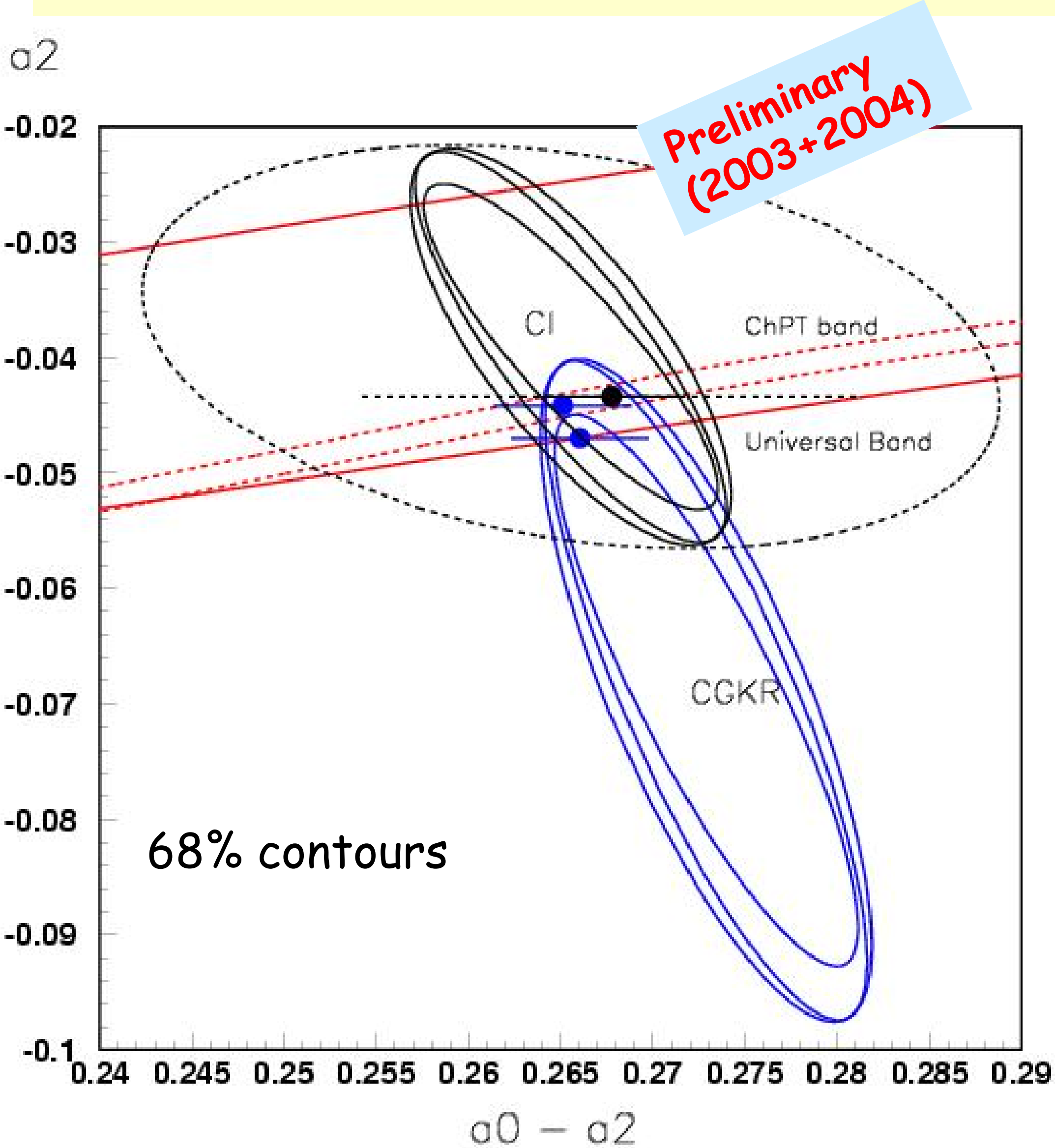
- k'_0 is extracted from a 2-dimensional fit
- Other parameters are fitted including a fixed k'_0 value

Preliminary (2003+2004)

$k'_0 = 0.0095 \pm 0.0002$stat. ± 0.0005syst

	g_0	h'_0
CI model	0.652 ± 0.001stat ± 0.003syst	-0.039 ± 0.001stat ± 0.003syst
CGKR model	0.621 ± 0.001stat ± 0.003syst	-0.049 ± 0.001stat ± 0.003syst

Cusp: scattering lengths results



Black =CI fit results with:

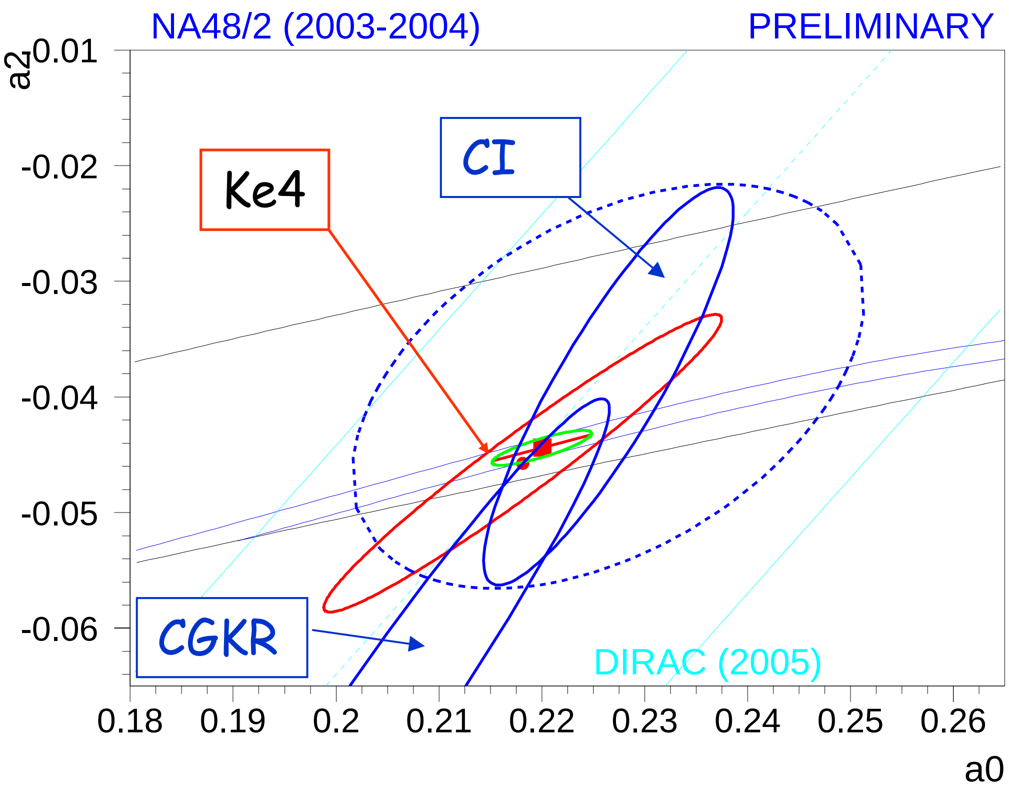
-stat error only
-stat + syst
-stat + syst + external
- - all including theoretical error

Black symbol = CI fit with ChPT constraint

Blue =CGKR fit results with:
-stat error only
-stat + syst
-stat + syst + external

Blue symbol = CGKR fits with ChPT constrain

Scattering lengths measurements by NA48



Two statistically independent measurements by NA48/2:

Cusp in $K^{\pm}$ ($\pi^{\pm} \pi^0 \pi^0$) :

- 2 free a_i fit within 2 models (CI, CGKR)
- 1free a_i fit with ChPT constraint and 2 models

Ke4 : "Bern" Isospin corrections ON

- 2p fit using Roy equations
- 1p fit with extra ChPT constraint

also DIRAC ππ atom life time

(PLB 619 (2005))

Impressive agreement with ChPT predictions

Comparing with other experimental measurements

Ke4 : apply **isospin corrections** to published phase points of all experiments and perform a_0 **ChPT fit**

Note : E865 number dominated by highest energy data point, otherwise compatible

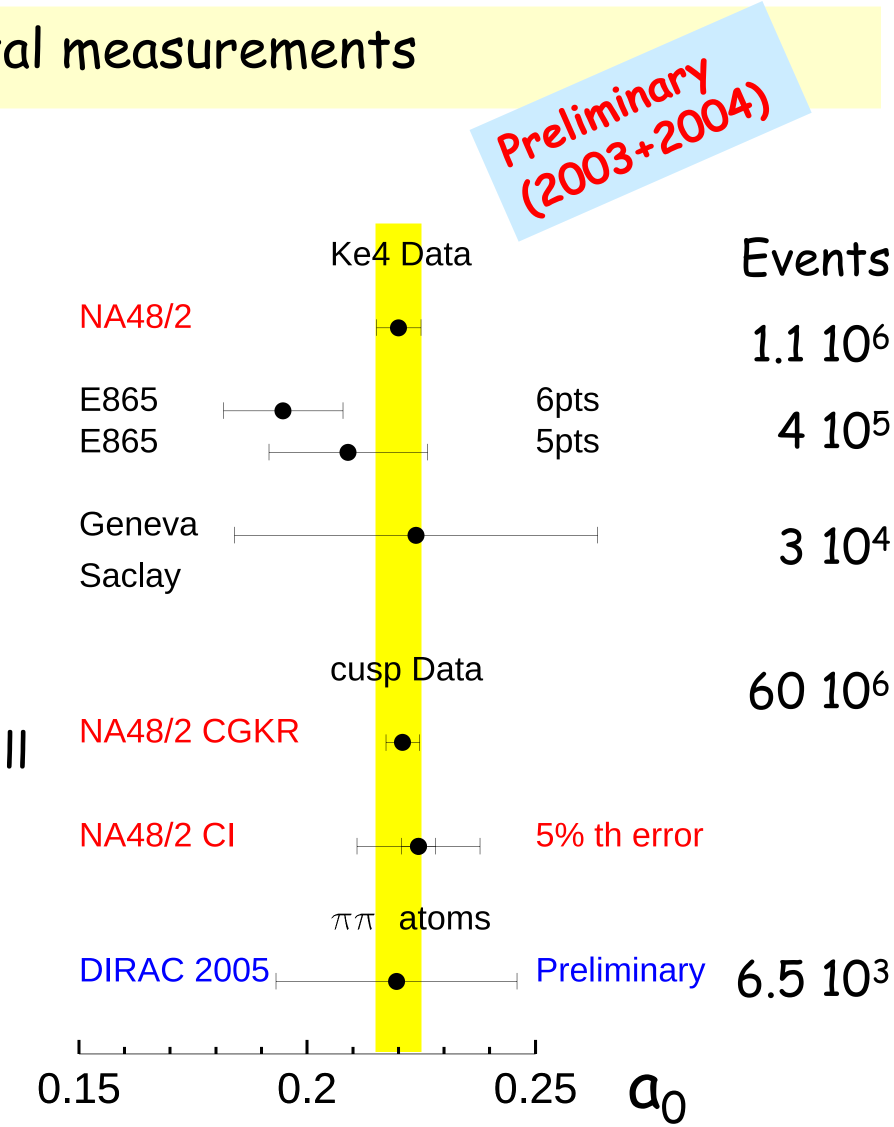
Cusp : $(a_0\text{-}a_2)$ ChPT fit with 2 models

ππ atoms DIRAC: $|a_0\text{-}a_2|$ errors from PLB619 (2005), use ChPT constraint (still being revisited + more Data)

Yellow band is ChPT prediction

$$a_0 = 0.220 \pm 0.005$$

NA48/2 experimental precision now at the same level !



NA48/2 : Summary and prospects

NA48/2: ~60 M K3π and ~1.15 M Ke4 events recorded in (2003+2004):

• Form Factors (Ke4) and Dalitz plot slopes (K3π) measured with an improved precision

• using recent theoretical calculations (one to two-loop evaluation for re-scattering effects), the scattering lengths can be extracted, giving a consistent experimental picture and an impressive agreement with predictions from ChPT: $a_0 = 0.220 \pm 0.005$

• The achieved experimental precision on a_0 is now competitive with the theoretical precision ($\pm$ 0.005) in both decay modes.

• Work is going on to implement the most recent theoretical refinements and improve the experimental systematic uncertainties.

The very positive collaboration with many theory groups (Bern, Bonn, Dubna, Orsay, Madrid, Marseille, Rome ..) was/still is invaluable in understanding how to extract scattering lengths from NA48 Data.

We are particularly grateful to CGKR and CI for their interest and investment in the analyses. J.Stern has been among our best "supporters" for many years.

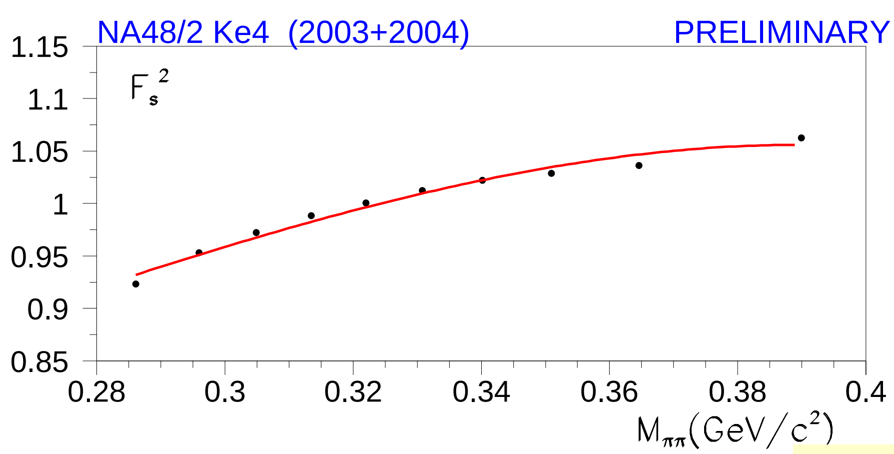
spares

Ke4 decays : getting F,G,H form factors and phase shift

Ten independent fits, one in each $M_{\pi\pi}$ bin, assuming ~constant form factors over each box. This allows a model independent analysis.

Without the overall normalization (Branching fraction), one can quote relative form factors and their variations with q^2,q^4 (q^2= $(S_\pi/4m_\pi^2$ -1) and $Se/4m_\pi^2$

- Fs^2 is obtained from the relative bin to bin normalization Data/MC after fit
- if projected along $Me\nu$, a residual variation is observed.
- a 2-dimension fit of the normalization is performed to get the 3 slopes



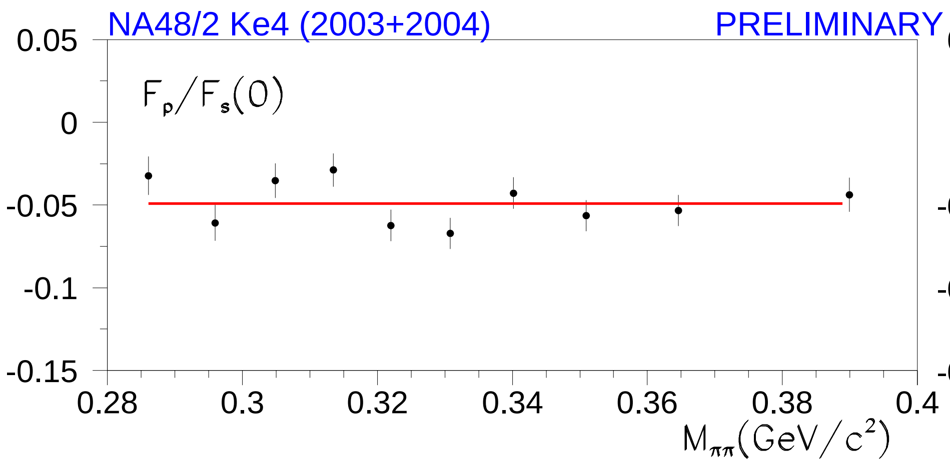
The 3 slopes are correlated

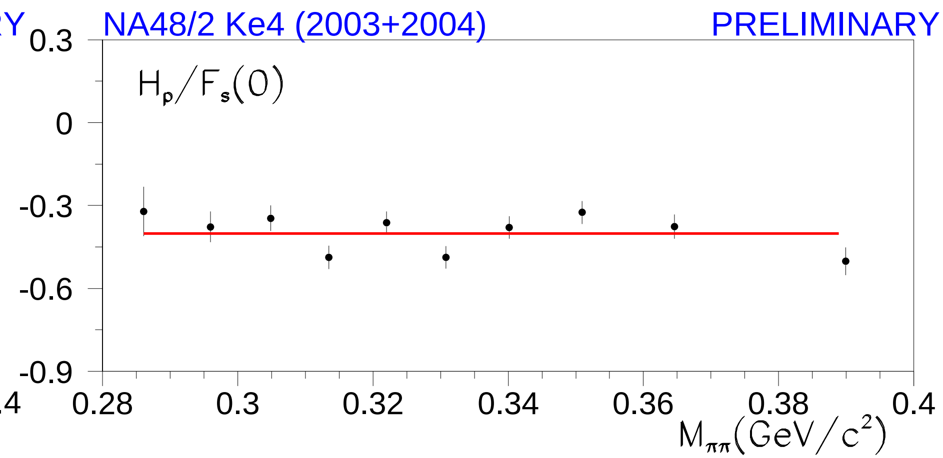
$Fs^2 \propto (1 + f's\, q^2 + f''s\, q^4 + f'e\, Se/4m\pi^2\,)^2$

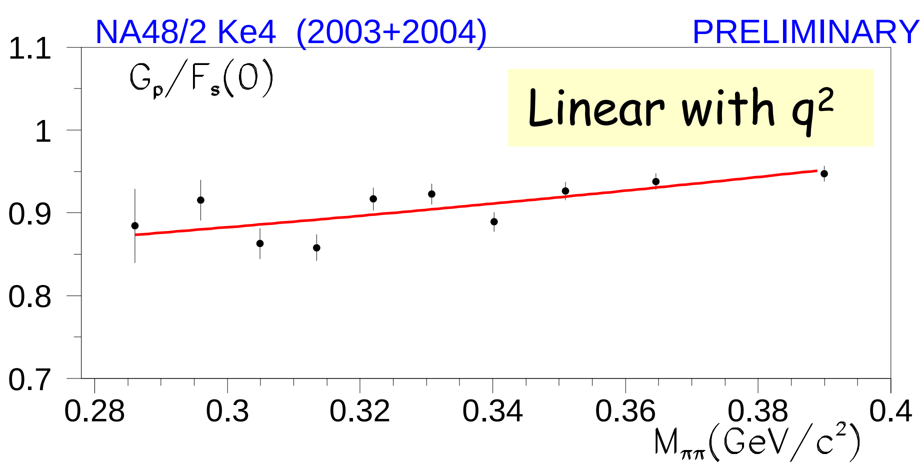
	f"s	f'e
f's	-0.95	0.08
f"s		0.02

Other parameterizations could be easily tried if the Taylor expansion does not apply..

Getting Fp, Gp, Hp



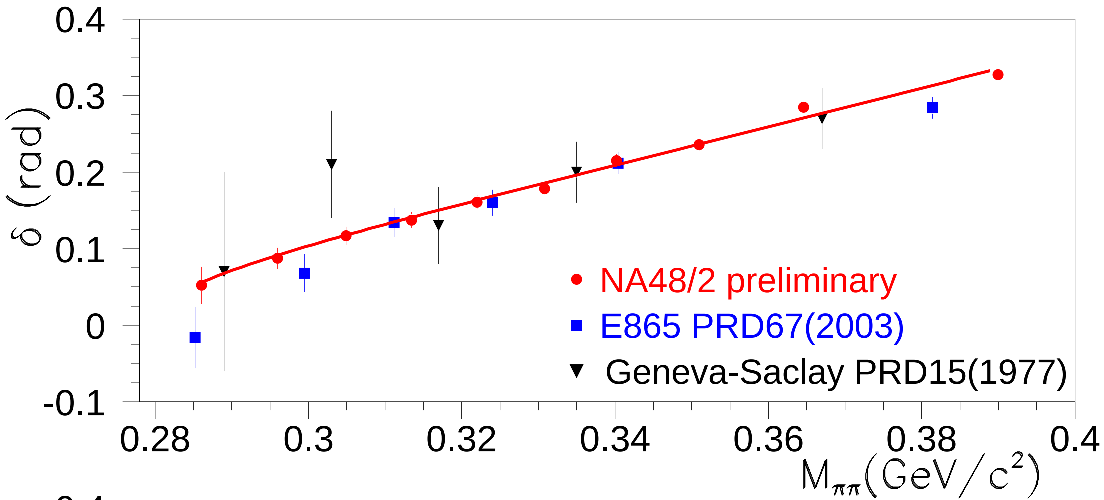




Correlation

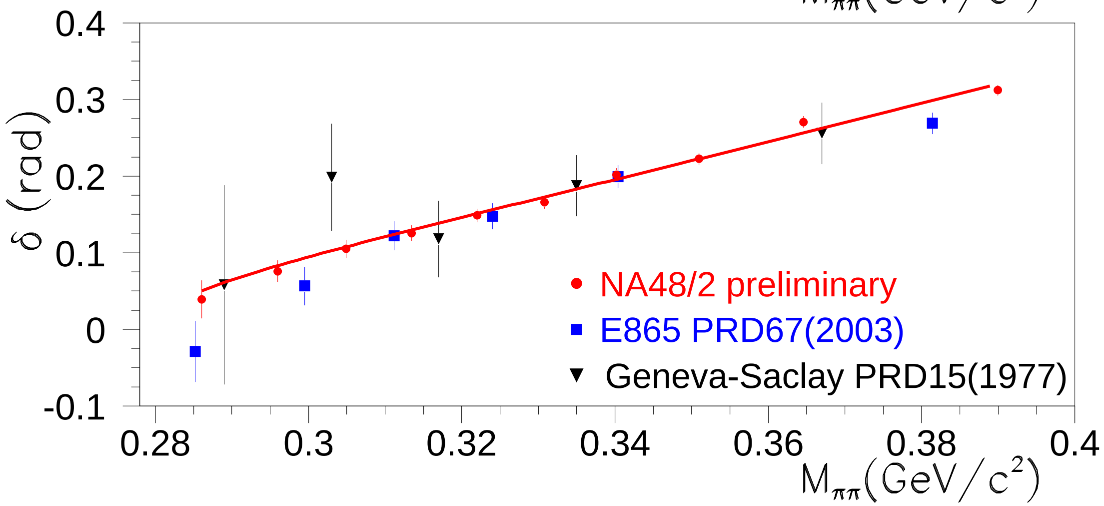
	g'_p
$g_p(0)$	-0.914

Comparison of Ke4 phase experimental measurements



Phase points without isospin corrections

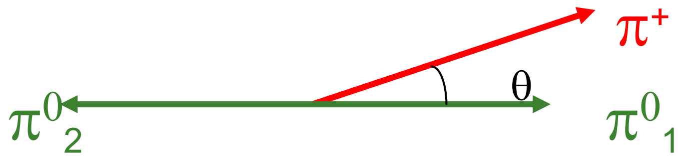
Line from a 2p fit to NA48 data alone



All Phase points corrected for isospin mass effects

K3π : measurement of the k' term

Going to a 2D fit would imply to use (M^2_{00} ,M^2_{+0}) variables. An alternate choice is (M^2_{00} ,cosθ) where θ is the angle between the charged π and the direction of the π^0's in their rest frame.

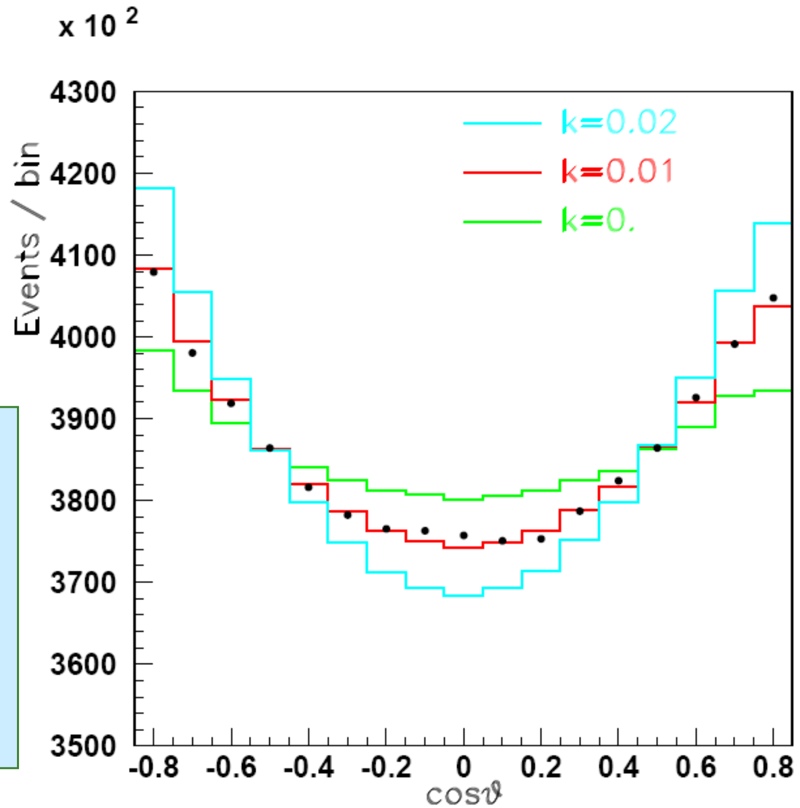


Use a modified matrix element :

$$M_0 = A_0\left(1 + \frac{1}{2}g_0 u + \frac{1}{2}h' u^2 + \frac{1}{2}k' v^2\right)$$

re-fit in M^2_{00} range [0.082, 0.097] $(GeV/c^2)^2$)

no incidence on previous **(a_0-a_2)** result.

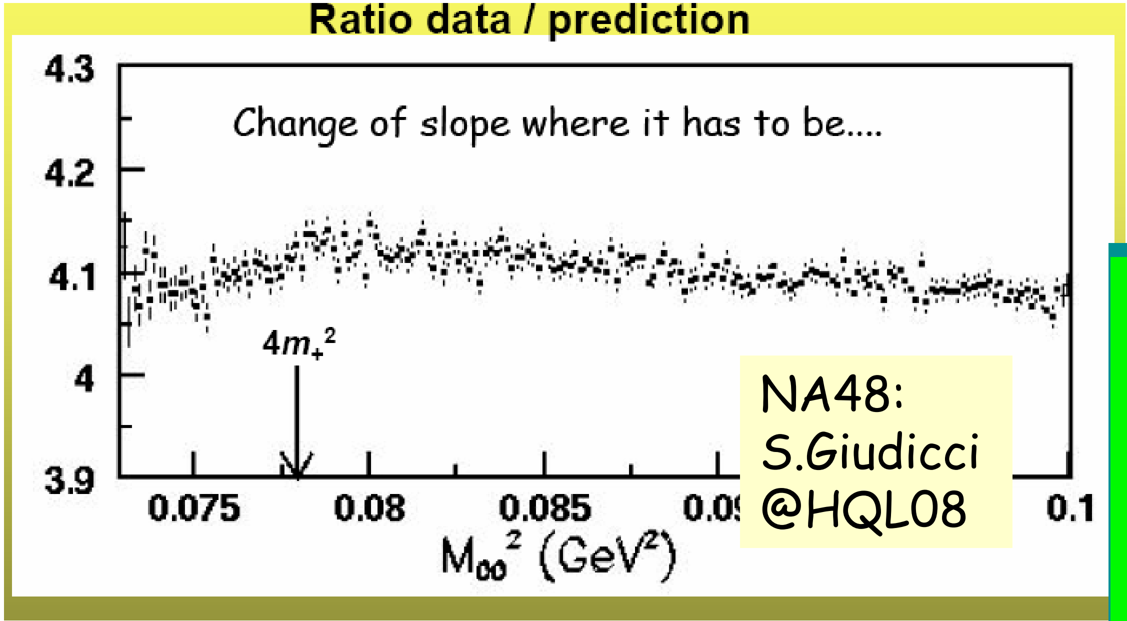


Preliminary result (2003+2004 data, K^+ and K^-)

$k' = 0.0095 \pm 0.0002_{stat} \pm 0.0005_{syst}$

Note: the different meaning (g_0,h',k') wrt PDG (g_0,h,k)

The cusp in $K_L \to 3\pi^0$ decays: work in progress



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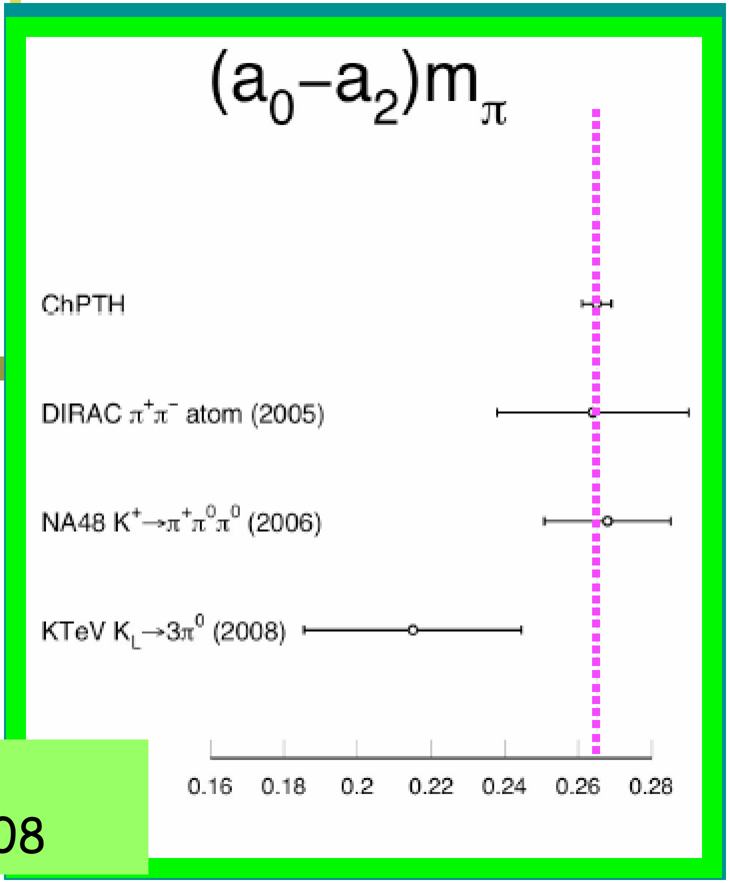

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