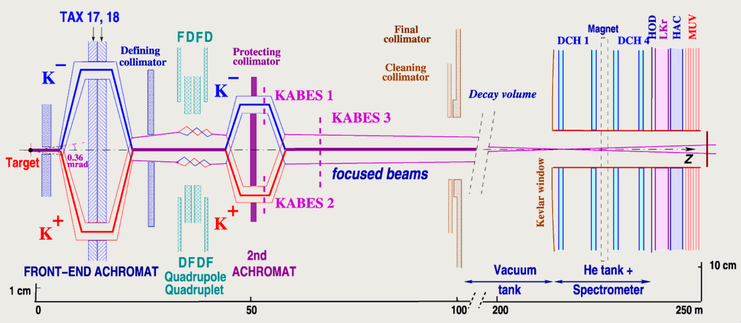
DETAILED STUDY OF K_{e4} DECAY PROPERTIES BY THE NA48/2 EXPERIMENT AT CERN

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Florence, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Turin, Vienna

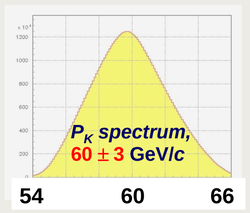
Beam Line schematics

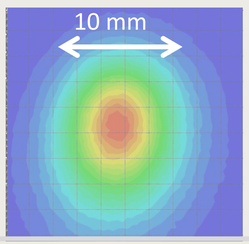


Primary Beam: SPS protons (400 GeV/*c*) impinging on a Be target

Secondary Beams:

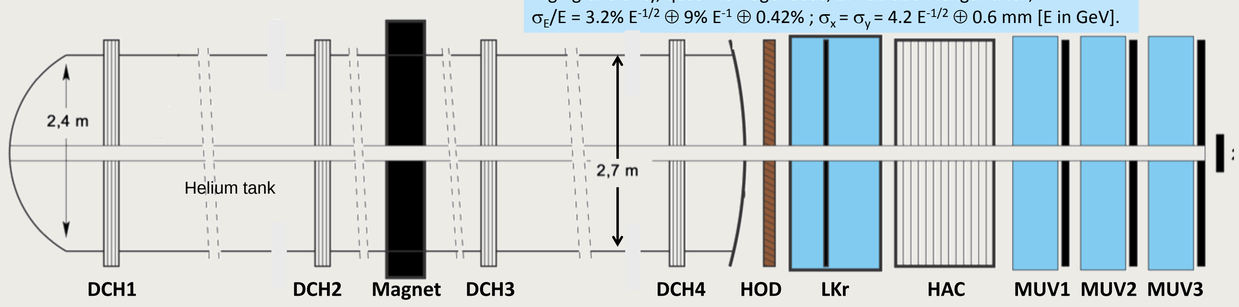
- **simultaneous** K^+ and K^- (60 ± 3) GeV/*c* momentum;
- **focused** at DCH1 with ~10mm transverse size;
- **superimposed** beam axes within 1 mm.





NA48/2 Detector Layout

Liquid Krypton Calorimeter (LKr) :
High granularity, quasi-homogeneous, 27 radiation length thick;
$\sigma_E/E = 3.2\% E^{-1/2} \oplus 9\% E^{-1} \oplus 0.42\%$; $\sigma_x = \sigma_y = 4.2 E^{-1/2} \oplus 0.6$ mm [E in GeV].



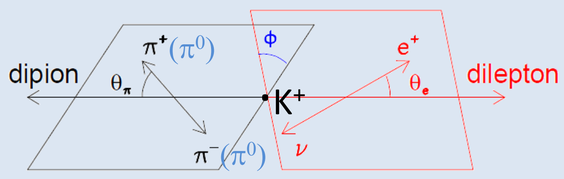
Magnetic spectrometer (4DCHs):
4 views/DCH: redundancy/high efficiency ;
$\sigma_p/p = 1.02\% \oplus 0.044\%$ p [p in GeV/*c*].

Hodoscope (HOD) :
2 orthogonal planes of scintillators;
Fast trigger signal; σ_t = ~150 ps.

Muon Veto (MUV) :
Three planes of scintillator strips;
80 cm iron shielding before each.

K^{+-}_{e4} Formalism $K^\pm \to \pi^+\pi^- e^\pm \nu$

- Four-body final state described by five kinematic variables:
  $S_\pi\ (M^2_{\pi\pi}),\ S_e\ (M^2_{e\nu}),\ \cos\theta_\pi,\ \cos\theta_e,\ \Phi$
  (Cabibbo-Maksymowicz, Phys. Rev. 137(1965))
- Partial wave expansion of the decay amplitude into S- and P-waves (Pais-Treiman, Phys. Rev. 168 (1968))
  Two complex axial form factors: $F = F_s e^{i\delta_s} + F_p e^{i\delta_p}\cos\theta_\pi$, $G = G_p e^{i\delta_g}$
  One complex vector form factor: $H = H_p e^{i\delta_h}$
- Map the distribution of the Cabibbo-Maksymowicz variables in the five-dimensional space with 4 real form factors and 1 phase $\delta = \delta_s - \delta_p$,assuming identical phase for the P-wave ffs (F_p , G_p , H_p).



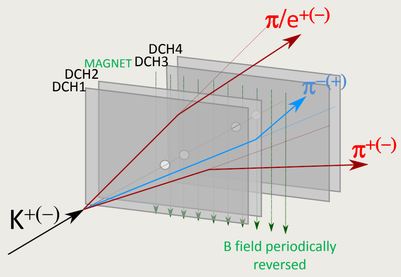
- 10 independent fits (1 per S_π bin) of F_s , F_p , G_p , H_p and δ
- Extract scattering lengths from δ variation with S_π using Roy equations solutions (Phys.Rep. 353 (2001)) and Isospin breaking mass corrections (Eur.Phys.J. C59 (2009))

K^{00}_{e4} Formalism $K^\pm \to \pi^0\pi^0 e^\pm \nu$

- Four-body final state with two identical particles described by three kinematic variables
  $S_\pi\ (M^2_{\pi\pi}),\ S_e\ (M^2_{e\nu}),\ \cos\theta_e$
- Dipion $\pi^0\,\pi^0$ system only in S-wave state: one complex form factor $F = F_s\, e^{i\delta_s}$
- Decay amplitude is proportional to F_s^2, determined in 12 x10 bins in the (S_π , S_e) plane
- Parameterize F_s , F_p , G_p , H_p variations with S_π and S_e as series expansions of the dimensionless variables: $q^2 = S_\pi/4m^2_{\pi^+} - 1$, $S_e/4m^2_{\pi^+}$

$$F_s = f_s + f_s'\, q^2 + f_s''\, q^4 + f_e'\, S_e/4m^2_{\pi^+} + \cdots$$
$$F_p = f_p + \cdots,\ G_p = g_p + g_p'\, q^2 + \cdots,\ H_p = h_p + \cdots$$

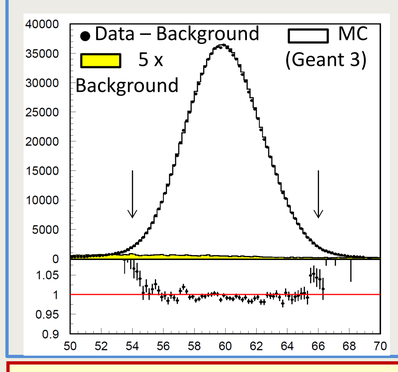
K^{+-}_{e4} Selection and Reconstruction



Decay mode for normalization : $K^\pm \to \pi^\pm\pi^+\pi^-$
Event reconstruction: require
- 3 tracks, reconstructed by the magnetic spectrometer, forming a vertex within the decay volume;
- No MUV hit associated with tracks.

Background
- $K_{3\pi}$ with fake-electron or $\pi^\pm \to e^\pm \nu$ decay
- Accidental track
- $K^\pm \to \pi^\pm\pi^0$ (π^0) followed by $\pi^0 \to e^+e^-\gamma$ decay with fake-pion and undetected γ

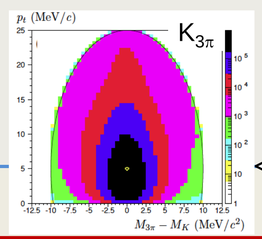
Kaon momentum reconstruction (both modes):
Under energy-momentum conservation, assuming kaon mass and missing neutrino, get the two solutions for kaon momentum, pick the closest to the nominal beam momentum (60 GeV/c) between 54 and 66 GeV/c.



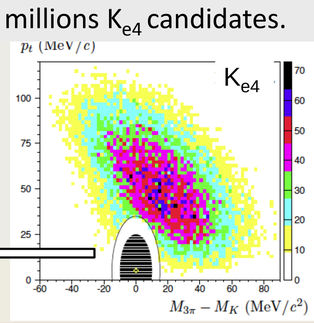
Decay mode for normalization : $K^\pm \to \pi^\pm\pi^0\pi^0$
Event reconstruction: require
- 2 cluster pairs (γ1, γ2) and (γ3, γ4) satisfying π^0 mass constraint and following conditions:
  $|Z_{12} - Z_{34}| < 500$ cm ; $Z_n = (Z_{12} + Z_{34})/2$ within decay volume
- Z_c at CDA of charged track to the beam line with $|Z_c - Z_n| < 800$ cm

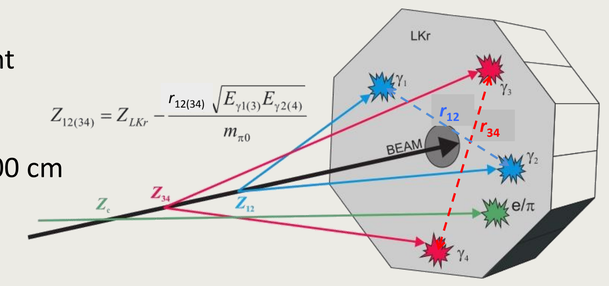
Background
- $K_{3\pi}$ with fake-electron or $\pi^\pm \to e^\pm \nu$ decay
- Accidental track/photon

Signal / normalization kinematic separation (both modes):
Compute the invariant mass and p_t (wrt the beam line) in pion hypothesis. An elliptic cut separates 1.9 (0.1) billions $K_{3\pi}$ candidates from 1.1 (0.07) millions K_{e4} candidates.

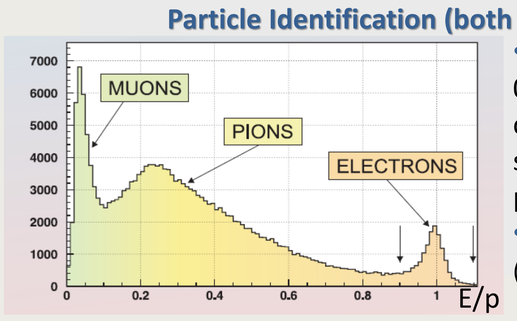


Zoom of $K_{3\pi}$ area





Particle Identification (both modes):

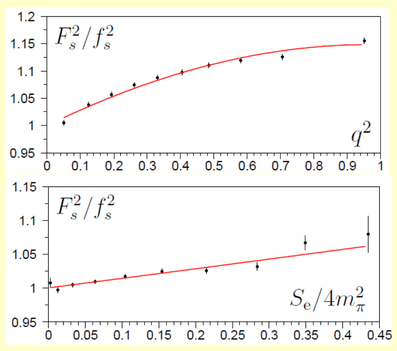


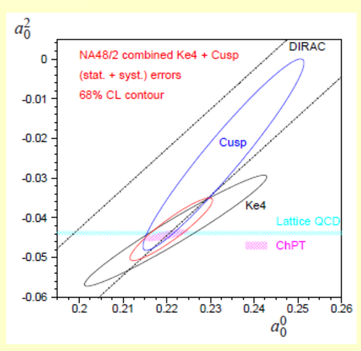
- **electron-ID:** 0.9 < E/p < 1.1 complemented by shower-shape properties
- **charged pion-ID:** (K^{+-}_{e4} mode) E/p < 0.8

K^{+-}_{e4} Form Factors & scattering lengths

f_s'/f_s	$= 0.152$	$\pm 0.007_{stat}$	$\pm 0.005_{syst}$
f_s''/f_s	$= -0.073$	$\pm 0.007_{stat}$	$\pm 0.006_{syst}$
f_e'/f_s	$= 0.068$	$\pm 0.006_{stat}$	$\pm 0.007_{syst}$
f_p/f_s	$= -0.048$	$\pm 0.003_{stat}$	$\pm 0.004_{syst}$
g_p/f_s	$= 0.868$	$\pm 0.010_{stat}$	$\pm 0.010_{syst}$
g_p'/f_s	$= 0.089$	$\pm 0.017_{stat}$	$\pm 0.013_{syst}$
h_p/f_s	$= -0.398$	$\pm 0.015_{stat}$	$\pm 0.008_{syst}$

Final Eur. Phys. J. C70 (2010) 635

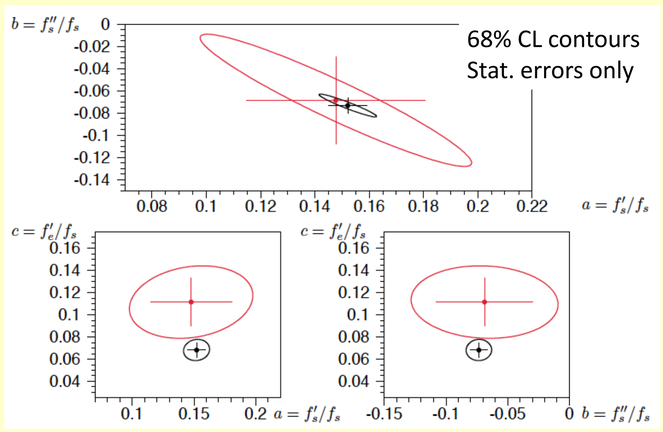




NA48/2 K_{e4} form factors & scattering lengths: Eur.Phys.J. C70 (2010)
NA48/2 $K_{3\pi}$ scattering lengths: Eur.Phys.J. C64 (2009)
DIRAC Pionium scattering lengths: Phys.Lett. B704 (2011)
ChPT prediction Colangelo,Gasser,Leutwyler: Phys.Lett. B488 (2000)
Lattice QCD: ETM Phys.Lett. B684 (2010); NPLQCD Phys.Rev. D77 (2008)

F_s Form Factor comparison ($q^2 > 0$)

K^{+-}_{e4} $F_s^{+-} = f_s^{+-} + f_s' q^2 + f_s'' q^4 + f_e' y^2$ | $F_s^{00} = f_s^{00}(1 + aq^2 + bq^4 + cy^2)$ K^{00}_{e4}



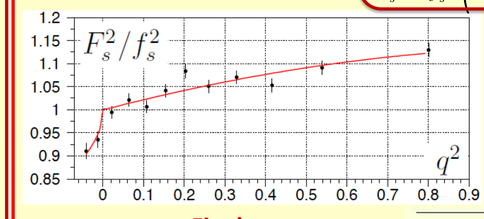
- similar $q^2 = S_\pi/4m^2_{\pi^+} - 1$ and $y^2 = S_e/4m^2_{\pi^+}$ dependence
- same correlations
- consistent within statistical errors

$$\mathrm{BR}(K_{e4}) = \frac{N_s - N_b}{N_n} \cdot \frac{A_n\ \varepsilon_n}{A_s\ \varepsilon_s} \cdot \mathrm{BR}(n)$$

$BR(K^{+-}_{e4})$	Input quantities	$BR(K^{00}_{e4})$
1.1×10^6	N_s = K_{e4} candidates	65210
0.95% x N_s	N_b = background to K_{e4}	1.00% x N_s
1.9×10^9 $(\pi^+\pi^+\pi^\pm)$	N_n = $K_{3\pi}$ candidates	$(\pi^0\pi^0\pi^\pm)$ 94×10^6
18.22%	A_s = Acceptance for K_{e4}	1.93%
24.18%	A_n = Acceptance for $K_{3\pi}$	4.05%
98.3%	ε_s = Trigger efficiency for K_{e4}	96.1%
97.5%	ε_n = Trigger efficiency for $K_{3\pi}$	97.4%
(5.59 ± 0.04)%	**BR($K_{3\pi}$) = normalization BR**	**(1.761±0.022)%**

K^{00}_{e4} Form Factor

$F_s^{00} = f_s^{00}(1 + aq^2 + bq^4 + cy^2)$ $q^2 \geq 0$
$F_s^{00} = f_s^{00}(1 + d\sqrt{|q^2|/(1+q^2)} + cy^2)$ $q^2 < 0$



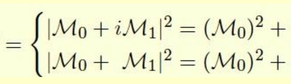
$q^2 > 0$: series expansion in q^2 and y^2 as in the K^{+-}_{e4} mode
$q^2 < 0$: cusp-like function and linear y^2 dependence

Final arXiv:1406.4749 [hep-ex]
interpretation in progress

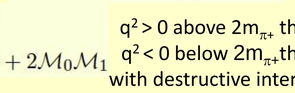
a	$= 0.149$	$\pm 0.033_{stat}$	$\pm 0.014_{syst}$
b	$= -0.070$	$\pm 0.039_{stat}$	$\pm 0.013_{syst}$
c	$= 0.113$	$\pm 0.022_{stat}$	$\pm 0.007_{syst}$
d	$= -0.256$	$\pm 0.049_{stat}$	$\pm 0.016_{syst}$

Charge exchange process $\pi^+\pi^-$ to $\pi^0\pi^0$ in the K^{+-}_{e4} mode:

unperturbed amplitude: $\mathcal{M}_0 = F_s^{00}(q^2)$



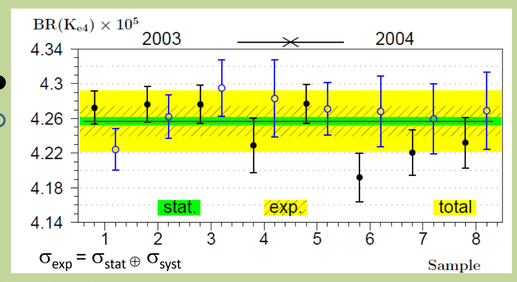
one-loop contribution: $\mathcal{M}_1 = -\frac{2}{3}(a_0^0 - a_0^2)\sqrt{\frac{|q^2|}{1+q^2}}\, F_s^{+-}(q^2)$



$|\mathcal{M}|^2 = \begin{cases} |\mathcal{M}_0 + i\mathcal{M}_1|^2 = (\mathcal{M}_0)^2 + (\mathcal{M}_1)^2 \\ |\mathcal{M}_0 + \mathcal{M}_1|^2 = (\mathcal{M}_0)^2 + (\mathcal{M}_1)^2 + 2\mathcal{M}_0\mathcal{M}_1 \end{cases}$

$q^2 > 0$ above $2m_{\pi^+}$ threshold
$q^2 < 0$ below $2m_{\pi^+}$ threshold with destructive interference

$BR(K^{+-}_{e4}) = (4.257 \pm 0.016_{exp} \pm 0.031_{ext}) \times 10^{-5}$



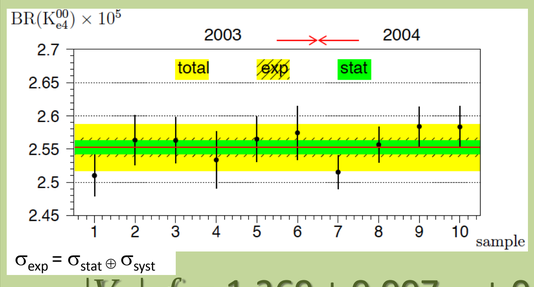
World average (2012) precision of 2.4% improved by a factor of 3, now 0.8%, dominated by external error (0.7%)

$|V_{us}| \cdot f_s = 1.285 \pm 0.004_{exp} \pm 0.005_{ext}$
and $f_s = 5.705 \pm 0.017_{exp} \pm 0.031_{ext}$

systematic uncertainties (%)	δBR/BR	$\delta f_s/f_s$	systematic uncertainties (%)	δBR/BR	$\delta f_s/f_s$
Acceptance	0.18	0.23	Particle-ID	0.09	0.05
Muon vetoing	0.16	0.08	Background	0.07	0.03
Accidental	0.21	0.10	Radiative effects	0.08	0.06
Trigger efficiency	0.11	0.06	External sources	0.72	0.54

Final Phys. Lett. B715 (2012)

$BR(K^{00}_{e4}) = (2.552 \pm 0.014_{exp} \pm 0.032_{ext}) \times 10^{-5}$



World average (2012) precision of 18% improved by more than one order of magnitude, now 1.4%, dominated by external error (1.25%).

$|V_{us}| \cdot f_s = 1.369 \pm 0.007_{exp} \pm 0.009_{ext}$
and $f_s = 6.079 \pm 0.030_{exp} \pm 0.046_{ext}$

systematic uncertainties (%)	δBR/BR	$\delta f_s/f_s$	systematic uncertainties (%)	δBR/BR	$\delta f_s/f_s$
Acceptance	0.23	0.42	Particle-ID	0.25	0.13
Trigger	0.05	0.03	Background	0.02	0.01
Radiative effects	0.19	0.14	External sources	1.25	0.75

NA48/2 Final CERN-PH-EP-2014-145, arXiv:1406.4749 [hep-ex]
NA62 **NA62 prospects** 5 to 10 X more statistics in 2015-2017

©The NA48/2 collaboration June 2014
created by Brigitte Bloch-Devaux / presented by Michal Zamkovsky