Precise Measurements of $\pi\pi$ Scattering Lengths from Kaon Decays

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P326 NA62

Kaon Physics in the New Millennium

Since about a decade a new generation of high-statistics kaon experiments are in operation

NA48, KTeV, KLOE, ISTRA+, E787/949, NA62, ...

$\Downarrow$

Significant progress in many fields in the last few years:

- Direct CP violation (ε'/ε)
- Precision determination of $|V_{us}|$
- **Tests of Chiral Perturbation Theory** ← **This Talk**
- Search for New Physics phenomena (LFV, ...)

Outline

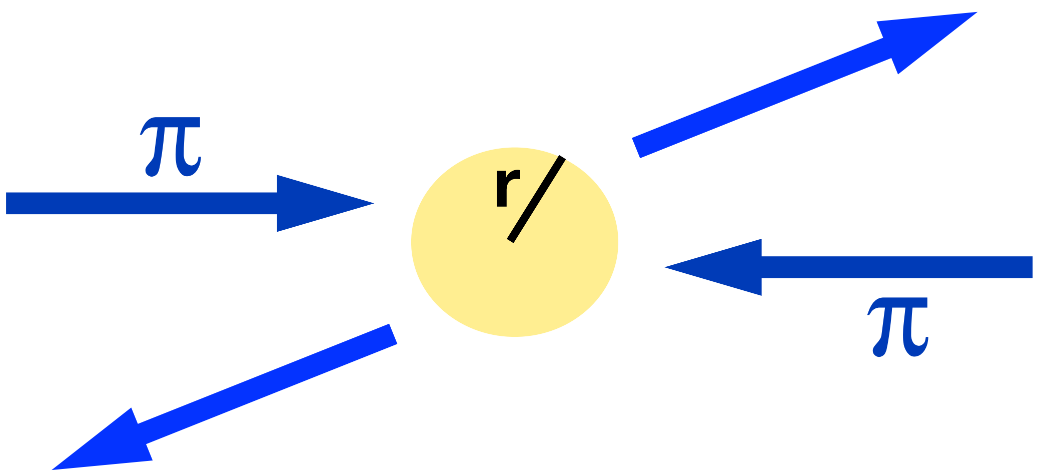
Introduction

Measurement of ππ scattering lengths

- **Wigner-cusp in $K^\pm \to \pi^\pm \pi^0 \pi^0$**
- **Measurement of $K^\pm \to \pi^+ \pi^- e^\pm \nu$ (K_{e4}) decays**
- **Future Prospects**

Conclusions

ππ Scattering Lengths



$$\mathbf{k} = \frac{\sqrt{\mathbf{2mE}}}{\hbar}$$

At low energy *k r << 1*: **S-wave** dominates scattering amplitude.
Isospin *I = 0,2* because of Bose statistics.

- Scattering matrix $\mathbf{S}|\pi\pi\rangle = \mathbf{e}^{\mathbf{2i}\delta}|\pi\pi\rangle$ parametrized by two phases:

$$\delta_{\mathbf{0,2}} = -\mathbf{a}_{\mathbf{0,2}} \cdot \mathbf{k} + \mathcal{O}(\mathbf{k}^2)$$

- At low energy **S-wave scattering lengths a_0, a_2** are essential parameters of **Chiral Perturbation Theory (ChPT).**

(In the following: $a_{0,2}$ quoted in units of m_π)

Chiral Symmetry Breaking

Chiral symmetry: (considering only *u,d* quarks)

- Vacuum ground state not invariant under chiral symmetry.

$\Longrightarrow$ $SU(2)_L \otimes SU(2)_R$ becomes $SU(2)_{L+R}$.

$\Longrightarrow$ Three massless Goldstone bosons are created. (π^+, π^-, π^0)

Spontaneous symmetry breaking:

- Assumption: u, d, s quarks massless.

$\Longrightarrow$ Lagrange function $\mathcal{L}_{QCD}$ invariant under $SU(2)_L \otimes SU(2)_R$ transformations.

Chiral Symmetry Breaking

Symmetry breaking by finite quark masses:

- Symmetry broken by quark masses $m_u,\, m_d \neq 0$

  $\Longrightarrow$ Pions acquire masses.

- **Gell-Mann, Oakes, Renner:** (PR 175 (1968) 2195)

$$\boxed{\boldsymbol{M_\pi^2 \;\approx\; \frac{1}{F_\pi^2}\,(m_u + m_d)\,|\langle 0|q\bar{q}|0\rangle|}}$$

  with: F_π = pion decay constant $= 92.4$ MeV

  and: $\langle 0|q\bar{q}|0\rangle$ = ***quark condensate.***

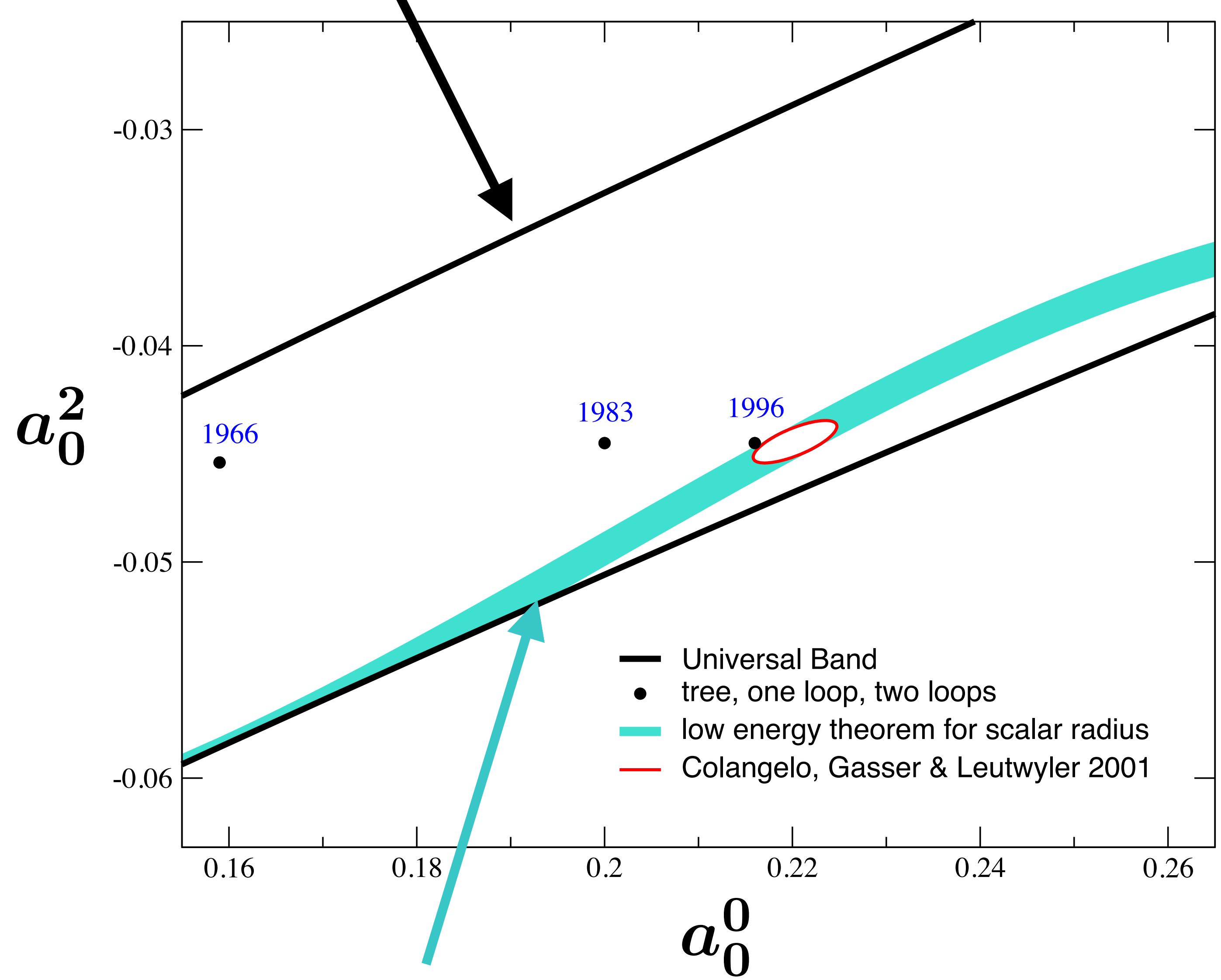
- **Stern *et al.*:** $\langle 0|q\bar{q}|0\rangle$ not necessarily $\neq 0$ for $m_u,\, m_d \neq 0$, (NP B457 (1995) 513) as expansion in the quark masses is:

$$M_\pi^2 \;=\; \frac{1}{F_\pi^2}\,(m_u + m_d)\,|\langle 0|q\bar{q}|0\rangle| \;-\; \frac{\bar{l}_3}{32\pi^2 F_\pi^2} M^4 \;+\; \mathcal{O}(M^6)$$

  with $\bar{l}_3$ = coupling constant, could be unexpectedly large.

Theory predictions for a_0 and a_2

Universal band (from Roy equations)



ChPT constraint:

- $\mathbf{a_0, a_2 = 0}$ for $m_d, m_u = 0$ (chiral limit)
- Scattering lengths $\mathbf{a_0, a_2}$ are directly connected to $\mathbf{m_\pi}$:

$$\boxed{\begin{aligned} a_0 &\sim \frac{7\, m_\pi^2}{32\pi\, F_\pi^2} = 0.16 \\ a_2 &\sim \frac{-\, m_\pi^2}{16\pi\, F_\pi^2} = -0.045 \end{aligned}}$$

(Weinberg, PRL 17 (1966) 216)

- Precise prediction within Chiral Perturbation Theory:

$$\boxed{\begin{aligned} a_0 &= 0.220 \pm 0.005 \\ a_2 &= -0.0444 \pm 0.0010 \end{aligned}}$$

(Colangelo, Gasser, Leutwyler, PRL 86 (2001) 5008)

$$a_2 = -0.0444(8) + 0.236\,(a_0 - 0.22) - 0.61\,(a_0 - 0.22)^2 - 9.9\,(a_0 - 0.22)^3$$

Measuring $\pi\pi$ Scattering Lengths

Three kinds of measurements have been performed:

- **Pionium lifetime $(\pi^+\pi^-)_{atom}$:** Measurement of $|a_2 - a_0|$

  ➜ DIRAC experiment

- **Cusp in $K \rightarrow \pi\pi\pi$ ($K_{3\pi}$) decays:** Measurement of a_2-a_0, a_2
  - $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ ➜ **NA48/2:** ~60 million events
  - $K_L \rightarrow \pi^0 \pi^0 \pi^0$ ➜ KTeV + NA48/2: (70+100) million events

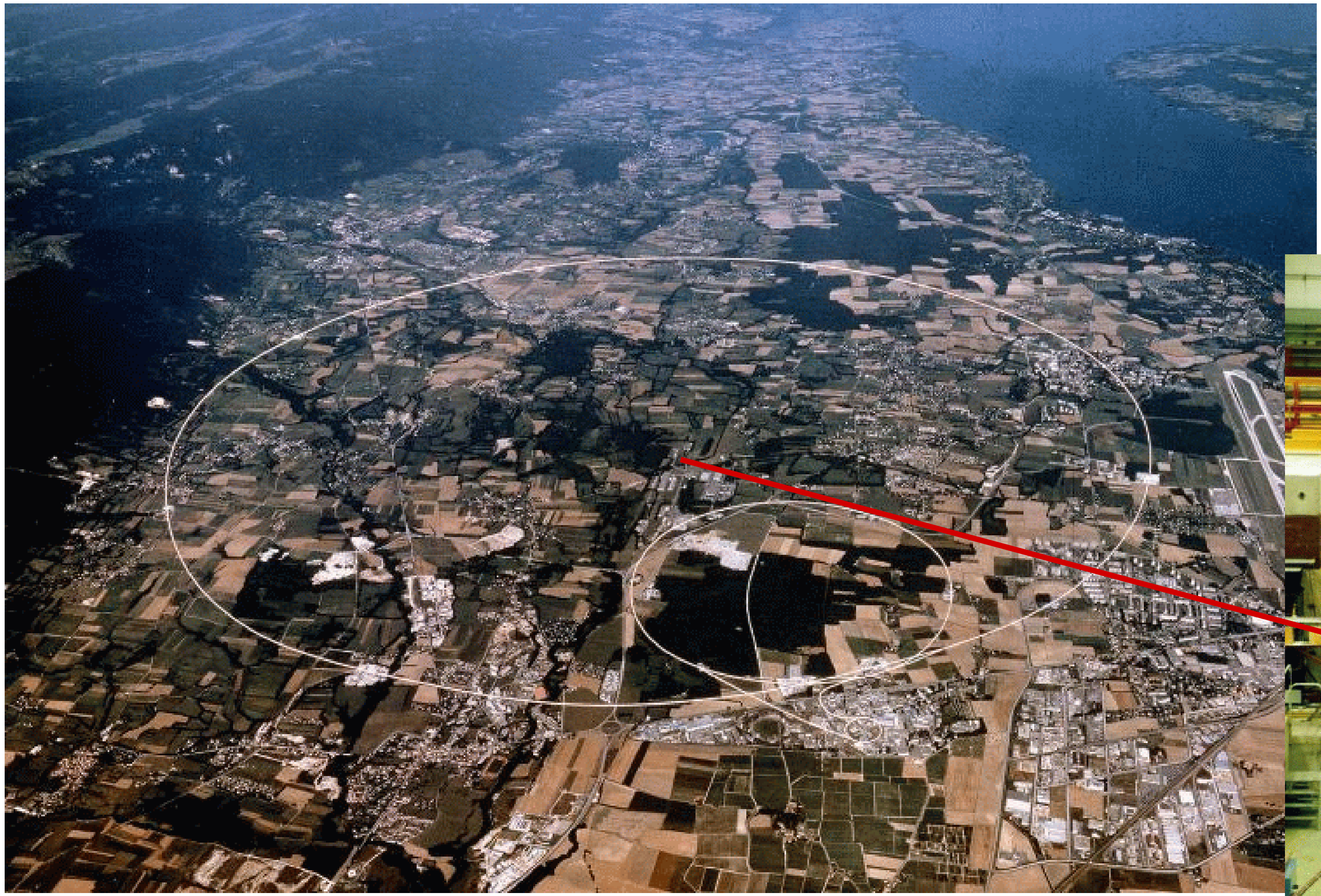
- **$K^\pm \rightarrow \pi^+ \pi^- e^\pm \nu$ (K_{e4}) decays:** Measurement of a_0, a_2

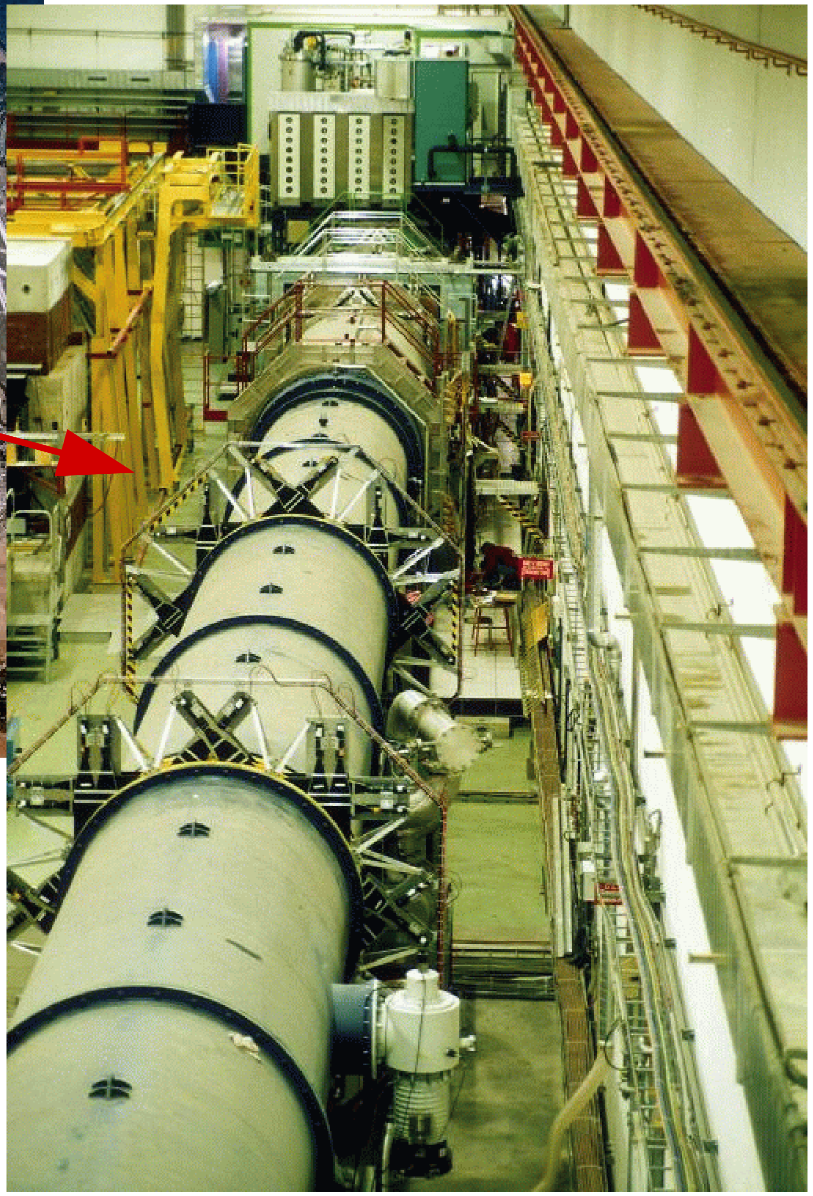
  ➜ S118 (Geneva-Saclay, 1977): ~30 000 events

  BNL E685 (2003): ~400 000 events

  NA48/2 (2009): ~1.1 million events

The NA48/2-Experiment

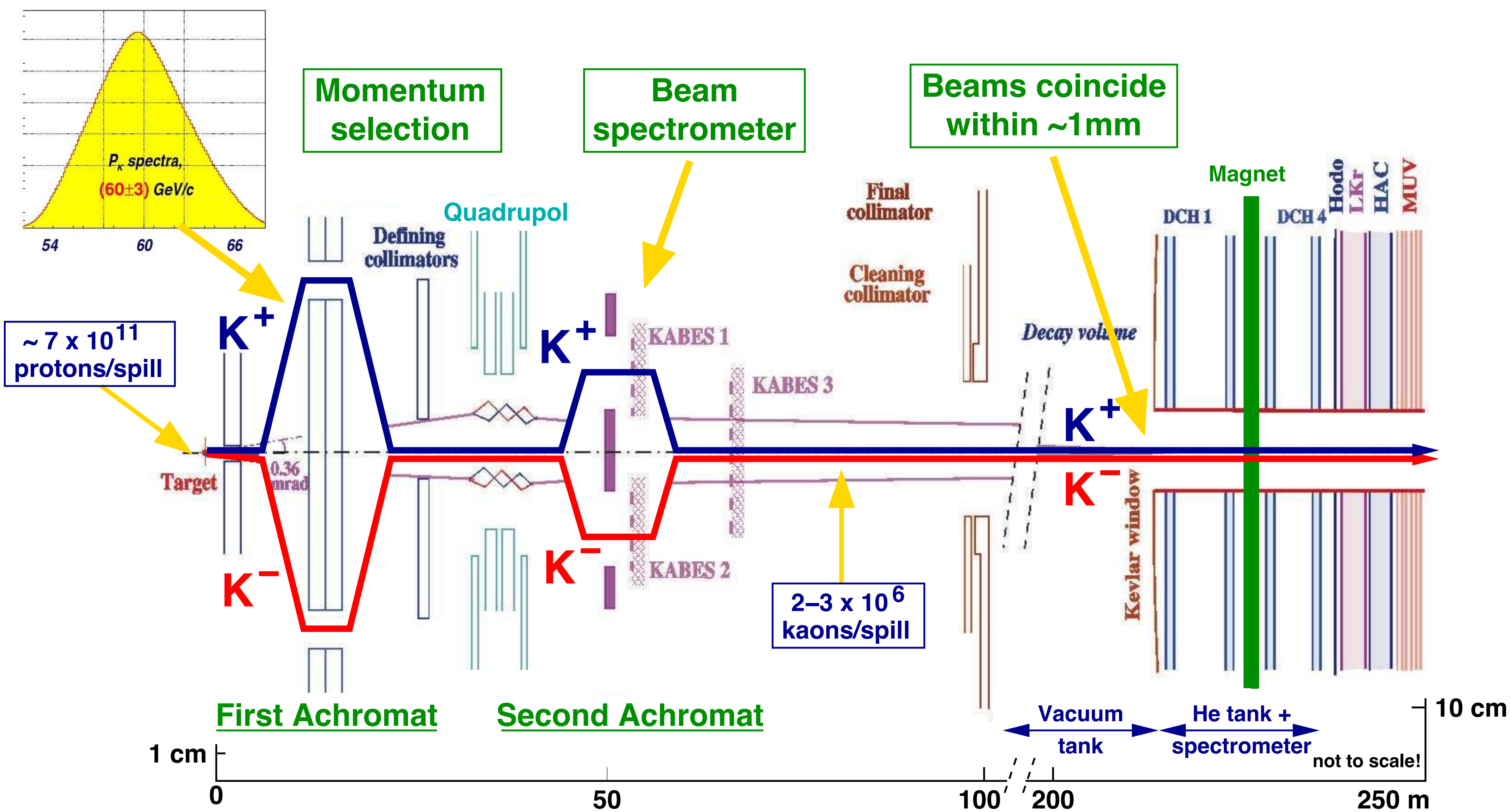




- **NA48:** 48^{th} experiment in the CERN North Area
- Fixed-target experiment with 400 GeV/c proton beam from the SPS

NA48/2 in 2003/2004

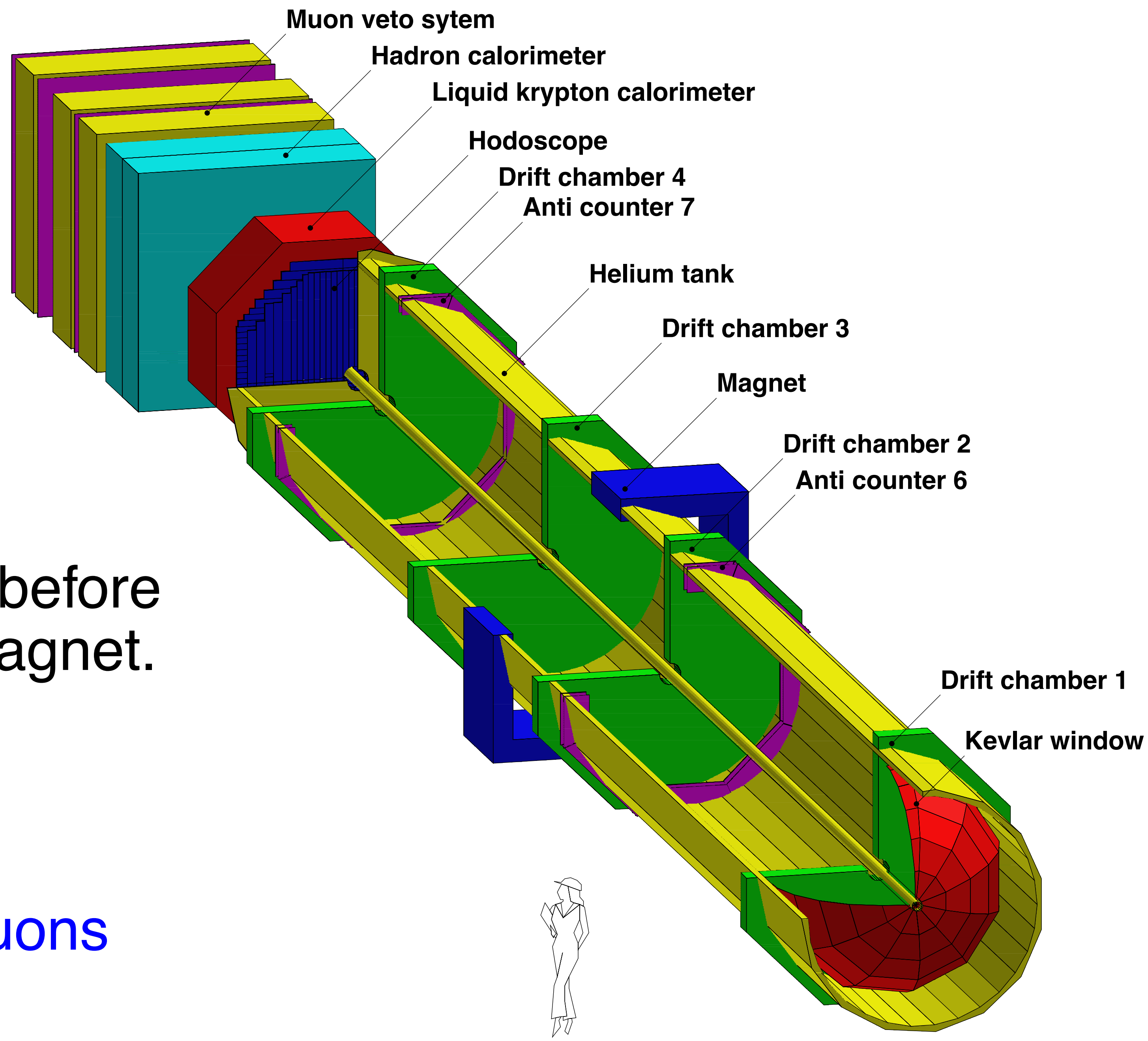
- **Simultaneous K^+ and K^- beams** with $p_{K^\pm} = (60 \pm 3)$ GeV/c.



Trigger:
- **3 charged tracks** or
- **1 charged track + missing p_T**

➜ **Efficiencies > 99 %**

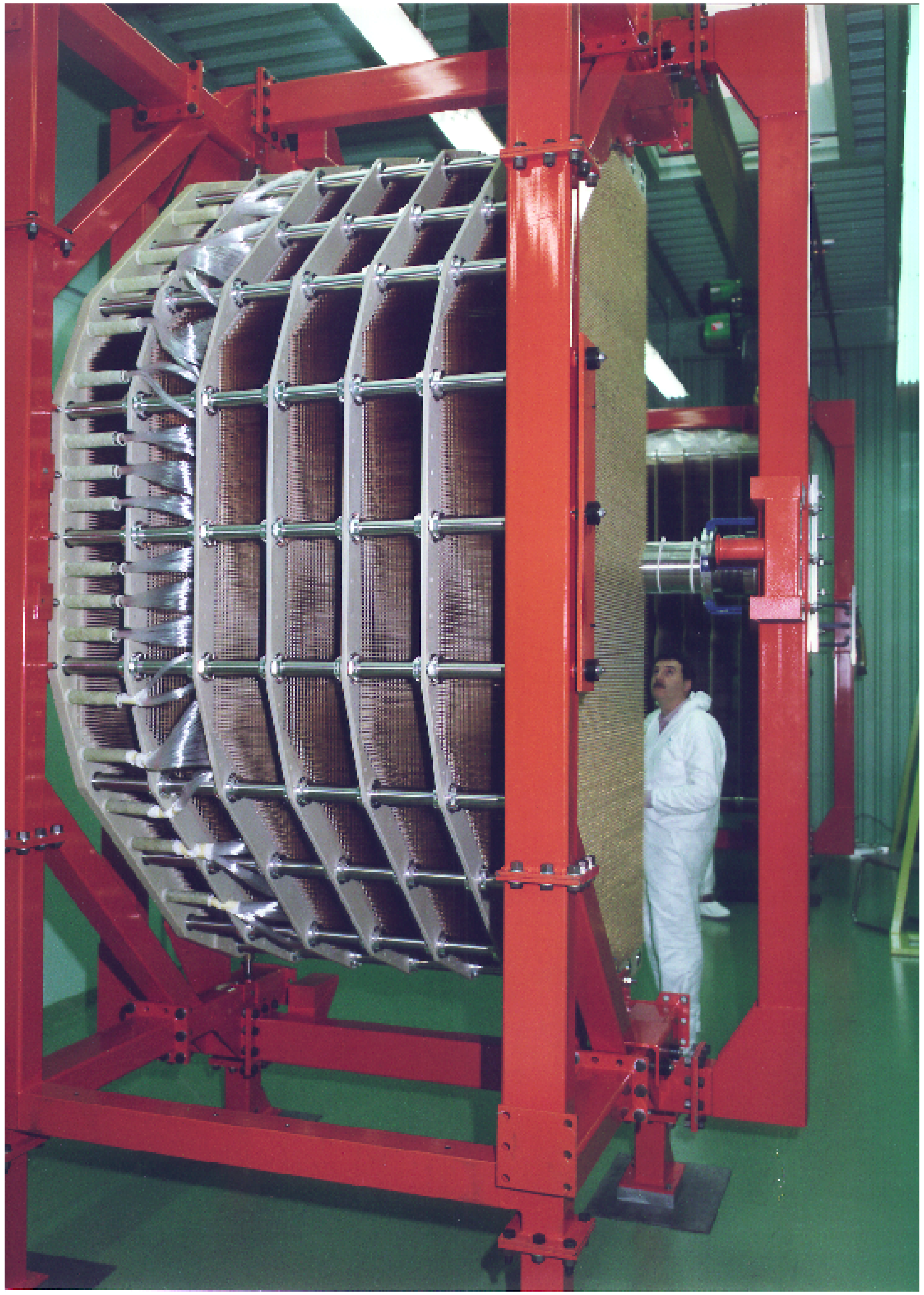
NA48/2 Detector



Main detector components:

- **Magnet spectrometer**
  - Two drift chambers each before and after spectrometer magnet.
  - Momentum resolution: $\leq 1\%$ for 20 GeV/c tracks.
- Anti-counters for photons, muons
- **Liquid Krypton Calorimeter**

Liquid-Krypton Calorimeter (LKr)



- **Structure:**

  13212 cells of 2×2 cm^2 along beam axis in ~ 10 m^3 liquid krypton.

- **Energy resolution:**

$$\boxed{\frac{\Delta E}{E} = \frac{3.2\%}{\sqrt{E[\text{GeV}]}} \oplus \frac{90\ \text{MeV}}{E} \oplus 0.42\%}$$

  $\Longrightarrow\ \sim 1\%$ for 20 GeV photons.

- **Position resolution:**

$$\boxed{\sigma_x = \sigma_y = \frac{4.2\ \text{mm}}{\sqrt{E[\text{GeV}]}} \oplus 0.6\ \text{mm}}$$

  $\Longrightarrow\ \sim 1$ mm for 20 GeV photons.

Cusp in $K^{\pm} \rightarrow \pi^{\pm} \pi^{0} \pi^{0}$ Decays

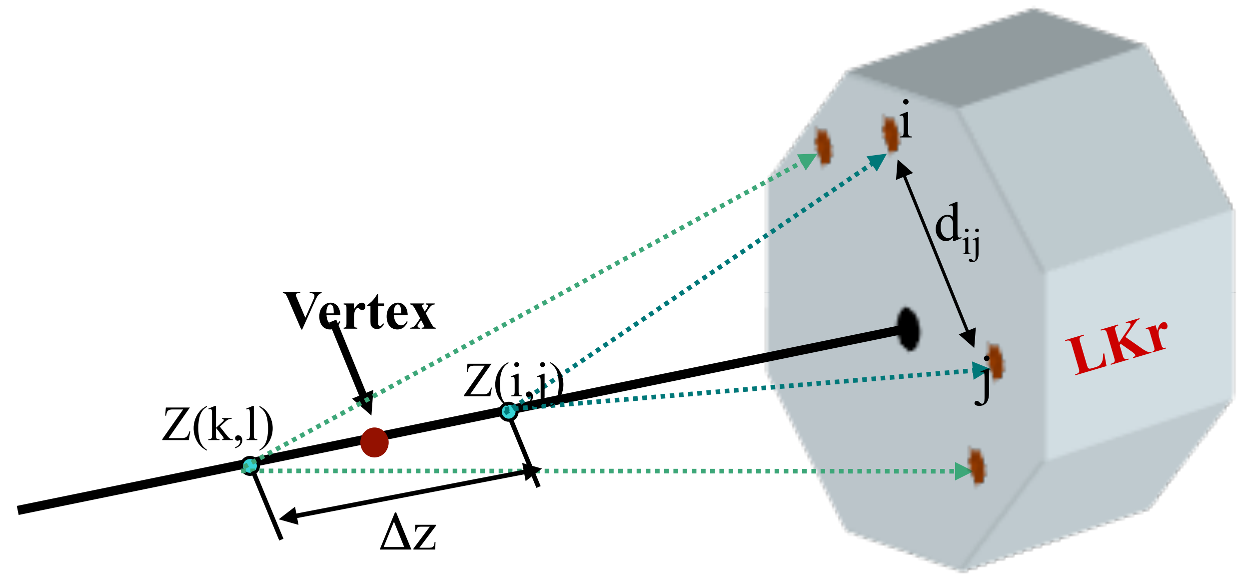
Reconstruction of $K \to \pi^{\pm}\pi^0\pi^0$ Decays

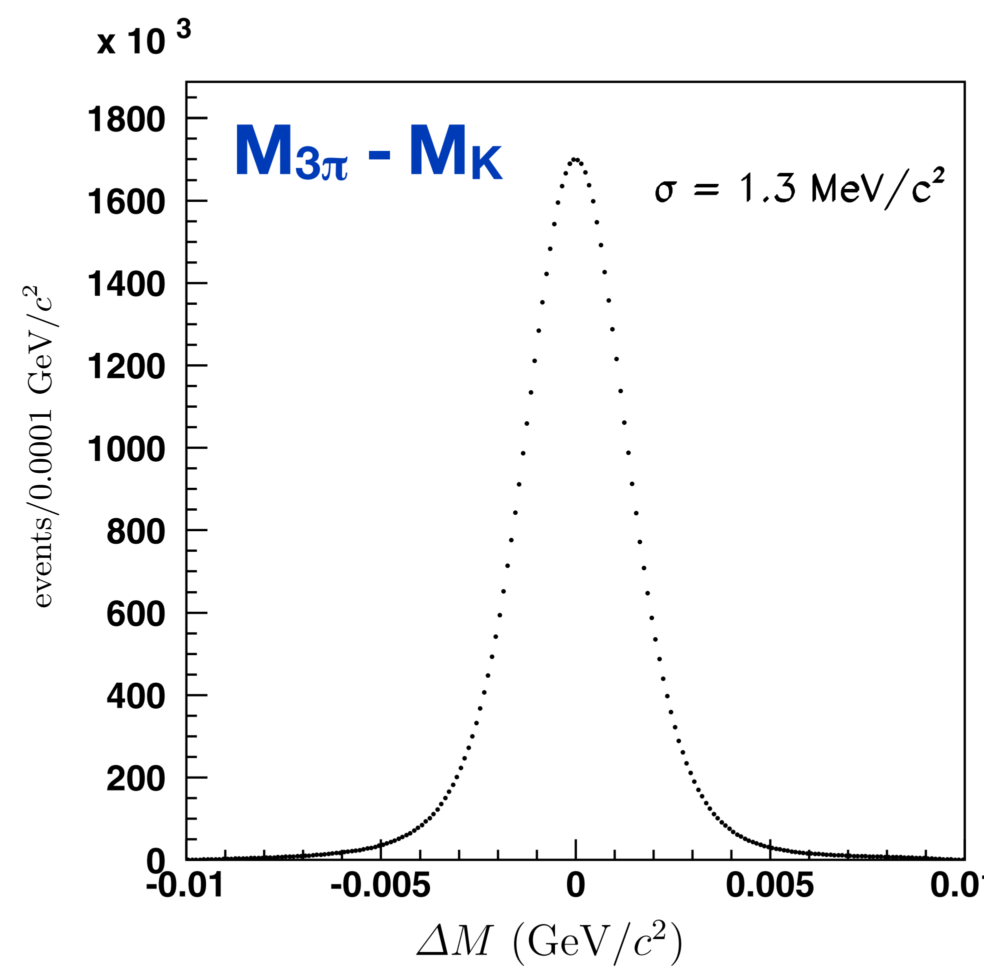
$K \to \pi^{\pm}\pi^0\pi^0$ selection:

- 1 charged track +
  4 e.m. calorimeter clusters
- $\pi^0 \to \gamma\gamma$ selection:
  consider all 3 pairings and
  minimize vertex difference Δz
- **invariant $\pi^0\pi^0$ mass $M(\pi^0\pi^0)$:**
  only calorimeter and vertex
  information used

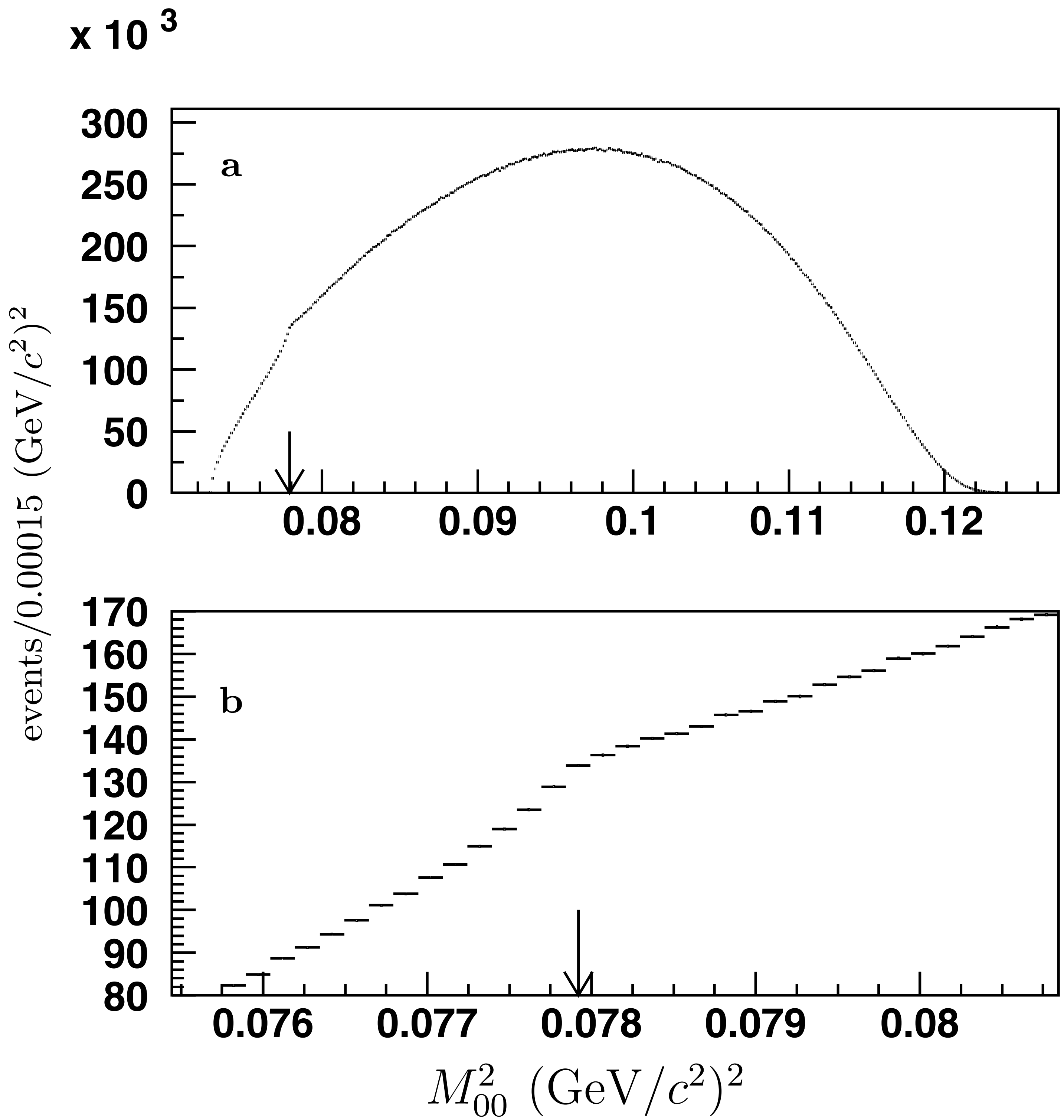
➜ **60 million events**

(mass resolution 1.3 MeV,
negligible background)





Cusp in $K^{\pm} \to \pi^{\pm}\,\pi^0\,\pi^0$ Decays



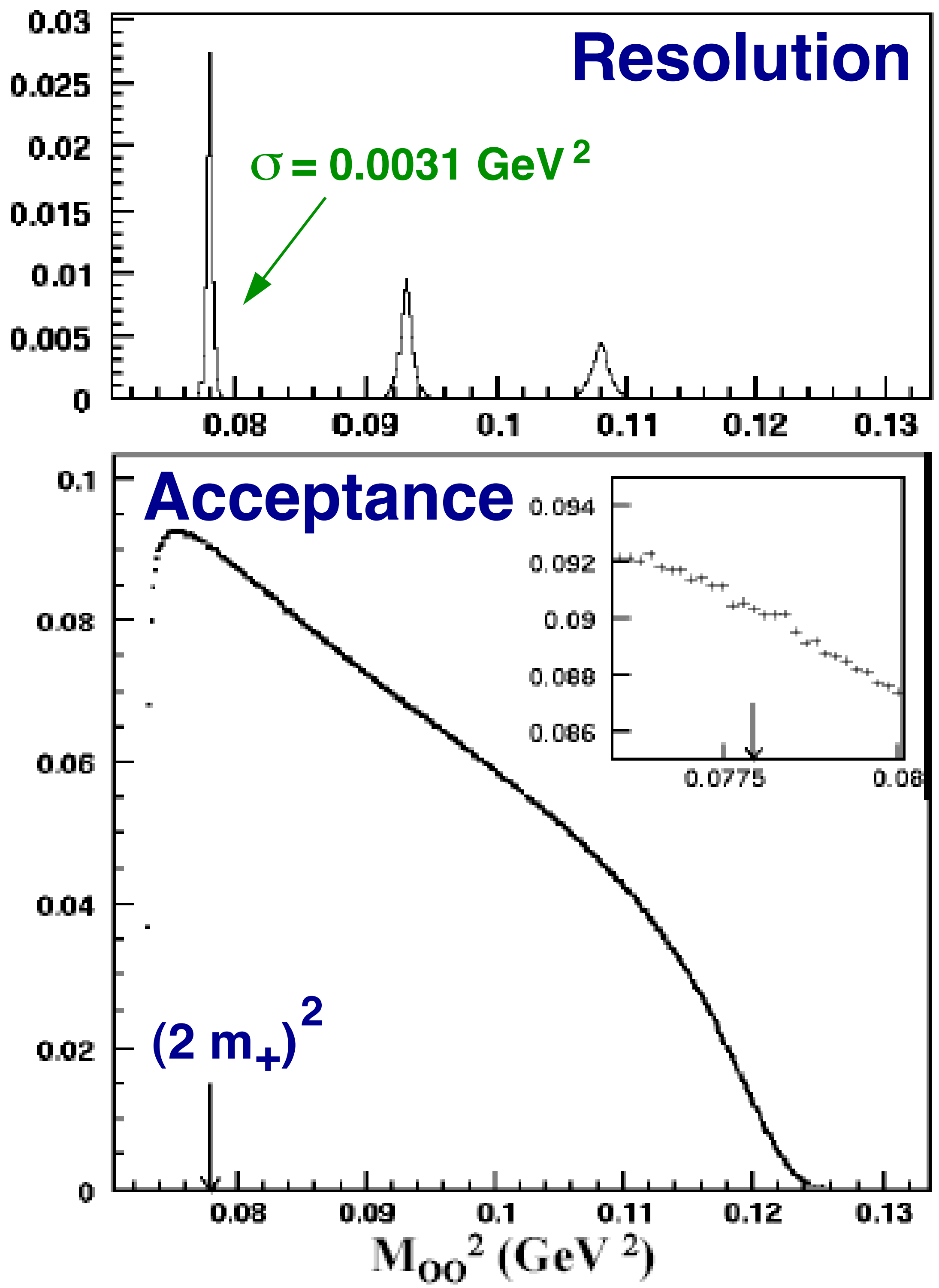
M($\pi^0\pi^0$) distribution:

Clear cusp at

M($\pi^0\pi^0$) = 2 m($\pi^{\pm}$)

(were expecting peak from pionium formation
$K \to \pi^{\pm}\,(\pi\pi)_{atom} \to \pi^{\pm}\,\pi^0\,\pi^0$)

$K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ Acceptance and Resolution



Resolution on $m^2_{\pi^0\pi^0}$:

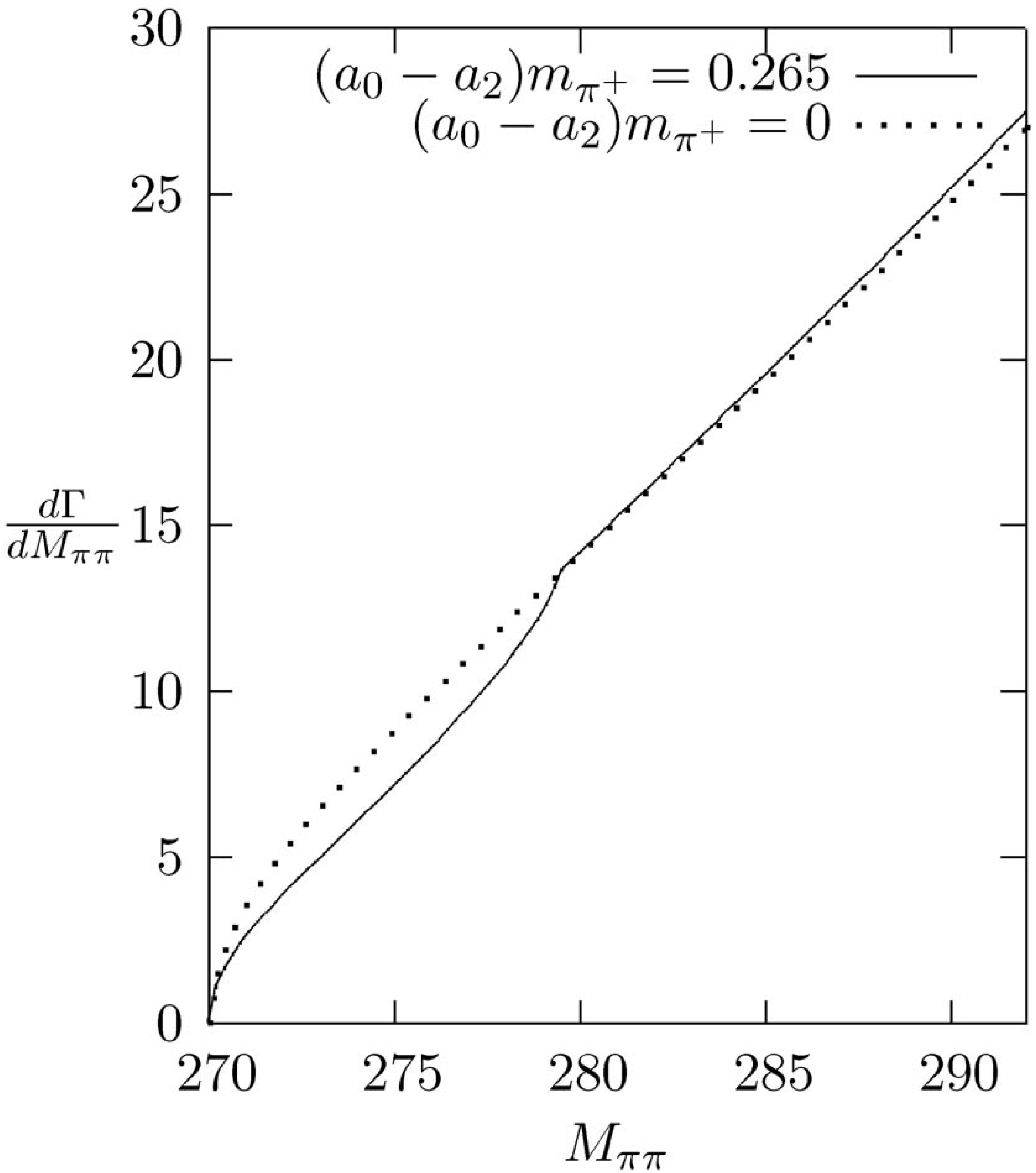
- Determined by **LKR resolution.**
- Best at low $m^2_{\pi^0\pi^0}$ (due to kinematical constraints).
- At $\mathbf{m^2_{\pi^0\pi^0} = (2\,m_{\pi^+})^2}$:

  $\sigma = 0.0031$ **GeV^2**

Acceptance:

- Acceptance $\approx$ linearly varying around $(2\,m_{\pi^+})^2$.
- Modelled by **Monte Carlo simulation.**

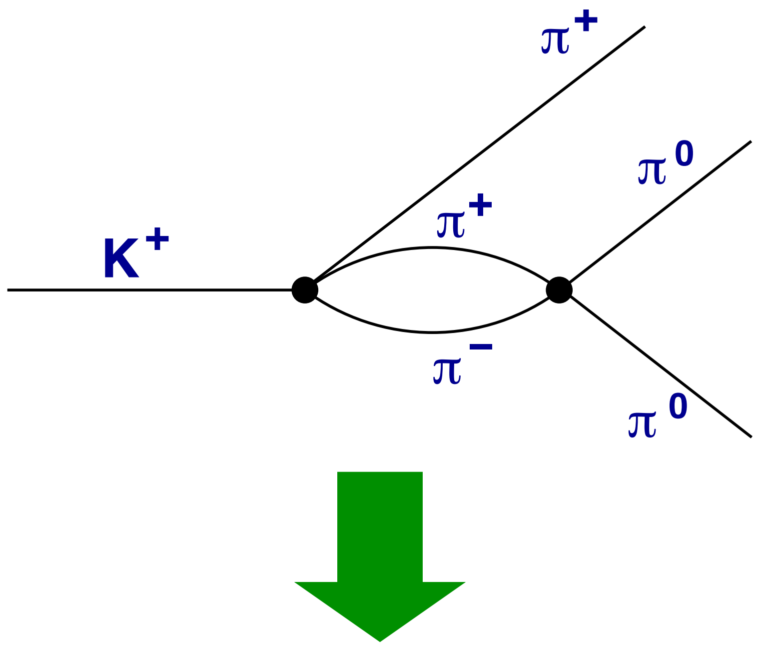
Cusp Explanation



N. Cabibbo (2004):

(PRL 93 (2004) 121801)

Rescattering of $K^+ \to \pi^+\pi^+\pi^-$ to $K^+ \to \pi^+\pi^0\pi^0$.



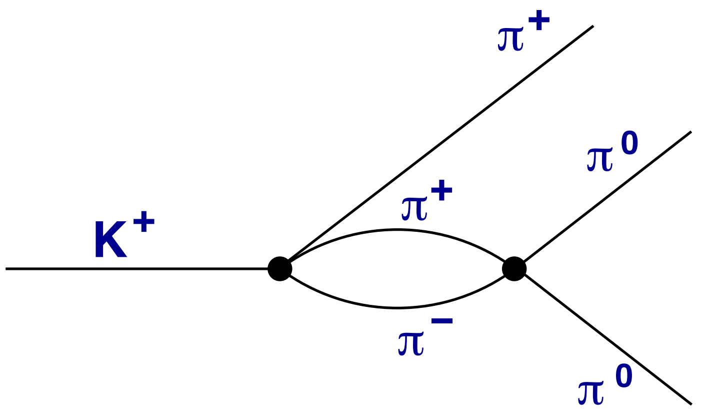
Measurement of $\pi\pi$ scattering length $a_0 - a_2$ possible!

(Effect already predicted much earlier: Budini, Fonda, PRL 6 (1961) 419)

a_0-a_2 from $K^\pm \to \pi^\pm \pi^0 \pi^0$ Decays

- **$K^\pm \to \pi^\pm\pi^0\pi^0$ decays:**

  $K^+ \to \pi^+\pi^+\pi^-$ amplitude
  contributes to $K^+ \to \pi^+\pi^0\pi^0$ via
  charge exchange $\pi^+\pi^- \to \pi^0\pi^0$.



- **Matrix element:** $\mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = \mathcal{M}_0 + \mathcal{M}_1$

  with: $\mathcal{M}_0 =$ "unperturbed" amplitude
  $\mathcal{M}_1 =$ rescattered amplitude

- **Above threshold:** $\boxed{s_\pi = m^2_{\pi^0\pi^0} > 4\, m^2_{\pi^+}}$ **$\mathcal{M}_1$ imaginary**

$$\mathcal{M}_1 = -\tfrac{2}{3}\,(\mathbf{a_0} - \mathbf{a_2})\, m_{\pi^+}\, \mathcal{M}^{\text{threshold}}_{K^+\to\pi^+\pi^+\pi^-} \cdot \pi \sqrt{4m^2_{\pi^+}/s_\pi - 1}$$

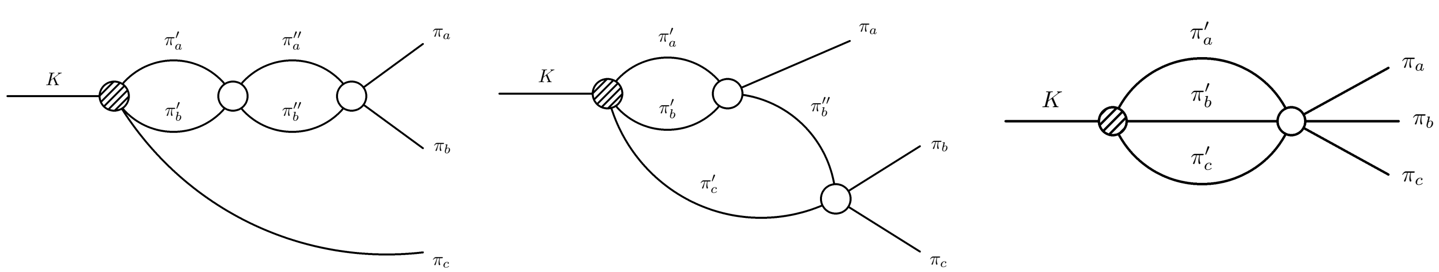
Below threshold: $\boxed{s_\pi < 4\, m^2_{\pi^+}}$ **$\mathcal{M}_1$ real and negative**

$$\mathcal{M}_1 = -\tfrac{2}{3}\,(\mathbf{a_0} - \mathbf{a_2})\, m_{\pi^+}\, \mathcal{M}^{\text{threshold}}_{K^+\to\pi^+\pi^+\pi^-} \cdot (-i)\pi \sqrt{1 - 4m^2_{\pi^+}/s_\pi}$$

Theoretical Approach (CI)

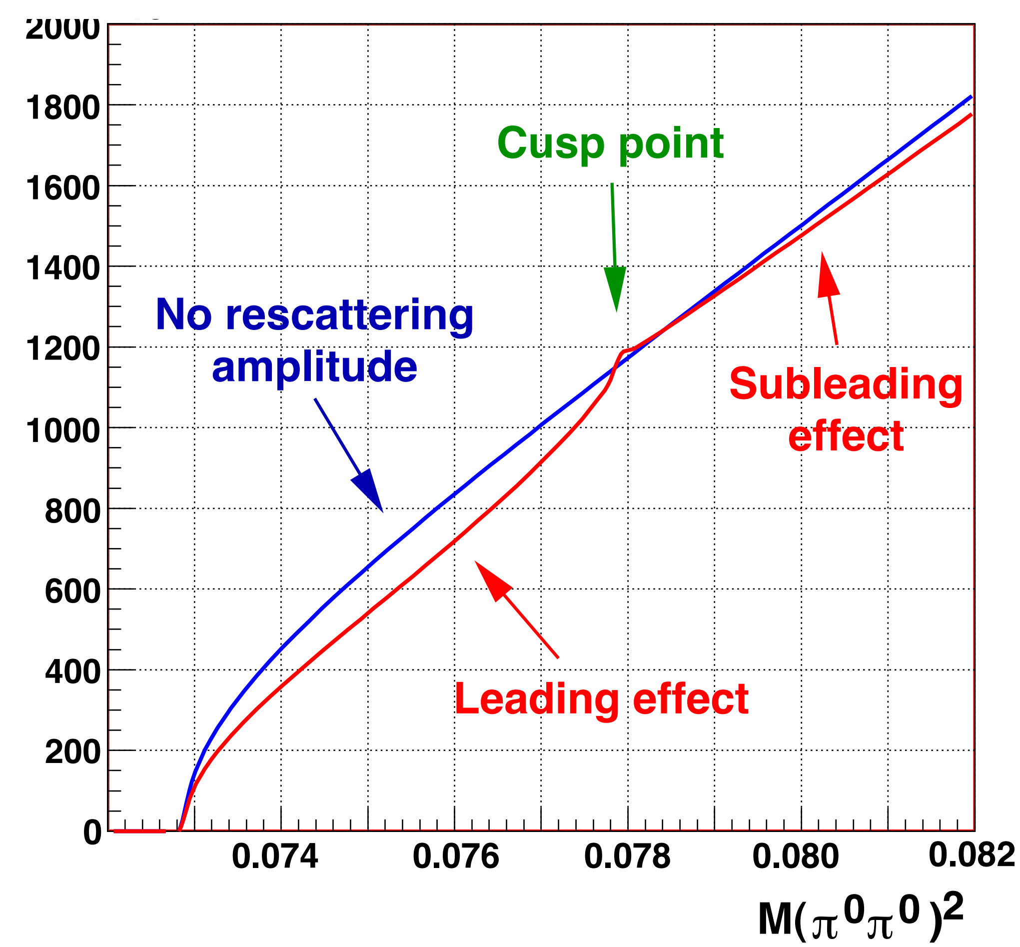
Second order computations: (Cabibbo, Isidori, JHEP03 (2005) 21)

- Other rescattering corrections: $\pi^0\pi^0 \to \pi^0\pi^0,\ \pi^+\pi^0 \to \pi^+\pi^0, \ldots$
- Two-loop level $\mathcal{O}(a_i^2)$ corrections:



$\Longrightarrow$ Separate determination of a_2 possible.

Also: Coverage of all $K \to \pi\pi\pi$ **decays** (e.g. $K_L \to \pi^0\pi^0\pi^0$).



- $\mathcal{O}(a_i^3)$ corrections.
- Radiative corrections.

$\Longrightarrow$ About 5% uncertainty on $a_0 - a_2$.

Theoretical Approach (BB)

Approach by the Bern-Bonn group:

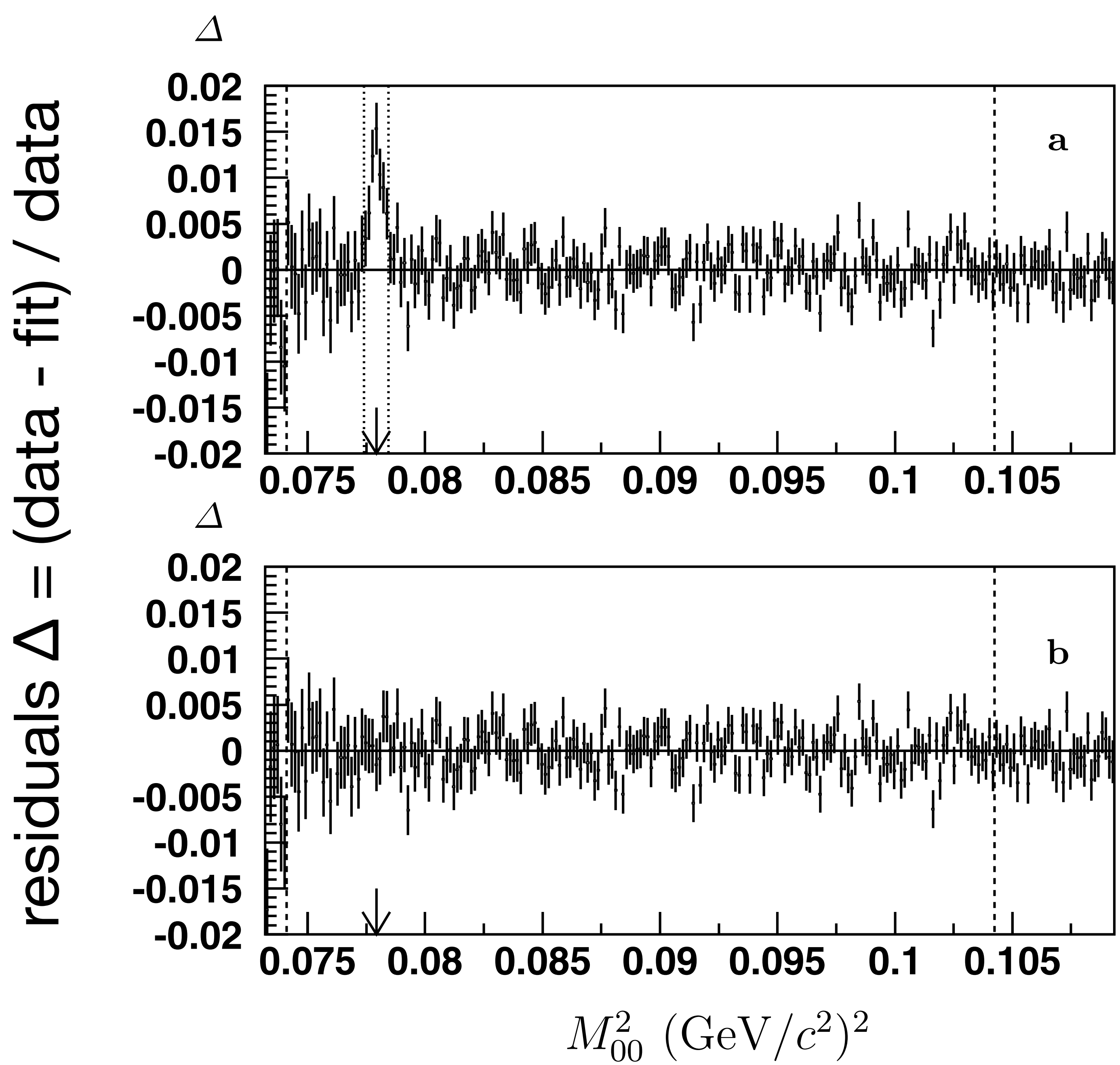
(Colangelo, Gasser, Kubis, Rusetsky, PLB 638 (2006) 187;
Bissinger, Fuhrer, Gasser, Kubis, Rusetsky, PLB 659 (2008) 576; NPH B806 (2009) 178)

- based on an effective non-relativistic lagrangian
- different structure of the expansion (w.r.t. CI)
- simultaneous fitting of neutral and charged amplitudes to extract Dalitz plot slope parameters (modified w.r.t. PDG parametrization)
- electromagnetic effects and radiative corrections outside the cusp point are included

➜ **provides so far most complete description of rescattering effect**

Fit to the $M(\pi^0\pi^0)$ Spectrum (BB Model)

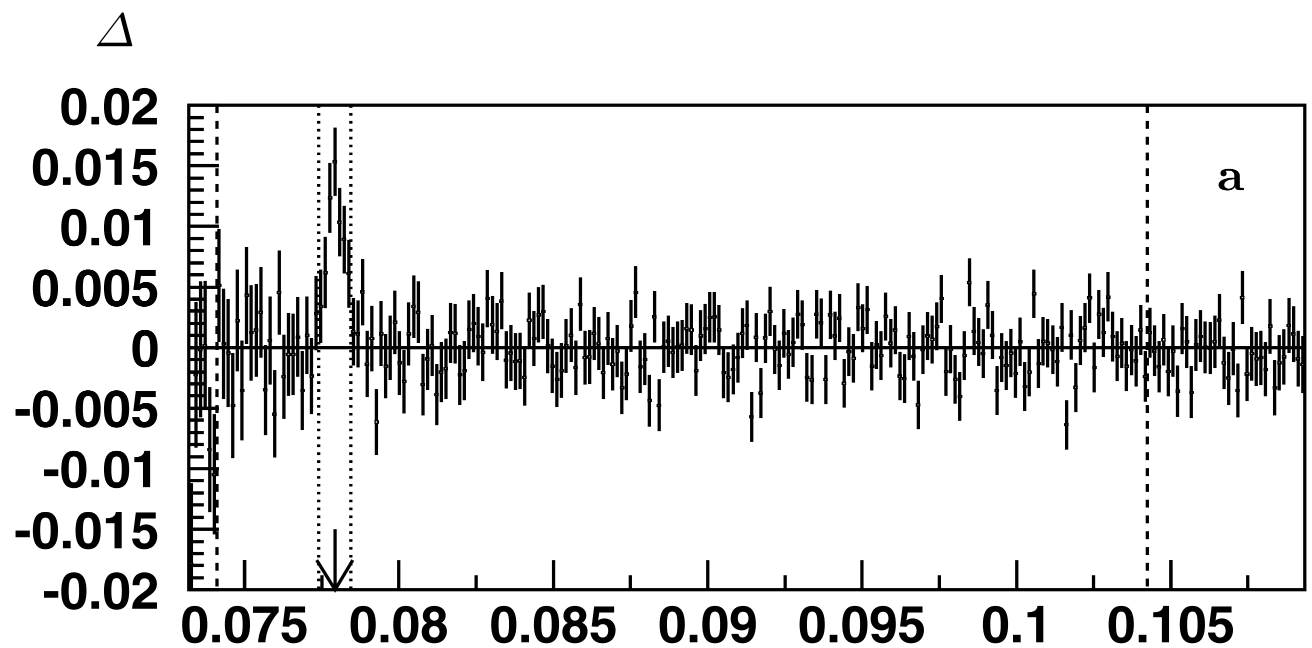
Free fit parameters: a_0-a_2, a_2, Dalitz plot parameters, normalizations
(fit also includes $K \to \pi^{\pm}\pi^{+}\pi^{-}$ decays)



7 bins around
cusp/pionium excluded

pionium fraction f_{atom}
left free in the fit

Brief Excursion: Pionium



Pionium $\pi^+\pi^-$ atoms decaying to $\pi^0\pi^0$?

- Observe an excess of events at cusp point $m_{\pi^0\pi^0} = 2m_{\pi^+}$ of $(\mathbf{1.8 \pm 0.2}) \times \mathbf{10^{-5}}$ per $K^\pm \to \pi^\pm\pi^+\pi^-$ decay.
- Prediction for pionium formation in $K^\pm \to \pi^\pm\pi^+\pi^-$ (Silagadze, '94): $\mathbf{0.8 \times 10^{-5}}$
- But: have to take electromagn. corrections to FSI into account!
  $\Longrightarrow$ $\pi^+\pi^-$ unbound states with resonant structure.
  (Gevorkian, Tarasov, Voskresenskaya, PLB 649 (2007) 159)

Fit Results on $a_0 - a_2$ and a_2

fit	χ^2/ndf	a_0-a_2	a_2	f_{atom}
CI	206.3/195	0.2727(46)	-0.0392(80)	0.0533(91)
CI (a)	201.6/189	0.2689(50)	-0.0344(86)	0.0533
CI (c)	210.6/196	0.2749(21)	-0.0413	0.0441(76)
CI (a,c)	207.6/190	0.2741(21)	-0.0415	0.0441
BB	462.9/452	0.2815(43)	-0.0693(136)	0.0530(95)
BB (a)	458.5/446	0.2775(48)	-0.0593(142)	0.0542
BB (c)	467.3/453	0.2737(26)	-0.0417	0.0647(76)
BB (a,c)	459.8/447	0.2722(27)	-0.0421	0.0647
CI	205.6/195	0.2483(45)	-0.0092(91)	0.0625(92)
CI (a)	202.9/189	0.2461(49)	-0.0061(98)	0.0625
CI (c)	222.1/196	0.2646(21)	-0.0443	0.0420(77)
CI (a,c)	219.7/190	0.2645(22)	-0.0444	0.0420
BB	**477.4/452**	**0.2571(48)**	**-0.0241(129)**	**0.0631(97)**
BB (a)	474.4/446	0.2544(51)	-0.0194(132)	0.0631
BB (c)	479.8/453	0.2633(24)	-0.0447	0.0538(77)
BB (ac)	478.1/447	0.2627(25)	-0.0449	0.0538

Rad. corr. off

a: pionium f_{atom} fixed

c: with ChPT constraint

a,c: both

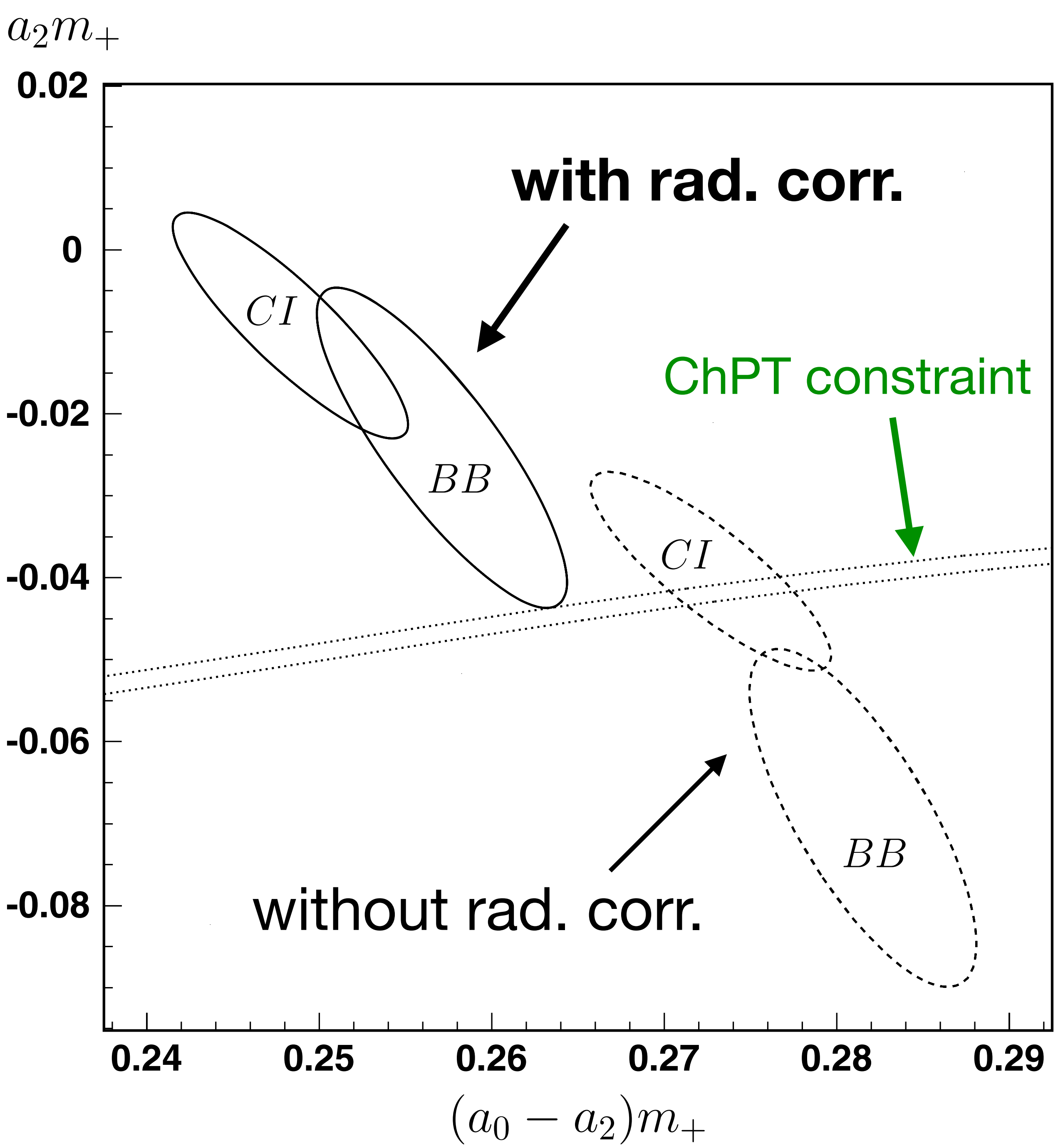
(...): statistical error

Rad. corr. on

← **final result**

(f_{atom}, a_0-a_2, a_2 free in the fit)

Cusp Results on $a_0 - a_2$ and a_2



(only statistical uncertainties shown)

Final result: (EPJC 64 (2009) 589)

$a_0 - a_2 = 0.257(5)_{stat}(3)_{sys}(1)_{ext}$

$a_2 = -0.024(13)_{stat}(9)_{sys}(2)_{ext}$

(statistical correlation -0.839)

With ChPT constraint:

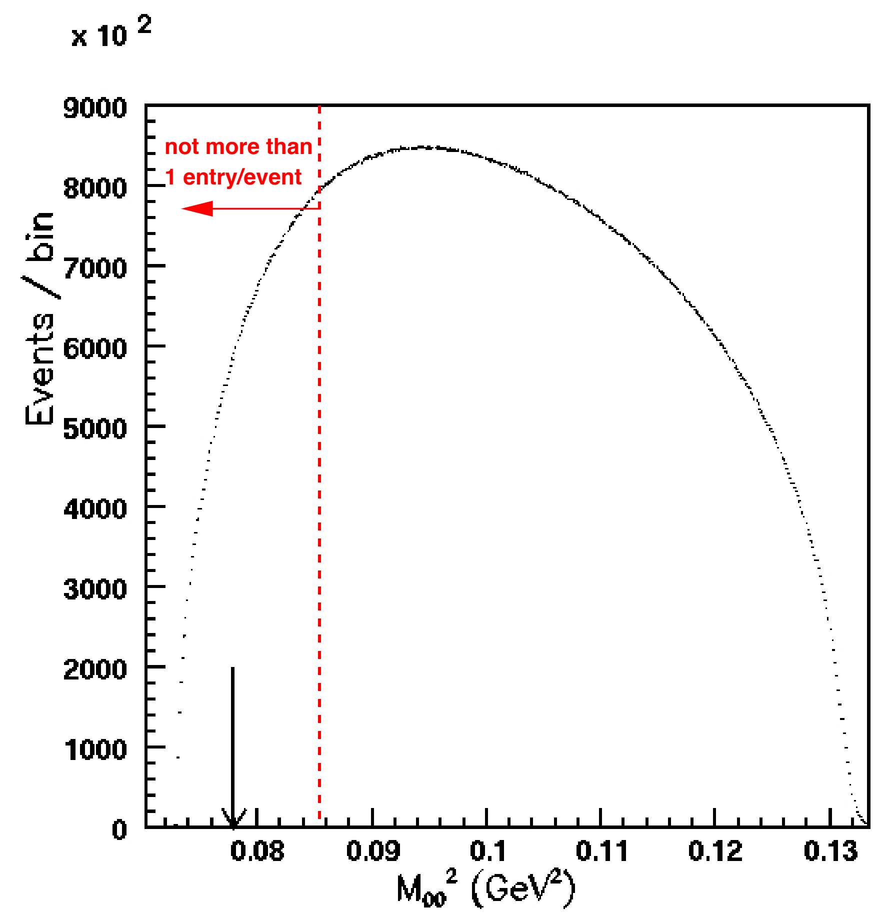
$a_0 - a_2 = 0.2633(24)_{stat}(14)_{sys}(19)_{ext}$

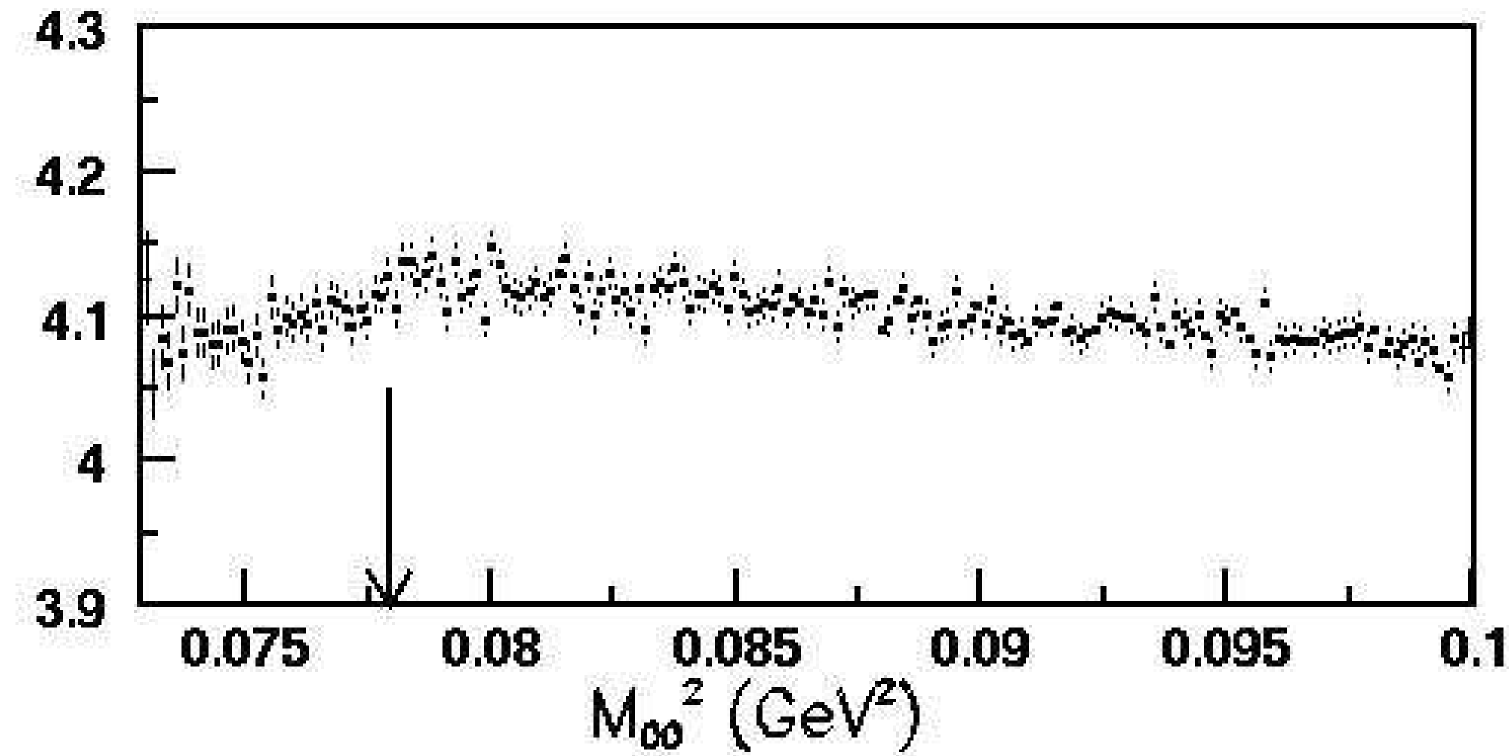
ChPT prediction:

$a_0 - a_2 = 0.265(4)$

Rescattering in $K_L \rightarrow \pi^0\pi^0\pi^0$ (NA48/2)

NA48/2 data taking in 2000:

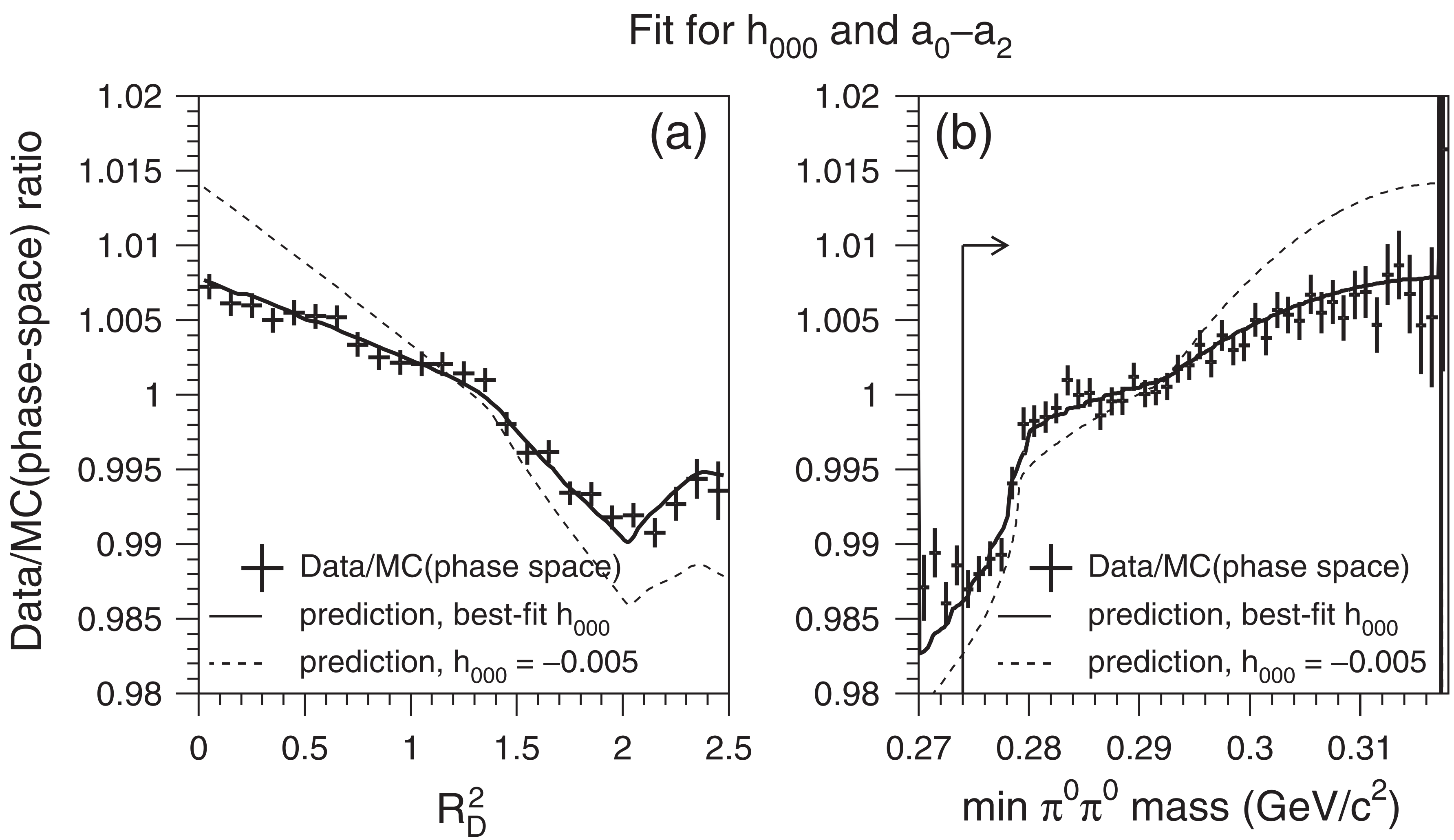




$\Longrightarrow$ **Evidence for a change in slope near the cusp point.**

Rescattering in $K_L \to \pi^0\pi^0\pi^0$ (KTeV)

(PRD 78 (2008) 032009)



$$m_{\pi^+}(a_0 - a_2) = 0.215 \pm 0.014_{\text{stat}} \pm 0.025_{\text{syst}} \pm 0.006_{\text{ext}}$$

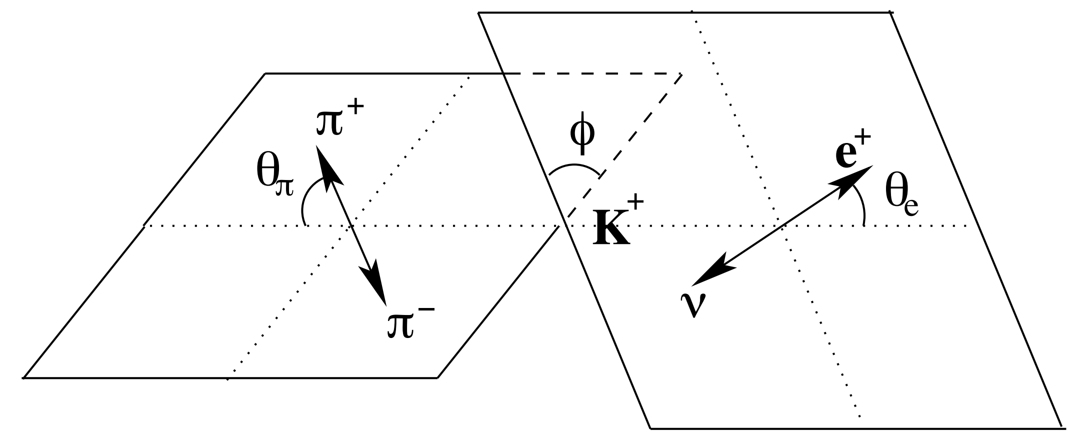
$$h_{000} = (-2.09 \pm 0.62_{\text{stat}} \pm 0.72_{\text{syst}} \pm 0.28_{\text{ext}}) \times 10^{-3}$$

Scattering Lengths from $K^{\pm} \to \pi^{+} \pi^{-} e^{\pm} \nu$ Decays

$\pi\pi$ Scattering Lengths from K_{e4} Decays

K_{e4} decay:

- $\mathbf{K_{e4} = K^{\pm} \rightarrow \pi^{+} \pi^{-} e^{\pm} \nu}$
- Very rare: **$Br(K_{e4}) \sim 4 \times 10^{-5}$**

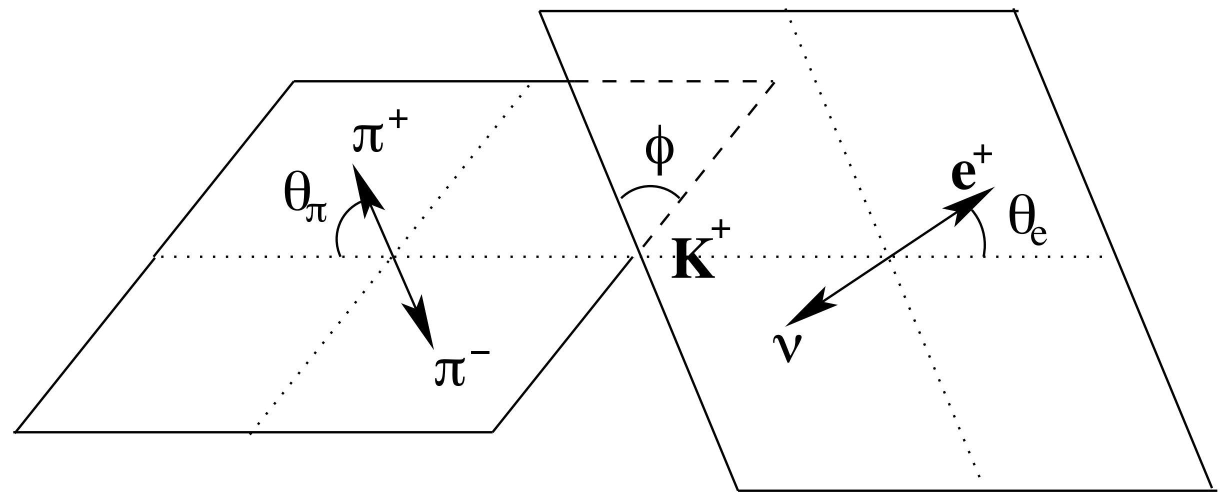


Measurement of $\pi\pi$ scattering in $\mathrm{K_{e4}}$:

- K_{e4} decay amplitude depends on two complex phases:
  - $\delta_{\mathbf{0}} = \pi\pi$ scattering phase shift for $I = 0,\ l = 0$ (S-wave)
  - $\delta_{\mathbf{1}} = \pi\pi$ scattering phase shift for $I = 1,\ l = 1$ (P-wave)

  ($I = 2$ suppressed by $\Delta I = \frac{1}{2}$ rule)
- Decay rate depends on difference $\delta = \delta_{\mathbf{0}} - \delta_{\mathbf{1}}$, with $\delta = \delta(m_{\pi\pi})$.
- $\delta \neq \mathbf{0}$ implies asymmetric distribution of lepton w.r.t. $\pi\pi$ plane.

Kinematic Variables in K_{e4}



$\mathbf{K_{e4}}$ is 4-body decay $\Longrightarrow$ 5 independent kinematic variables.
(Cabibbo-Maksymowicz variables)

$\mathbf{s_\pi = M^2_{\pi\pi}}$	Invariant di-pion mass squared.
$\mathbf{s_e = M^2_{e\nu}}$	Invariant di-lepton mass squared.
θ_π	Angle of π^+ w.r.t. $\pi\pi$ direction of flight in $\pi\pi$ rest frame.
$\theta_\mathbf{e}$	Angle of e^+ w.r.t. $e\nu$ direction of flight in $e\nu$ rest frame.
ϕ	Angle of $\pi\pi$ decay plane w.r.t. $e\nu$ decay plane.

K_{e4} Selection

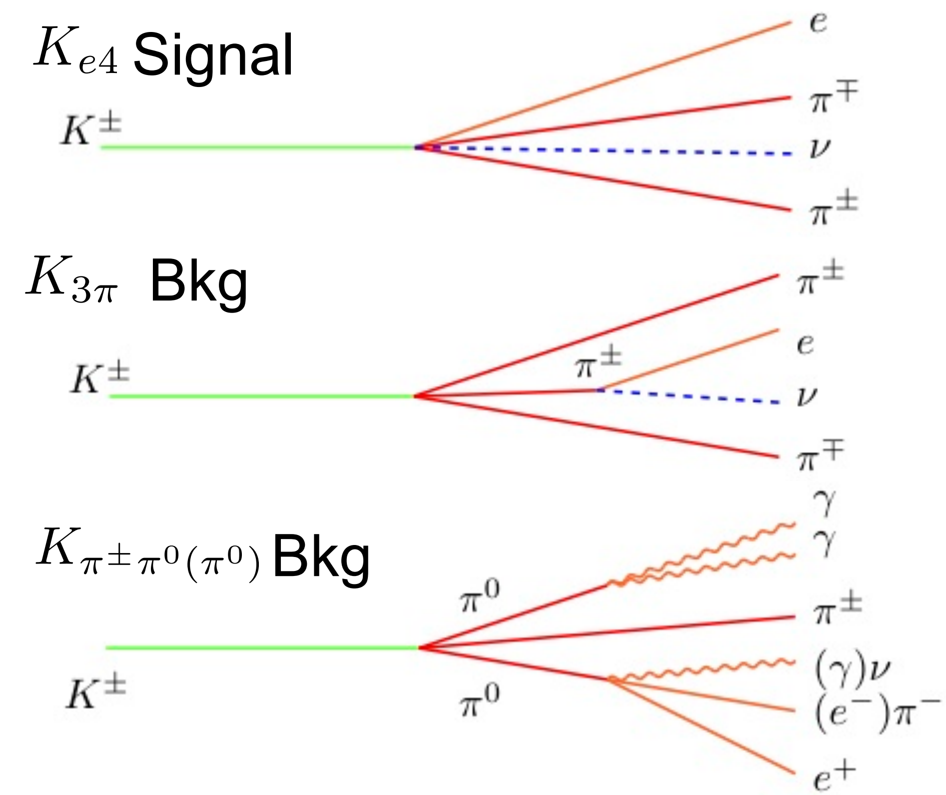
K_{e4} selection:

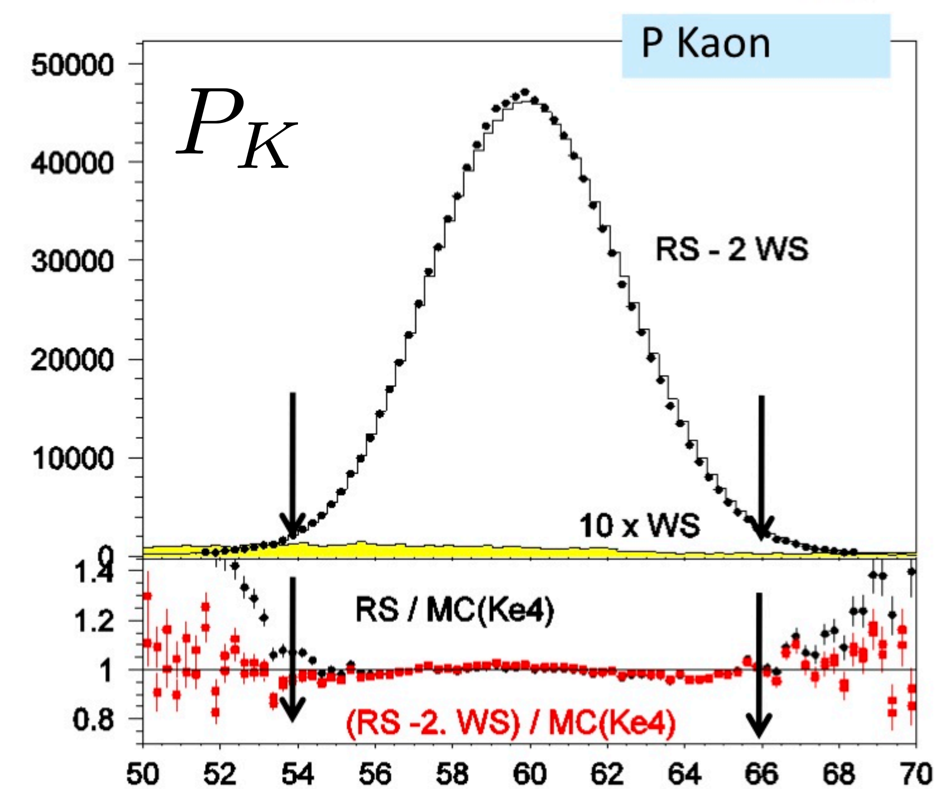
- 3 charged tracks and 1 good vertex
- 2 opposite-sign pions, 1 electron (E/p ~ 1)
- missing transverse momentum
- kaon momentum close to 60 GeV/c

Background:

- $\mathbf{K^{\pm} \rightarrow \pi^{\pm} \pi^{+} \pi^{-}}$ with $\pi \rightarrow e\nu$ or mis-identified pion
- $\mathbf{K^{\pm} \rightarrow \pi^{\pm} \pi^{0} (\pi^{0})}$ with $\pi^{0} \rightarrow e^{+} e^{-} \nu$ and mis-identified electron
- **Background estimation** from wrong-sign $\pi^{+} \pi^{+} e^{-}$ events

➜ **Background ~ 0.6 %**





K_{e4} Fitting Procedure

- **Full event sample** (2003+2004): **1.13 million K_{e4} decays**
- **Fit** in ***iso-populated boxes*** in the 5-dim. CM variables:

$$\mathbf{10}(\mathrm{M}_{\pi\pi}) \times \mathbf{5}(\mathrm{M}_{\mathbf{e}\nu}) \times \mathbf{5}(\cos\theta_{\mathbf{e}}) \times \mathbf{5}(\cos\theta_{\pi}) \times \mathbf{12}(\phi) = \mathbf{15000}\ \mathrm{Boxes}$$

- Assuming constant form factors, K^+ and K^- samples fitted separately in **10 independent $M_{\pi\pi}$ bins** and then combined in each $M_{\pi\pi}$ bin.

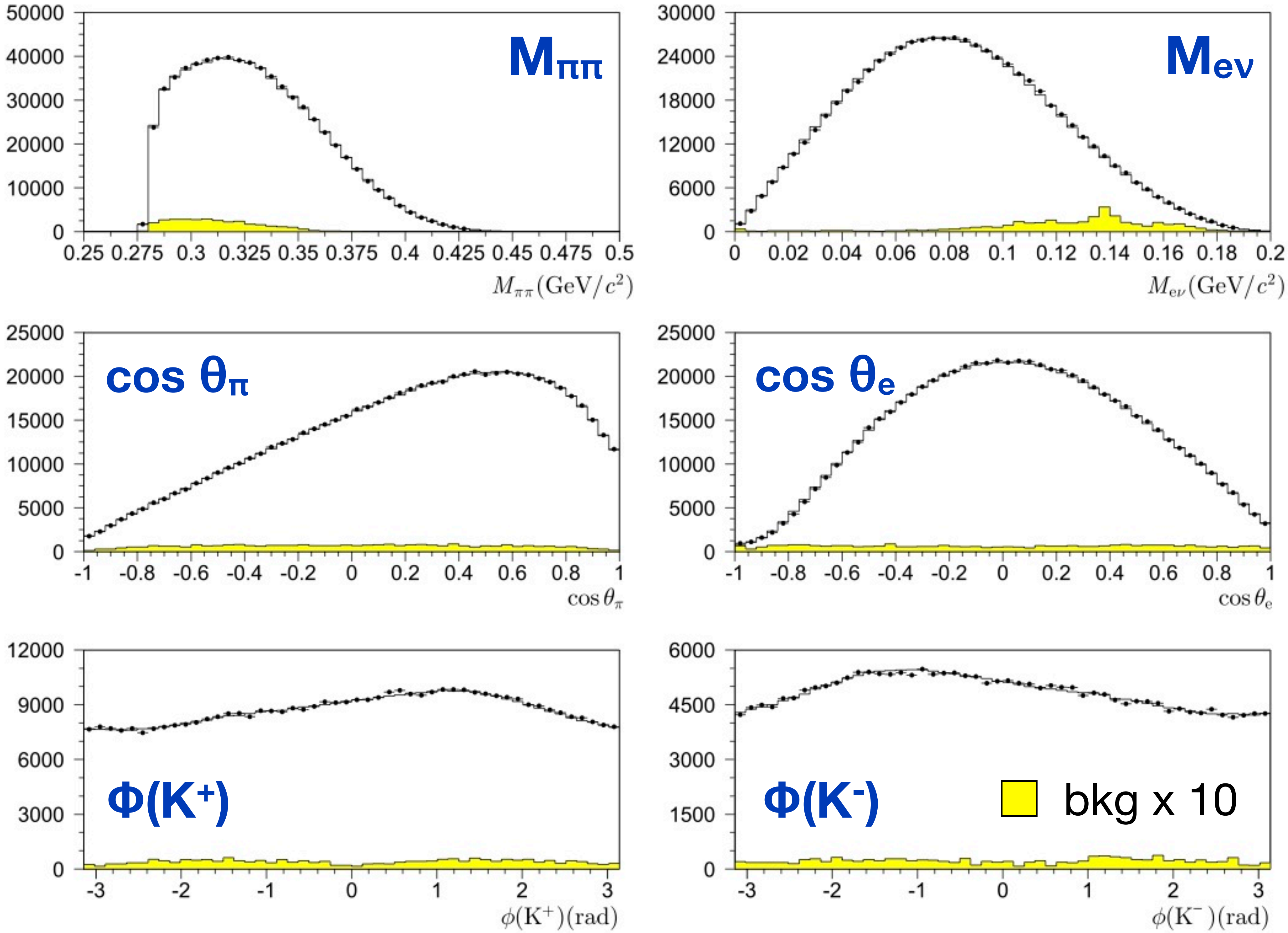
Data:

K^+ sample:	726 400 events	→	48 events/box
K^- sample:	404 400 events	→	27 events/box

MC:

K^+ sample:	17.4×10^6 events	→	1160 events/box
K^- sample:	9.7×10^6 events	→	650 events/box

K_{e4} Fit Results



K_{e4} Form Factor Results

Partial wave expansion of form factors:

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} \cos\theta_\pi + d \text{ wave} \ldots$$
$$G = G_p e^{i\delta_g} + d \text{ wave} \ldots$$
$$H = H_p e^{i\delta_h} + d \text{ wave} \ldots$$

Single form factors parametrized in Taylor expansion:

$$F_s = f_s + f_s' q^2 + f_s'' q^4 + f_e' S_e / 4m_\pi^2 + \ldots$$
$$F_p = f_p + f_p' q^2 + \ldots$$
$$G_p = g_p + g_p' q^2 + \ldots$$
$$H_p = h_p + h_p' q^2 + \ldots$$
$$\delta(q^2) = \delta_s - \delta_p \,.$$

$$q^2 = (S_\pi / 4m_\pi^2) - 1$$

- All form factors measured w.r.t. f_s
- Systematics from acceptance and background control

$$
\begin{aligned}
f_s'/f_s &= 0.152 \pm 0.007_{\text{stat}} \pm 0.005_{\text{syst}} \\
f_s''/f_s &= -0.073 \pm 0.007_{\text{stat}} \pm 0.006_{\text{syst}} \\
f_e'/f_s &= 0.068 \pm 0.006_{\text{stat}} \pm 0.007_{\text{syst}} \\
f_p/f_s &= -0.048 \pm 0.003_{\text{stat}} \pm 0.004_{\text{syst}} \\
g_p/f_s &= 0.868 \pm 0.010_{\text{stat}} \pm 0.010_{\text{syst}} \\
g_p'/f_s &= 0.089 \pm 0.017_{\text{stat}} \pm 0.013_{\text{syst}} \\
h_p/f_s &= -0.398 \pm 0.015_{\text{stat}} \pm 0.008_{\text{syst}}
\end{aligned}
$$

➜ **First evidence of non-zero f'_e and f_p !**

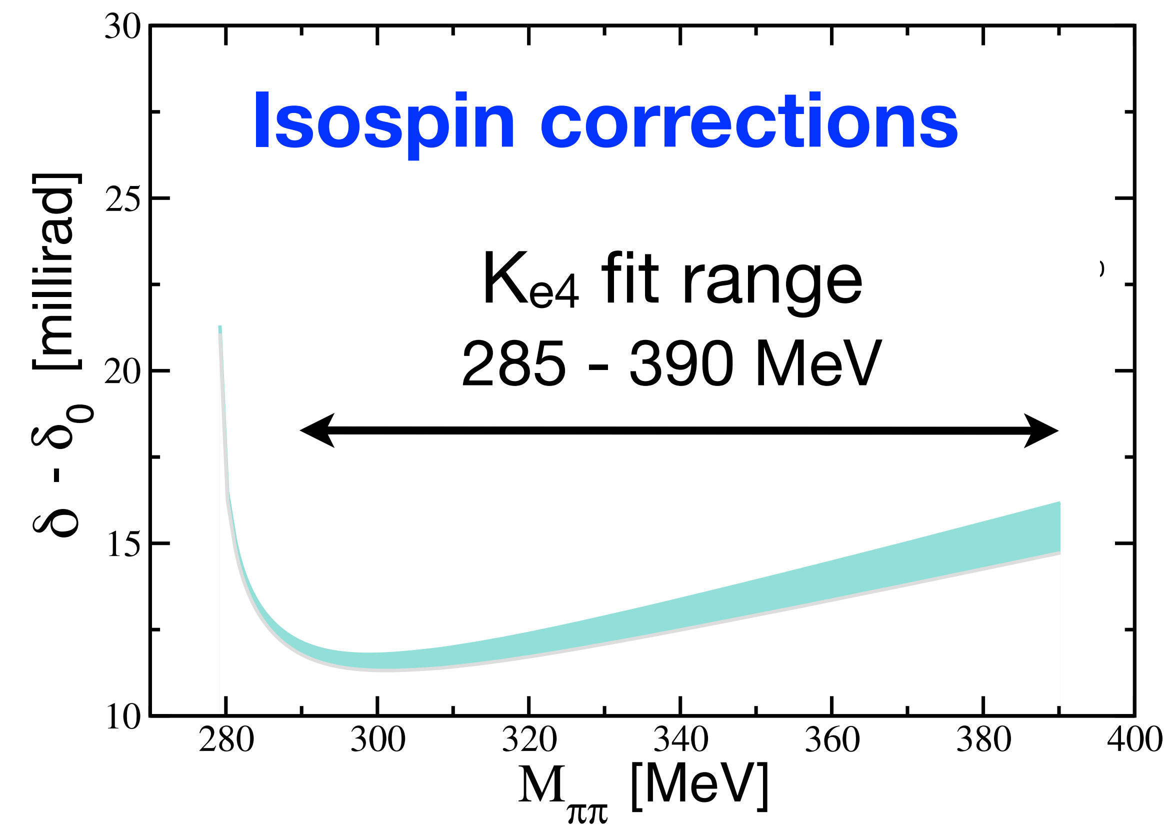
From phase shifts to scattering lengths

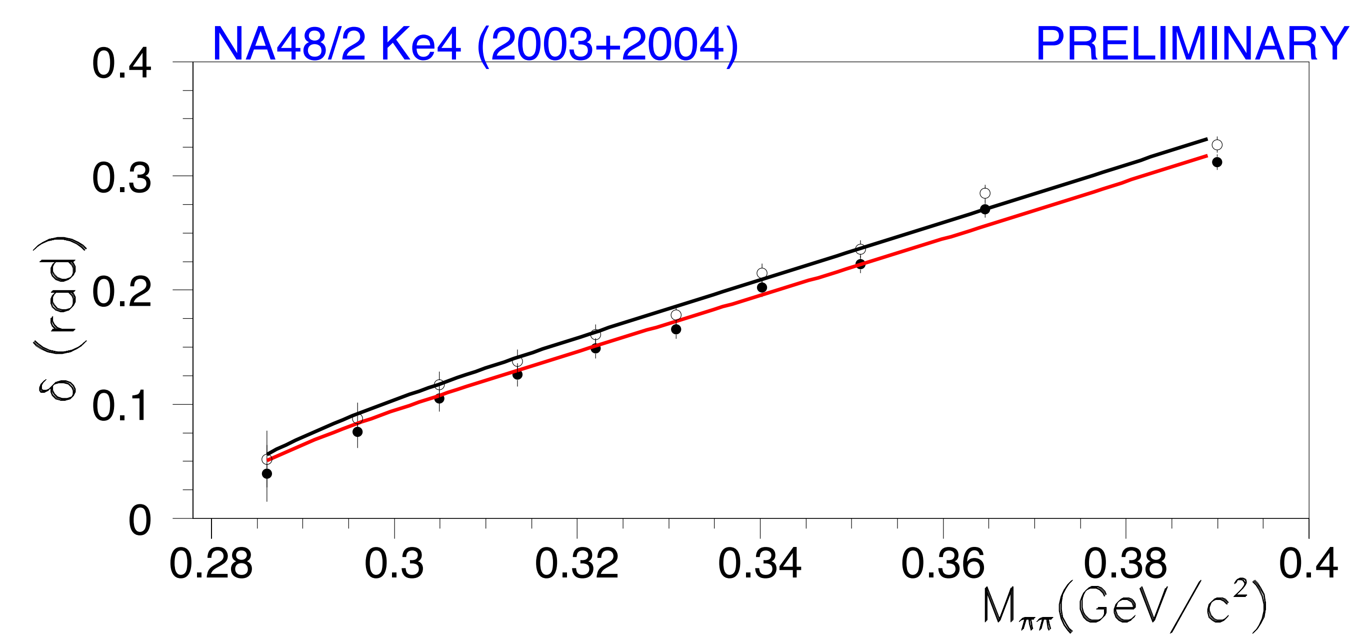
Corrections to be applied:

- **Radiative effects:**
  Included in the simulation (Coulomb attraction, IB).

- **Mass effects:**
  Isospin corrections have to be applied to δ.
  Developed in close collaboration with NA48/2.
  (Colangelo, Gasser, Rusetsky, EPJC 59 (2009) 777)

Effect of 10-15 mrad on δ
(stat. precision ~7-8 mrad)





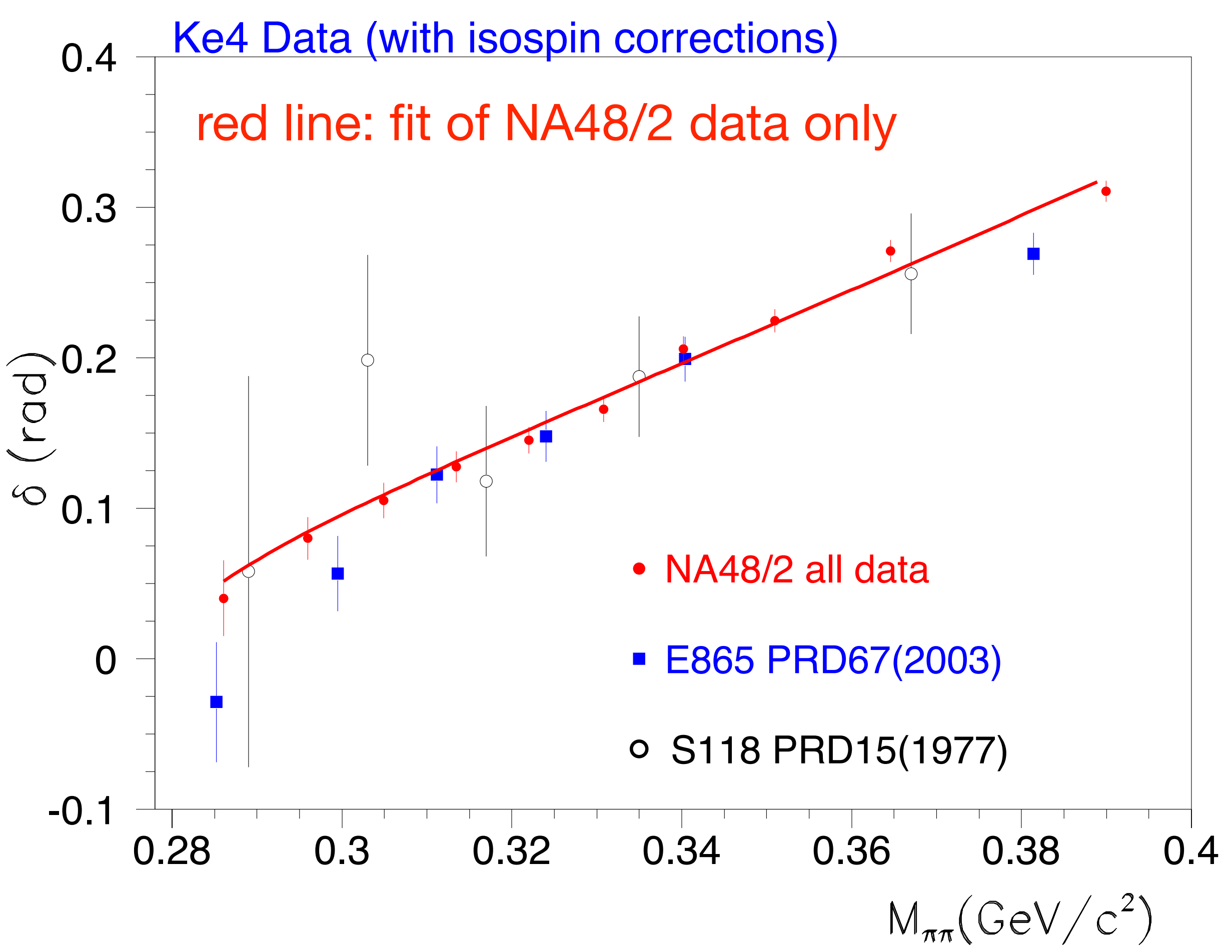
NA48/2 Fit of Phase Shift $\delta = \delta_0 - \delta_1$

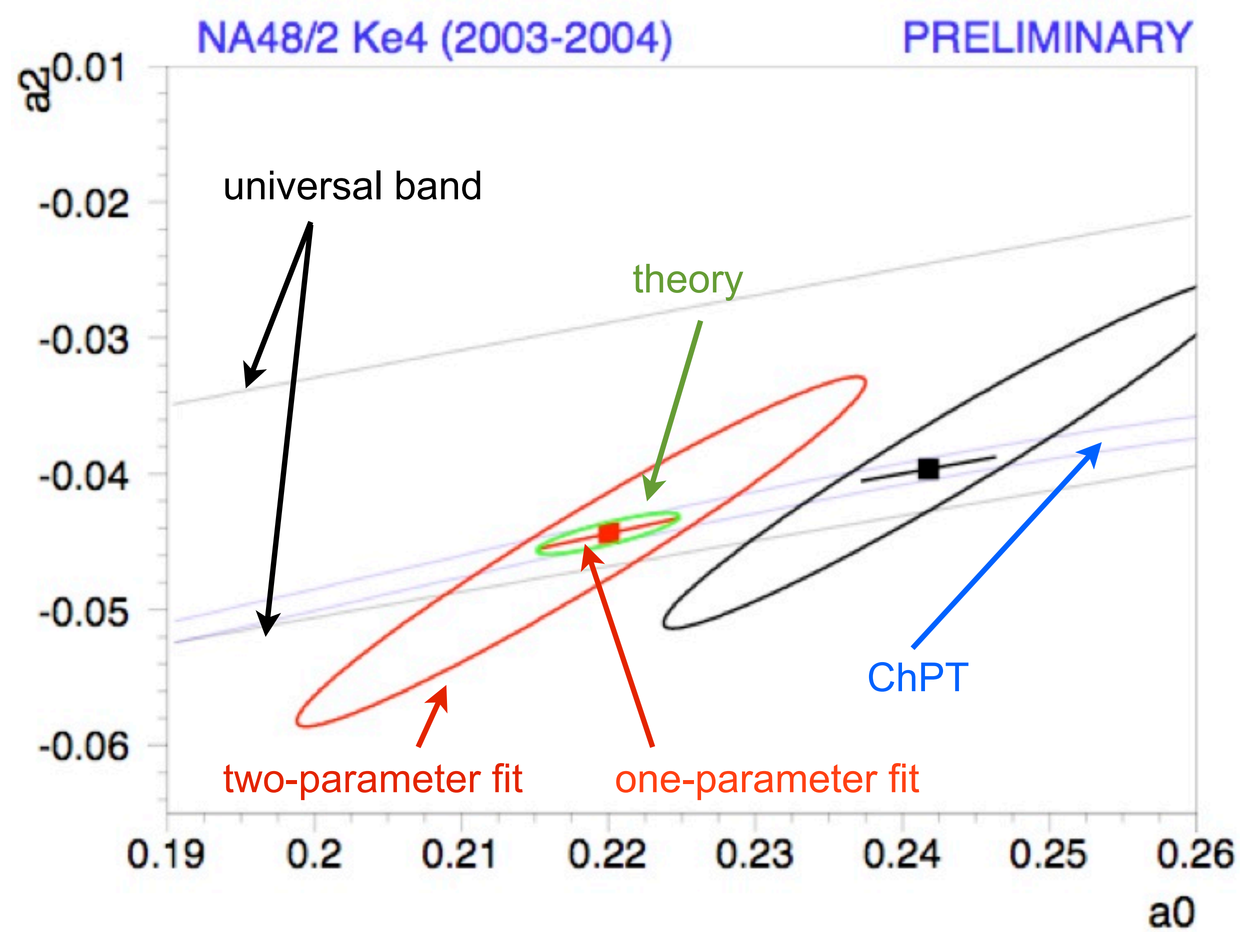
Two-parameter fit:

$$\begin{aligned} a_0 &= 0.2220 \pm 0.0128_{\rm stat} \pm 0.0050_{\rm syst} \pm 0.0037_{\rm theo} \\ a_2 &= -0.0432 \pm 0.0086_{\rm stat} \pm 0.0034_{\rm syst} \pm 0.0028_{\rm theo} \end{aligned}$$

One-parameter fit:
(with ChPT constraint)

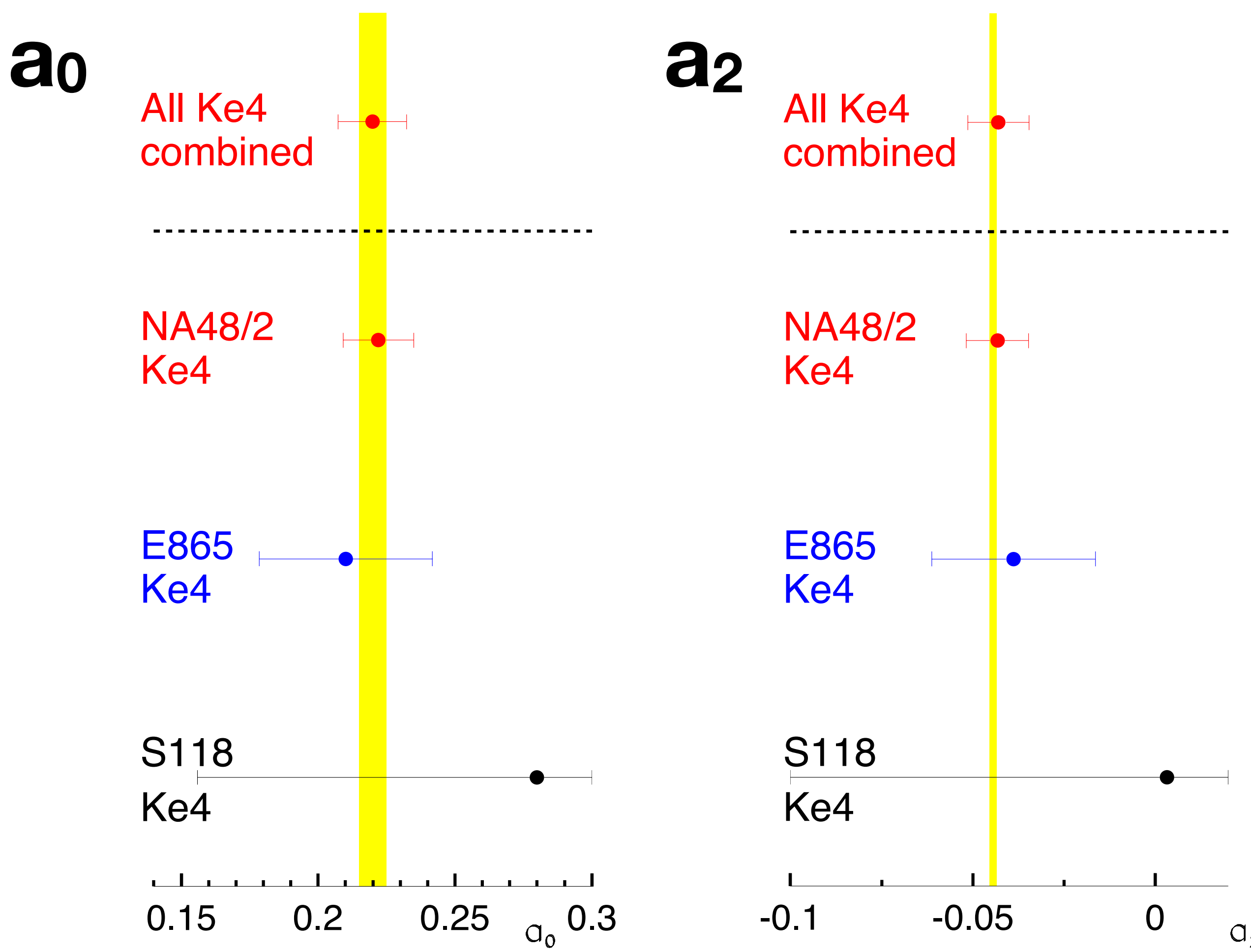
$$a_0 = 0.2206 \pm 0.0049_{\rm stat} \pm 0.0018_{\rm syst} \pm 0.0064_{\rm theo}$$





Comparison with previous K_{e4} Results

Comparison of **K_{e4} measurements**
(without ChPT constraint, old experiments isospin corrected):



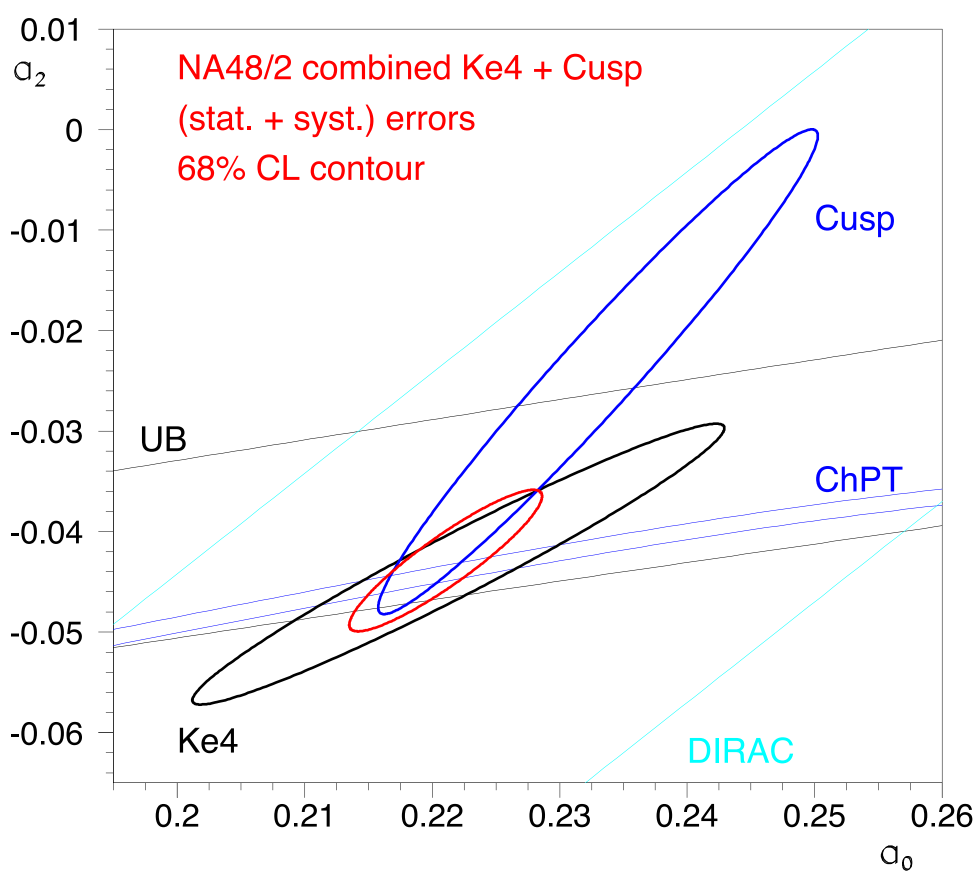
- NA48/2 dominates K_{e4} measurements
- **Perfect agreement with ChPT prediction!**

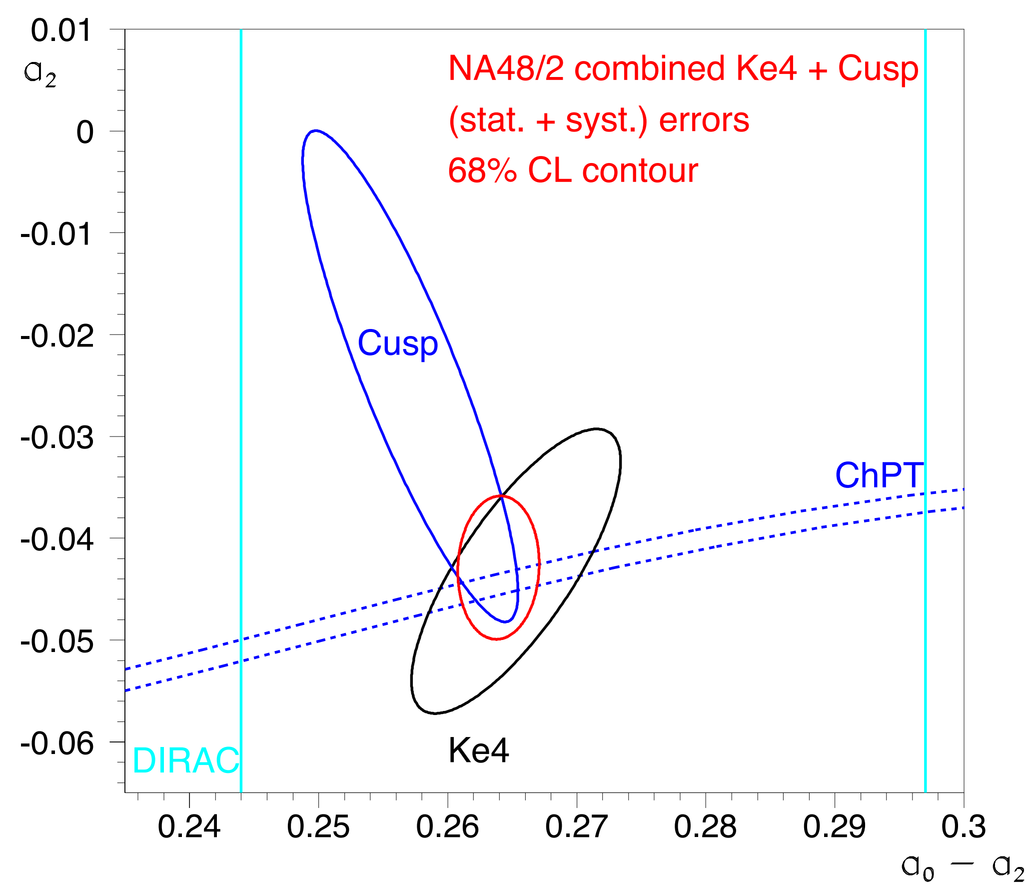
Yellow lines: Theory prediction
(*not* combination of measurements)

Combination of Cusp and K_{e4}

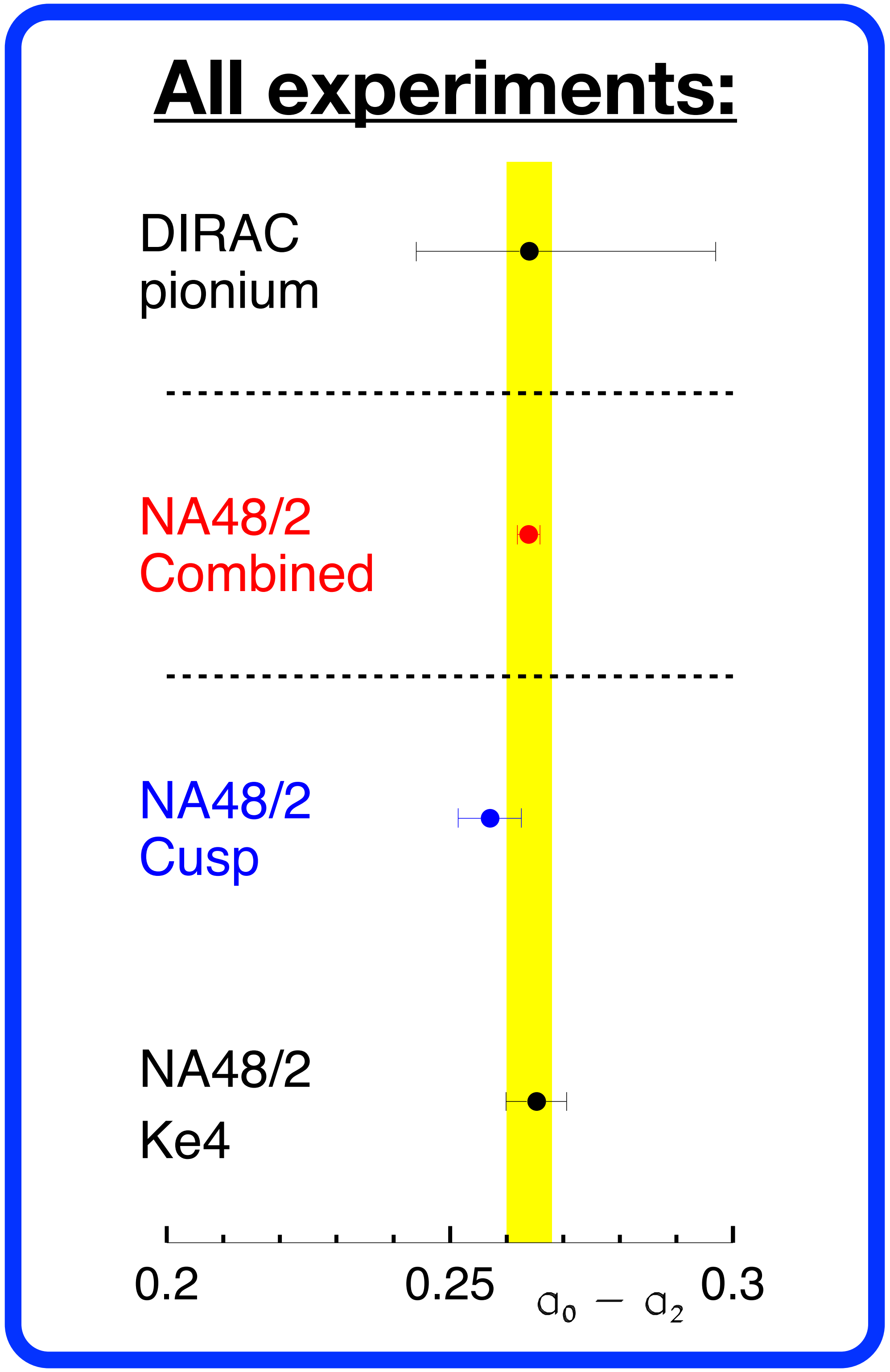
Two independent measurements with different samples, different systematics and different theory:

$$\begin{aligned} a_0 &= 0.2210 \pm 0.0047_{\mathrm{stat}} \pm 0.0040_{\mathrm{syst}} \\ a_2 &= -0.0429 \pm 0.0044_{\mathrm{stat}} \pm 0.0028_{\mathrm{syst}} \\ a_0 - a_2 &= 0.2639 \pm 0.0020_{\mathrm{stat}} \pm 0.0015_{\mathrm{syst}} \end{aligned}$$





Combination with other Measurements



Experimental data has reached theoretical precision

Perfect agreement with ChPT prediction

Yellow line: Theory prediction

(*not* combination of measurements)

(no uncertainty from theory on cusp result)

Future Prospects

Future for Scattering Lengths from Kaons

NA48/2: Results still not systematically limited, but need more kaons. Single event sensitivity ~10^{-10} for NA48/2.

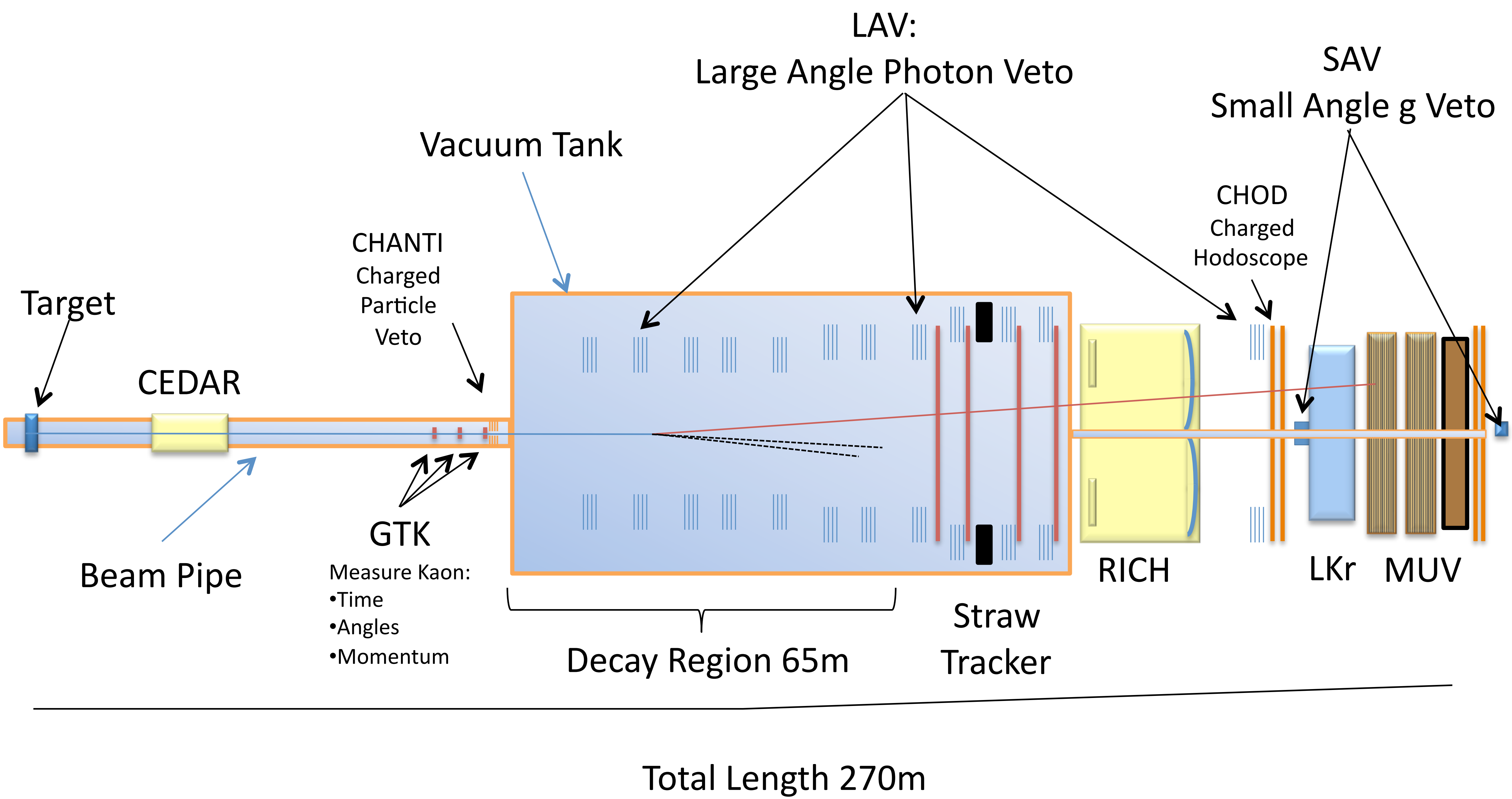
➜ Need more than ~100 billion kaon decays to top this!
(if acceptance is about 10% and trigger efficiency ~1)

NA62: New experiment for measuring 100 $K^{\pm} \rightarrow \pi^{\pm} \nu \nu$ events, starting in 2013 (Br ~10^{-10}).

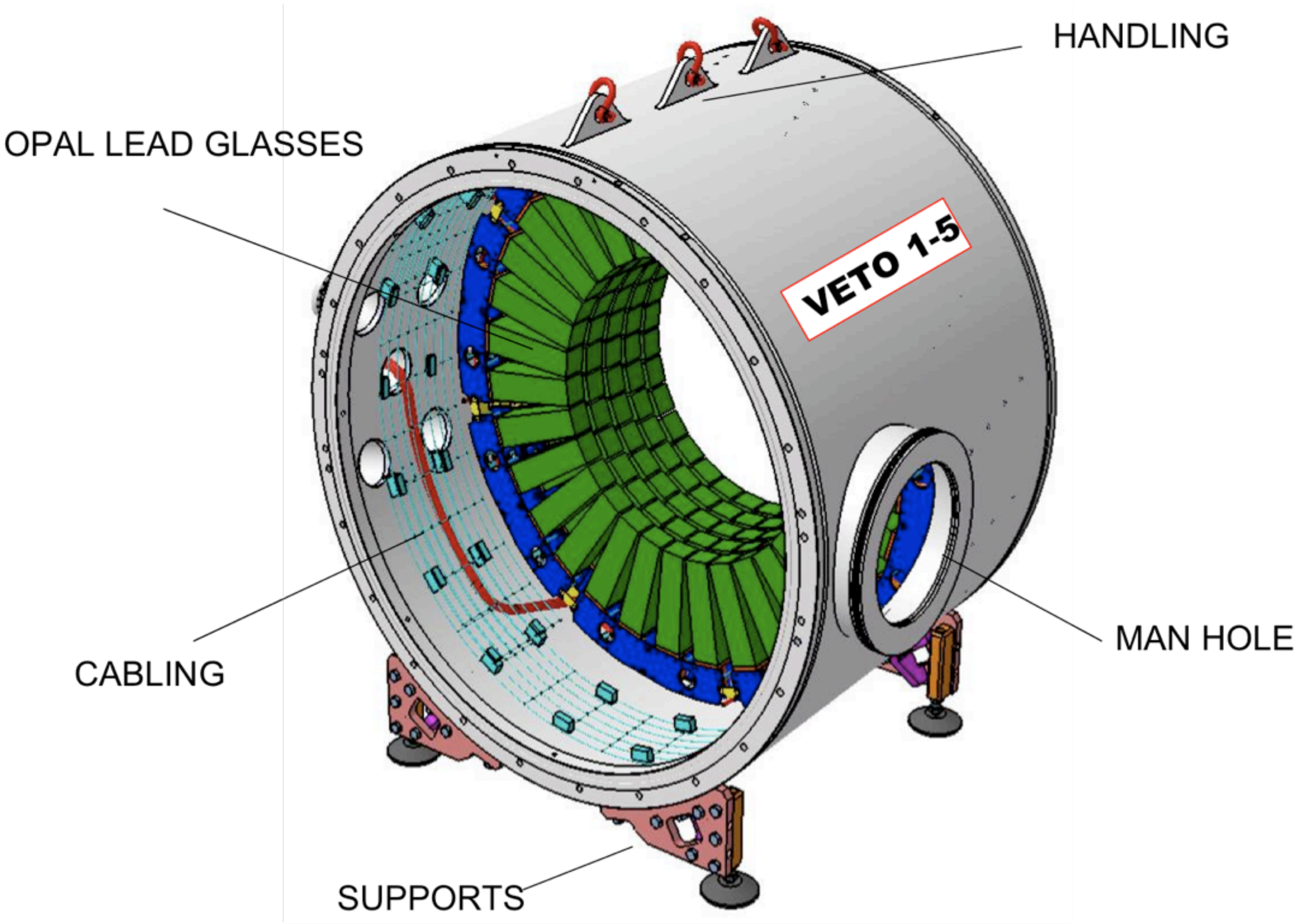
➜ Single event sensitivity ~10^{-12}!
However: trigger conditions are not selecting $K_{3\pi}$, K_{e4}

➜ Special running periods needed.

The NA62 Detector



First Detectors



Conclusions

- **Kaon decays** give unique possibility to study low-energy hadronic interactions with high precision.
- With very high statistics, **NA48/2** has checked ChPT predictions on ππ scattering lengths with high accuracy using both $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ and K_{e4} decays.
  ➜ **very strong test of the theory**
- **NA62:** Measurement of **100 $K^{\pm} \to \pi^{\pm}\nu\nu$ events,** starting in **2013,** also possibility to collect even more statistics for scattering length measurements.

Spares

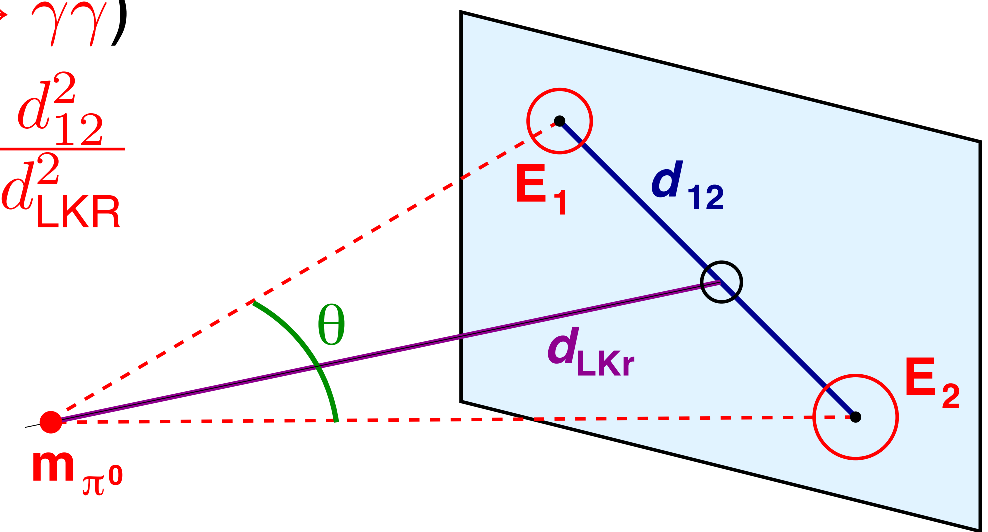
$K^\pm \to \pi^\pm\pi^0\pi^0$: $M(\pi^0\pi^0)$ Reconstruction

- **Decays in two photons:** (z.B. $\pi^0 \to \gamma\gamma$)

$$m_{\pi^0}^2 = 2\,E_1\,E_2\,(1-\cos\theta) = E_1\,E_2\,\frac{d_{12}^2}{d_{\mathrm{LKR}}^2}$$

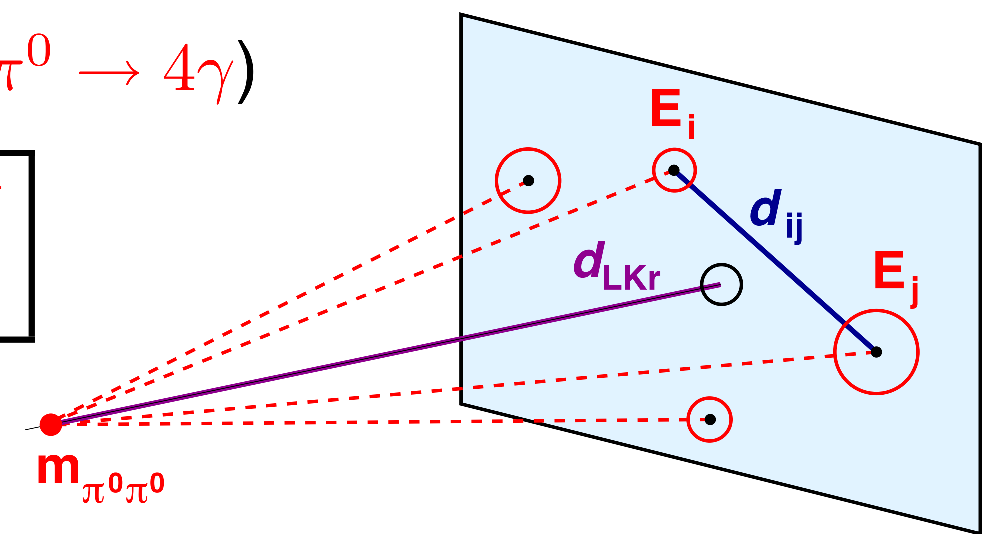
If mass m_{π^0} known:

$$\boxed{d_{\mathrm{LKR}} = \frac{1}{m_{\pi^0}}\sqrt{E_1\,E_2\,d_{12}^2}}$$



- **Decays in many photons:** (z.B. $\pi^0\pi^0 \to 4\gamma$)

$$\boxed{d_{\mathrm{LKR}} = \frac{1}{m_{\pi^0\pi^0}}\sqrt{\sum_{i,j;i>j} E_i\,E_j\,d_{ij}^2}}$$



- **Turning it around:**

Vertex d_{LKR} always the same!

$\Longrightarrow$ Invariant mass $m_{\pi^0\pi^0}$ only from LKr information:

$$\boxed{m_{\pi^0\pi^0} = m_{\pi^0}\sqrt{\sum_{i,j;i>j} E_i\,E_j\,d_{ij}^2 \Big/ \tfrac{1}{2}\left(\sqrt{E_1\,E_2\,d_{12}^2} + \sqrt{E_3\,E_4\,d_{34}^2}\right)}}$$