ECT* workshop
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Trento

Pion scattering lengths from NA48/2 experiment at CERN

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NA48/2 collaboration

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

Overview

- The NA48/2 experiment
  - performance in photon measurements: the η mass
- Recent results (CPV in $K^{\pm} \rightarrow 3\pi$ decays)
  - charged and neutral asymmetry measurements
- Scattering lengths in $K^{\pm} \rightarrow \pi^{\pm}\pi^{o}\pi^{o}$
  - observation of a structure
  - theoretical interpretation
  - extraction of the scattering lengths
- Scattering lengths in $K_L \rightarrow \pi^{o}\pi^{o}\pi^{o}$
- Experimental status of the scattering lengths

History of NA48 physics

Run	Beam	Physics goals
1997	K_L+K_S	ε'/ε
1998	K_L+K_S	ε'/ε Rare K_L decays
1999	K_L+K_S	ε'/ε Rare K_L decays
	K_L	Ke3/Kμ3
	HI K_S	K_S/hyperon decays
2000 NA48 and NA48/1	K_L	ε'/ε checks Neutral K_L decays
	η	ε'/ε checks
	HI K_S	Neutral K_S decays η_{000}

No spectrometer

Run	Beam	Physics goals
2001 NA48	K_L+K_S	ε'/ε Ke3/Kμ3
2002 NA48/1	HI K_S	Rare K_S decays Hyperon decays
2003 NA48/2	K^+/K^-	Direct CPV in $K^\pm \rightarrow (3\pi)^\pm$ Rare K decays
2004 NA48/2	K^+/K^-	Direct CPV in $K^\pm \rightarrow (3\pi)^\pm$ Rare K decays

FUTURE(>2008) K very rare decays

Direct CP violation program
I. Neutral kaons (1997-2001)

Simultaneous detection of the 4 decay modes:

$K_S \to \pi^+\pi^-$, $K_S \to \pi^o\pi^o$, $K_L \to \pi^+\pi^-$, $K_L \to \pi^o\pi^o$

$K_L \to \pi^o\pi^o$ statistics: 4×10^6

Measurement with the double ratio technique:

$$R = \frac{N(K_S \to \pi^-\pi^+)\,N(K_L \to \pi^o\pi^o)}{N(K_S \to \pi^o\pi^o)\,N(K_L \to \pi^+\pi^-)} = 1 - 6\,\Re(\epsilon'/\epsilon)$$

Final result (PLB 554 2002): $\Re(\epsilon'/\epsilon) = (14.7 \pm 2.2) \times 10^{-4}$

Experimental key features:

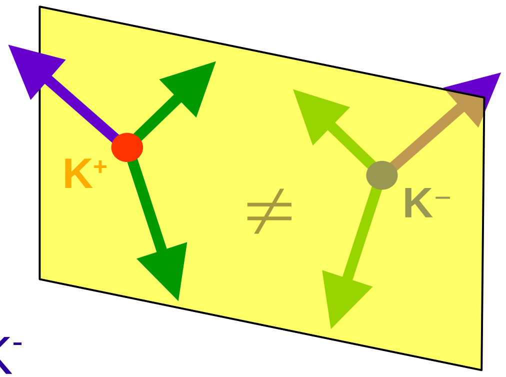
- Liquid Krypton EM Calorimeter: status of the art for high resolution in energy, position, time and remarkable trigger and read-out capabilities

- Tagged KS beam

Direct CP violation program
II. Charged kaons (2003-2004)

Simultaneous detection of the 4 decay (in flight) modes:

$K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ and $K^{\pm} \rightarrow \pi^{\pm}\pi^{o}\pi^{o}$



Looking for a difference in Dalitz plot distribution of K^+ vs K^-
(more sensitive than decay width (Γ) difference)

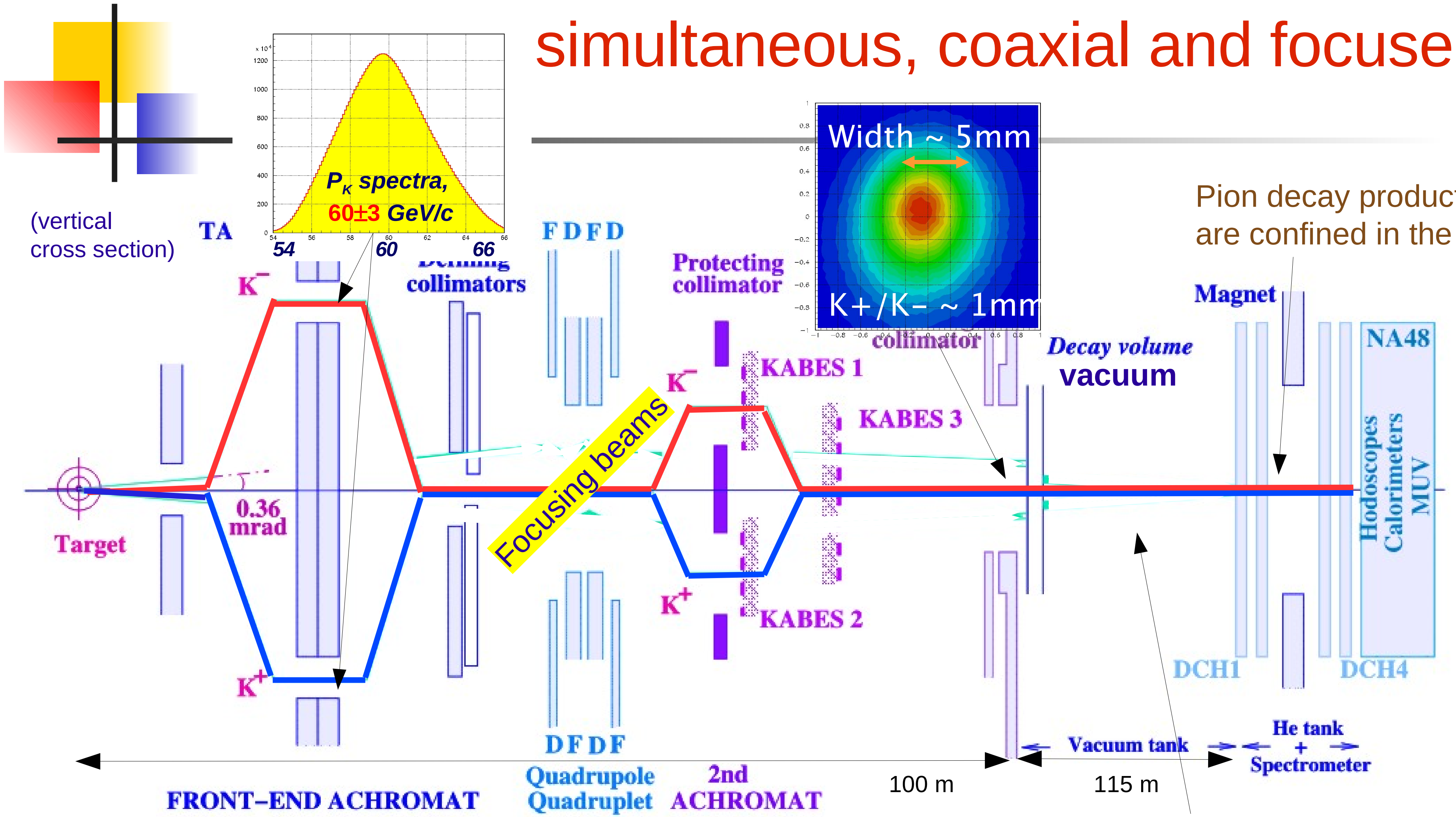
Experimentally:

* huge statistics possible:
  more than 3 x 10^9 $K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ and 0.1 x 10^9 $K^{\pm} \rightarrow \pi^{\pm}\pi^{o}\pi^{o}$ collected in 2 years
  ie the hugest statistics ever collected for these channels
* no need for K+/K- relative flux measurement
* simple selection
* negligible background

... and the proper detector was already there !!! Only change the beam ...

The NA48/2 experimental setup

The NA48/2 K+/K- narrow band beam: simultaneous, coaxial and focused



Pion decay products are confined in the pipe

Beams overlap within ~1mm through the 115m long decay volume (vacuum 10^{-5} mbar)

- Split +/-
- Select momentum
- Recombine +/-

- Focusing
- Muon sweeping

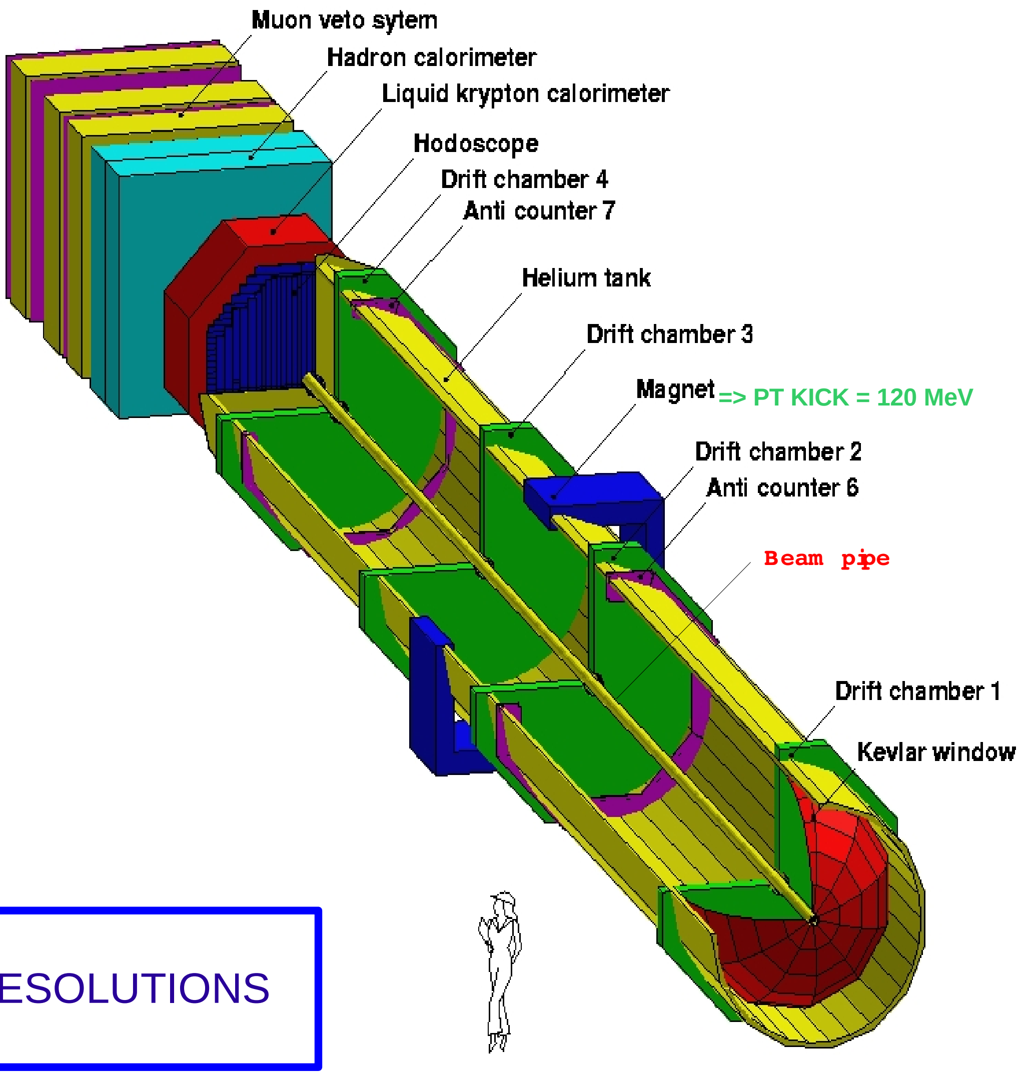
- Cleaning
- Beam spectrometer (resolution 0.7 %)

The NA48 central detector

Main detector components:

- Magnetic spectrometer (4 DCHs):
  redundancy ⇒ high efficiency;
  $\Delta p/p = 1.0\% + 0.044\% * p$ [GeV/c]
- Hodoscope
  fast trigger;
  precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogenious;
  $\Delta E/E = 3.2\%/\sqrt{E} + 9\%/E + 0.42\%$ [GeV].
- Hadron calorimeter, muon veto counters, photon vetoes.

HIGH RATES, HIGH RESOLUTIONS



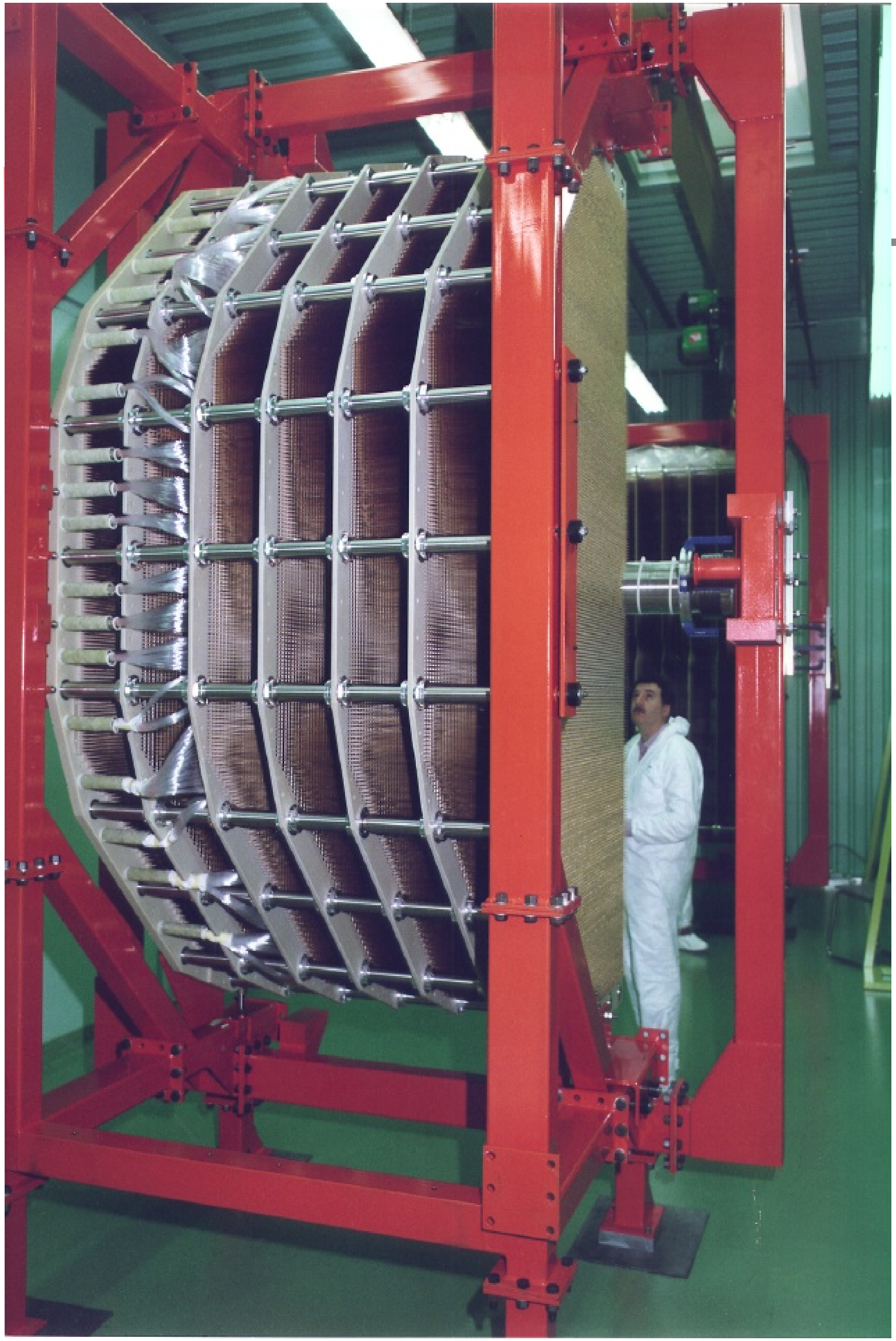
NA48/2 Two level Trigger

- 3-track events (topology)
  - $K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$
  - $K^{\pm} \rightarrow e^{\pm}\nu\pi^{+}\pi^{-}$
  - Dalitz decays, $K \rightarrow ll l\nu$

L1 = fast hodoscope + LKR info
L2 = hardware coordinate builder
+
farm of microprocessors for

on-line reconstruction of kinematics

- 1-track events with $(P_K-P_\pi)^2 > m_{\pi^0}^2$ (missing mass) + photons
  - $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$
  - $K^{\pm} \rightarrow e^{\pm}\nu\pi^{0}\pi^{0}$
  - $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\gamma$, $K^{\pm} \rightarrow \pi^{\pm}\gamma\gamma$
  - Rejected dominant mode $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}$
- Minimum Bias (control) triggers (downscaling factor D~100)
  - Trigger efficiencies
  - Many 1-track channels: Ke2(γ), Ke3(γ), ... of physical interest



Liquid Krypton electromagnetic calorimeter

13248 projective cells, 2 x 2 cm²

Energy resolution:

$$\frac{\sigma(E)}{E} = \frac{0.032}{\sqrt{E}} \oplus \frac{0.09}{E} \oplus 0.0042$$

$\sigma(E) \approx 142$ MeV for E = 10 GeV

Space resolution:

$$\sigma_x = \sigma_y = \frac{0.42}{\sqrt{E}} \oplus 0.06\,\mathrm{cm}$$

$\sigma_x = \sigma_y \approx 1.5$ mm for E = 10 GeV

Time resolution better than 500ps

Non linearity < 0.1%

Very stable over 9 years

The η mass case

Intermezzo: back to the
first NA48 phase with neutral K beams
to illustrate Lkr performances

The η mass by NA48

Special running conditions: K_L and η beams

↳ analysis of $K_L \rightarrow \pi^o\pi^o\pi^o$ and $\eta \rightarrow \pi^o\pi^o\pi^o$

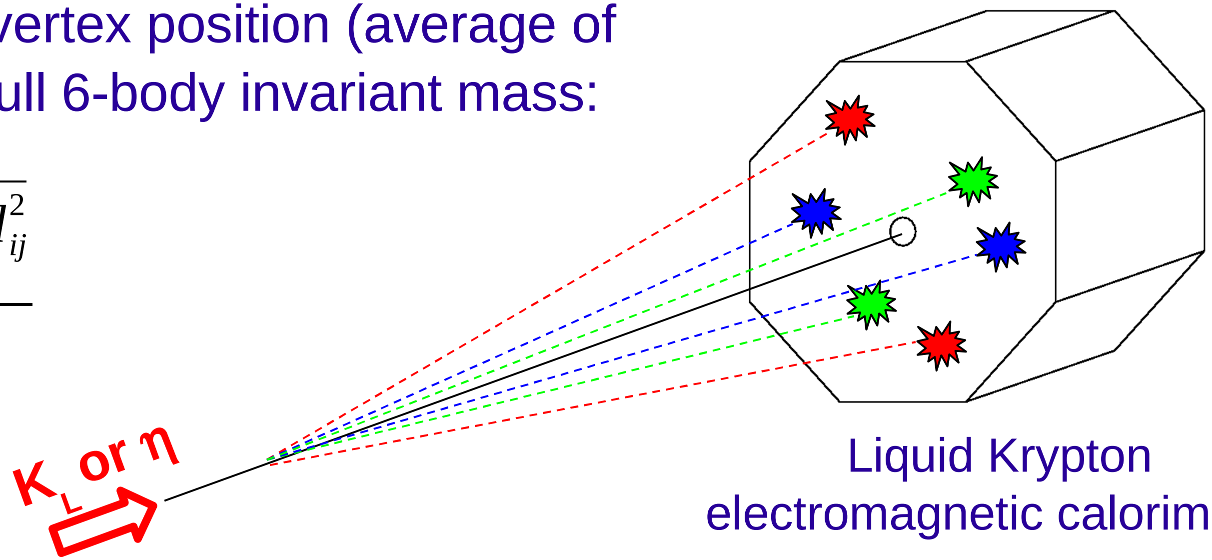
For each photon pair (*i*,*k*) reconstruct common vertex along beam axis under the assumption of $\pi^\circ \rightarrow \gamma\gamma$ decay (mass constraint)

$$m_{\pi_o}^2 = 2E_1E_2(1-\cos\theta) \approx E_1E_2\theta^2 = \frac{E_1E_2d_{12}^2}{z_{12}^2} \quad \longrightarrow \quad z_{12} = \frac{\sqrt{E_1E_2}\,d_{12}}{m_{\pi_o}}$$

From the 3 π°'s get the vertex position (average of the 3 z) and obtain the full 6-body invariant mass:

$$M = \frac{\sqrt{\sum_{i,j=1,6;i<j} E_iE_jd_{ij}^2}}{z_{average}}$$

- Excellent resolution
- Unbiased Mass



Liquid Krypton electromagnetic calorimeter

The η mass by NA48

NA48: $M_\eta = (547.843 \pm \underset{\text{Stat}}{0.030} \pm \underset{\text{Syst}}{0.041})\ \mathrm{MeV}/c^2$ PLB 533 (2002) 196

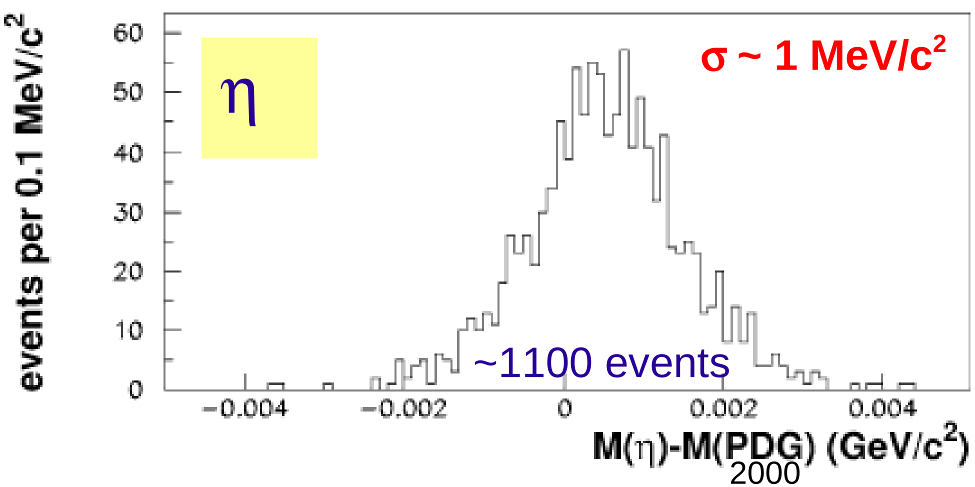
The $3\pi^o$ mode is bakground free

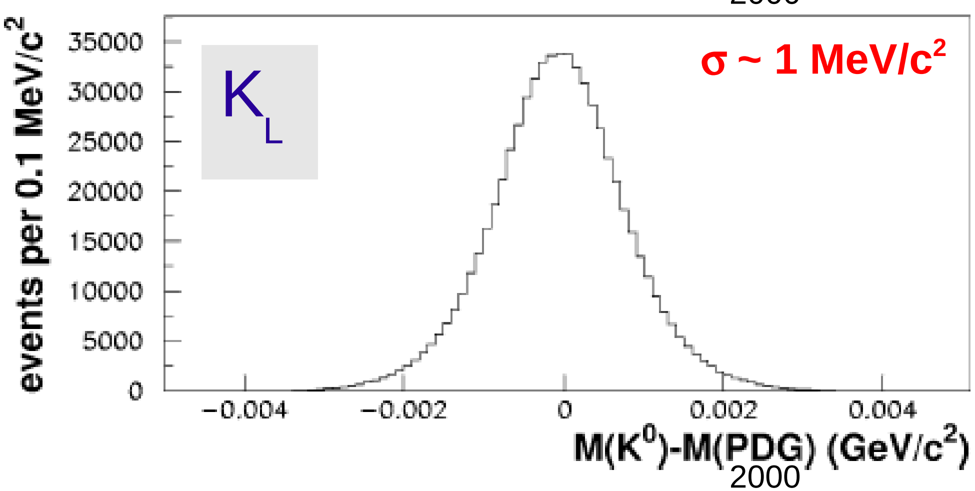
Thanks to the π^o constraint the resolution in M is better than $1\mathrm{MeV}/c^2$

The method is applied also to K_L whose huge statistics allows systematics cross checks.

No sensitivity to absolute E scale: the measured quantity is the ratio $M\eta$ / $M\pi^o$

Main residual systematics from:
- non linearities
- shower overlap





The η mass by GEM

GEM: $M_\eta = (547.311 \pm \underset{\text{Stat}}{0.028} \pm \underset{\text{Syst}}{0.032})\ \mathrm{MeV}/c^2$ PLB 619 (2005) 281

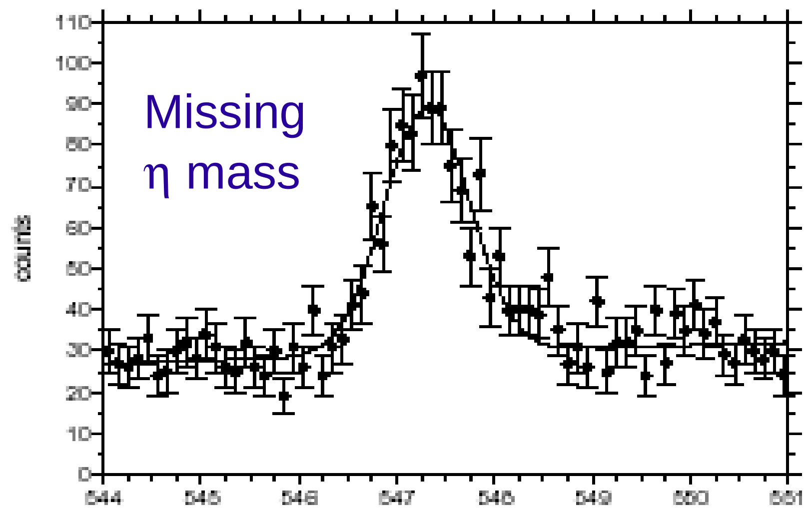
Measurement method:

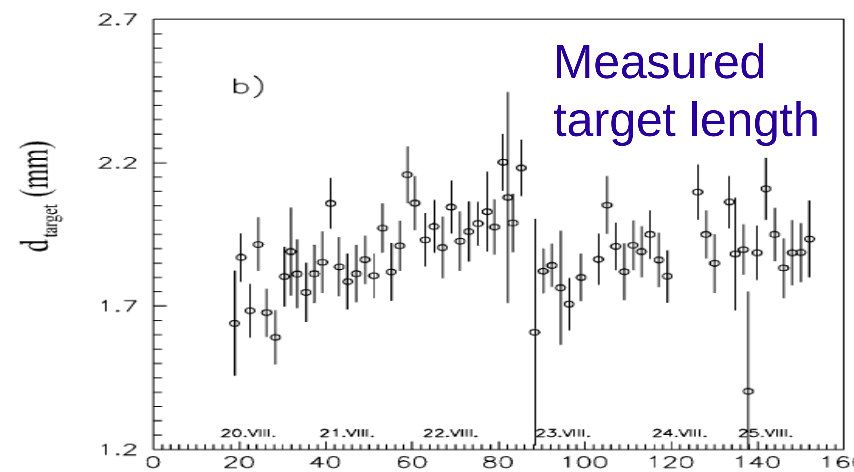
High resolution spectrometer

Missing mass peak on a (varying) backgroung in reaction $p + d \rightarrow {}^3He + X$

Calibration of spectrometer and measurement of the beam momentum by series of measurements on different nuclear reactions

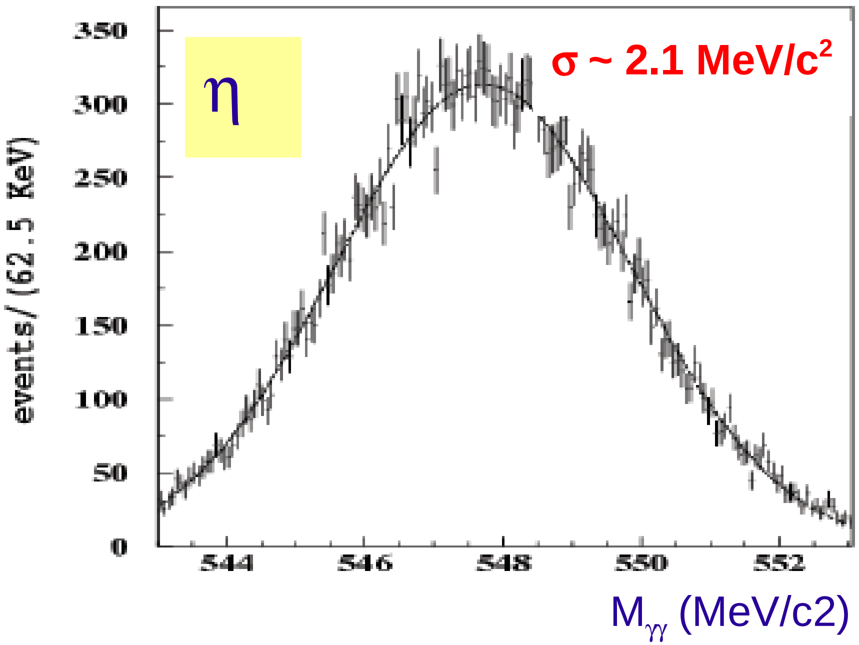
Systematics:
very accurate knowledge of
- beam momentum
- energy released (dE/dx) in the liquid deuterium target
- energy released (dE/dx) in the spectrometer





The η mass by KLOE

KLOE: $M_\eta = (547.822 \pm 0.005_{\text{Stat}} \pm 0.069_{\text{Syst}})$ MeV/c^2 PRELIMINARY 2005



Measurement:

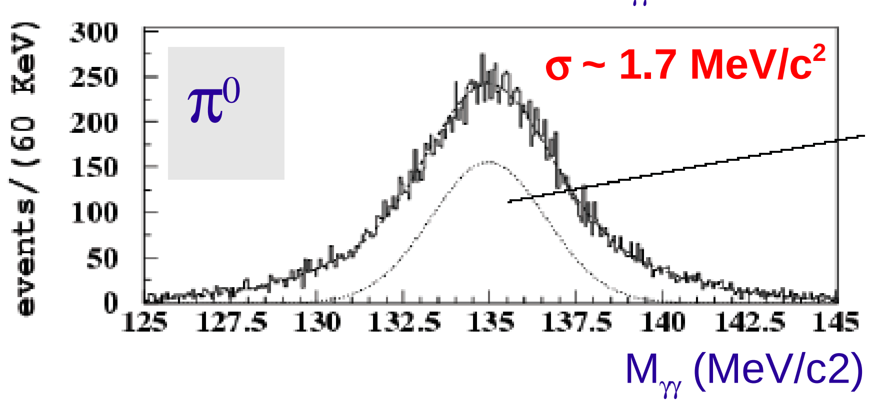
$$\phi \to \eta\gamma, \quad \eta \to \gamma\gamma$$

Cross check:

$$\phi \to \pi^0\gamma, \quad \pi^0 \to \gamma\gamma$$

Kinematic fit applied on $\phi \to \gamma\gamma\gamma$ events ($\mathbf{p}$, E_{tot}, cluster times)

η and π^0 selected by looking at different Dalitz plot regions



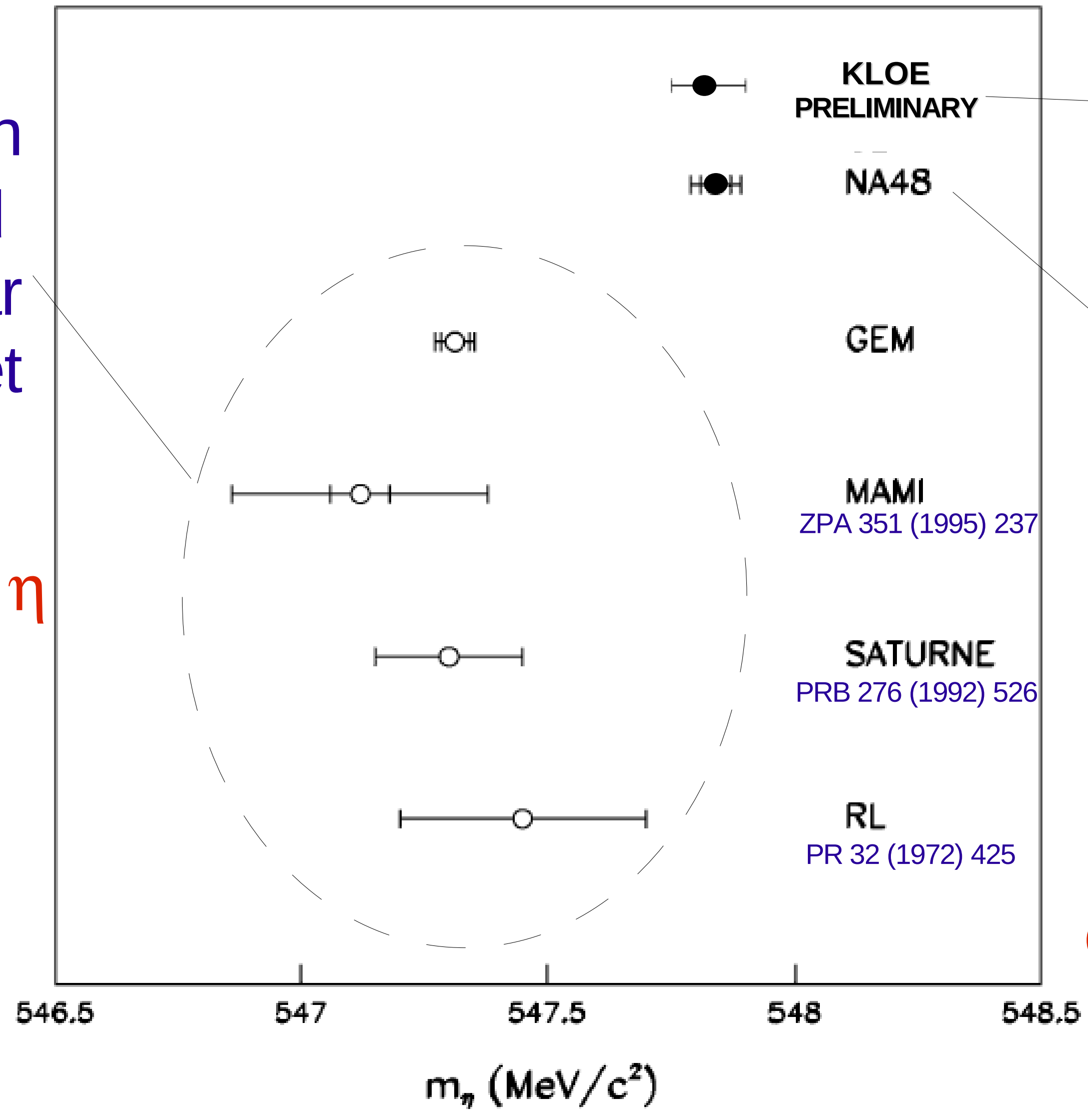
The mass measurement is almost independent from the energy of the clusters, it is dominated by the cluster positions.

Main residual systematics
- $\sqrt{s}$
- vertex position

Status of η measurement

η production at threshold on nuclear target

Idirect measurent of η



$\phi \to \eta\gamma$

$\hookrightarrow \gamma\gamma$

In vacuum

$\eta \to 3\pi^o$

Direct measurement of η

KLOE and NA48 compatible at 0.24 σ

Study of the Dalitz plot for the $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ and $K^{\pm} \to \pi^{\pm}\pi^{\circ}\pi^{\circ}$ decays

CPV slope asymmetries

$K^{\pm} \to 3\pi$ decays in Dalitz plot

"charged" BR($K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$) = 5.57%

"neutral" BR($K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$) = 1.73%

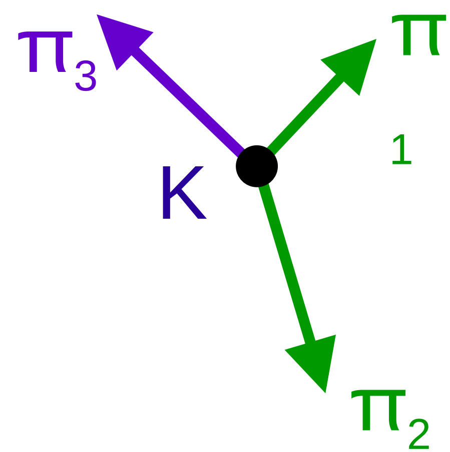
Kinematics:

$$s_i = (P_K - P_{\pi i})^2 \quad i=1,2,3 \text{ (3=odd } \pi)$$

$$s_0 = (s_1+s_2+s_3)/3$$

$$u = (s_3 - s_0)/m_\pi^2 = 2m_K (m_K/3 - E^*_{odd})/m_\pi^2$$

$$v = (s_2 - s_1)/m_\pi^2 = 2m_K (E^*_1 - E^*_2)/m_\pi^2$$



Matrix element:

$$|M(u,v)|^2 \sim 1 + gu + hu^2 + kv^2 + \ldots$$

$K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ $g = -0.2154 \pm 0.0035$

$K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ $g = 0.652 \pm 0.031$

$|h|, |k| \ll |g|$

Just a polynomial expansion

Phase space is small → the expansion in u,v converges rapidly

Linear slope g dominates over quadratic terms h,k

Direct CP violation in $K^{\pm} \rightarrow 3\pi$

by Comparing the SHAPE of distributions in the Dalitz plot

mix with $\Delta I=3/2$

$$A(K \rightarrow 3\pi) = a + b \times u$$

Two $\Delta I=½$ amplitudes with different weak and strong phases → interference effects on

$$|A(K \rightarrow 3\pi)|^2 \propto 1 + g \times u$$

Search DCPV by comparing the linear slopes of K+ and K-

$$A_g^{c,n} \stackrel{\text{def}}{=} \frac{(g_+ - g_-)}{(g_+ + g_-)} = \frac{(\Delta g)}{2g}$$

C : charged mode
N: neutral mode

If asymmetries $A_g^{C,N} \neq 0$
→direct CP violation

Direct CP violation in $K^{\pm} \rightarrow 3\pi$

Method and RECENT (2006) RESULTS

Two simultaneous K+ and K- beams with narrow band momentum overlapping both in space and time

Detect asymmetry exclusively by considering the slopes of ratios of u distributions of K+ and K-

Equalize averaged K+ and K- acceptances by alternating frequently the polarities of the relevant magnets along the particle paths

$K^{\pm} \rightarrow \pi^{\pm}\pi^{+}\pi^{-}$ mode full statistics:
3.1×10^{9} decays

Preliminary result on
full statistics (2003 + 2004):

$$A^{C}_{g} = (-1.3 \pm 2.3) \cdot 10^{-4}$$

result on 2003 data published in
PLB 634 (2006) 474

$K^{\pm} \rightarrow \pi^{\pm}\pi^{o}\pi^{o}$ mode full statistics:
0.11×10^{9} decays

Result on 2003 data
(50% full statistics):

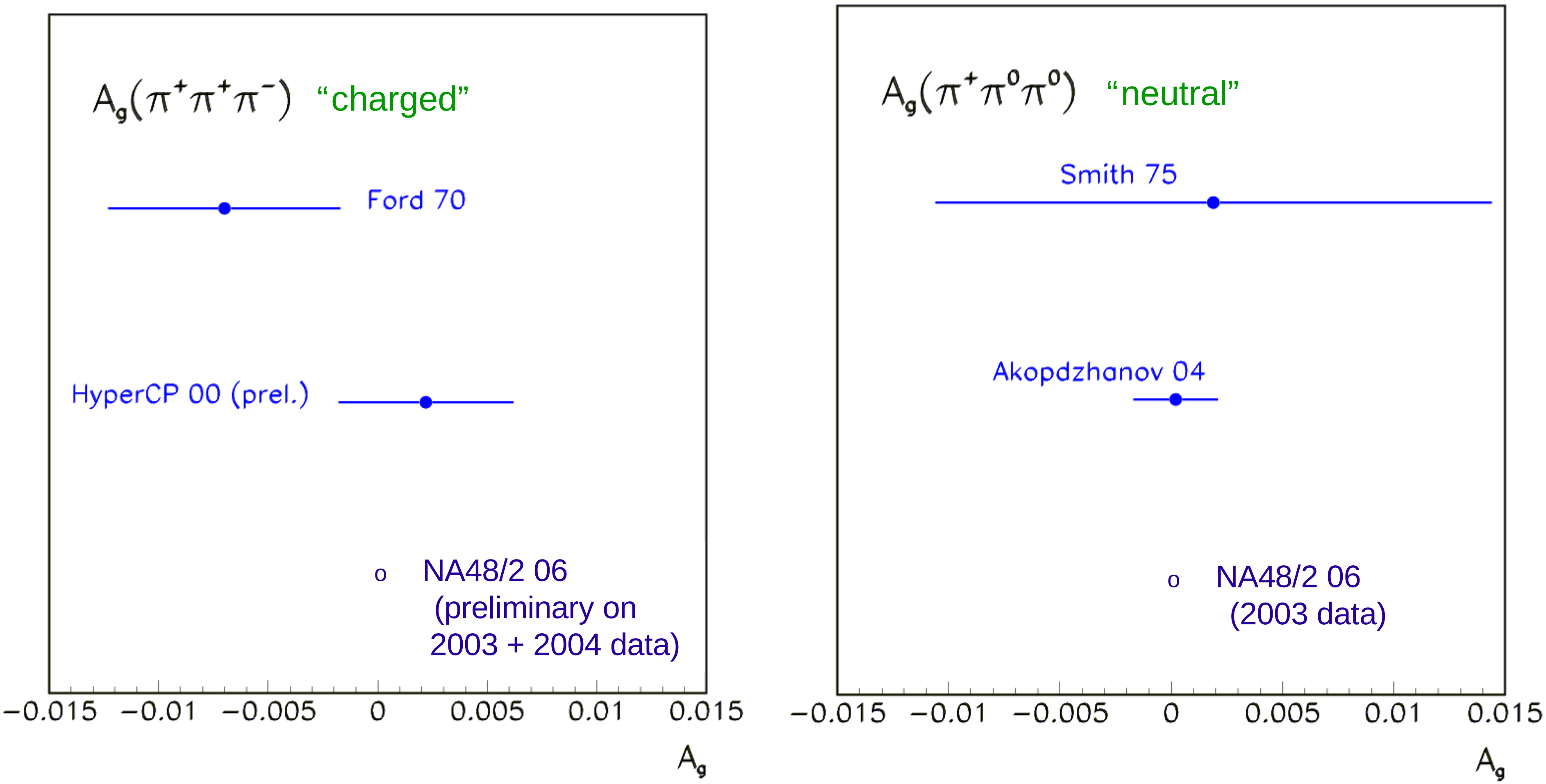
$$A^{N}_{g} = (1.8 \pm 2.6) \cdot 10^{-4}$$

PLB 638 (2006) 22

Errors dominated by the statistics

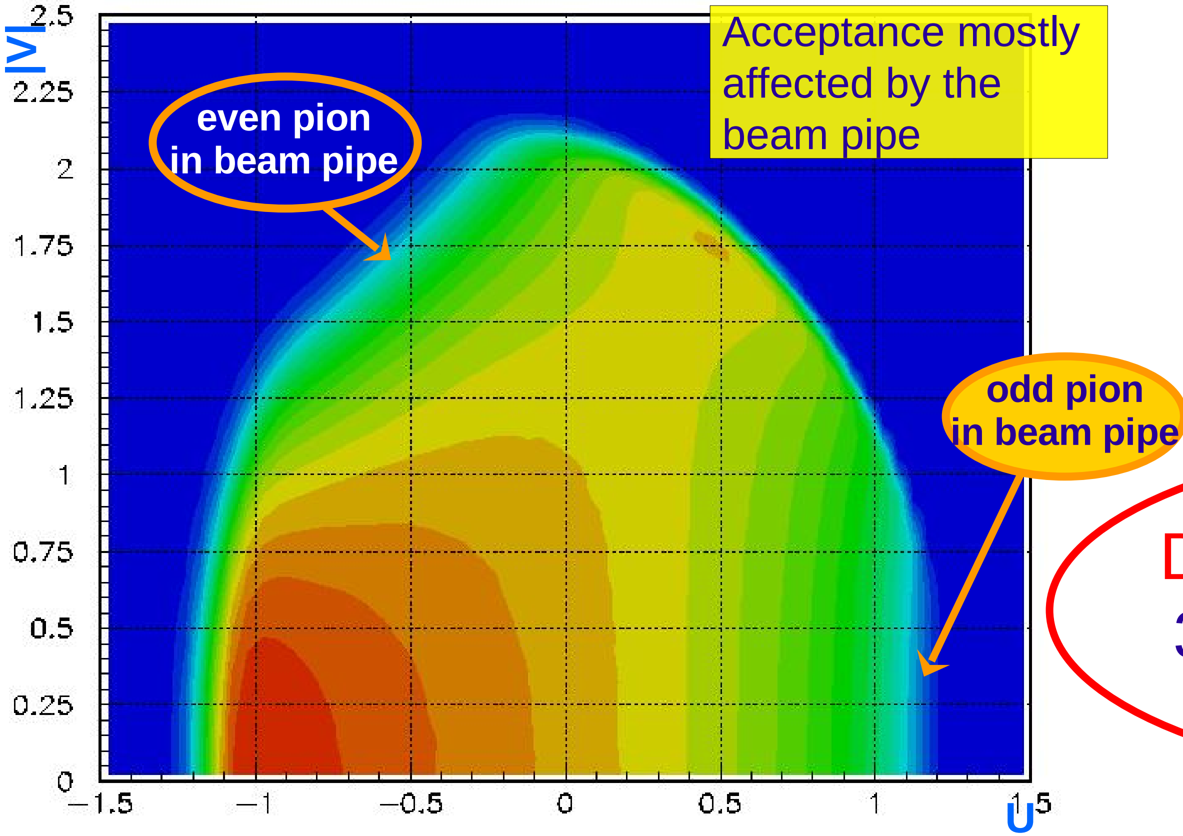
Asymmetry measurements

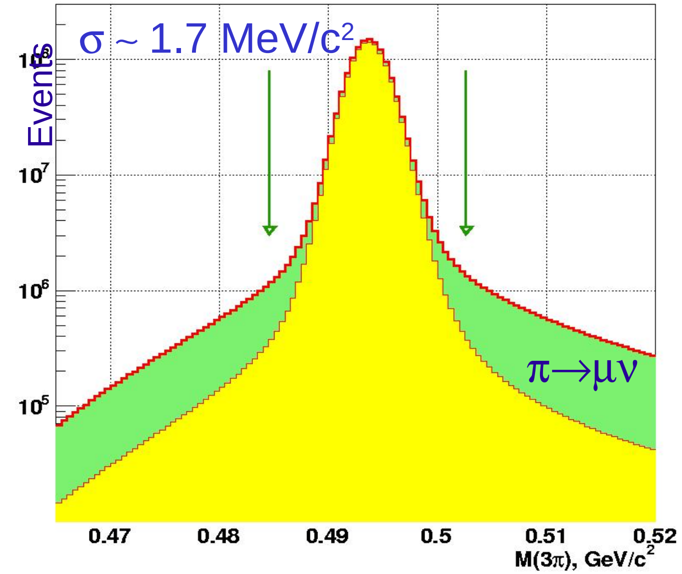
NO DCPV FOUND ... but with a sensitivity x10 better than the previous experiments



Dalitz plot of $K^{\pm}\rightarrow\pi^{\pm}\pi^{+}\pi^{-}$

Simple selection based only on the spectrometer
Negligible background





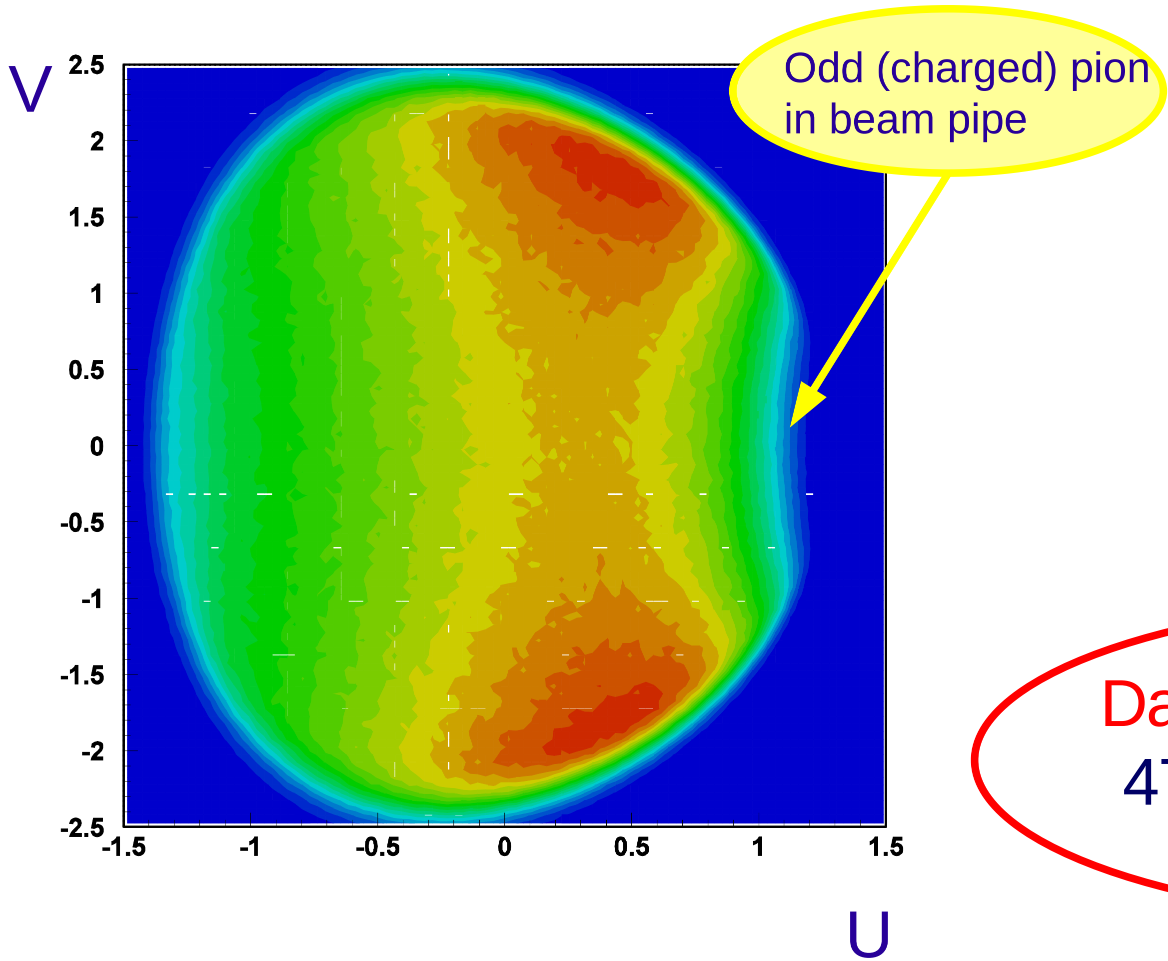
$\pi^+\pi^-\pi^+$ invariant mass (GeV/c^2)

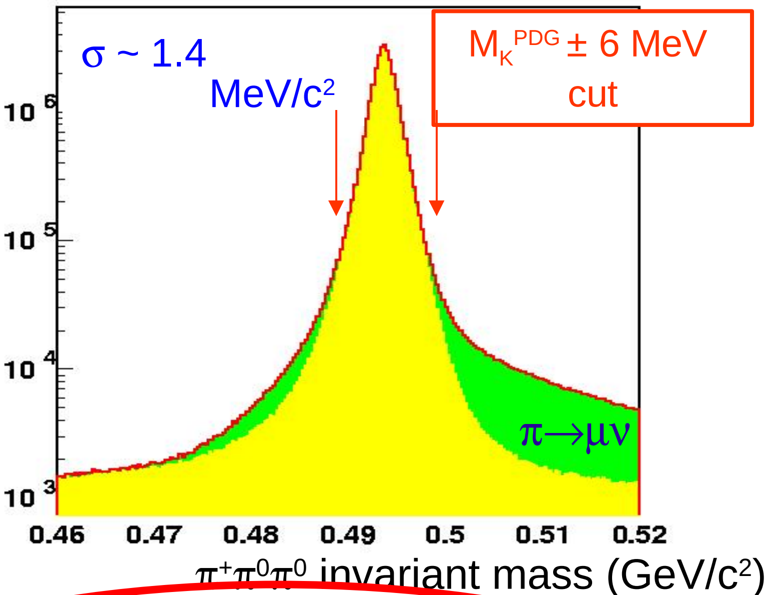
Data-taking 2003 + 2004:
3.1×10^9 events selected
($K^+ / K^- \sim 1.8$)

Dalitz plot of $K^{\pm} \rightarrow \pi^{\pm}\pi^{o}\pi^{o}$

The $\pi^{o}\pi^{o}$ invariant mass can be reconstructed independently from

- the information of LKr Calorimeter ONLY (very good resolution at low u)
- the spectrometer + KABES information ONLY (good resolution at high u)





Data-taking 2003 + 2004:

47 x 10^6 events selected

($K^+ / K^- \sim 1.8$)

Observation of a “cusp” in the decay $K^{\pm} \rightarrow \pi^{\pm}\pi^{0}\pi^{0}$

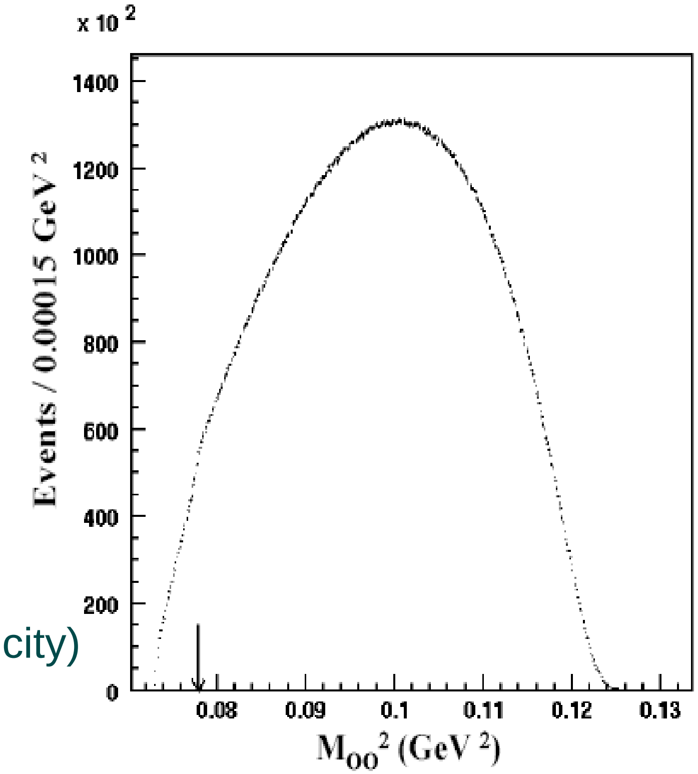
Observation of a "cusp" in $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$ ... a bit of history

1) I.Mannelli: search for pionium atoms in the $K^\pm \to \pi^\pm\pi^o\pi^o$ channel as a resonance at $\pi^o\pi^o$ invariant mass $M^2_{oo} = 4\ m^2_+$ (threshold for $\pi^\pm\pi^\pm$ production) by exploiting the very good LKr calorimeter resolution
   + proper M_{oo} reconstruction strategy
   + high statistics

2) Data reveal a structure in the $M^2_{oo} = 4\ m^2_+$ region.
   The structure has a physical origin: whatever torture is made to data it keeps sitting there

3) N.Cabibbo realized that it was a "clean and beautiful" example of a general cusp-like behaviour of cross sections next to threshold for new channels (Wigner 1948)

   He provided a method based on first principles (unitarity, analiticity) for extracting information on strong interaction at low energy.



- First observation of $\pi\pi$ scattering effects in the Dalitz plot
- Precise and model independent measurement of a_0-a_2

Event selection

Quite simple selection criteria. No relevant background !

- At least one charged particle with momentum $p < 5\text{GeV/c}$
- At least 4 photons with $E_\gamma > 3\text{GeV}$ detected in the Lkr Calorimeter
- Geometrical cuts to eliminate detector edge effects (near beam tube and near outer edges of drift chambers and LKr calorimeter)
- Distance between photons at LKr >10 cm
- Distance between photons and charged particle at LKr >15 cm

Good events selected: 23×10^6 $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$

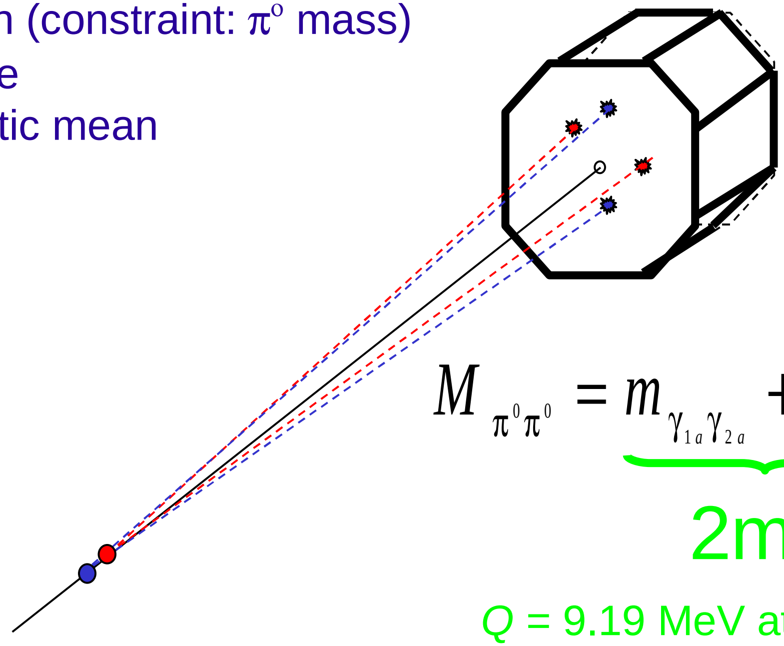
NOTE: these are only 20% of full statistics good events,
ie the final analysis will be performed on a x5 bigger sample !

Invariant mass of the $\pi^o\pi^o$ pair

1) for each photon pair obtain a vertex position (constraint: π^o mass)
2) choose the two pairs with minimum distance
3) measure the decay vertex from the arithmetic mean of the two Z

$$z_a = \frac{\sqrt{E_1 E_2}\, d_{12}}{m_{\pi_o}} \qquad z_b = \frac{\sqrt{E_3 E_4}\, d_{34}}{m_{\pi_o}}$$

$$z_{\pi^0\pi^0} = \frac{z_a + z_b}{2} \qquad \Delta z = \frac{z_a - z_b}{2}$$



$$M_{\pi^0\pi^0} = \underbrace{m_{\gamma_{1a}\gamma_{2a}} + m_{\gamma_{1b}\gamma_{2b}}}_{2m_{\pi^o}} + Q$$

Q = 9.19 MeV at $\pi^+\pi^-$ threshold

4) invariant mass of the 4 photons is the invariant mass of the $\pi^o\pi^o$ pair

$$\frac{M^2_{oo}}{4m^2_{\pi_o}} = \frac{\sum_{i<j=1,4;\,i<j} E_i E_j d^2_{ij}}{\left(\sqrt{E_1 E_2}\, d_{12} + \sqrt{E_3 E_4}\, d_{34}\right)^2} > 1$$

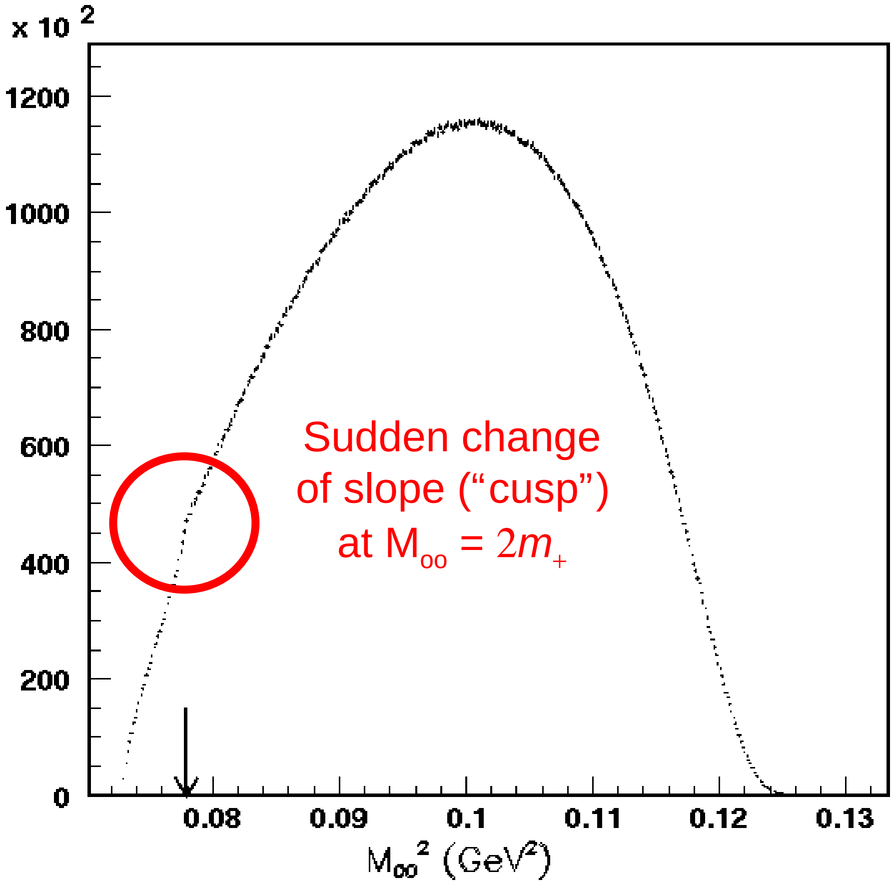
- **Kinematic limit respected by construction**
- **Least biased M_{oo} estimatior : Cancellation of systematic effects (E scale, ...)**
- **Best resolution in the cusp region**

Observation of a "cusp" in $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$

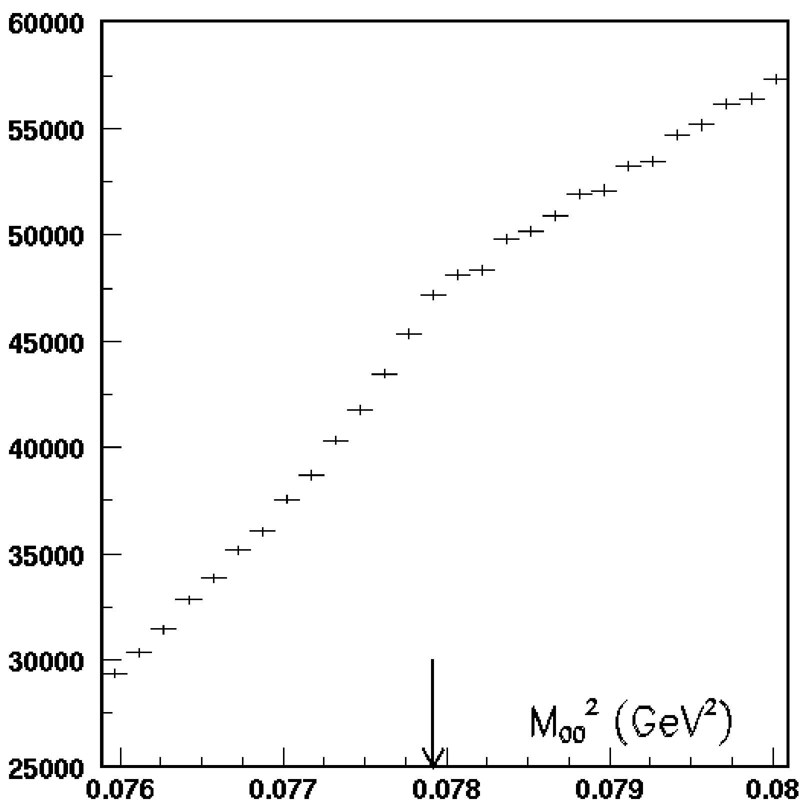
$S3 \equiv M_{oo}^2 \equiv \pi^{\circ}\pi^{\circ}$ invariant mass square

Arrow: $M_{oo} = 2m_+$; m_+ : π^+ mass

Experimental M_{oo}^2 distribution
for 23 x 10^6 $K^{\pm} \rightarrow \pi^{\pm}\pi^{\circ}\pi^{\circ}$ decays



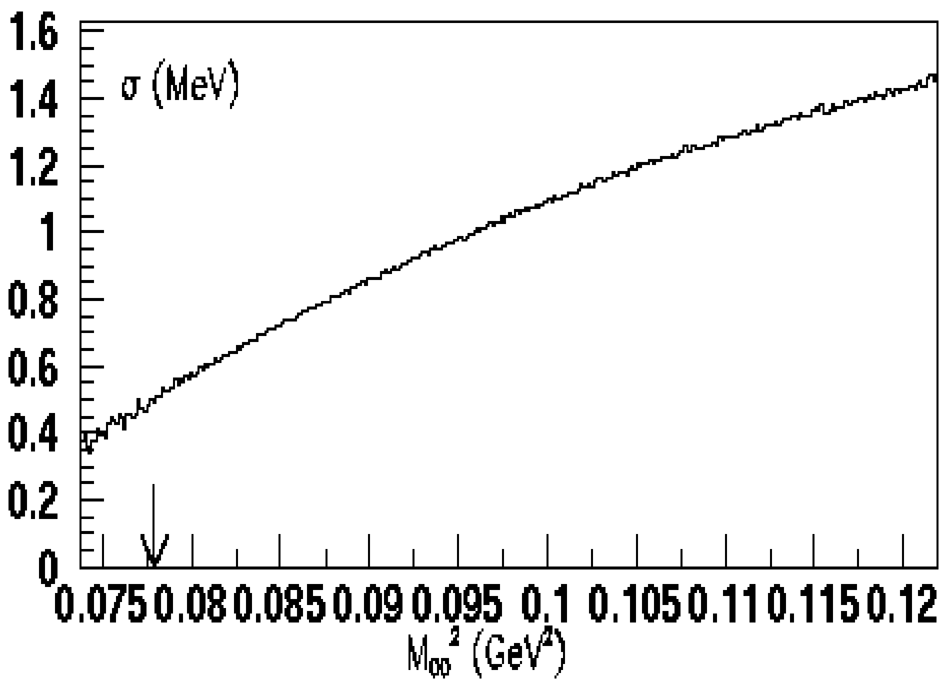
Experimental M_{oo}^2 distribution
"Zoom" on the cusp region



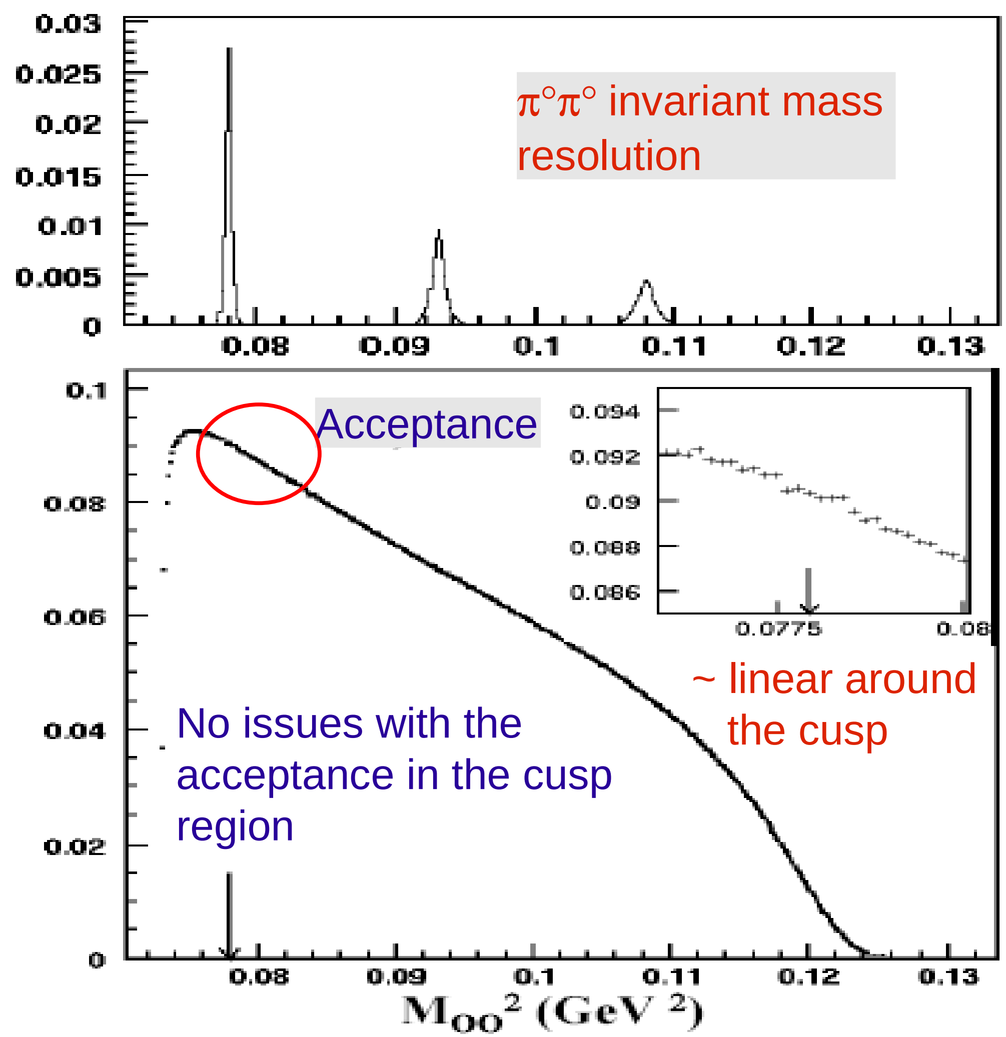
Physical cusp or instrumental effect ? Resolution and acceptance: simulation

$\pi^\circ\pi^\circ$ invariant mass resolution (σ) versus M_{oo}^2

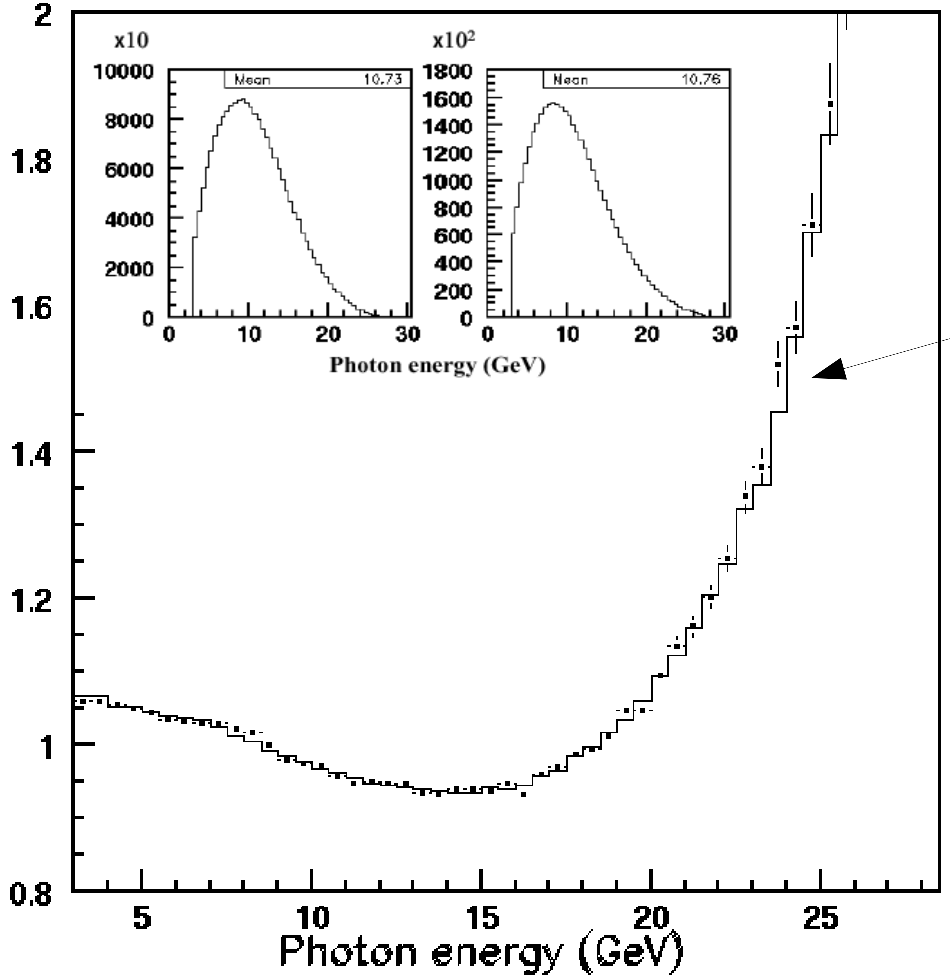
(from MonteCarlo simulation)



$\sigma \approx 0.5$ MeV at $M_{oo} = 2m_+$



Physical cusp or instrumental effect ? photon energies below/above threshold

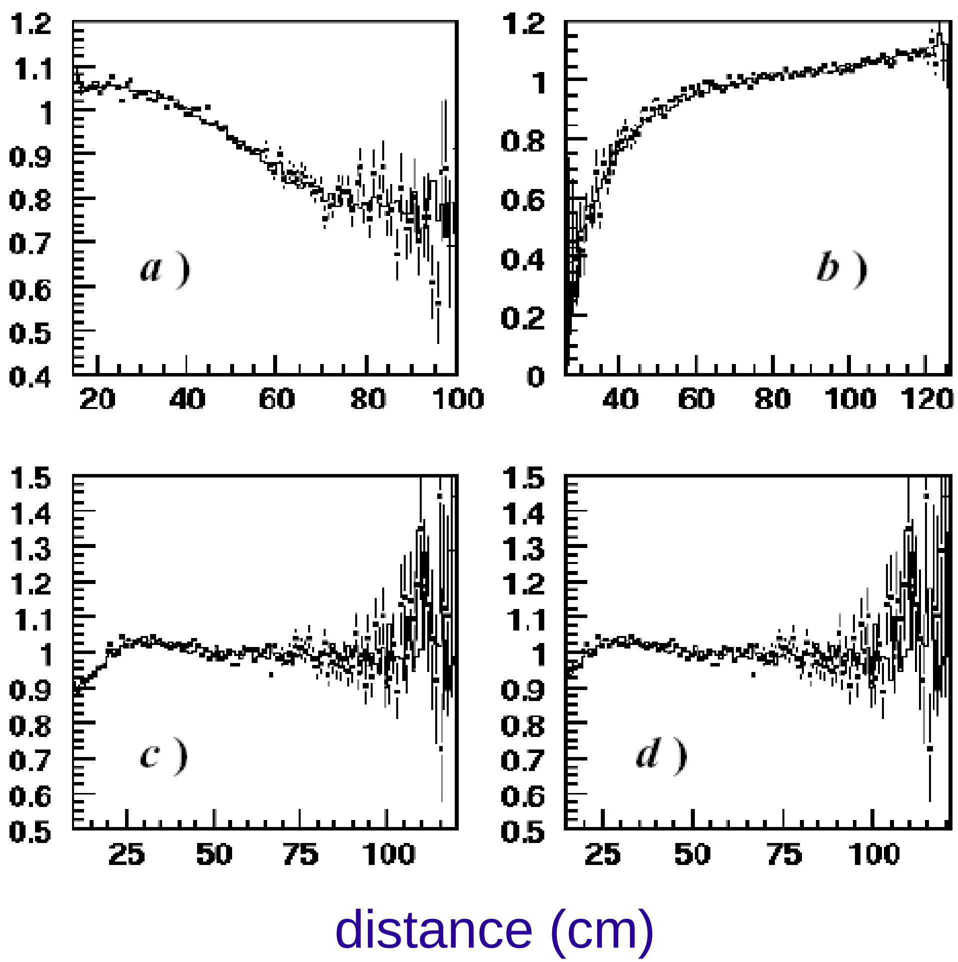


Study data in two small M_{oo} intervals just below and above $2m_+$

Ratio data above/below
VS same ratio in MC (solid line)

VARIATION OF SHAPE OF PHOTON ENERGY DISTRIBUTION ACROSS "CUSP" AGREES WITH MONTECARLO PREDICTION WITHOUT CUSP

Physical cusp or instrumental effect ? Event topology below/above threshold



GOOD AGREEMENT BETWEEN DATA AND MC ALSO FOR DISTRIBUTIONS OF VARIOUS PHOTON DISTANCES MEASURED AT LKr

Good agreement for both:

1) data above/below

2) data/MC Which is important because the extraction of scattering lengths is MC dependent

a) min. γ distance from axis ; b) max. γ distance from axis;

c) min. $\gamma - \gamma$ distance; d) min. $\gamma - \pi^{\pm}$ distance

Theoretical interpretation

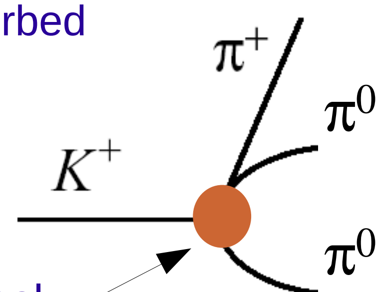
Ingredients to extract the scattering lenghts

How does the cusp arises: FSI effects in $K^{\pm} \to 3\pi$ decays

The singularity in $\pi^{\circ}\pi^{\circ}$ invariant mass spectrum at $\pi^{+}\pi^{+}$ threshold is mainly caused by the (destructive) interference of the charge exchange $\pi^{+}\pi^{-} \to \pi^{\circ}\pi^{\circ}$ re-scattering from $K^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}$ with the direct emission $K^{\pm} \to \pi^{\pm}\pi^{o}\pi^{o}$

The effect of interference is present (at first order) below the threshold and not above threshold

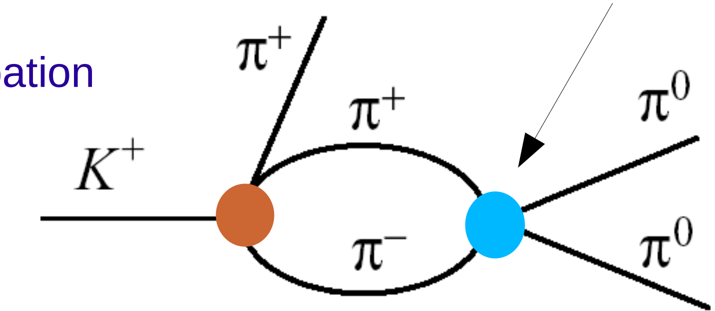
Unperturbed



Real amplitude

+

Perturbation



Real 1-loop coupling

Amplitude ~ i $(M^2_{oo} - 4m^2_{+})^{1/2}$

Imaginary above threshold

Real amplitude below threshold

I. Cabibbo rescattering model [O(a) terms]

Cabibbo PRL **93** (2004) 121801

Amplitude of direct emission $K^{\pm}\to\pi^{\pm}\pi^{o}\pi^{o}$
large, real>0

$$M_0 = A^0(1+g^0(s_3-s_0)/2m^2_{\pi+})$$

$$M_{K^+\to\pi^+\pi^o\pi^o} = M_0 + M_1$$

$$M_+^{thr} = A^0(1+g^+(M_K^2-9m^2_{\pi+})/12m^2_{\pi+})$$

Amplitude of decay $K^{\pm}\to\pi^{\pm}\pi^{+}\pi^{-}$
at threshold

Contribution from the re-scattering channel:

- AMPLITUDE KNOWN ABOVE THRESHOLD
- EXTENDED BY ANLICITY BELOW THR.

Above threshold
Imaginary for $M_{oo}^2 > 4m_+^2$
→no interference

$$s_3 > 4m^2_{\pi+}$$

$$M_1 = i\,2\frac{(a_0-a_2)m_{\pi+}}{3}M_+^{thr}\sqrt{\frac{s_3-4m^2_{\pi+}}{s_3}}$$

$$|M^2| = (M_0)^2 + |M_1{}^2|$$

Below threshold
Real<0 for $M_{oo}^2 < 4m_+^2$
→ interfere destructively

$$s_3 < 4m^2_{\pi+}$$

$$M_1 = -2\frac{(a_0-a_2)m_{\pi+}}{3}M_+^{thr}\sqrt{\frac{4m^2_{\pi+}-s_3}{s_3}}$$

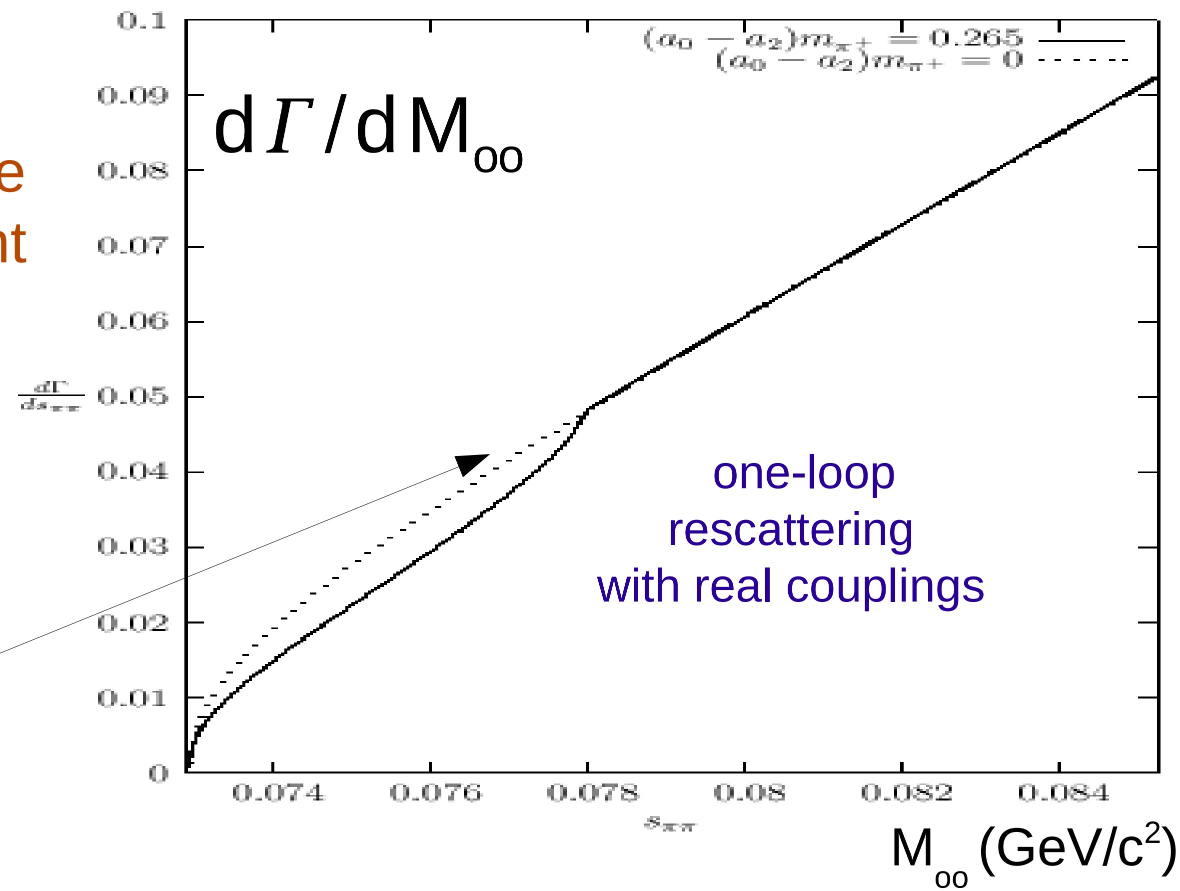
$$|M^2| = (M_0)^2 + (M_1)^2 + 2M_0M_1$$

I. Cabibbo rescattering model

The cusp is proportional to the S-wave $\pi^+\pi^-$ charge exchange scattering length $a_x = (a_2 - a_0)/3$

(valid in the limit of exact isospin)

Experimentally we have to extract the coefficient in front of the $(M^2_{oo} - 4m^2_{\pi})^{1/2}$ term



Square root singularity $(M^2_{oo} - 4m^2_{\pi})^{1/2}$

II. Cabibbo-Isidori approach [O(a^2) terms]

Cabibbo, Isidori JHEP 0503 (2005) 21

- In order to deal with an experimental measurement of a_0-a_2 at few % by NA48 the theory has to be good to few 10^{-3} (since the cusp is a 10% effect)

→ higher order rescattering effects, and radiative corrections have to be included.

"It is possible to set up a systematic computation of the singular parts of an amplitude in terms of its non-singular parts. This leads to a development of the $K \to 3\pi$ amplitudes in powers of the $\pi\pi$ scattering lengths a0,a2. The development is useful because the scattering lengths are small, which is a general consequence of the fact that pions act as pseudo Goldstone bosons for chiral symmetry breaking." (N.Cabibbo)

- The development to the second order in powers of a_0,a_2 reveals a second cusp above the $\pi^+\pi^-$ threshold.
  Theoretical uncertainty level up to now ~ 5%

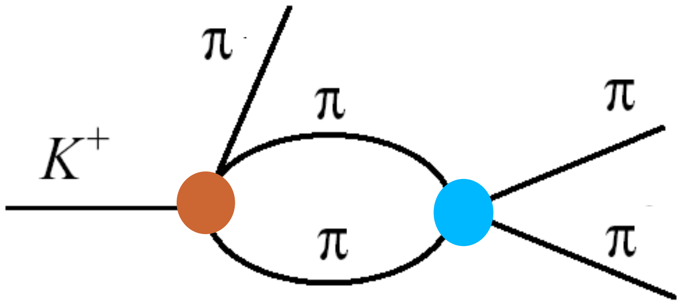
- But O(a_i^3) + radiative corrections can be computed:
  the theoretical prediction can reach the 1% uncertainty level and be almost independent of CHPT

III. Cabibbo-Isidori approach

One-loop diagrams

$\pi^+\pi^\circ \to \pi^+\pi^\circ \qquad \pi^+\pi^- \to \pi^\circ\pi^\circ$

$\pi^\circ\pi^\circ \to \pi^\circ\pi^\circ$



The discontinuities of these diagrams determine the coefficients of the perturbation terms ~ $(M^2_{oo} - 4m_+^2)^{1/2}$

in the matrix elements.

Those coefficients are calculated in powers of the scattering lengths, up to second order.

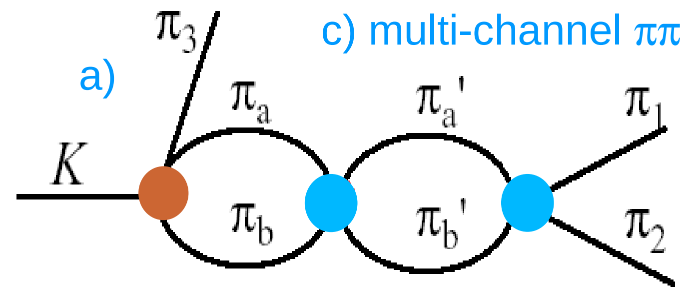
Two-loop diagrams

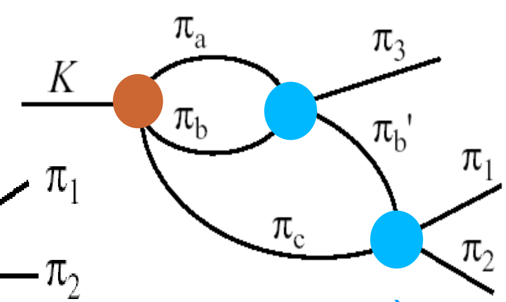
a) single channel $\pi\pi$ scattering

b) irreducible $3\pi \to 3\pi$

c) multi-channel $\pi\pi$ scattering

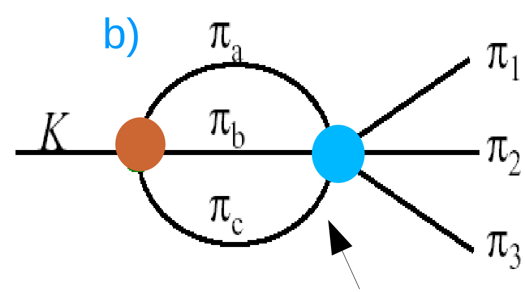
a)





c)

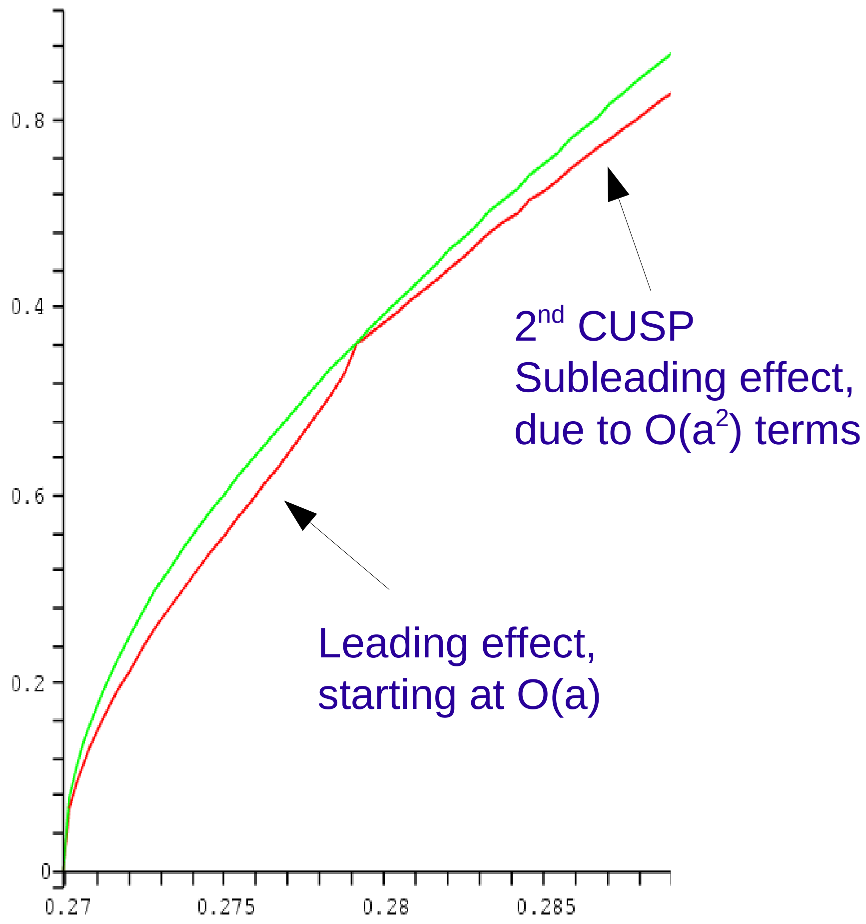
b)



The irreducible term b) is the only one that needs CHPT. It results negligible.

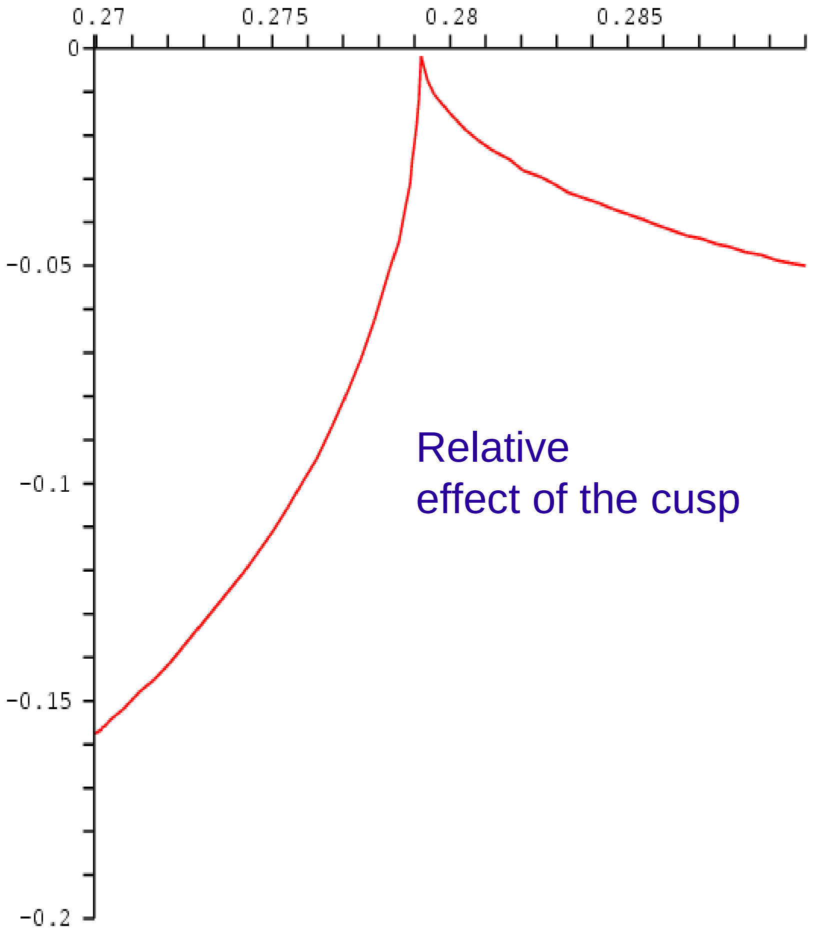
II. Cabibbo Isidori approach

$d\Gamma / dM_{oo}$



M_{oo} (GeV/c^2)

M_{oo} (GeV/c^2)



II. Cabibbo Isidori approach

The unperturbed matrix elements for $K \rightarrow 3\pi$ include quadratic terms

Two loop approximation produces much longer formulae:
they involve not only s_3 and a_x but also

- s_1 and s_2 (invariant mass $M_{\pi+\pi o}$) and
- the other scattering lengths (a_{++},a_{+-},a_{+o},a_{oo}) describing all the S-wave $\pi\pi$ elastic scattering channels

Note: a_x,a_{++},a_{+-},a_{+o},a_{oo} can expressed as linear combinations of the scattering lengths a_0 and a_2 and (following J.Gasser) the isospin breaking parameter $\varepsilon = (m_+^2 - m_o^2)/m_o^2 \sim 0.065$

eg: $a_x = (1 + \varepsilon /3)\ (a_0 - a_2)/3$

III. The electromagnetic ingredients Coulomb and radiative corrections

Corrections due to virtual and real photons are missing up to now.

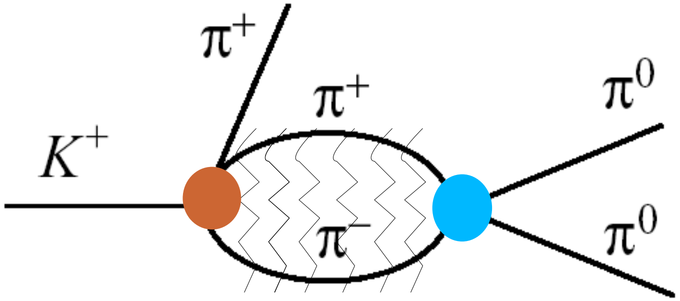
The effects are expected to be small (% level) except possibly next to the cusps, e.g. due to the presence of bound states.
For this reason we exclude few bins around the cusp when fitting the data to extract a_0-a_2

We investigate separately the presence of bound states on top of the cusp.

III. The electromagnetic ingredients
Bound states: pionium

Following Silagadze (JETP Lett. 60 (1994) 689) we expect a contribution from $\pi^+\pi^-$ bound state with dominant decay mode $\pi^\circ\pi^\circ$ proportional to

$$\frac{\Gamma(\mathrm{K}^+ \to \pi^+ + \mathit{pionium})}{\Gamma(\mathrm{K}^+ \to \pi^+\pi^o\pi^o)} \approx 2.6 \times 10^{-5}$$



(recalculated according to the latest PDG B.R.'s)

The expected contribution to the M^2_{oo} bin centered at $(2m_+)^2$ is ~2.6%

IV. Effective field theory approach

Colangelo, Gasser, Kubis and Rusetsky hep-ph/0604084

Recently CGKR calculated the K→ 3π amplitudes within a non relativistic effective Lagrangian framework, by a double expansion in a_i (scattering lengths) and e (kinetic energies) up to term a^2e^2.

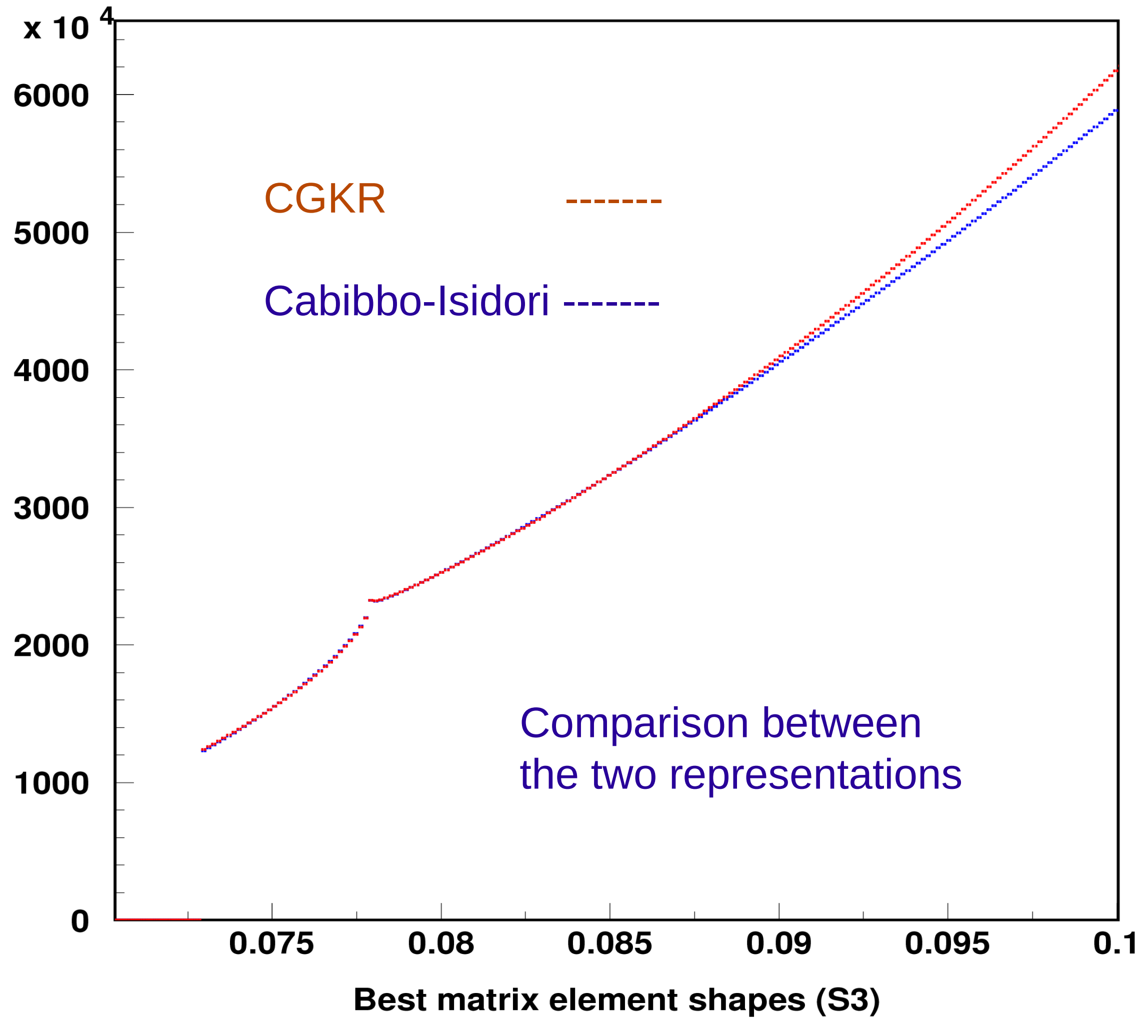
CGKR representation is valid in the whole physical region.

Experimental work is in progress for:

1) analyzing data with the CGKR representation extending the fit to a wider range

2) investigating the additional cusps which are present at the border of the Dalitz plots, also for the channel $K^+ \to \pi^+\pi^+\pi^-$

Note: at border resolution tails and quality of the simulation for acceptance are crucial.



Fits to the experimental invariant mass distribution

Fit procedure

- The matrix element for the $K^{\pm}\rightarrow\pi^{\pm}\pi^{o}\pi^{o}$ decay depends on both s3 $\equiv M_{oo}$ and s3 $\equiv M_{o+}$ but instead of fitting the 2D Dalitz plot we perform an approximate 1D fit as a function only of s3, by substituting s1 with its average for each value of s3 (<s1> = <s2> = (3s0 – s3) / 2)

- The fit has 5 free parameters:
  - * g_o, h' → linear and quardatic terms in decay amplitude
  - * $(a_0\text{-}a_2)m_+$, a_2m_+ →scattering lengths (isospin breaking effects included)
  - * N → normalization factor

- Generate theoretical s3 distribution G_i (420 bins of 0.00015 GeV^2) with MC simulation
- From MC obtain a 420 x 420 matrix T_{ik} = probability that an event generated with s3 in bin ***i*** is detected and measured in bin ***k***
  T_{ik} includes both acceptance and resolution effects
- Produce "reconstructed" s3 distribution R_k

$$\mathbf{R}_k = \sum_i \mathbf{T}_{ik}\mathbf{G}_i$$

- Fit distribution R_k to experimental S3 distribution

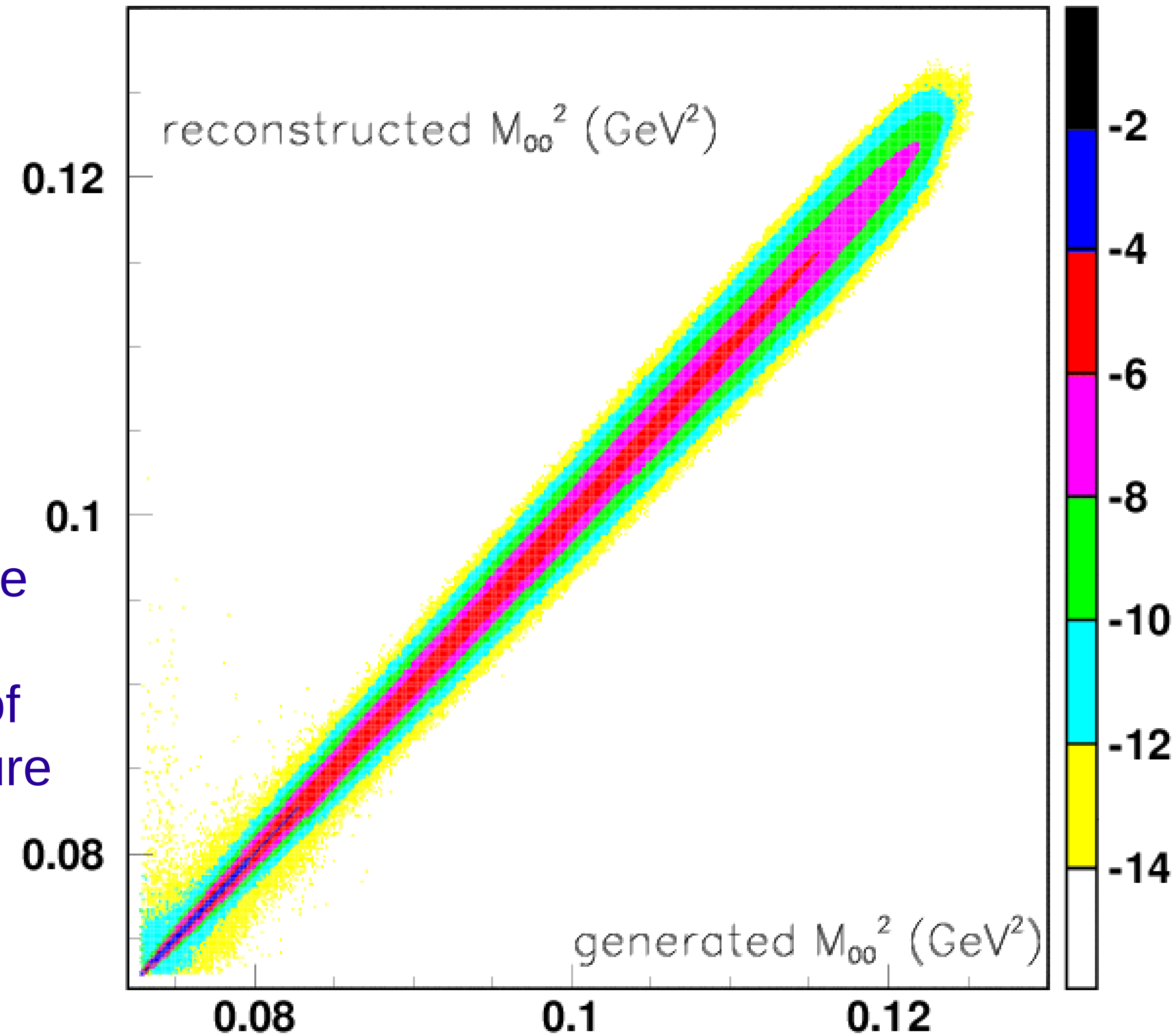
Fit procedure

$\mathrm{Log}(T_{ik})$

(from MonteCarlo simulation)

The “projection” matrix which relates the generated kinematic variable s3 to the reconstructed variable $s3_{rec}$

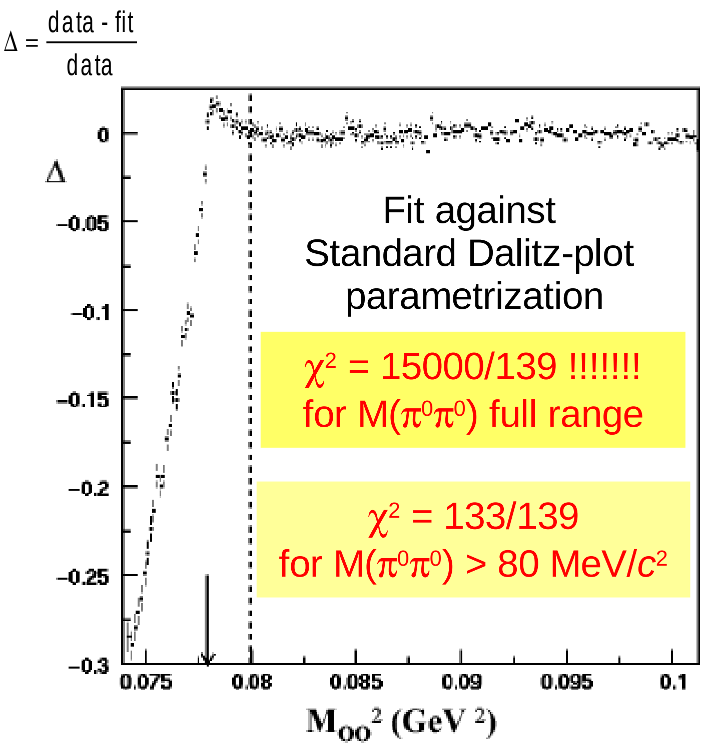
This allows a fast generation of the reconstructed s3 distribution from the generated one, for every set of parameters, during the fit procedure

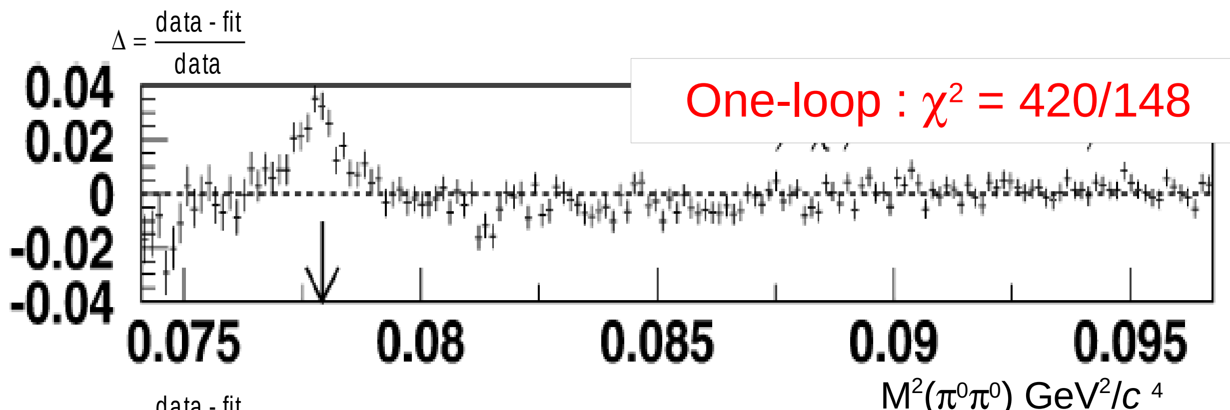


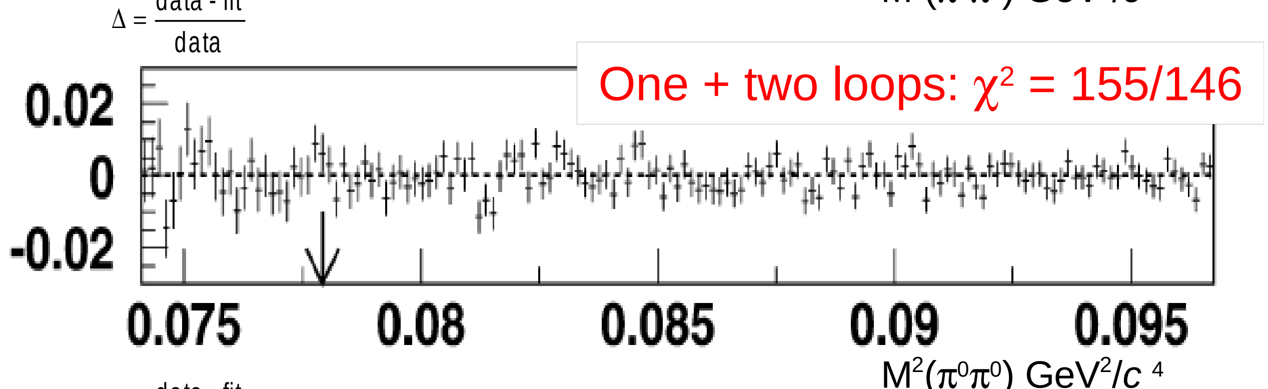
Fits data against different models

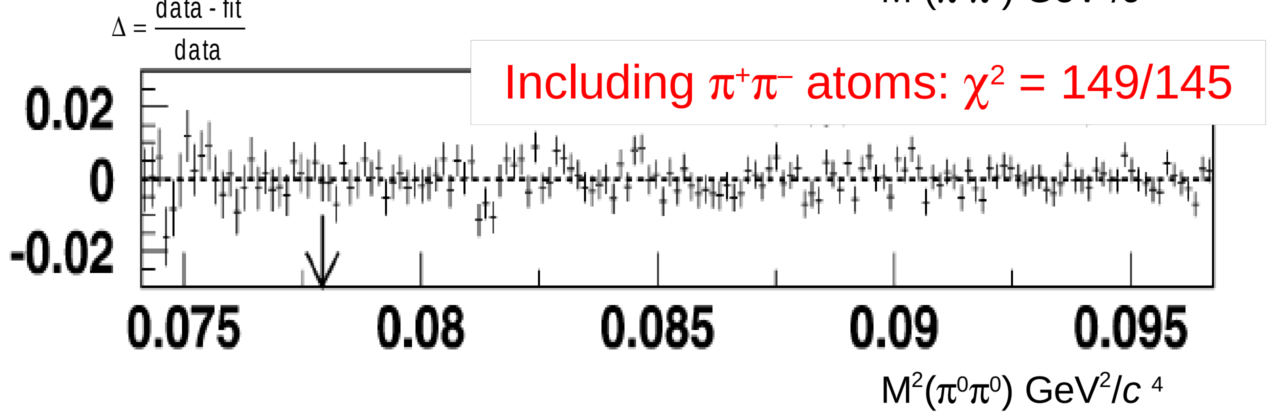
5 FIT PARAMETERS:

- g, h: 1st 2nd order parameters
  linear and quardatic
  Dalitz plot param.s
- $(a_0-a_2)m_+$, a_2m_+
- normalization









Pionium contribution

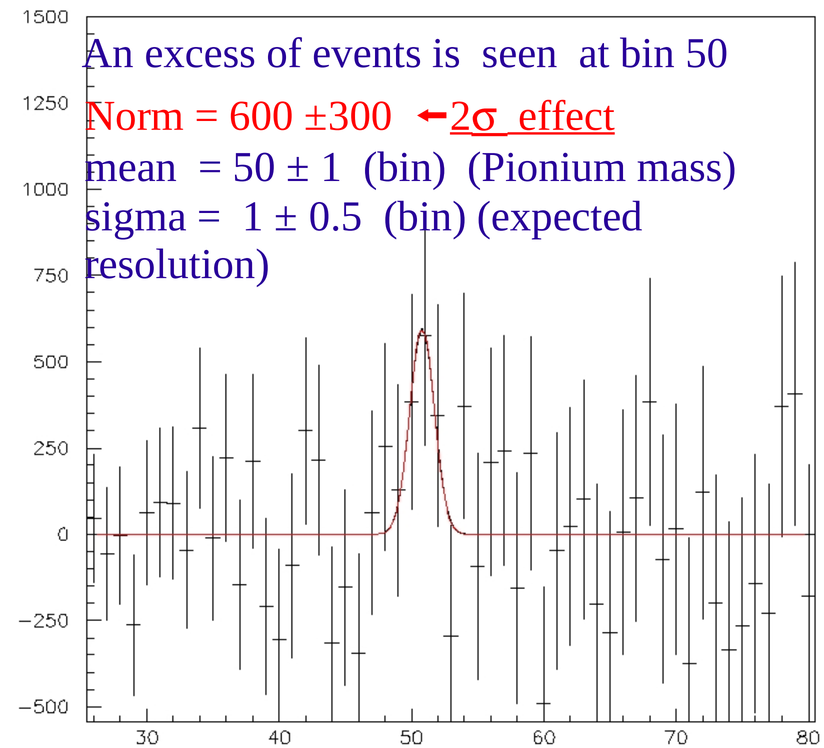
Fit including $\pi^+\pi^-$ atoms: $\chi^2 = 149 / 145$ d.f.

$$\frac{\Gamma(K^+ \to \pi^+ + pionium)}{\Gamma(K^+ \to \pi^+ \pi^+ \pi^-)} \approx (1.6 \pm 0.7) \times 10^{-5}$$

In reasonable agreement with the prediction value ~ 0.8×10^{-5} by Silagadze JETP Lett 60 (1994)

NOTE 1: Pionium is a delta function (convoluted with detector resolution via the T matrix) which sits on top of two (left/right) cusps: it will be very difficult to measure its branching ratio. In any case all the em contributions must be included in the fit.

Data – Fit

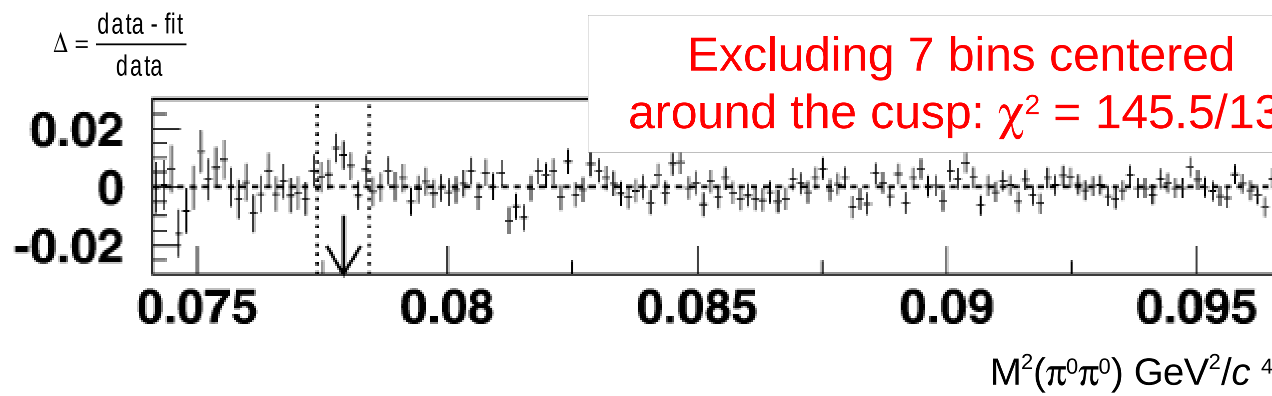


NOTE 2: At the moment no radiative corrections are included in the models, thus we excluded from the final fit the group of 7 bins centered at $s3=2m_+$

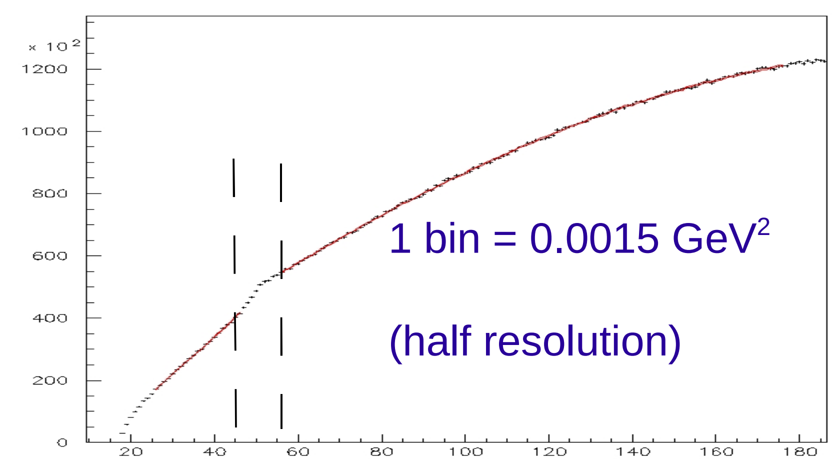
Best fit parameters

Three independent analysis were performed, two based on a “professional toy” MC, the third exploiting the full GEANT-based MC (also used for the asymmetry analysis)

Best fit excluding 7 bins around the cusp to reduce sensitivity on radiative corrections and pionium



		Stat.	Syst.
$(a_0 - a_2)m_+$	= 0.268	± 0.010	± 0.004
$a^2 m_+$	= -0.041	± 0.022	± 0.014
g	= 0.645	± 0.004	± 0.009
h’	= -0.047	± 0.012	± 0.011



NOTE 1: isospin breaking (π mass differences) included

NOTE 2: the two statistical errors are strongly correlated: correlation coefficient = -0.86

Main systematic errors

	$(a_0 - a_2)m_+$	$a_2 m_+$
Acceptance	±0.001	±0.012
Trigger efficiency	±0.001	±0.005
Fit interval	±0.0025	±0.006
others (*)	±0.002	-
Total	**±0.004**	**±0.014**
External (**)	**±0.013**	**-**

* mainly due to effect of non linear response of the calorimeter and to the effects of the $\pi^{\pm}$ shower in Lkr

** The main component (±0.013) of the external error is estimated by Cabibbo and Isidori as the result of neglecting higher order terms and radiative corrections in the rescattering model.
While a negliglble contribution (±0.003) is due to the uncertainty on the ratio between the weak amplitudes of $K^{\pm}\rightarrow\pi^{\pm}\pi^{+}\pi^{-}$ and $K^{\pm}\rightarrow\pi^{\pm}\pi^{o}\pi^{o}$ decays

Systematic checks

photon isolation and Z vertex

Cut on transverse distance at the calorimeter to avoid photon energy mis-measurement

min(γ–γ distance) > 10 cm

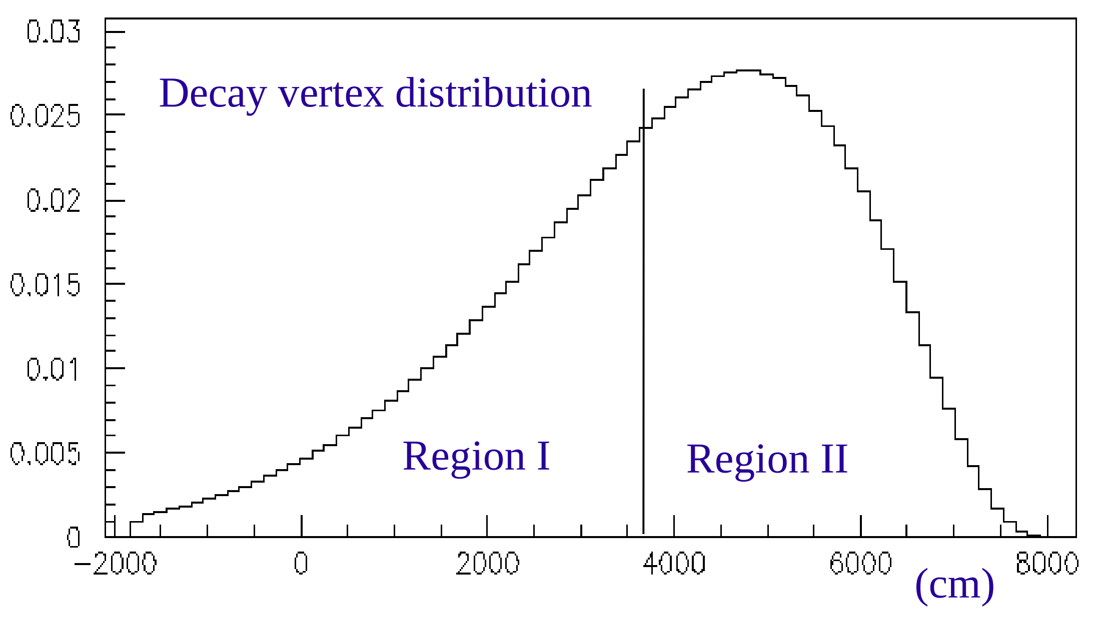
min (γ $\pi^{\pm}$ distance) > (10 + d) cm

If $\pi^{\pm}$ interacts with Kr, energy may be deposited even at large distance from the impact point

Default cut d=5 cm: 95% of the extra-shower associated to the $\pi^{\pm}$ is contained

Trying d=5,10,15 cm

systematic on $(a_0 - a_2) \cdot m_+$ $\pm$ 0.002



Measurement from two different Z decay regions

NO systematic on $(a_0 - a_2) \cdot m_+$

Effect of k term on the cusp parameters

The $K \rightarrow \pi^+\pi^o\pi^o$ decay matrix element depends on two independent variables s_3 and s_1-s_2:

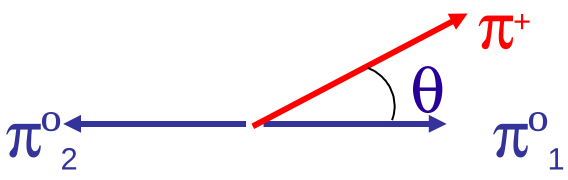
$$M_{+oo} = 1 + 1/2\, g_o (s_3 - s_0)/m_+^2 + 1/2\, h'(s_3 - s_0)^2/m_+^4 + 1/2\, k (s_1 - s_2)^2/m_+^4 + \ldots$$

By performing a 2D fit of the Dalitz plot

NA48 finds evidence for a k>0 term: order of magnitude ~1%

Few details about the preliminary fit for the k term:

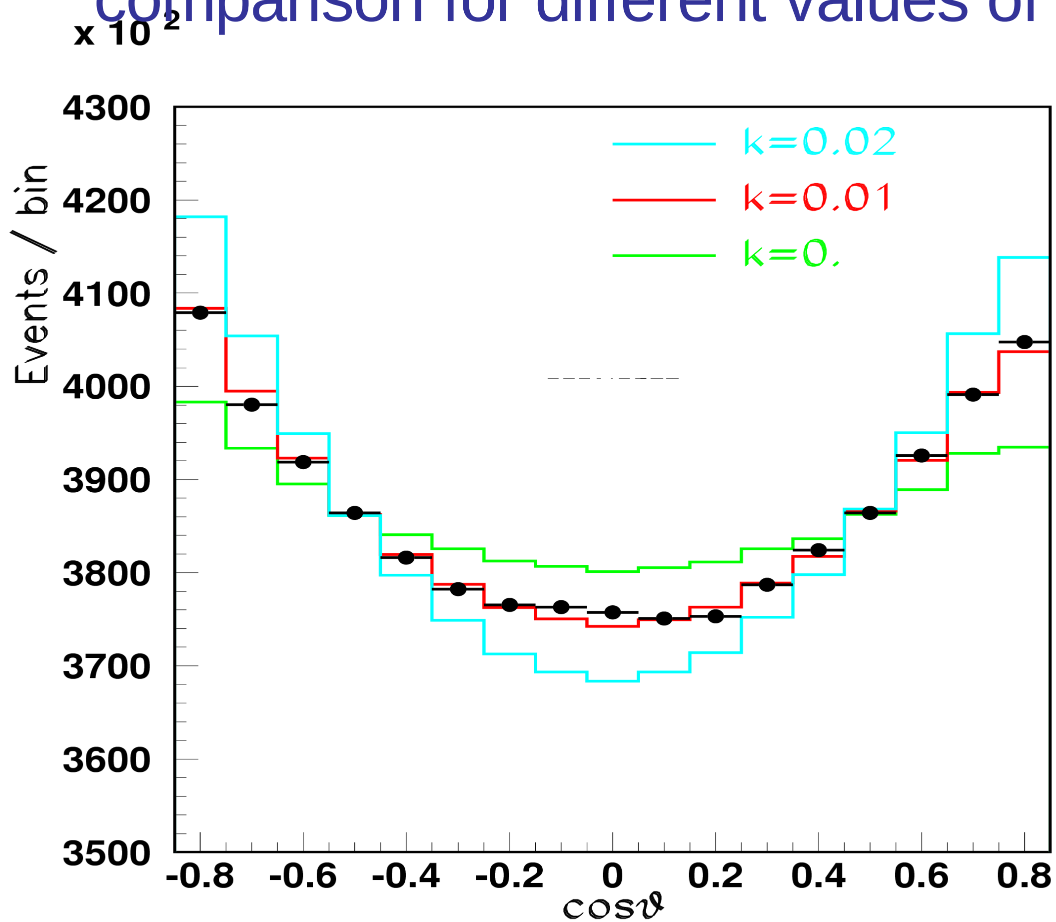
- Instead of (s3,s1-s2) the two alternative variables (s3,cosθ) are used, where cosθ is the angle between the odd π and the direction of the even pions in their CM reference system.
- The resulting rectangular shaped Dalitz plot (-1<cosθ<+1 independent of s3) allows for binnig without crossing the physical boundaries and thus a simpler 2D fit.



- At the moment we fit only in a region beyond (s3 > M^2_{oo}) the cusp.

Effect of k term on the cusp parameters

Data – MC
comparison for different values of k



Effect of k term on the cusp parameters

Concerning the fit to the cusp the strategy is to keep the 1D fit procedure in order not to spoil the excellent resolution in s_3 by a mix with other variables (s_1,s_2 or cosθ).

Determine cusp parameters by fitting one-dimensional s_3 distribution as usual after correcting |matrix element|^2 (≡ $|M|^2$) for non-zero k value in generation

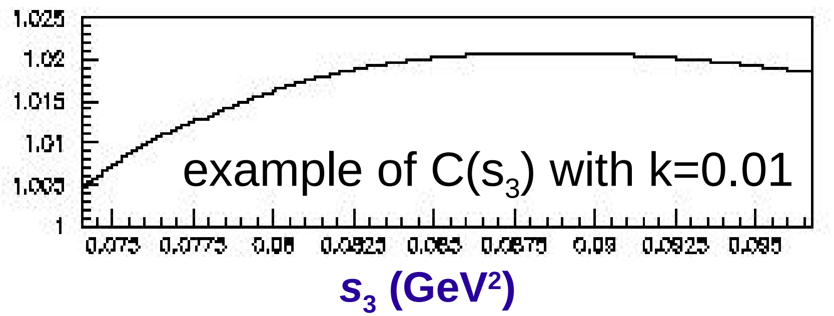
$$|M(s_3, s_1; k=0)|^2 \rightarrow |M(s_3, s_1; k=0)|^2 \cdot C(s_3)$$

$$C(s_3) = \frac{\int_{s_1 min}^{s_1 max} ds_1 |M(s_3, s_1; k=k_{meas})|^2 \;\; \Phi(s_3, s_1) \;\; A(s_3, s_1)}{\int_{s_1 min}^{s_1 max} ds_1 |M(s_3, s_1; k=0)|^2 \;\; \Phi(s_3, s_1) \;\; A(s_3, s_1)}$$

Φ = phase space
A = acceptance

Effect of k term on the cusp parameters

Deviations
from 1 of $C(s_3)$ are ~ $O(k)$



We find negligible effect of k ~ O(0.01)
on the values of (a_0-a_2) and a_2 : solid measurement

The χ^2 of the fit is unchanged.

Of course the parameters g_0 and h' move.

Work in progress.

Results for the scattering lengths

NA48/2 PLB 633 (2006) 173

The final result of unconstrained fit to the rescattering model are:

		Stat.	Syst.	Ext.
$(a_0 - a_2)m_+$	$= 0.268$	± 0.010	± 0.004	± 0.013
$a_2 m_+$	$= -0.041$	± 0.022	± 0.014	

Agreement with theory and other experiments

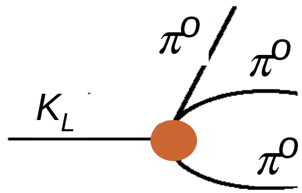
Performing the fit with constraints imposed by analyticity and chiral symmetry (after Colangelo etal. PRL 86 2001) leads to the following:

		Stat.	Syst.	Ext.
$(a_0 - a_2)m_+$	$= 0.264$	± 0.006	± 0.004	± 0.013
$a_0 m_+$	$= 0.220$	± 0.006	± 0.004	± 0.011

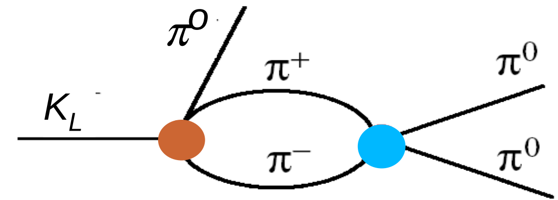
The "cusp" in $K_L \to \pi^\circ\pi^\circ\pi^\circ$

Expected extent of cusp effect in K_L vs K^+

K_L

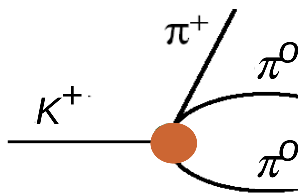


$M_0 \propto 1$

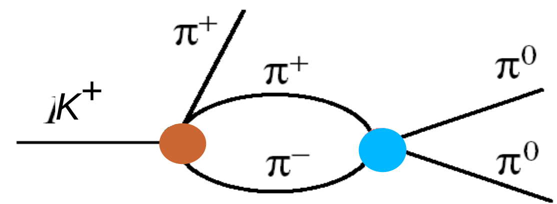


$M_1 \propto (1 + \mathbf{g_{+-0}}\, u)$ (a0-a2)

K^+



$M_0 \propto (1 + \mathbf{g_{+00}}\, u)$



$M_1 \propto (1 + \mathbf{g_{++-}}\, u)$ (a0-a2)

$$\frac{(M_1/M_0)_{K^+}}{(M_1/M_0)_{K_L}} = 2\sqrt{2}\,\frac{1+g_{++-}u}{1+g_{+00}u} \times \frac{1}{1+g_{+-0}u} \approx 7$$

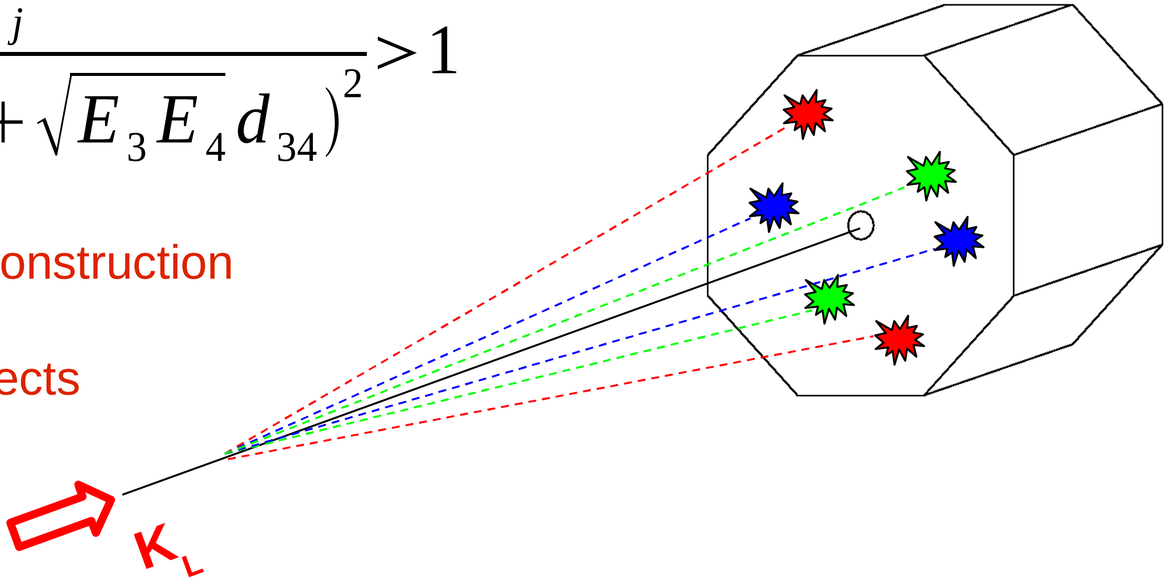
The cusp effect for K_L is a factor 7 smaller (at the $2m_\pi$ threshold)

Invariant mass of the $\pi^o\pi^o$ pairs in $K_L \to 3\pi^o$

1) z vertex by imposing the K mass to the 6 photons
2) Photon pairing by minimizing $\chi^2 = \Sigma\,(z - z_i)^2$ i=1,3;
   where z_i = vertices from each π^0 mass
3) $m_{\pi\pi}$ as in $K^+ \to \pi^+\pi^0\pi^0$ analysis

$$z_i = \frac{\sqrt{E_j E_k}\, d_{jk}}{m_{\pi_o}}$$

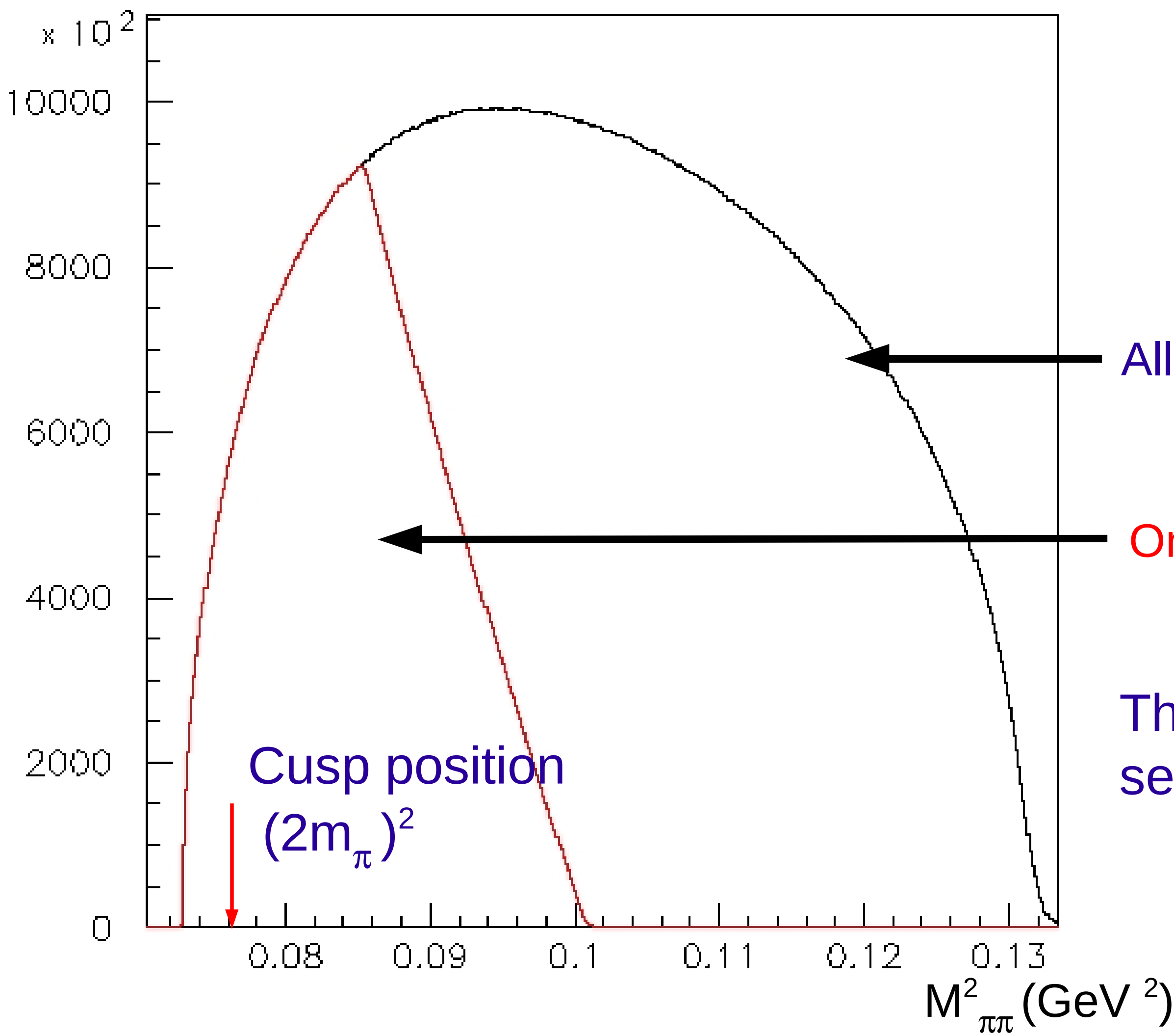
$$\frac{M_{oo}^2}{4m_{\pi_o}^2} = \frac{\sum_{i<j=1,4;\,i<j} E_i E_j d_{ij}^2}{\left(\sqrt{E_1 E_2}\, d_{12} + \sqrt{E_3 E_4}\, d_{34}\right)^2} > 1$$



- Kinematic limit respected by construction
- Cancellation of systematic effects (E scale, non linearities, ...)
- Best resolution in the cusp region

Invariant mass of the $\pi^o\pi^o$ pairs in $K_L \to 3\pi^\circ$

Statistics: 0.1 x 10^9 events collected in year 2000



$K_L \to \pi_1^\circ\pi_2^\circ\pi_3^\circ$

$$m_{12}^2 + m_{13}^2 + m_{23}^2 = M_K^2 + 3\mathrm{m}_\pi^2$$

All (3) pairs plotted simultaneously

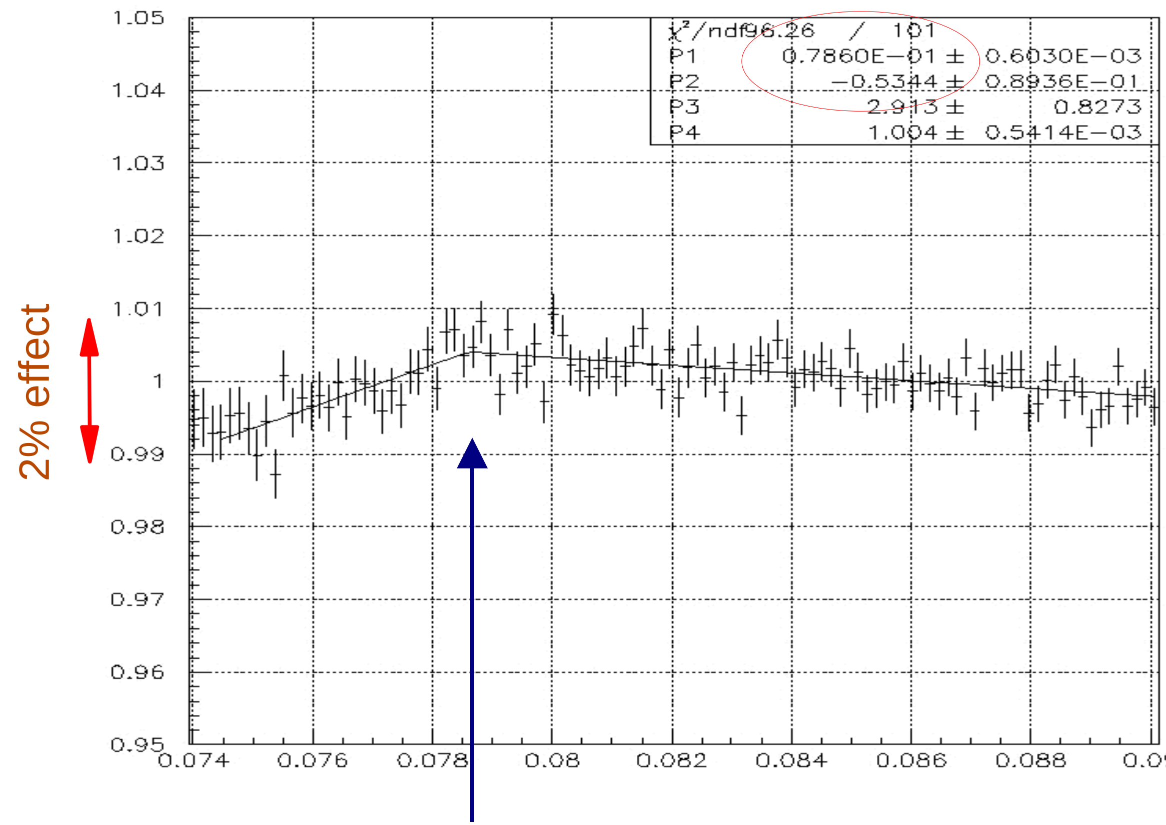
Only the lightest mass m_{12}

The cusp is small and hard to be seen by eye !

The "cusp" in $K_L \to \pi^\circ\pi^\circ\pi^\circ$

Looking for a cusp: fit the position on data to MC ratio

(pure phase space in MC and no rescattering effects)



Cusp position fit at **0.0786** GeV^2
close to the expected value $4m_\pi^2$ = 0.07728

... could this be the source of h < 0 in PDG ?

Experimental status of pion scattering lengths

Scattering lengths measurements

$K^{\pm} \rightarrow \pi^+\pi^- e^{\pm}\nu$ (Ke4): FSI of $\pi\pi \rightarrow$ asymmetry of electron direction w.r.t. plane of $\pi\pi$

* extraction of form factors and phase shift difference $\delta^0_0 - \delta^1_1$ phase shift difference
  as a function of $\pi\pi$ energy.
* scattering lengths extracted in a model dependent way with theoretical
  input $a_2 = f(a_0)$. A general constraint is the "universal band" [NP B10 (1969) 261].
  Much more stringent constraints might be used, eg PRL 86 (2001) 5008.
* available C.M. energy range $2m_\pi < M_{\pi\pi} < m_K - m_\pi$ reduced by acceptance.

Pionium lifetime: τ proportional to $(a_0 - a_2)^2$

* very short time (3 10^{-15}) → very sophisticated/complicated technique:
  detection of $\pi\pi$ pairs resulting from pionium atoms ionized in the production target
* need accurate knowledge of breakup probability:
  detailed description of cross sections and atom interaction dynamics
* insensitive to the sign of $a_0 - a_2$

Scattering $\pi N \rightarrow \pi\pi N$ near threshold : fit of double differential cross section

* model dependent
* additional hadrons in the final state

Cusp in $K^{\pm} \rightarrow 3\pi$: direct measurement of scattering lengths at threshold

* very accurate, sensitive to the sign of $a_0 - a_2$
* model independent; only general assumptions of unitarity + analyticity
* radiative+Coulomb corrections needed to enhance the sensitivity of to $a_0 - a_2$

Scattering lengths measurements

$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ (Ke4)

$a_0 m_+ = 0.280 \pm 0.050$ — Rosselet et al. PRD 15 (1977) 574 (Geneva-Saclay)

$a_0 m_+ = 0.216 \pm 0.013 \pm 0.003$ — Pislak et al. PRD 67 (2003) 072004 (E865)

$a_2 m_+ = -0.0454 \pm 0.0031 \pm 0.0013$ — (using narrow band constraint a2 = f(a0) from PR 353 (2001) 207)

NA48 results on Ke4 coming soon (QCD06): 2003 statistics same as full E856 but more acceptance for higher $M\pi\pi$: longer lever arm than E865

Pionium lifetime

$| a_0 - a_2 | m_+ = 0.264^{+0.033}_{-0.020}$ — Adeva et.al. PLB 619 (2005) 50 (DIRAC)

(improvement in analysis: expected error ~3%)

Scattering $\pi N \to \pi\pi N$ near threshold

$a_0 m_+ = 0.260 \pm 0.050$ — Froggatt et al. NPB 129 (1977) 89

$a_0 m_+ = 0.204 \pm 0.014 \pm 0.008$ — Kermani et al. PRC 58 (1998) 3431

Cusp in $K^{\pm} \to \pi^{\pm}\pi^{o}\pi^{o}$

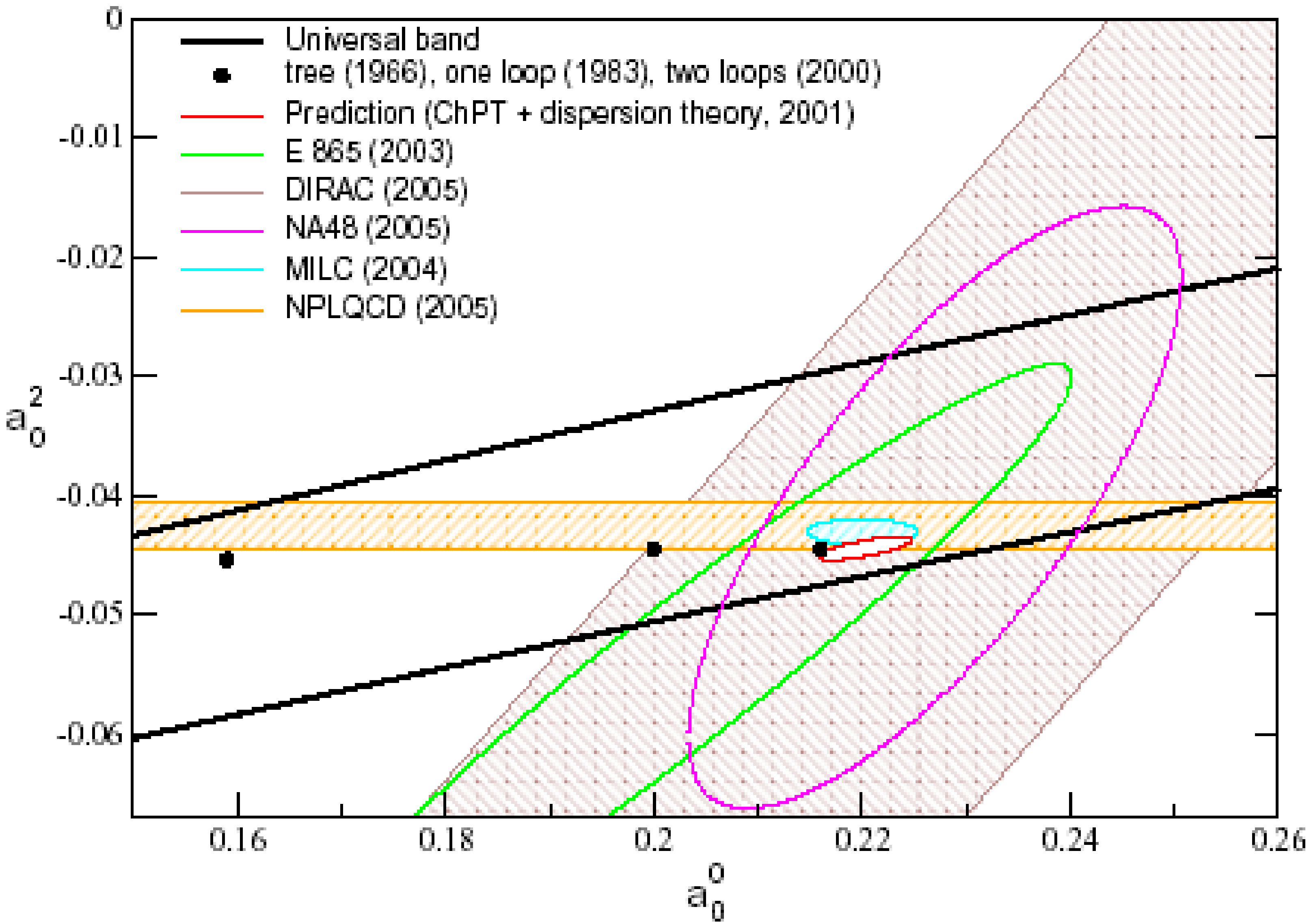
$(a_0 - a_2)\, m_+ = 0.268 \pm 0.010 \pm 0.013$ — Batley et al. PLB 633 (2006) 173 (NA48)

$a_2 m_+ = -0.041 \pm 0.022 \pm 0.014$ — (improvement in analysis: statistics x5)

$(a_0 - a_2)\, m_+ = 0.264 \pm 0.006 \pm 0.013$ — When fit includes a narrow band constraint a2 = f(a0)

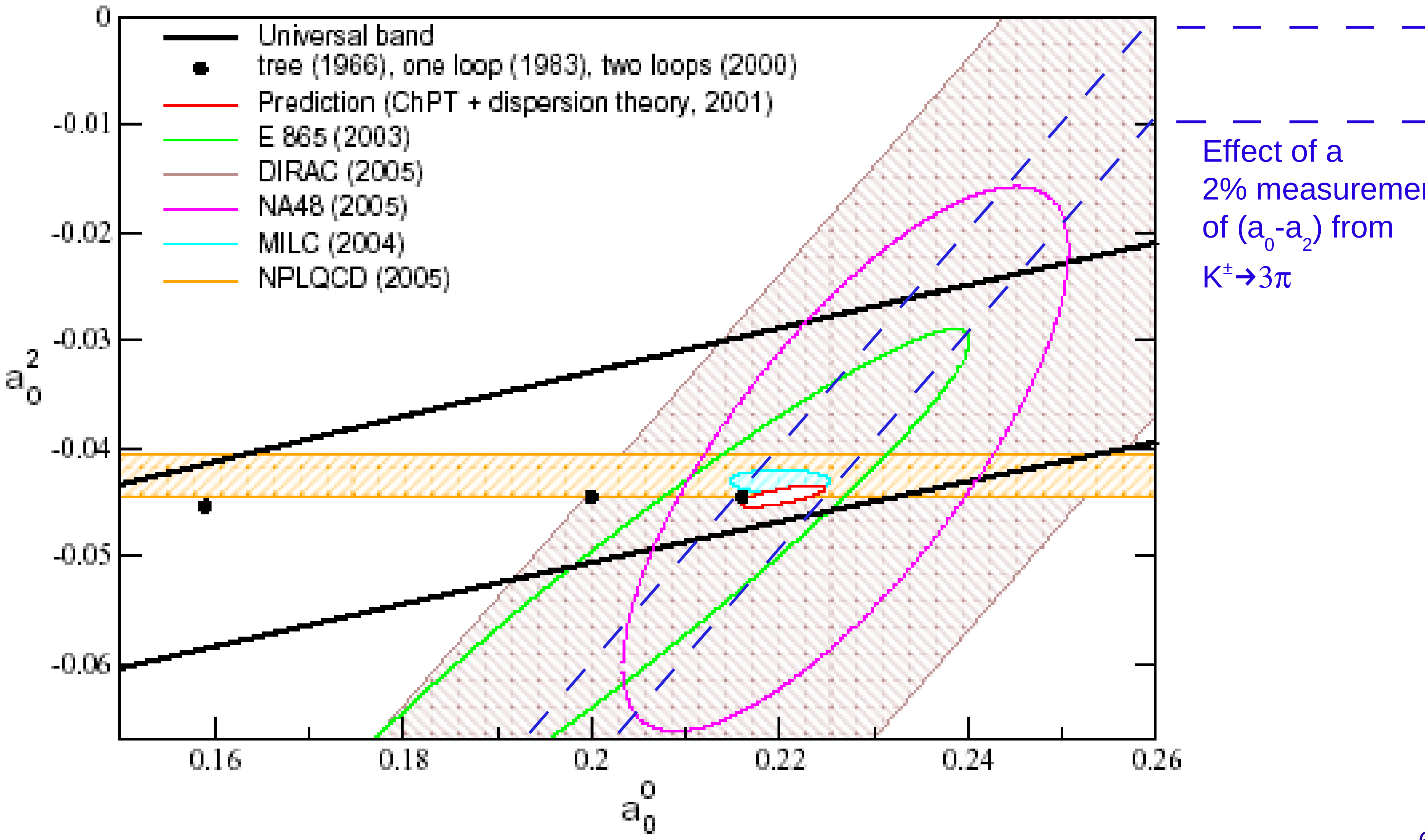
$a_0 m_+ = 0.220 \pm 0.006 \pm 0.012$ — as proposed in PRL 86 (2001) 5008

Measurements and predictions for a_0 and a_2



From
H.Leutwyler
Symposium
in Bern 5/5/2006

Measurements and predictions for a_0 and a_2



Effect of a 2% measurement of $(a_0 - a_2)$ from $K^{\pm} \rightarrow 3\pi$

Pion scattering lengths
... a direct probe of the QCD vacuum

Weinberg (1966)
* Effective field theory for strong interaction at low E

$$a_0 m_{\pi^+} = \frac{7 m_{\pi^+}^2}{16\pi f_\pi^2} = 0.159$$

$$a_2 m_{\pi^+} = \frac{-m_{\pi^+}^2}{8\pi f_\pi^2} = -0.045$$

Colangelo et al. (2001)
* CHPT + dispersion relations

$$a_0 m_{\pi^+} = 0.220 \pm 0.005$$

$$a_2 m_{\pi^+} = -0.0444 \pm 0.0010$$

$$(a_0 - a_2) m_{\pi^+} = 0.265 \pm 0.004$$

Pelaez and Yndurain (2005)
* phase shift analysis of data, using unitarity, analyticity and dispersion relations (no CHPT)

$$(a_0 - a_2)\, m_\pi = 0.278 \pm 0.016$$

High precision (1.5%) reached is quite unusual for hadronic physics predictions
→ experiments didn't yet reached the same level of accuracy ... but are on the way

Conclusions

- A new cusp structure in $K^{\pm}\rightarrow\pi^{\pm}\pi^{o}\pi^{o}$ and $K^{\pm}\rightarrow 3\pi^{o}$ decays was observed

- The cusp has been interpreted by Cabibbo as due to
  $\pi\pi$ final state charge exchange scattering

- This provides a new method for a precise determination of
  $a_0 - a_2 = 0.268 \pm 0.010\ (\text{stat.}) \pm 0.013\ (\text{syst.})$
  where the systematic error is mainly due to theoretical uncertainties.

- The measured value is in agreement with theory and with the other measurements.

- For the first time the parameter a_2 was directly measured, even though with low accuracy

- Pionium bound state is also found but further investigation is needed

Conclusions

- By analyzing the full data sample we expect an increase in statistics by a factor of 5.
- An experimental error below 1.5% seems not to be out of reach.

- At the moment the external uncertainty related to the theoretical method is ~5% and the data quality calls for additional theoretical effort in order to extract precise values of the $\pi\pi$ scattering parameters (higher orders + electromagnetic corrections).

- The fit according to different amplitudes representation (CGKR) is in progress together with a fit the full Dalitz plot and in particular at the border, also for $K^{+} \rightarrow \pi^{-}\pi^{+}\pi^{+}$

- The study of cusp effects in $K_L \rightarrow 3\pi^{\circ}$ on the year 2000 NA48 data is going on

- At QCD06 (Montpellier) the NA48 preliminary results on $K^{\pm}e4$ decays will be presented. They will provide additional precise information about $\pi\pi$ scattering.