Probing non perturbative QCD
Test with K_{e4} decays from NA48/2

Bernard Peyaud
CEA Saclay, IRFU-SPP

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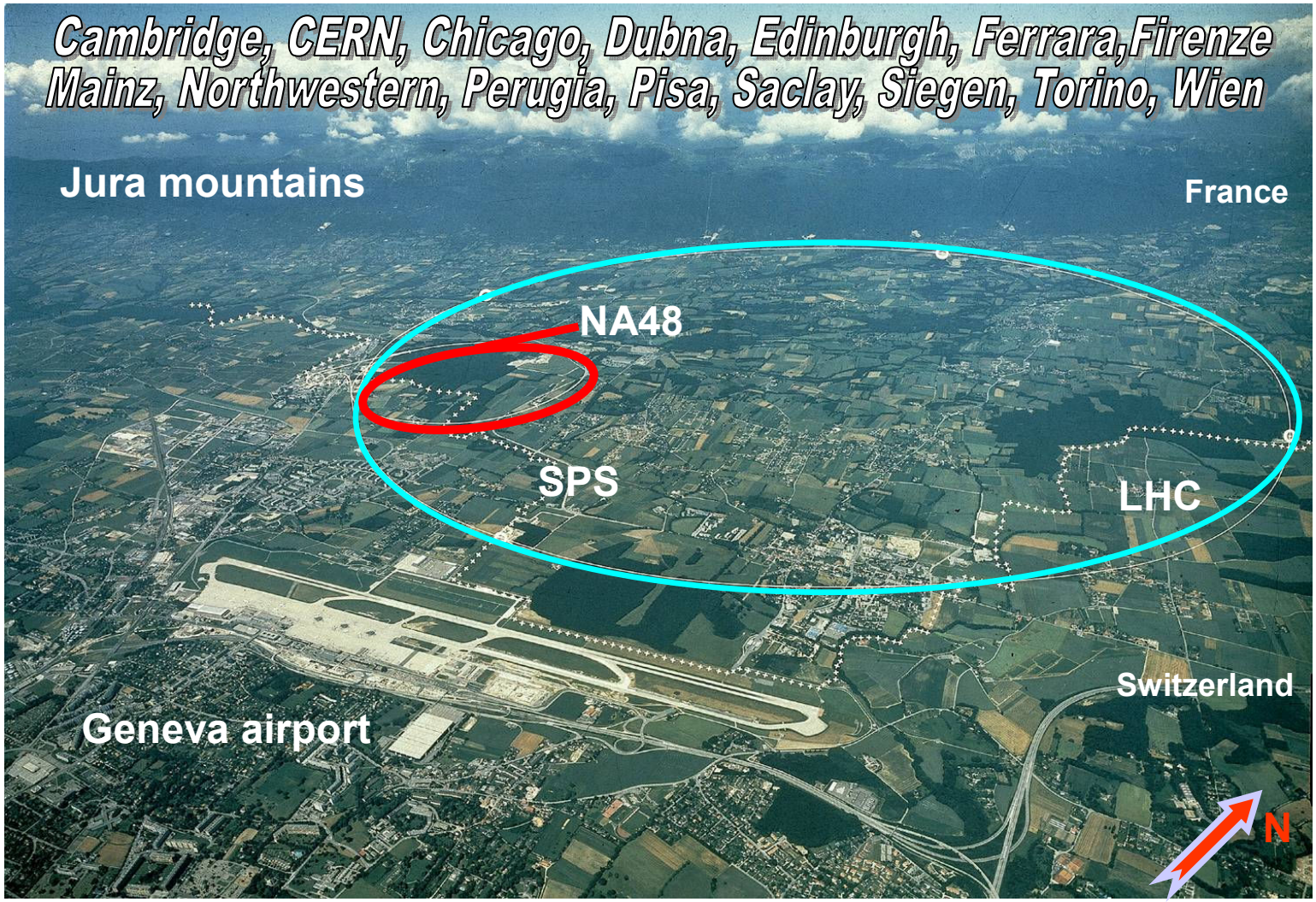
Florida State University

Outline

- **Na48/2**: beams, detector and performances
- **Low energy QCD** predictions for $\pi\pi$ scattering lengths
- **K_{e4} decays ($K^{\pm} \rightarrow e^{\pm} \nu \pi^{+}\pi^{-}$)** : measurements of Form Factors, phase shifts $\Rightarrow$ $\pi\pi$ scattering lengths
- **Comparison with theory and other measurements**
- **Summary**

NA48/2 : CERN experiment dedicated to Kaon physics

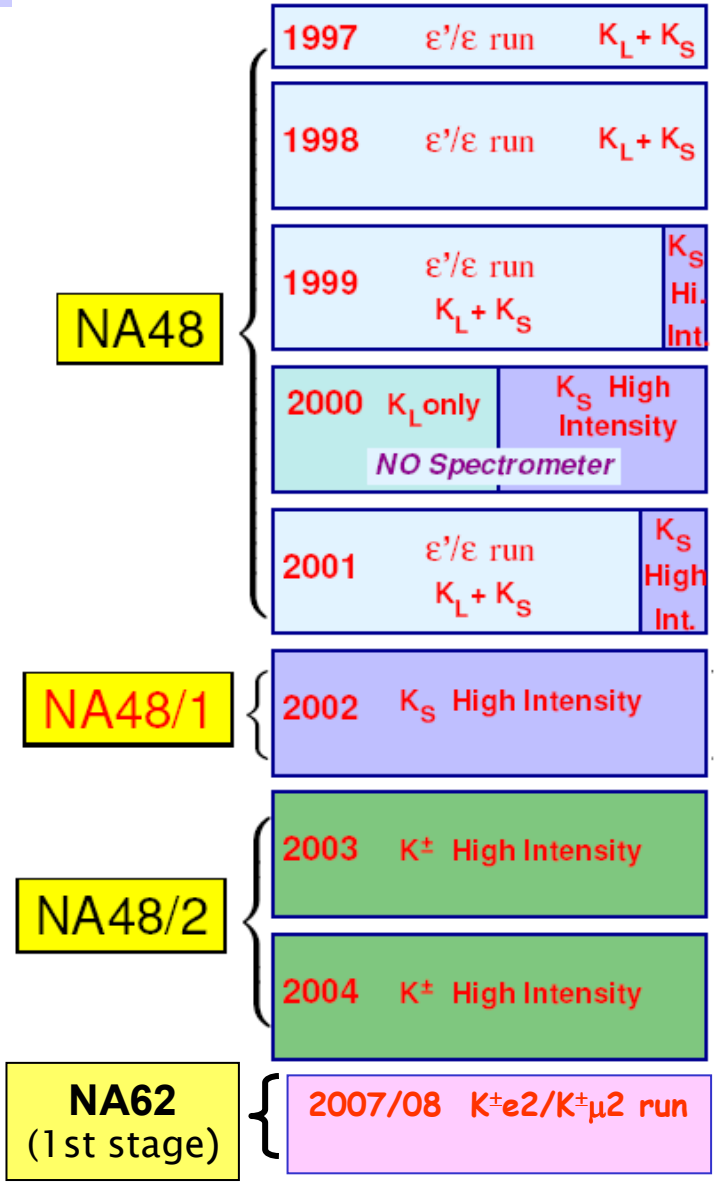
NA48/2 collaboration: ~100 physicists from 15 Institutes in 8 countries



The NA48 epochs

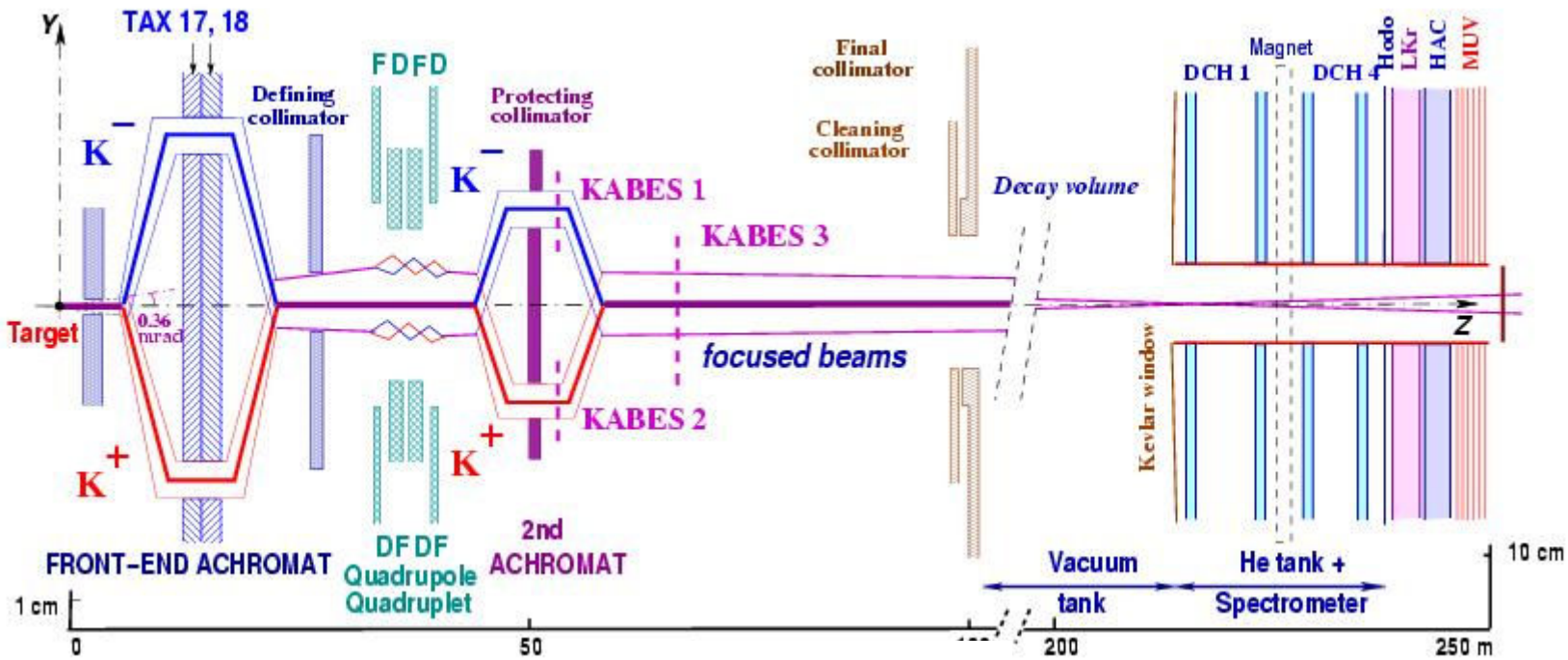
NA48: experiment dedicated measurements of CPV quantities in kaon decays:

- **NA48 1997-2001** : ε'/ε measurement
- **NA48/1 2002 : Ks** High intensity
- **NA48/2 2003-2004** : search for direct CP violating charge asymmetries A_g in $K^{\pm}\to 3\pi$ decays + many rare decays
- **NA62** (1st stage) 2007-2008: study of **$K^{\pm}e2/K^{\pm}\mu2$ ratio**
- NA62 2011-2012: search for $K^{+}\to\pi^{+}\nu\,\nu bar$

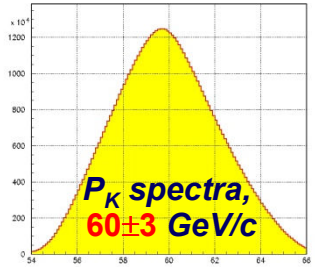


NA48/2 beam line at CERN-SPS

2003 run: ~ 50 days + 2004 run: ~ 60 days



Simultaneous K^+ and K^- beams: large charge symmetrization of experimental conditions



Beams coincide within ~1mm all along the 114m decay volume flux ratio K^+/K^- ~1.8

NA48/2 detector and performances

Magnetic spectrometer

- 4 high-resolution DCH's + dipole magnet
- **$\Delta p/p$ = (1.0 $\oplus$ 0.044 p)% (p in GeV/c)**
- Very good resolution
  e.g. **σ (M3$\pi^{\pm}$) = 1.7 MeV/c^2**

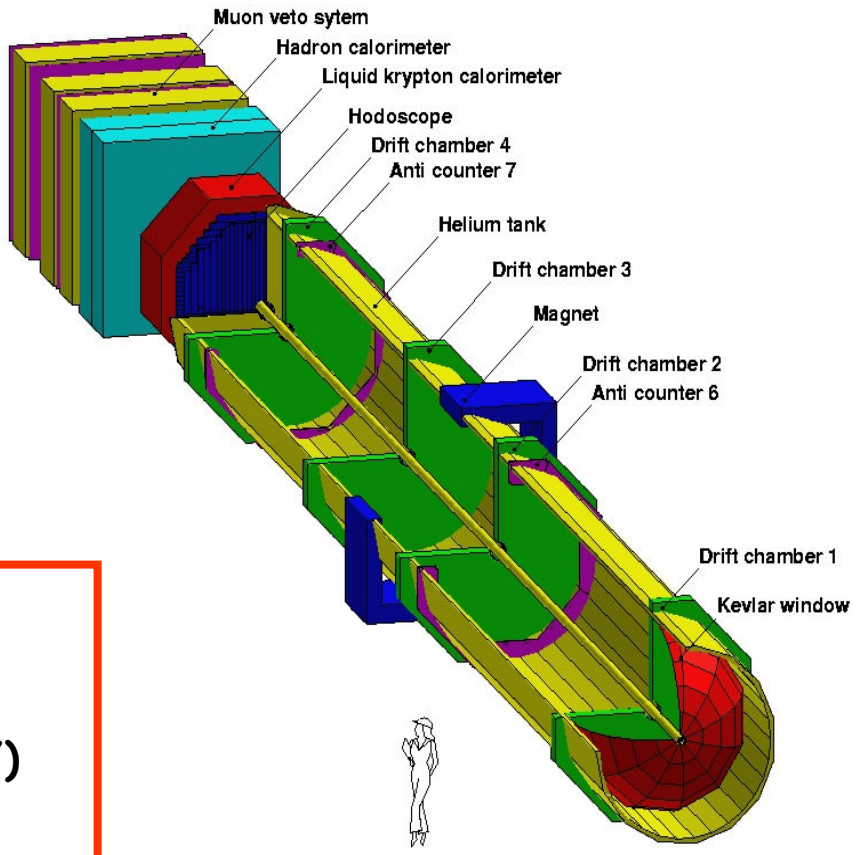
Hodoscope for charged fast trigger

Time resolution: σt = 150 ps

LKr electromagnetic calorimeter

quasi-homogenous and high granularity
$\Delta E/E$ = (3.2/$\sqrt{E}$ $\oplus$ 9.0/E $\oplus$ 0.42)% (E in GeV)
σx = σy ~ 1.5 mm for E=10 GeV
Very good resolution for neutrals ($\pi^0 \rightarrow \gamma\gamma$)
e.g. **σ (M$\pi\pi^0\pi^0$) = 1.3 MeV/c^2**

E/p ratio used for e /π discrimination



NA48/2 data taking in 2003-2004 : (50 + 60) days

Several billions triggers and > 200 TB of data recorded

Total statistics in 2 years:

$K^{\pm} \to \pi^{+}\,\pi^{-}\,\pi^{\pm}$: ~ 4. 10^9 evts

$K^{\pm} \to \pi^{0}\,\pi^{0}\,\pi^{\pm}$: ~1. 10^8 evts

$K^{\pm} \to \pi^{+}\,\pi^{-}\,e^{\pm}\,\nu$: ~1.1 10^6 evts

$K^{\pm} \to \pi^{0}\,\pi^{0}\,e^{\pm}\,\nu$: ~3.1 10^4 evts

and many more rare decays

Published:

Ag_+(2003+2004) Phys.Lett.B, 634(2006)474

Dalitz plot param. Phys.Lett.B, 649(2007)349

Ag_0(2003+2004) Phys.Lett.B, 638(2006) 22

common method EPJC 52 (2007) 875

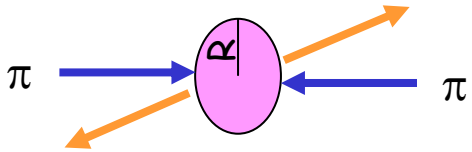
Cusp 2003 Phys.Lett.B, 633(2006) 173

See R. Arcidiacono's talk session 4A

Ke4 2003 EPJC 54 (2008) 411

Results from the full Ke4 samples (2003+2004) are presented

$\pi\pi$ scattering lengths: interesting?



angular wave number: $k = \frac{\sqrt{2mE}}{\hbar}$

- low energy ⇨ kR <<1 S-wave with I = 0, 2 (Bose statistics) dominates
- dispersion relations & analyticity connect amplitude & phases

  ⇨ $\pi\pi$ scattering amplitudes $S|\pi\pi\rangle = \exp(2i\delta)|\pi\pi\rangle$

  $\delta_{0,2} = a_{0,2}\, k + O(k^2)$: 2 phases relate to scattering lengths

  a_0, a_2 are essential parameters of ChPT
- $\langle q\,\bar{q}\rangle$ condensate provides *medium* for dynamic generation of m_π

  see M. Pennington's talk session-II
- $m_{u,d}/\Lambda_{QCD}$ ~few percent ⇒ precise predictions
- progress in experiments allows a stringent test of ChPT predictions

Hadronic physics and K_{e4} decays

$K^{\pm} \rightarrow e^{\pm} \nu \pi^{+}\pi^{-}$: $\pi\pi$ system at threshold to probe low energy QCD

- perturbative regime does not apply below 1 GeV ….. but
- effective field theory at work ⇒ ChPT
- isospin symmetry limit ($m_u = m_d$ and no γ)
- Chiral symmetry breaking treated with Low Energy Constants $\overline{\ell}_i$

  See J. Pelaez's talk session-VI
- Lattice QCD also used to compute LEC's

Predictions for F_π and M_π in term of quark masses:

$$M_\pi^2 = M^2\left(1 - \frac{M^2}{32\pi^2 F^2}\overline{\ell}_3 + O(p^4)\right)$$

$$M^2 \equiv -\frac{m_u + m_d}{F^2}\langle 0|\bar{q}q|0\rangle$$

Gell-Mann, Oakes, Renner (68)

$$F_\pi = F\left(1 + \frac{M^2}{16\pi^2 F^2}\overline{\ell}_4 + O(p^4)\right)$$

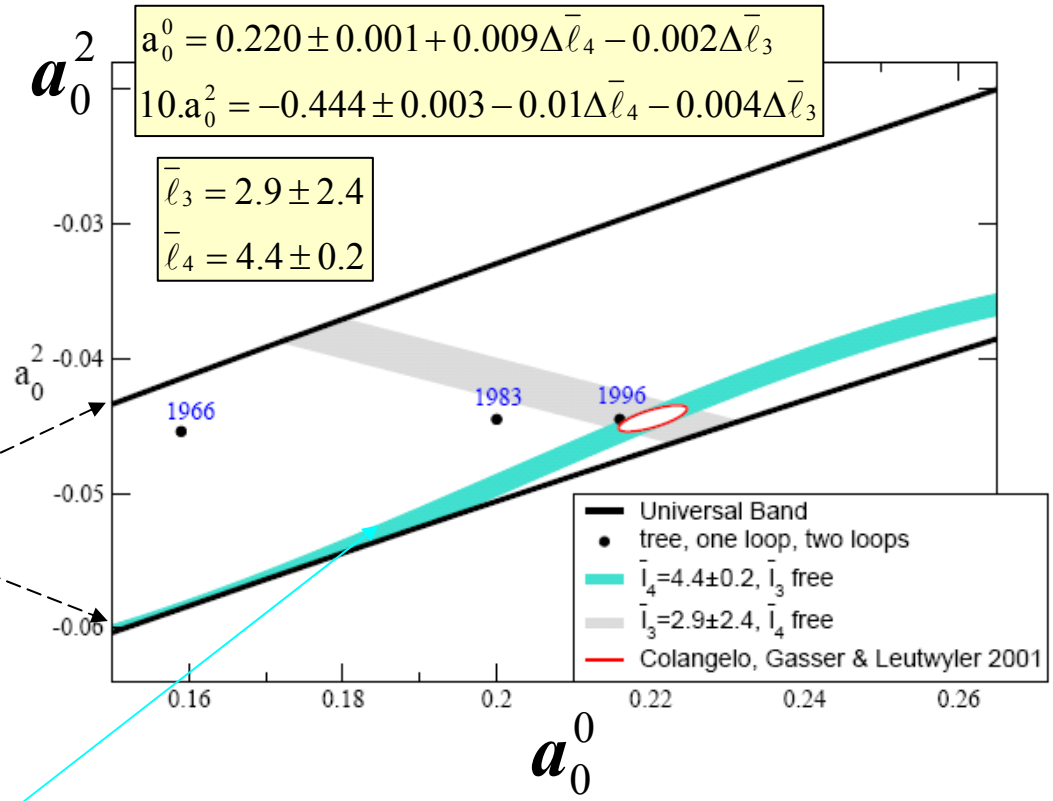
ππ scattering lengths

ππ scattering described theoretically by Roy equations ⇒ 2 "constants" = a_0, a_2

Numerical solutions developed in Bern ACGL(2001) and Orsay DFGS (2002)

Universal Band :

$a_0^2 = (-0.0849 + 0.232\, a_0^0 - 0.0865\, (a_0^2)^2) \pm 0.0088$



for low energy ππ interaction ChPT introduces constrains between a_0 and a_2

⇒ChPT band

$a_2 = (-0.0444 + 0.236\,(a_0 - 0.22) - 0.61\,(a_0 - 0.22)^2 - 9.9\,(a_0 - 0.22)^3) \pm 0.0008$

Accurate predictions by matching $O(p^6)$ ChPT to Roy Eq.(Colangelo, Gasser, Leutwyler 2001)

$a_0^0 = 0.220 \pm 0.005$

$10.a_2^2 = -0.444 \pm 0.010$

Measurements of $\pi\pi$ scattering lengths

3 kinds of measurements have been performed

$K_{3\pi}$ mode: $K^{\pm} \rightarrow \pi^0\pi^0\pi^{\pm}$ BR = $(1.757 \pm 0.024)\times10^{-2}$

Sensitivity to (a_0-a_2) and a_2 from cusp structure observed in $\pi^0\pi^0$ invariant mass

NA48/2 CERN/SPS 60M **(see R. Arcidiacono's talk session 4A)**

Pionium atoms : DIRAC CERN/PS Atom($\pi\pi$) life time measurement

Sensitivity to $(a_0-a_2)^2$, PLB 619 (2005) ~40% data analyzed

K_{e4} decay is rare but offers

very clean environment for the study of $\pi\pi$ system (no other hadron)

- Sensitivity to a_0 and a_2 from angular distributions
- Known for long but limited statistics (BR = $(4.09 \pm 0.09)\times10^{-5}$)

Geneva-Saclay CERN/PS S118 experiment: 30 000 K^+ (1977)

BNL E865 experiment: 400 000 K^+ (2003)

CERN/SPS NA48/2 : 1 130 000 $K^{\pm}$ (2009)

K_{e4} rare decay

$$K^{\pm} \longrightarrow \pi^{+}\ \pi^{-}\ e^{\pm}\ \nu$$

$$BR = (4.09 \pm 0.09) \times 10^{-5}$$

Ke4 experiments are rare also …..

269 events 1969

PHYSICAL REVIEW VOLUME 180, NUMBER 5 25 APRIL 1969

Study of K_{e4} Decays

ROBERT P. ELY, JR., GEORGE GIDAL, VASKEN HAGOPIAN,* AND GEORGE E. KALMUS
Lawrence Radiation Laboratory, University of California, Berkeley, California 94720

AND

KELVIN BILLING, FREDERICK W. BULLOCK, MICHAEL J. ESTEN, M. GOVAN, CYRIL HENDERSON,†
WILLIAM L. KNIGHT,‡ F. RUSSELL STANNARD, AND ORTWIN TREUTLER
Department of Physics, University College London, London, England

AND

UGO CAMERINI, DAVID CLINE, WILLIAM F. FRY, HERMANN HAGGERTY, ROBERT H. MARCH,
AND WILLIAM J. SINGLETON
Department of Physics, University of Wisconsin, Madison, Wisconsin 53706

(Received 16 December 1968)

A study of 13.3×10^6 stopped K^+ in a heavy-liquid bubble chamber yielded 269 K_{e4} decays of the type $e^+\pi^+\pi^-\nu$, a total greater by a factor of 4 than the number found in a previous experiment. No examples of $e^-\pi^+\pi^+\bar{\nu}$ were found. With 95% confidence, the upper limit for the decay rate of $K_{e4}(e^-)$ was found to be 56 $\sec^{-1}$. The rate for $K_{e4}(e^+)$ was found to be $(2.60\pm0.30)\times10^3$ $\sec^{-1}$. The angular distributions and the dipion invariant-mass plot have been fitted by varying the form factors f_s, f_p, g, and h, and the difference between s- and p-wave π-π phase shift. Two acceptable solutions have been found. Both agree that the vector form factor h is significantly nonzero and that its sign is negative with respect to that of f_s. Also, it has been found necessary to include f_p in order to obtain a good fit. Although both solutions give the magnitude of the phase difference to be 25 ± 9 deg, the two estimates have opposite sign. No evidence of a σ resonance is seen. The angular distributions are found to be consistent with time-reversal invariance, and with the locality of lepton production.

8141 events 1973

VOLUME 30, NUMBER 9 PHYSICAL REVIEW LETTERS 26 FEBRUARY 1973

Pion-Pion Phase Shift and Form Factors in the Decays $K^\pm \to \pi^+\pi^-e^\pm\nu_e$

E. W. Beier, D. A. Buchholz,* A. K. Mann, S. H. Parker,† and J. B. Roberts‡
Department of Physics, University of Pennsylvania,§ Philadelphia, Pennsylvania 19104

(Received 27 November 1972)

We present the results of an analysis of 8141 K_{e4} decays. These include (i) the form factors of the weak hadronic current and their dependences on the invariant di-lepton and di-pion masses, and (ii) the magnitude and dependence on invariant di-pion mass of the π-π scattering phase shift $\delta_s-\delta_p$.

30 000 events 1977

PHYSICAL REVIEW D VOLUME 15, NUMBER 3 1 FEBRUARY 1977

Experimental study of 30 000 K_{e4} decays*

L. Rosselet, P. Extermann, J. Fischer, O. Guisan, R. Mermod, and R. Sachot†
University of Geneva, Geneva, Switzerland

A. M. Diamant-Berger, P. Bloch, G. Bunce,‡ B. Devaux, N. Do-Duc, G. Marel, and R. Turlay
Département de Physique des Particules Elémentaires, Centre d' Études Nucléaires, Saclay, France

(Received 26 August 1976)

An experiment on the decay $K^+\to\pi^+\pi^-e^+\nu$ was performed at the CERN proton synchrotron with spark-chamber and counter techniques. The K_{e4} branching ratio has been measured relative to the τ decay. The $\pi\pi$ phase-shift difference $\delta_0^0-\delta_1^1$ and the form factors of the hadronic current have been determined as functions of the $\pi\pi$ energy. The $\pi\pi$ scattering length a_0^0 has been evaluated from the phase shifts with a phenomenological model. The results are compared with the theoretical predictions of current algebra and other models.

400 000 events 2003

PHYSICAL REVIEW D **67**, 072004 (2003)

High statistics measurement of K_{e4} decay properties

S. Pislak,[1,2] R. Appel,[2,3] G. S. Atoyan,[4] B. Bassalleck,[5] D. R. Bergman,[2,*] N. Cheung,[3] S. Dhawan,[2] H. Do,[2] J. Egger,[6] S. Eilerts,[5,†] H. Fischer,[5,‡] W. Herold,[6] V. V. Issakov,[4] H. Kaspar,[6,2] D. E. Kraus,[3] D. M. Lazarus,[7] P. Lichard,[3] J. Lowe,[5] J. Lozano,[2,§] H. Ma,[7] W. Majid,[2,∥] A. A. Poblaguev,[4] P. Rehak,[7] A. Sher,[3,¶] Aleksey Sher,[1] J. A. Thompson,[3] P. Truöl,[1,2] and M.E. Zeller[2]
[1]*Physik-Institut, Universität Zürich, CH-8057 Zürich, Switzerland*
[2]*Physics Department, Yale University, New Haven, Connecticut 06511*
[3]*Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260*
[4]*Institute for Nuclear Research of Russian Academy of Sciences, Moscow 117 312, Russia*
[5]*Department of Physics and Astronomy, University of New Mexico, Albuquerque, New Mexico 87131*
[6]*Paul Scherrer Institut, CH-5232 Villigen, Switzerland*
[7]*Brookhaven National Laboratory, Upton, New York 11973*

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We report experimental details and results of a new measurement of the decay $K^+\to\pi^+\pi^-e^+\nu_e(K_{e4})$. A sample of more than 400,000 K_{e4} events with low background has been collected by Experiment 865 at the Brookhaven Alternate Gradient Synchrotron. From these data, the branching ratio $(4.11\pm0.01\pm0.11)\times10^{-5}$ and the $\pi\pi$ invariant mass dependence of the form factors F, G, and H of the weak hadronic current as well as the phase shift difference $\delta_0^0-\delta_1^1$ for $\pi\pi$ scattering were extracted. Using constraints based on analyticity and chiral symmetry, a new value with considerably improved accuracy for the s-wave $\pi\pi$ scattering length a_0^0 has been obtained also: $a_0^0=0.216\pm0.013$ (stat)±0.002 (syst)±0.002 (theor).

DOI: 10.1103/PhysRevD.67.072004 PACS number(s): 13.20.Eb, 13.75.Lb

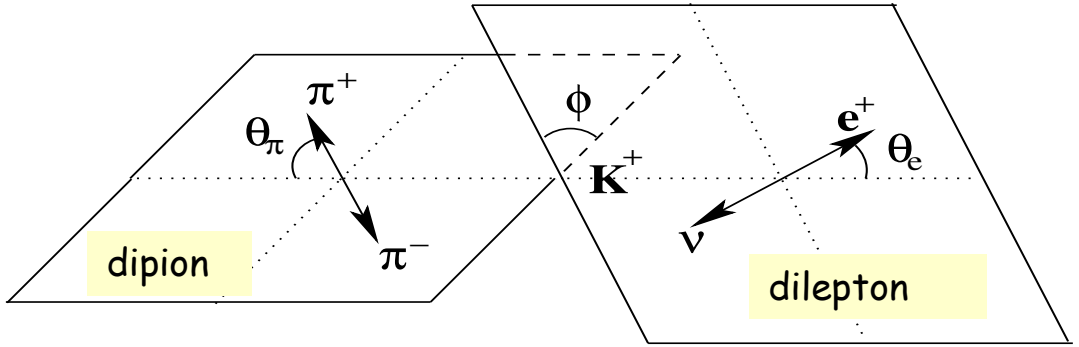
Description of K_{e4} decay

5 kinematic variables

(Cabibbo-Maksymowicz 1965)

$S_\pi = M^2_{\pi\pi}$), $S_e = M^2_{e\nu}$,

$\cos\theta_\pi$, $\cos\theta_e$ and ϕ.



Partial Wave expansion of the amplitude

S and P waves (Pais-Treiman 1968)

+ Watson theorem (T-invariance) for δ_l^I

$$\delta_0^0 \equiv \delta_s \text{ and } \delta_1^1 \equiv \delta_p$$

F, G = Axial Form Factors

$F = F_s\, e^{i\delta s} + F_p\, e^{i\delta p} \cos\theta_\pi$

$G = G_p\, e^{i\delta g}$

H = Vector Form Factor

$H = H_p\, e^{i\delta h}$

F, G, H are complex and dimensionless

Fit distributions in the **5D space** with 4 Form factors and one phase shift , assuming identical phases for the p-wave Form Factors F_p, G_p, H_p (T invariance)

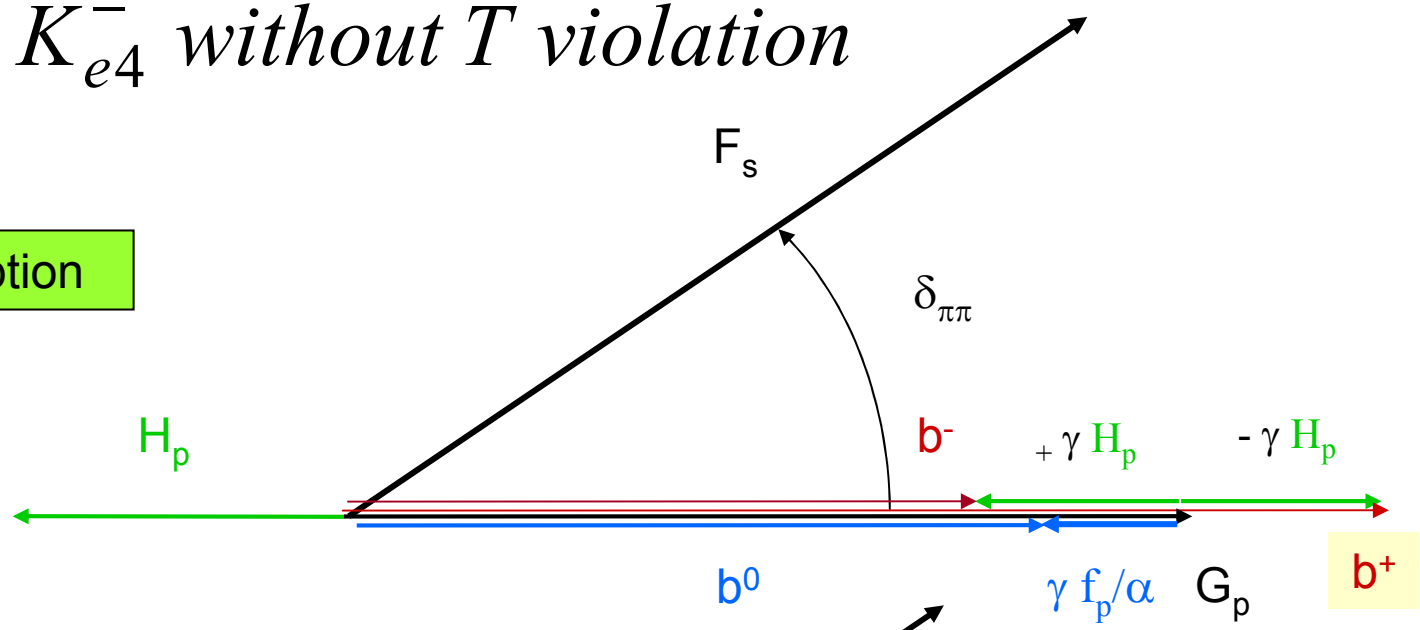
The fit parameters are :

F_s F_p G_p H_p and $\delta = \delta_s - \delta_p$

(F_s , F_p , G_p, , H_p are real)

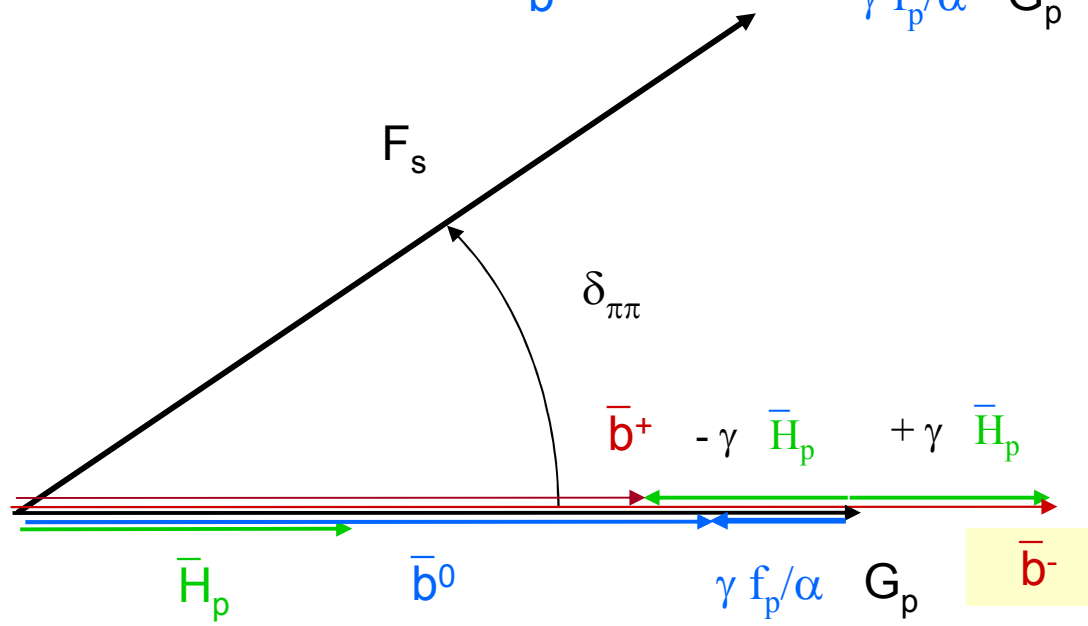
K^+_{e4} and K^-_{e4} without T violation

CPT assumption



no CP violation ⇨

1 phase $\delta_{\pi\pi} = \delta_S - \delta_P$



Ke4 decays: background rejection

Main background source:

$K^+ \rightarrow \pi^- \pi_2^+ \pi_1^+$

↳ $e^+\nu$ or mis-ident e^+ Right Sign

↳ $e^- \nu$ or mis-ident e^- Wrong sign

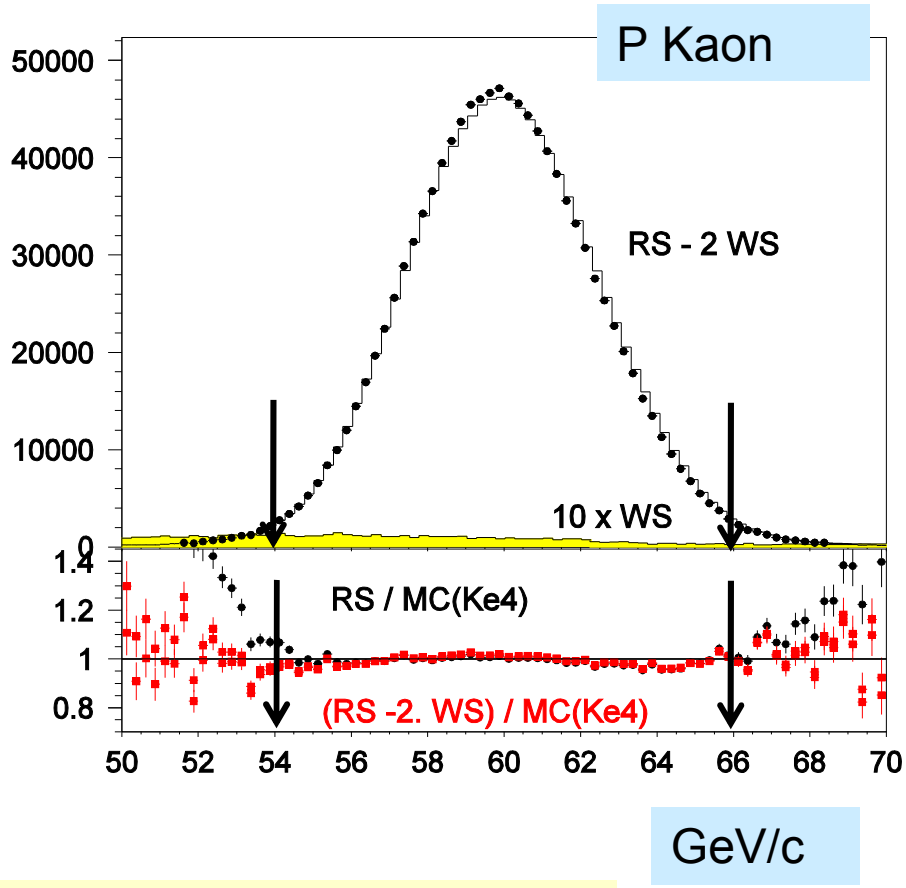
Bkg shape from **"Wrong Sign"** events

$K^\pm \rightarrow \pi^\pm \pi^\pm e^\mp \nu$

Rate (RS/WS) events:

2 if coming from K3π (dominant)

1 if coming from $K2\pi(\pi^0_D)$



Total background level is kept at ~ 0.6 % relative level estimated from WS events rate and checked with MC simulation

Ke4 decays : fitting procedure

Total (2003+2004) 1.13 Million K_{e4} decays

Use of 10x5x5x5x12=15000 variable size boxes with equal population in the 5-dimension space ($M_{\pi\pi}$, $M_{e\nu}$, $\cos\theta_\pi$, $\cos\theta_e$ and ϕ)

4 parameters fit to minimize the difference between data and MC predicted populations in each $M_{\pi\pi}$ "slice" (1500 bins)

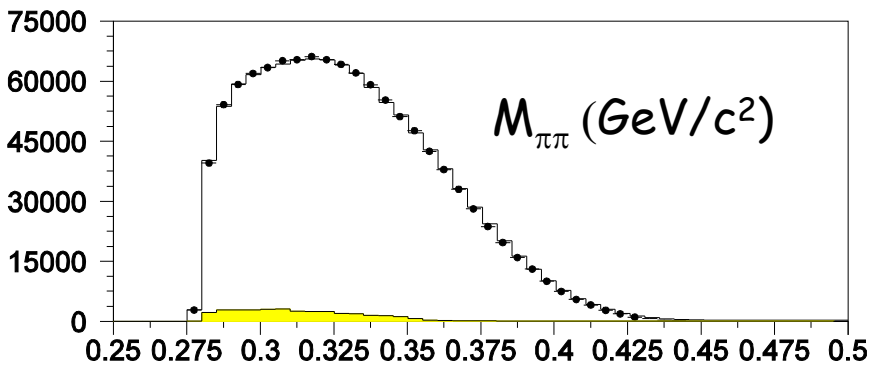
Data sample	MC sample
K^+ sample (726 400 events) 48 events/box	K^+ MC (17.4 Million events) 1160 events/box
K^- sample (404 400 events) 27 events/box	K^- MC (9.7 Million events) 650 events/box

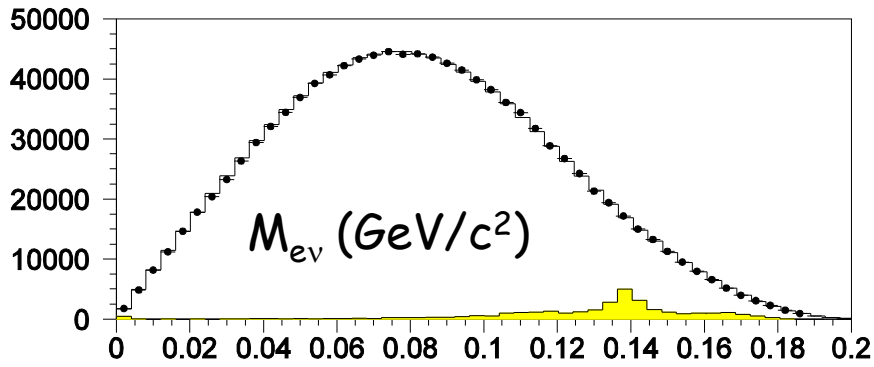
K^+ and K^- samples fitted separately in 10 independent $M\pi\pi$ bins & then combined

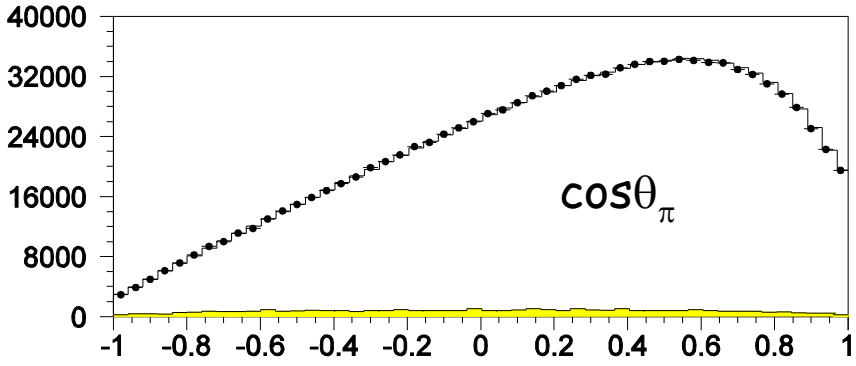
No assumption on the δ and FF variation from one $M_{\pi\pi}$ bin to the next

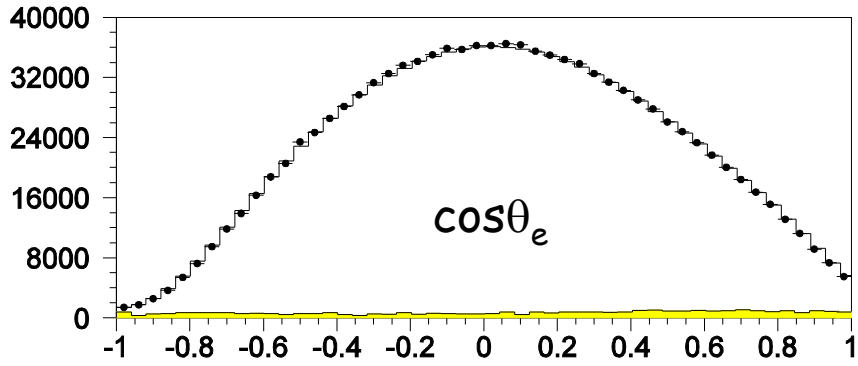
$\Rightarrow$ "model independent" analysis

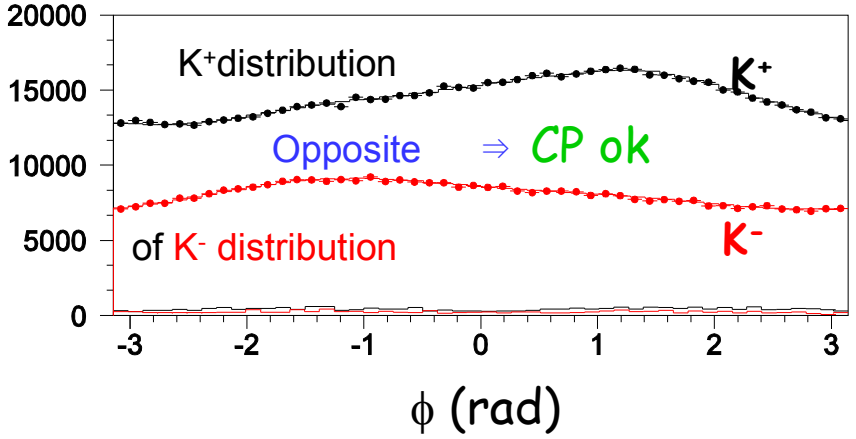
Comparing Data & MC after fit











↓	= Data
histogram	= Simulation after fit
Yellow hist	= WS Background (x10 to be visible)

Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)

$\pi\pi$ phases predicted from data above 0.8 GeV using **Roy equations** with 2 constants **a_0 and a_2**

Numerical solutions developed (ACGL, DFGS) assume **Isospin symmetry** ⇒ **Universal Band in the [a_2,a_0] plane**,

Needed in real world

factorization of electromagnetic and mass effects :

Gamow factor x PHOTOS generator × Isospin corrections

effects included in MC

Gamow factor : classical Coulomb attraction between the 2 charged pions

PHOTOS generator: real photon(s) are emitted and tracked in the simulation

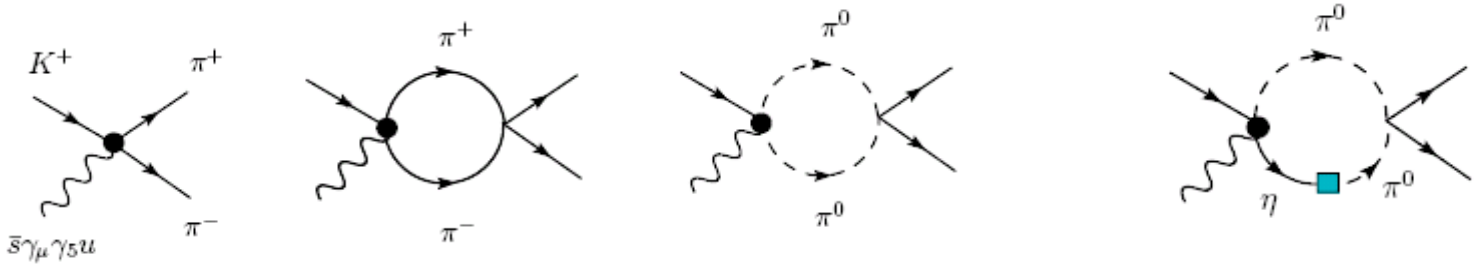
Mass effects: CGR EPJ C59 (2009) 777

- recently computed and used as corrections to measurements
- size is x2 experimental precision !

Ke4 charged decays : isospin corrections to δ

CGR EPJ C59 (2009) 777 formulation developed in close contact with NA48,

tree | one loop | π^0–η mixing

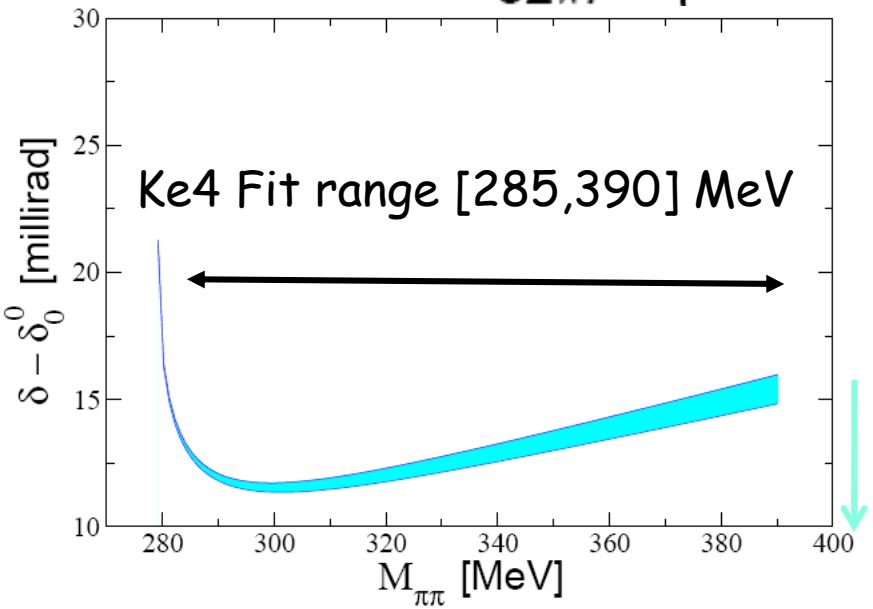


$\delta_0{}^0 \rightarrow$ $$\delta = \frac{1}{32\pi F^2}\left\{(4\Delta_\pi + s)\sigma + (s - M_{\pi^0}^2)\left(1 + \frac{3}{2R}\right)\sigma_0\right\}$$

$$\Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2,$$

$$\sigma = \sqrt{1 - \frac{4M_\pi^2}{s}},$$

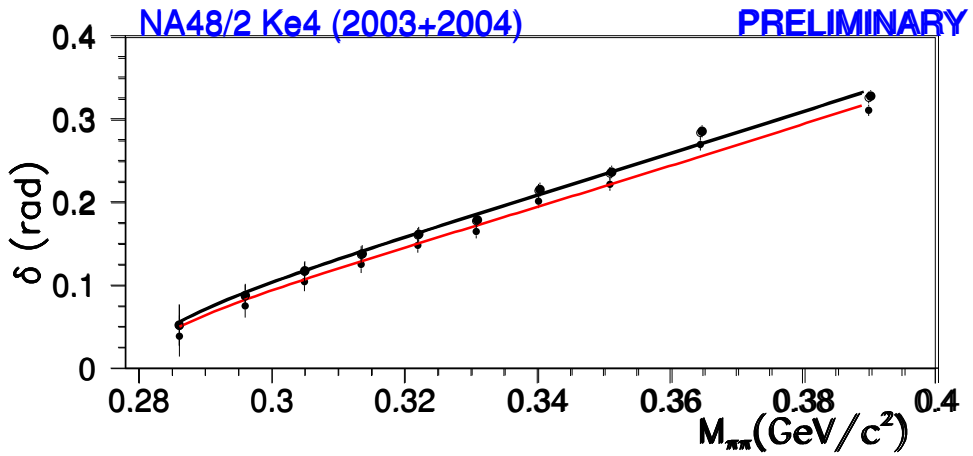
$$R = \frac{m_s - \hat{m}}{m_d - m_u}$$



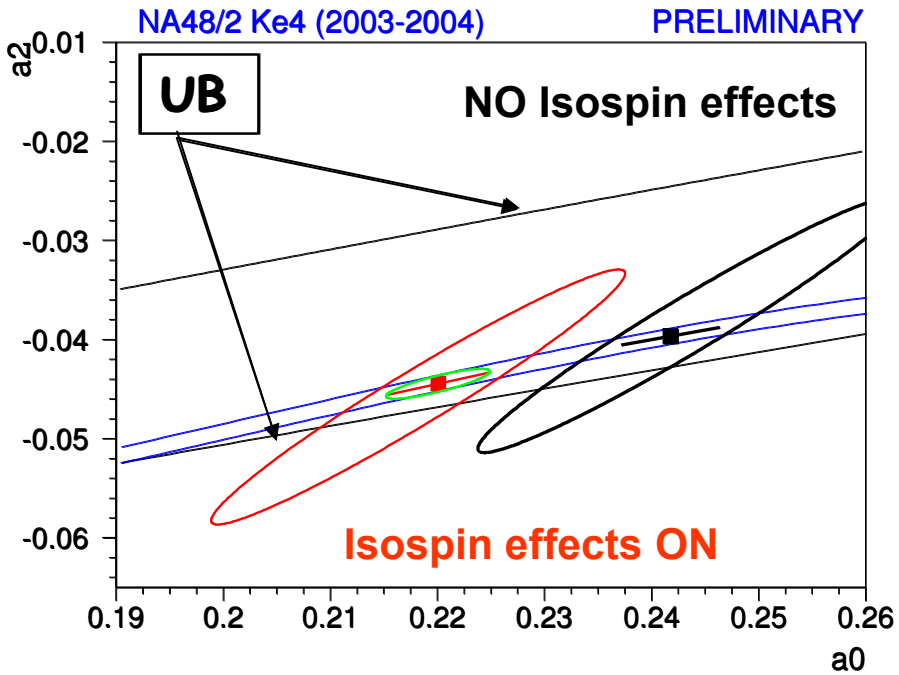
Size of correction ~ 10-15 mrad

Exp. Stat precision ~ 7-8 mrad

Ke4 decays: from phase shifts to scattering lengths (a_0,a_2)



isospin effects bring significant change in precise experimental measurement!



This **changes** (a_0,a_2) values

from a 2p fit	from a 1p fit
Δa_0 = -0.025, Δa_2 = -0.007	Δa_0 = -0.022
$\sigma(a_0)$: ± 0.013 ± 0.005	$\sigma(a_0)$: ± 0.005 ± 0.002
$\sigma(a_2)$: ± 0.0084 ± 0.0034	stat syst
stat syst	

Ellipses are 68% CL contours in 2p fits

Ke4 decays: comparison with theoretical predictions

THEORY prediction

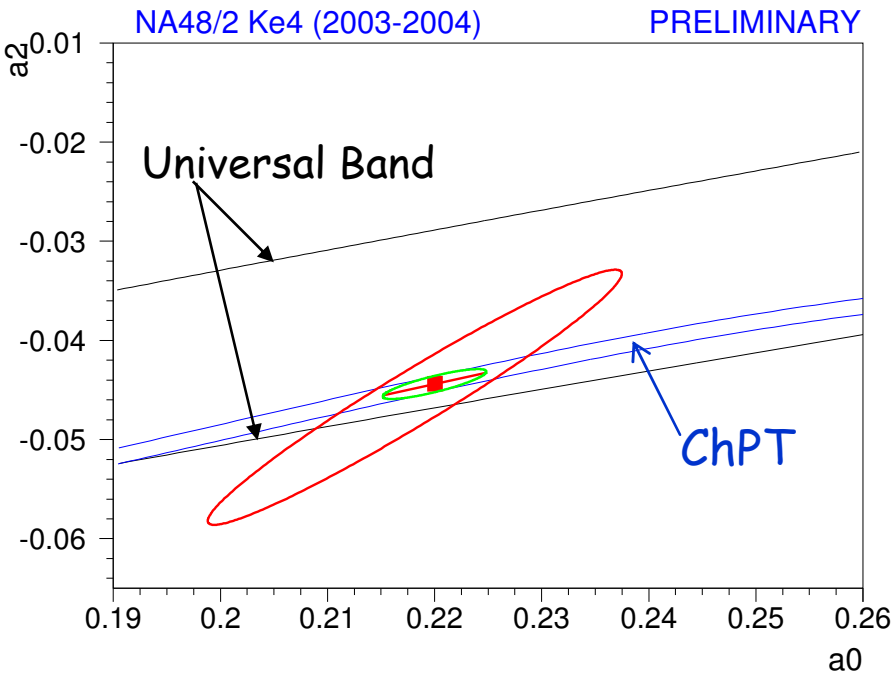
With inputs from ChPT, the prediction is constrained (CGL NPB603(2001),PRL86(2001)):

a_0 = 0.220 ± 0.005

a_2 = -0.0444 ± 0.0010

Experimental measurement

a_0 ChPT 1p fit	0.2206 ± 0.0049 stat ± 0.0018 syst ± 0.0064 theo *
a_0 free a_2 free 2p fit	0.2220 ± 0.0128 stat ± 0.0050 syst ± 0.0037 theo* -0.0432 ± 0.0086 stat ± 0.0034 syst ± 0.0028 theo* Correlation 96.7%



*evaluated from isospin corrections & inputs to numerical solutions Roy equation CGR EPJ C59 (2009)777

Comparison of Ke4 phase shift experimental measurements

Isospin corrections (10-15 mrad) applied to all published points :

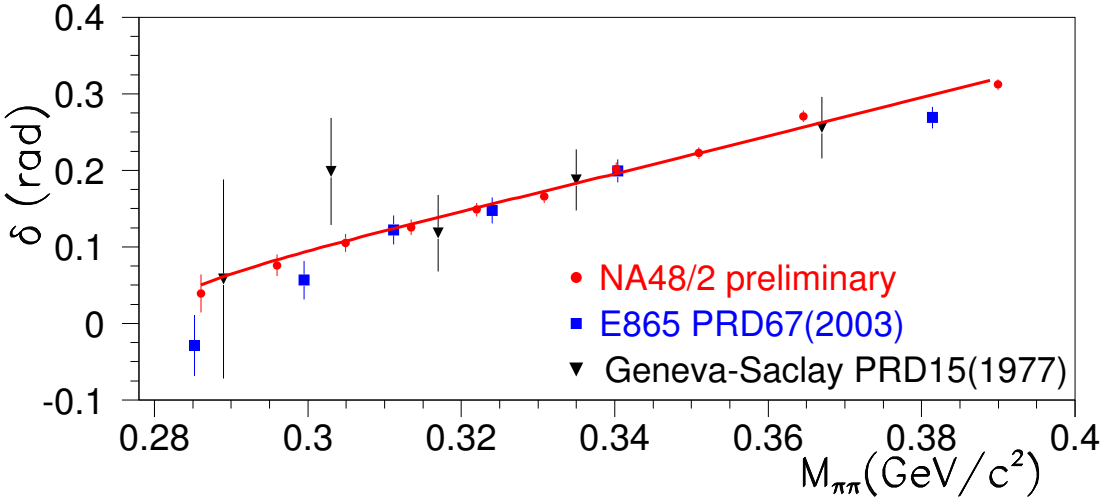
E865: typical error 15-20 mrad

S118 (Geneva-Saclay): typical error 40-50 mrad

Small isospin corrections shift data points downwards

NA48/2 typical error 7-8 mrad

improved precision due to both
-larger statistics ~3 x E865
-larger acceptance at high $\pi\pi$ mass



- All Phase points corrected for isospin mass effects
- Independent experiments
- Errors stat + syst

Line from a 2p fit to NA48 data alone

Fit to all data points (21 points) : dominated by NA48/2 measurements

2p fit:

$a_0 = 0.2199 \pm 0.0125_{exp} \pm 0.0037_{theo}$

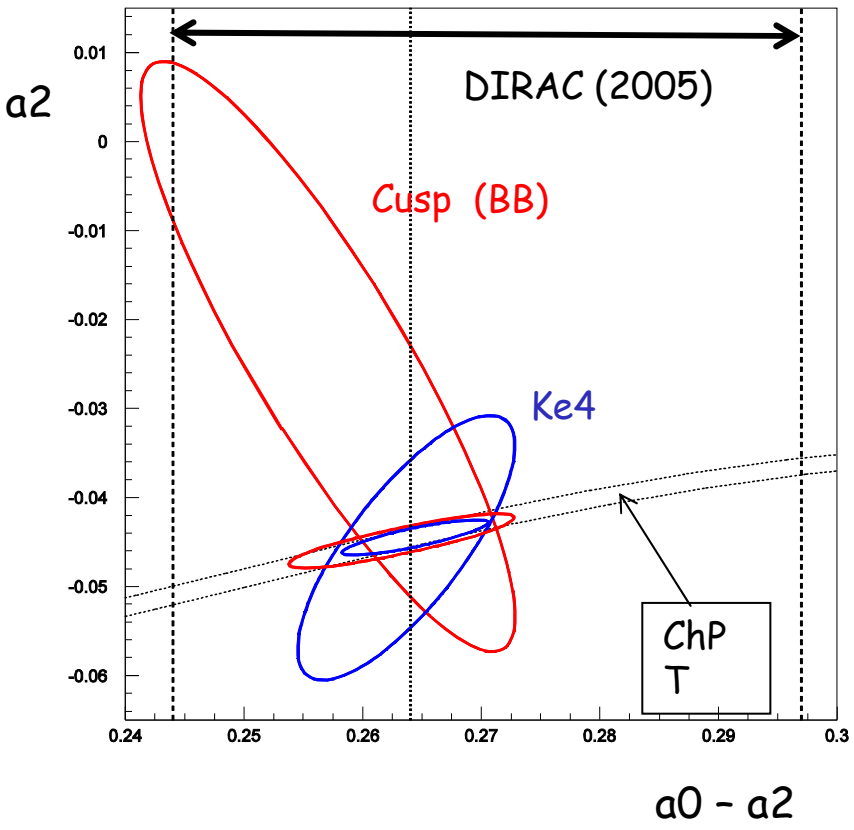
$a_2 = -0.0430 \pm 0.0083_{exp} \pm 0.0028_{theo}$

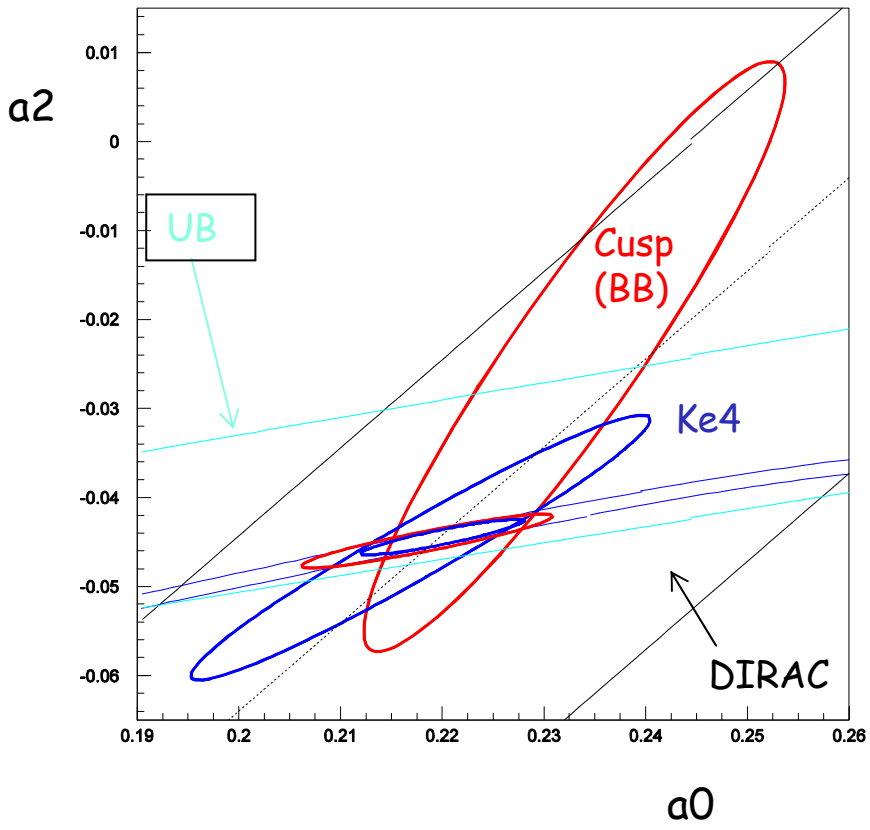
1p fit :

$a_0 = 0.2168 \pm 0.0048_{exp} \pm 0.0064_{theo}$

(theory error common to all expts)

Cusp and Ke4 : scattering lengths results





Two statistically independent measurements by NA48/2

Large overlap in the (a_0,a_2) plane in excellent agreement with precise ChPT predictions

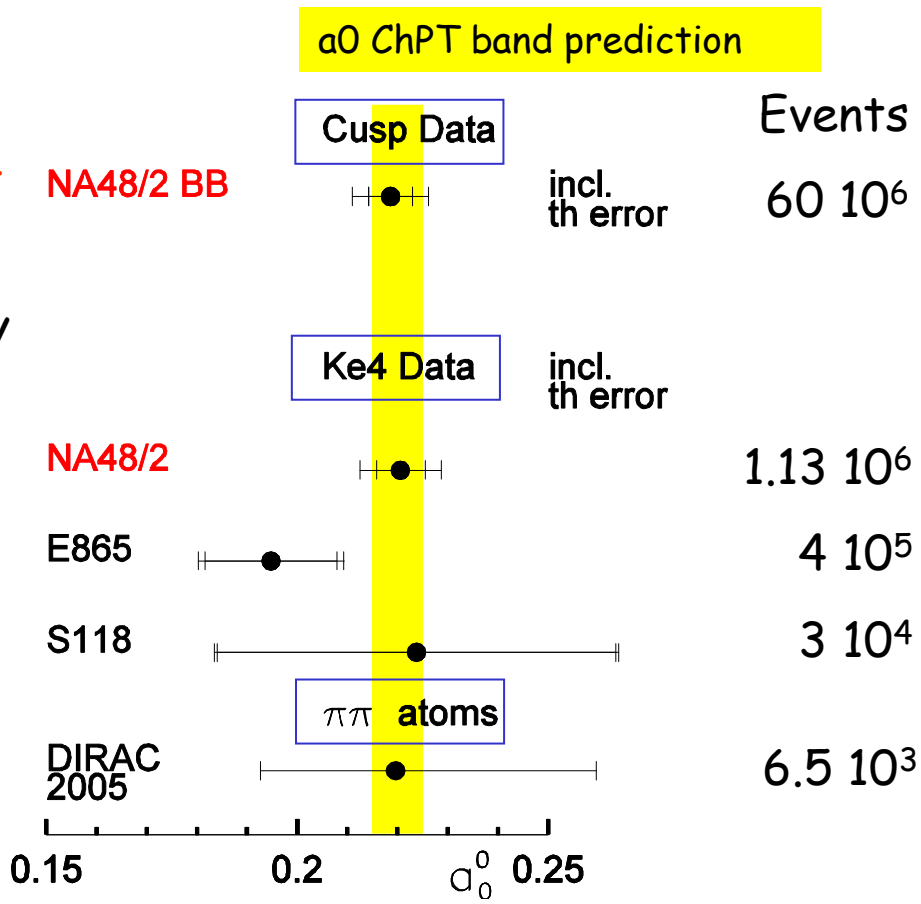
Comparison with other experimental measurements

Cusp : (a_0-a_2) ChPT fit with 2 models

Ke4 : isospin corrections applied to all experimental δ points before a_0 ChPT fit

$\pi\pi$ atoms DIRAC: $|a_0-a_2|$ errors from PLB619 (2005), use ChPT constraint (only 40% Data analyzed)

Yellow band is ChPT prediction



$a_0 = 0.220 \pm 0.005$

NA48/2 experimental precision and most precise theory prediction now at the same level!

NA48/2 : Summary

~1.13 M K_{e4} events recorded in (2003+2004) and analyzed

K_{e4} Form Factors measured with improved precision

- Detailed theoretical calculations to extract scattering lengths
- experimental results show striking agreement (<2.5%) with ChPT predictions

Theory $a_0 = 0.220 \pm 0.005$, $a_2 = -0.0444 \pm 0.0010$, $a_0 - a_2 = 0.264 \pm 0.004$

K_{e4} experiment $a_0 = 0.2206 \pm 0.0049_{stat} \pm 0.0018_{syst} \pm 0.0064_{th}$

- quark condensate accounts for ~95% of m_π

Joint work with theorists was invaluable in understanding the details to extract scattering lengths from NA48 Data