

INDUCED CHARGE ON DIAMOND DETECTOR

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An explicit analytical formula for the dependence of the induced charge on the thickness of diamond detector has been derived based on a simplified model, i.e., the so-called linear model, in which the local mean free path of the electrons and holes is assumed to increase linearly with the depth measured from the substrate side. The formula was found to reproduce the general trend of the charge measured on the latest sample at two different thickness reasonably well. It has been found that the induced charge is significantly dependent on the thickness of the diamond and which portion in the depth is used. Conversely the formula provides the basis for evaluating the growth process. The induced charge has been found to be independent of the relative orientation of the applied electric field. It is encouraging to find that the model predicts that the latest samples of diamond are close in their growth parameters to what is needed to create large enough signal for a microvertex detector when they are grown properly and lapped down to 400 μm which is equivalent to the canonical 300 μm of silicon microvertex detector sensors in terms of radiation length.

INTRODUCTION

For a diamond detector it is important to know the dependence of the induced charge signal on its thickness and also which part in the depth out of the original as-grown blank is used. Such information are needed to find out plausible recipe, a combination of many parameters, for diamond growth. The objective is to maximize the signal for a diamond detector with limited thickness. If it is for microvertex detector, the canonical thickness is 400 μm which is, in terms of radiation length, equivalent to 300 μm of silicon, the conventional material for microvertex detector.

In the following, we attempt to derive an analytical formula of the dependence of the induced charge in a diamond detector on its thickness. We take so-called *linear model* which is believed to be a reasonable assumption of how the electron/hole transmission increases with the thickness of CVD diamond layer.

Throughout the discussion, the following simplification is made:

- A) The electrode on both sides of a flat diamond layer are infinitely wide so that the transmission of the electrons and holes created by the passage of minimum ionizing particles is regarded to be perpendicular to the electrode planes.
- B) The diamond material is assumed to be laterally uniform.
- C) Minimum ionizing particles are passing through the diamond layer perpendicularly.
- D) The energy deposit per unit path length is constant ignoring particle-by-particle fluctuation and the fluctuation along the path length.

It is obvious that, under the simplified geometry postulated above, the induced charge due to charge motion within diamond layer is nothing but the *mirror image* charge moving from one electrode to another via the conductor between the two, as depicted in the cartoon shown in **Figs. 1 a** and **b**. The charge *induced* on the electrodes is the current flowed through the outside conductor that connects¹ the two electrodes integrated over the

¹ The *connection* can be via an amplifier and or through *ground*. Simply the two electrodes have to be ultimately connected electrically.

time period of the electron and or hole motion within the diamond. If charge Q travels the complete path inside the diamond layer from one electrode to the other as shown in **Fig. 1b**, the corresponding induced charge is Q .

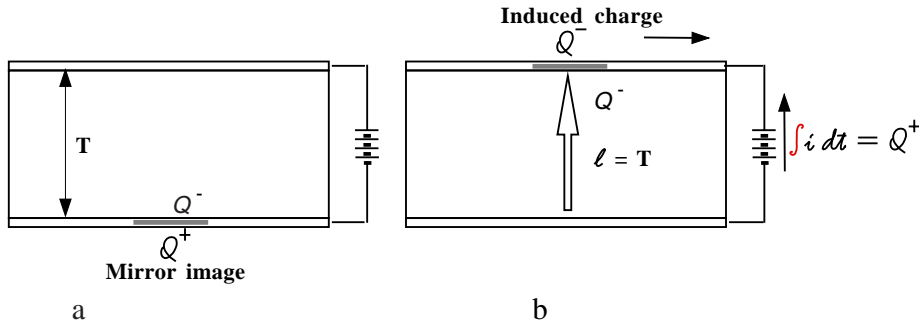


Fig. 1. Charge Q is deposited inside the diamond layer close to the bottom electrode. A mirror image Q^+ is induced on the bottom electrode. Due to the applied electric field, the charge Q drifts towards the top electrode. The mirror image Q^+ simultaneously flows through the connection between two electrodes towards the top electrode. When the charge motion is completed, the induced charge, i.e. the current i integrated over the period of the travel, is Q^- . The direction of the current is as if the charge Q^- inside the diamond has flowed out from the top electrode.

For the sake of the completeness of the argument, it is worth noting that in case of the tracking detector we can consider that the charge is deposited initially without making any signal. It is because the charges of the electrons and holes conveniently cancel each other locally at the initial moment of the ionization by the traversing charged particle. Only after they start moving apart from each other, the charge is induced on the electrodes.

Therefore the induced charge is a product of the charge inside the diamond layer and the distance it traveled divided by the distance between two electrodes. Because of the assumption made above, the field inside the diamond layer is uniform and therefore the induced charge is not a function of the depth where the charge is moving.

Since the charge is induced on the electrodes solely due to the charge motion within the material, whether or not the charge is actually *transmitted* to the electrode does not affect the induced signal. It is only a matter of long time constant effect if the charge stays at the surface of the electrode.

Another intriguing aspect of this argument is that the induced charge from either side of double-sided detector is the same as the charge from a single sided detector. It is because the charge is the result of the integration of the charge flowing through the conductor that connect the two electrodes and the “double-sided” is only a matter of picking-off the same current twice. The cartoon below illustrates such a configuration.

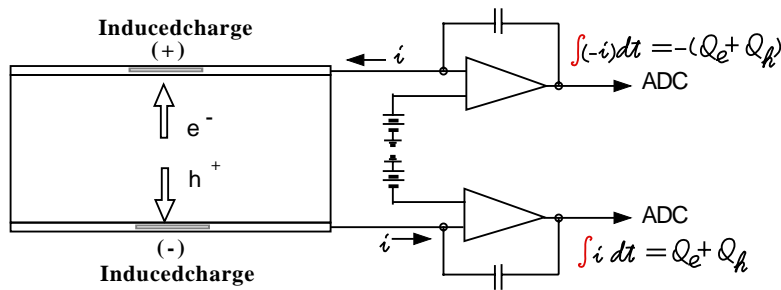


Fig. 2. Double-sided detector configuration.

Based on the above notion, the charge, $dQ_{ind}(x', x)$, induced on the electrodes by the motion by Δx of the charge, $dQ(x', x)$, created at depth x' and moving at x is expressed as follows:

$$dQ_{ind}(x', x) = C dV \quad (1)$$

$$= \frac{1}{T_r} \frac{dQ(x', x)}{dx dx'} \Delta x' \Delta x \quad (2)$$

where C = capacitance

T_r = thickness of diamond layer.

The charge could move towards the electrode on the *growth side* or towards the electrode on the *substrate side* depending on the sign of the charge and the direction of the applied voltage. First we consider the case in which the charge moves towards growth side electrode, or in other words, along with the growing charge absorption length.

We develop the formulae step by step although it is a matter of trivial integration.

There are three steps of integration:

- integration over the charge drift length to calculate the charge attenuation from the point of the charge creation and the point where the charge motion induces the signal charge on the electrode,
- integration over the charge drift length to calculate the induced charge corresponding to the entire drift of the charge from the point of creation to the electrode on the surface,
- integration over the penetrating minimum ionizing track.

LINEAR MODEL CALCULATION

A) Charge moves towards growth side electrode.

It is natural to assume locally an exponential behavior of the charge absorption by traps.

Then the charge created at depth x' , $dQ_0(x')$, is attenuated cascadedly by local absorption length $\lambda(x)$ ($x \geq x'$).

$$\begin{aligned} dQ(x', x) &= dQ_0(x') \lim_{\Delta x \rightarrow 0} \left[\exp\left\{-\frac{\Delta x}{\lambda(x')}\right\} \exp\left\{-\frac{\Delta x}{\lambda(x'+\Delta x)}\right\} \exp\left\{-\frac{\Delta x}{\lambda(x'+2\Delta x)}\right\} \cdots \exp\left\{-\frac{\Delta x}{\lambda(x)}\right\} \right] \\ &= dQ_0(x') \lim_{\Delta x \rightarrow 0} \exp\left[-\left\{ \frac{1}{\lambda(x')} + \frac{1}{\lambda(x'+\Delta x)} + \frac{1}{\lambda(x'+2\Delta x)} + \cdots + \frac{1}{\lambda(x)} \right\} \Delta x \right] \\ &= dQ_0(x') \exp\left\{ -\int_{l=x'}^x \frac{dl}{\lambda(l)} \right\} \end{aligned} \quad (3)$$

Taking “*linear model*”, we assume that the local charge absorption length $\lambda(x)$ is linearly dependent on the depth x measured from the original substrate side surface:

$$\lambda(x) = c x \quad (4)$$

with a constant c .

Then the above integration is executed as follows:

$$\begin{aligned} -\int_{l=x'}^x \frac{dl}{\lambda(l)} &= -\int_{l=x'}^x \frac{dl}{cl} \\ &= -\frac{1}{c} \left[\ln(l) \right]_{x'}^x \end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{c} \ln\left(\frac{x}{x'}\right) \\
&= \ln\left(\frac{x'}{x}\right)^{\frac{1}{c}}.
\end{aligned} \tag{5}$$

Therefore

$$\begin{aligned}
dQ(x', x) &= dQ_0(x') \exp\left\{ \ln\left(\frac{x'}{x}\right)^{\frac{1}{c}} \right\} \\
&= dQ_0(x') \left(\frac{x'}{x}\right)^{\frac{1}{c}}.
\end{aligned} \tag{6}$$

Define T = Total growth thickness².
 T_0 = thickness lapped-off from substrate side.

The charge induced on the electrodes from the charge created at depth x' , $Q_{ind}(x')$, is the result of integration of the local charge obtained above, $dQ(x', x)$, over the path length x of the created charge from x' up to the surface at T .

$$\begin{aligned}
dQ_{ind}(x') &= \frac{1}{T - T_0} \int_{x=x'}^T dQ(x', x) dx \\
&= \frac{1}{T - T_0} \int_{x=x'}^T dQ_0(x') \left(\frac{x'}{x}\right)^{\frac{1}{c}} dx \\
&= \frac{1}{T - T_0} dQ_0(x') (x')^{\frac{1}{c}} \int_{x=x'}^T (x)^{-\frac{1}{c}} dx
\end{aligned} \tag{7}$$

CASE a) $c \neq 1$:

$$\begin{aligned}
dQ_{ind}(x') &= \frac{1}{T - T_0} dQ_0(x') \left(\frac{1}{1 - \frac{1}{c}}\right) (x')^{\frac{1}{c}} \left[\{x'\}^{1 - \frac{1}{c}} \right]_{x'}^T \\
&= \frac{1}{T - T_0} dQ_0(x') \left(\frac{1}{1 - \frac{1}{c}}\right) (x')^{\frac{1}{c}} \left[\{T\}^{1 - \frac{1}{c}} - \{x'\}^{1 - \frac{1}{c}} \right] \\
&= \frac{1}{T - T_0} dQ_0(x') \left(\frac{1}{1 - \frac{1}{c}}\right) \left[\{T\}^{1 - \frac{1}{c}} (x')^{\frac{1}{c}} - x' \right]
\end{aligned} \tag{8}$$

where $dQ_0(x') = Q_0 dx'$ (9)
with Q_0 = charge created per unit path length of minimum ionizing particle.

The total induced charge Q_{ind} is then obtained by integrating the above formula over the path length x' of the traversing minimum ionizing particle.

² If the *growth side* is also lapped, T is the depth of the surface of the final sample measured from the substrate side.

$$\begin{aligned}
Q_{ind} &= \int_{x'=T_0}^T dQ_{ind}(x') \\
&= \frac{Q_0}{T-T_0} \left(\frac{1}{1-\frac{1}{c}} \right) \int_{x'=T_0}^T \left[\{T\}^{1-\frac{1}{c}} (x')^{\frac{1}{c}} - x' \right] dx' \\
&= \frac{Q_0}{T-T_0} \left(\frac{1}{1-\frac{1}{c}} \right) \left[\frac{\{T\}^{1-\frac{1}{c}}}{1+\frac{1}{c}} \left[(x')^{1+\frac{1}{c}} \right]_{T_0}^T - \frac{1}{2} [x'^2]_{T_0}^T \right] \\
&= \frac{Q_0}{T-T_0} \left(\frac{1}{1-\frac{1}{c}} \right) \left[\left\{ \frac{1}{1+\frac{1}{c}} - \frac{1}{2} \right\} T^2 - \frac{(T)^{1-\frac{1}{c}}}{1+\frac{1}{c}} (T_0)^{1+\frac{1}{c}} + \frac{1}{2} (T_0)^2 \right] \\
&= \frac{Q_0}{T-T_0} \frac{1}{\left(1-\frac{1}{c}\right)\left(1+\frac{1}{c}\right)} \left[\frac{1}{2} (T^2 + T_0^2) - \frac{1}{2c} (T^2 - T_0^2) - (T)^{1-\frac{1}{c}} (T_0)^{1+\frac{1}{c}} \right] \quad (10)
\end{aligned}$$

$$\begin{aligned}
\text{or} \quad &= \frac{Q_0 T}{1-\left(\frac{T_0}{T}\right)} \frac{1}{\left(1-\frac{1}{c}\right)\left(1+\frac{1}{c}\right)} \left[\frac{1}{2} \left\{ 1 + \left(\frac{T_0}{T}\right)^2 \right\} - \frac{1}{2c} \left\{ 1 - \left(\frac{T_0}{T}\right)^2 \right\} - \left(\frac{T_0}{T}\right)^{1+\frac{1}{c}} \right] \\
&= Q_0 T f\left(c, \frac{T_0}{T}\right) . \quad (11)
\end{aligned}$$

with a universal dimensionless shape factor $f\left(c, \frac{T_0}{T}\right)$ with a parameter c and a dimensionless quantity T_0/T :

$$f\left(c, \frac{T_0}{T}\right) = \frac{1}{\left(1-\frac{1}{c}\right)\left(1+\frac{1}{c}\right)} \frac{1}{1-\left(\frac{T_0}{T}\right)} \left[\frac{1}{2} \left\{ 1 + \left(\frac{T_0}{T}\right)^2 \right\} - \frac{1}{2c} \left\{ 1 - \left(\frac{T_0}{T}\right)^2 \right\} - \left(\frac{T_0}{T}\right)^{1+\frac{1}{c}} \right] . \quad (12)$$

Thus the induced charge Q_{ind} is linearly dependent on T modified by the universal function $f\left(c, \frac{T_0}{T}\right)$. $f\left(c, \frac{T_0}{T}\right)$ is plotted against T_0/T in **Fig. 3**.

The curves are bounded by the upper bound for $c = \infty$ indicated by a straight line

$$f\left(c = \infty, \frac{T_0}{T}\right) = \frac{1}{2} \left(1 - \frac{T_0}{T} \right). \quad (13)$$

The shape differences among the curves for different values of c is more apparent in the form $\left(1 + \frac{1}{c}\right) f\left(c, \frac{T_0}{T}\right)$ which is conveniently normalized to 0.5 at $\frac{T_0}{T} = 1$ as plotted against T_0/T in **Fig. 4** for various values of c . As intuitively expected, the contribution from deeper part of the layer grows with c .

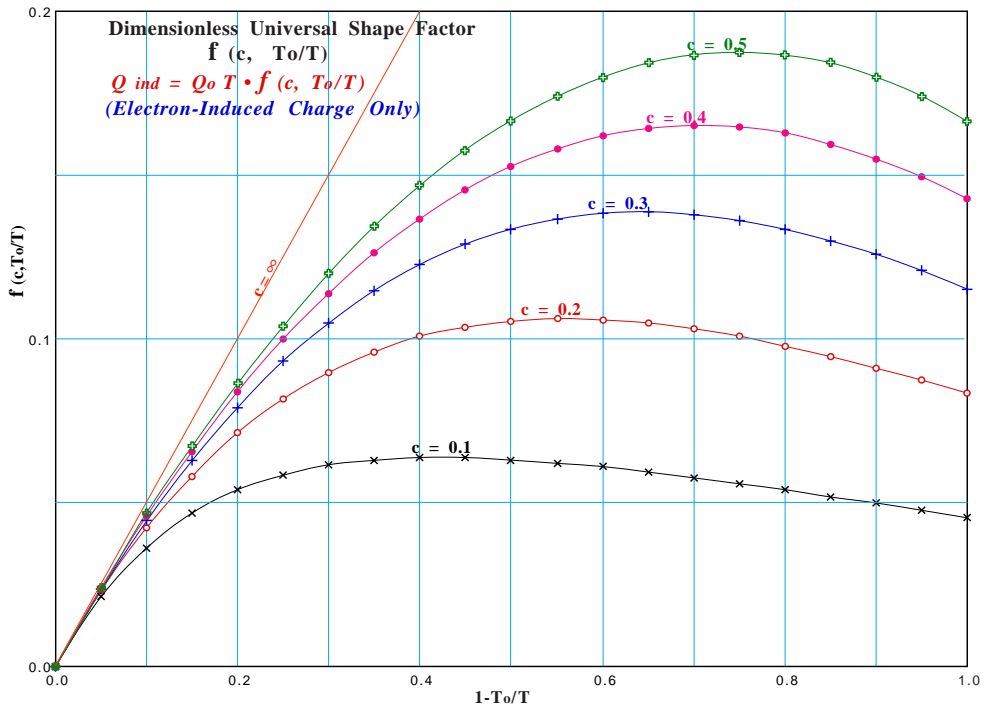


Fig. 3. Universal, dimensionless shape factor of induced charge.

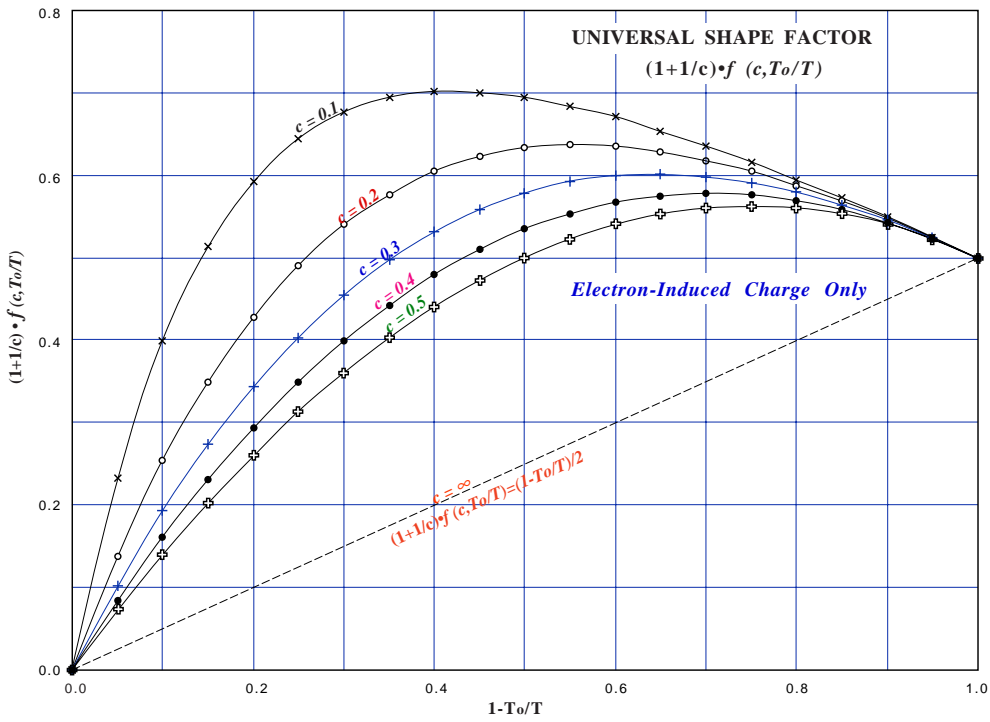


Fig. 4. Normalized shape factor of induced charge.

CASE b) $c = 1$:

$$\begin{aligned}
 dQ_{ind}(x') &= \frac{1}{T-T_0} dQ_0(x') x' \int_{x=x'}^T \frac{1}{x} dx \\
 &= \frac{1}{T-T_0} dQ_0(x') x' [\ln x]_{x'}^T \\
 &= \frac{1}{T-T_0} dQ_0(x') x' \{ \ln T - \ln x' \}.
 \end{aligned} \tag{14}$$

$$\begin{aligned}
 \therefore Q_{ind} &= \int_{x'=T_0}^T dQ_{ind}(x') \\
 &= \frac{Q_0}{T-T_0} \left\{ \frac{1}{2} (\ln T) [x'^2]_{T_0}^T - \int_{x'=T_0}^T x' (\ln x') dx' \right\} \\
 &= \frac{Q_0}{2(T-T_0)} \left\{ (\ln T) (T^2 - T_0^2) - \left[x'^2 \ln x' - \frac{1}{2} x'^2 \right]_{T_0}^T \right\} \\
 &= \frac{Q_0}{2(T-T_0)} \left\{ (\ln T) (T^2 - T_0^2) + \frac{1}{2} (T^2 - T_0^2) - (T^2 \ln T - T_0^2 \ln T_0) \right\}
 \end{aligned} \tag{15}$$

It should be noted that, even though the above formula appears different from formula (11), the result is continuous around $c = 1$.

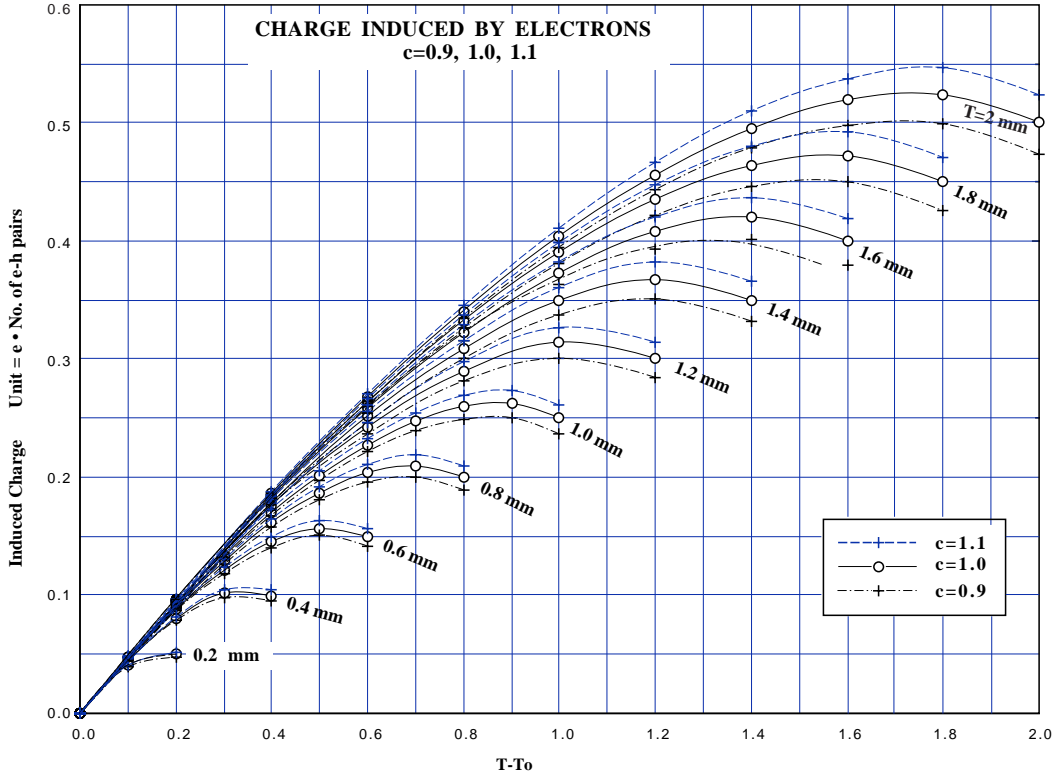


Fig. 5. Plot of the induced charge around the singularity at $c = 1$. Curves for $c = 1$ is from formula (15) while curves for $c = 0.9$ and 1.1 are calculated from formula (11).

Fig. 5³ is a plot of the induced charge for $c = 1$ derived from formula (15) together with the results for $c = 0.9$ and 1.1 derived from formula (11) derived for $c \neq 1$.

B) Charge moves towards substrate side electrode.

In this case, charge is created at depth x' and moves backward, i.e., $x \leq x'$, with the same coordinate system as the above in which x increases from 0 at substrate side surface and T at the growth side surface.

Therefore, the formula for the charge attenuation is now:

$$\begin{aligned}
 dQ(x', x) &= dQ_0(x') \lim_{\Delta x \rightarrow 0} \left[\exp\left\{-\frac{\Delta x}{\lambda(x')}\right\} \exp\left\{-\frac{\Delta x}{\lambda(x' - \Delta x)}\right\} \exp\left\{-\frac{\Delta x}{\lambda(x' - 2\Delta x)}\right\} \cdots \exp\left\{-\frac{\Delta x}{\lambda(x)}\right\} \right] \\
 &= dQ_0(x') \lim_{\Delta x \rightarrow 0} \exp\left[-\left\{ \frac{1}{\lambda(x')} + \frac{1}{\lambda(x' - \Delta x)} + \frac{1}{\lambda(x' - 2\Delta x)} + \cdots + \frac{1}{\lambda(x)} \right\} \Delta x \right] \\
 &= dQ_0(x') \exp\left\{ -\int_{l=x}^{x'} \frac{dl}{\lambda(l)} \right\}. \tag{16}
 \end{aligned}$$

Taking the same *linear* form of the charge absorption length $\lambda(x)$

$$\lambda(x) = cx$$

as formula (4), the integral becomes

$$\begin{aligned}
 -\int_{l=x}^{x'} \frac{dl}{\lambda(l)} &= -\int_{l=x}^{x'} \frac{dl}{cl} \quad (\text{Note: The integration is from } l=x \text{ to } l=x'.) \\
 &= \ln\left(\frac{x}{x'}\right)^{\frac{1}{c}}. \tag{17}
 \end{aligned}$$

Therefore

$$dQ(x', x) = dQ_0(x') \left(\frac{x}{x'}\right)^{\frac{1}{c}}. \quad : x \leq x'. \tag{18}$$

$$\begin{aligned}
 \therefore dQ_{ind}(x') &= \frac{1}{T - T_0} \int_{x=T_0}^{x'} dQ(x', x) dx \quad (\text{Note: The integration is from } x=T_0 \text{ to } x=x') \\
 &= \frac{1}{T - T_0} \int_{x=T_0}^{x'} dQ_0(x') \left(\frac{x}{x'}\right)^{\frac{1}{c}} dx \\
 &= \frac{1}{T - T_0} dQ_0(x') (x')^{-\frac{1}{c}} \int_{x=T_0}^{x'} (x)^{\frac{1}{c}} dx \tag{19}
 \end{aligned}$$

³ The unit for the induced charge in the graphs throughout this note is e , the electron charge, times the number of $e-h$ pairs / mm from a minimum ionizing particle. This unit is identical to “collection distance” in units of mm. Therefore a factor of 3,600 electrons / 0.1 mm translates this to the number of electrons.

CASE a) $c \neq 1$:

$$\begin{aligned}
dQ_{ind}(x') &= \frac{1}{T-T_0} dQ_0(x') \left(\frac{1}{1+\frac{1}{c}} \right) (x')^{-\frac{1}{c}} \left[(x')^{1+\frac{1}{c}} \right]_{T_0}^{x'} \\
&= \frac{1}{T-T_0} dQ_0(x') \left(\frac{1}{1+\frac{1}{c}} \right) (x')^{-\frac{1}{c}} \left[(x')^{1+\frac{1}{c}} - (T_0)^{1+\frac{1}{c}} \right] \\
&= \frac{1}{T-T_0} dQ_0(x') \left(\frac{1}{1+\frac{1}{c}} \right) \left[x' - (T_0)^{1+\frac{1}{c}} (x')^{-\frac{1}{c}} \right] \tag{20}
\end{aligned}$$

$$\begin{aligned}
\therefore Q_{ind} &= \int_{x'=T_0}^T dQ_{ind}(x') \\
&= \frac{Q_0}{T-T_0} \left(\frac{1}{1+\frac{1}{c}} \right) \int_{x'=T_0}^T \left[x' - (T_0)^{1+\frac{1}{c}} (x')^{-\frac{1}{c}} \right] dx' \\
&= \frac{Q_0}{T-T_0} \left(\frac{1}{1+\frac{1}{c}} \right) \left\{ \frac{1}{2} [x'^2]_{T_0}^T - \frac{(T_0)^{1+\frac{1}{c}}}{1-\frac{1}{c}} \left[(x')^{1-\frac{1}{c}} \right]_{T_0}^T \right\} \\
&= \frac{Q_0}{T-T_0} \left(\frac{1}{1+\frac{1}{c}} \right) \left[\frac{1}{2} (T^2 - T_0^2) - \frac{1}{1-\frac{1}{c}} \left\{ (T_0)^{1+\frac{1}{c}} (T)^{1-\frac{1}{c}} - (T_0)^2 \right\} \right] \\
&= \frac{Q_0}{T-T_0} \frac{1}{\left(1-\frac{1}{c}\right)\left(1+\frac{1}{c}\right)} \left[\frac{1}{2} (T^2 + T_0^2) - \frac{1}{2c} (T^2 - T_0^2) - (T)^{1-\frac{1}{c}} (T_0)^{1+\frac{1}{c}} \right] \tag{21}
\end{aligned}$$

The formula is exactly the same as (10) for the case in which the charge moves towards growth side electrode.

Therefore the polarity of the electrode does not affect the size of the signal⁴.

⁴ The effect of the direction of the electron motion on the position resolution with respect to the strips in a strip detector is out of the scope of this discussion.

C) Total induced charge from electron-hole pairs.

Naturally the charge induced from the holes moving in the direction opposite to the electron motion is additive to the charge induced by the electron motion. Based on the above finding, the added signal is not dependent on the relative polarity of the applied field and is described by two coefficients c_e and c_h , for electrons and holes, respectively.

Therefore the charge induced by electron-hole pairs ionized by minimum ionizing particles is a sum of two components as follows:

$$Q_{induced}^{e-h\ pairs} = \frac{Q_0}{T - T_0} \left[\frac{1}{\left(1 - \frac{1}{c_e}\right)\left(1 + \frac{1}{c_e}\right)} \left\{ \frac{1}{2}(T^2 + T_0^2) - \frac{1}{2c_e}(T^2 - T_0^2) - (T)^{1-\frac{1}{c_e}} (T_0)^{1+\frac{1}{c_e}} \right\} + \frac{1}{\left(1 - \frac{1}{c_h}\right)\left(1 + \frac{1}{c_h}\right)} \left\{ \frac{1}{2}(T^2 + T_0^2) - \frac{1}{2c_h}(T^2 - T_0^2) - (T)^{1-\frac{1}{c_h}} (T_0)^{1+\frac{1}{c_h}} \right\} \right] \quad (22)$$

or, using formula (12),

$$Q_{induced}^{e-h\ pairs} = Q_0 T \left\{ f\left(c_e, \frac{T_0}{T}\right) + f\left(c_h, \frac{T_0}{T}\right) \right\} \quad (23)$$

where c_e = Linear coefficient of the absorption length for electrons
 c_h = Linear coefficient of the absorption length for holes.

As discussed in the following section, a practical use of the above formula is to assume the same value of c for both of the components. In **Fig. 6**, as an example, curves for $c_e = c_h = 0.1, 0.2, 0.3,$ and 0.4 , for various as-grown thickness are plotted. It is seen that there is a mild peaking and the induced charge increases if the inefficient substrate side is lapped off.

An immediate question that follows is what condition is required to produce a large enough signal from ~400 μm thick diamond (As mentioned earlier, $400 \mu\text{m}$ of diamond is equivalent to a canonical $300 \mu\text{m}$ thick silicon layer I terms of radiation length.) Due to the smaller dielectric constant of diamond compared to that of silicon, we assume electronics noise of ~800 electrons. In order to achieve $S/N \approx 10/1$ at the most probable peak of the Landau-like distribution, the induced charge needs to be ~ 8000 electrons at the most probable peak. (Although the induced charge we have derived in the discussion so far can be interpreted as either the average or the most probable peak value, we take it as the former following the customary definition of *collection distance*.) The ratio of the average to the most probable peak of the true Landau distribution of the energy deposit by a minimum ionizing particle is ~ 1.2 based on GEANT simulation. Therefore the average signal for $S/N \approx 10/1$ is $\sim 10,000$ electrons. From **Fig. 6**, it is seen that such signal can be obtained from diamond layer grown to 1 mm to 2 mm depending on the value of c . Some examples are listed in **Table 2**.

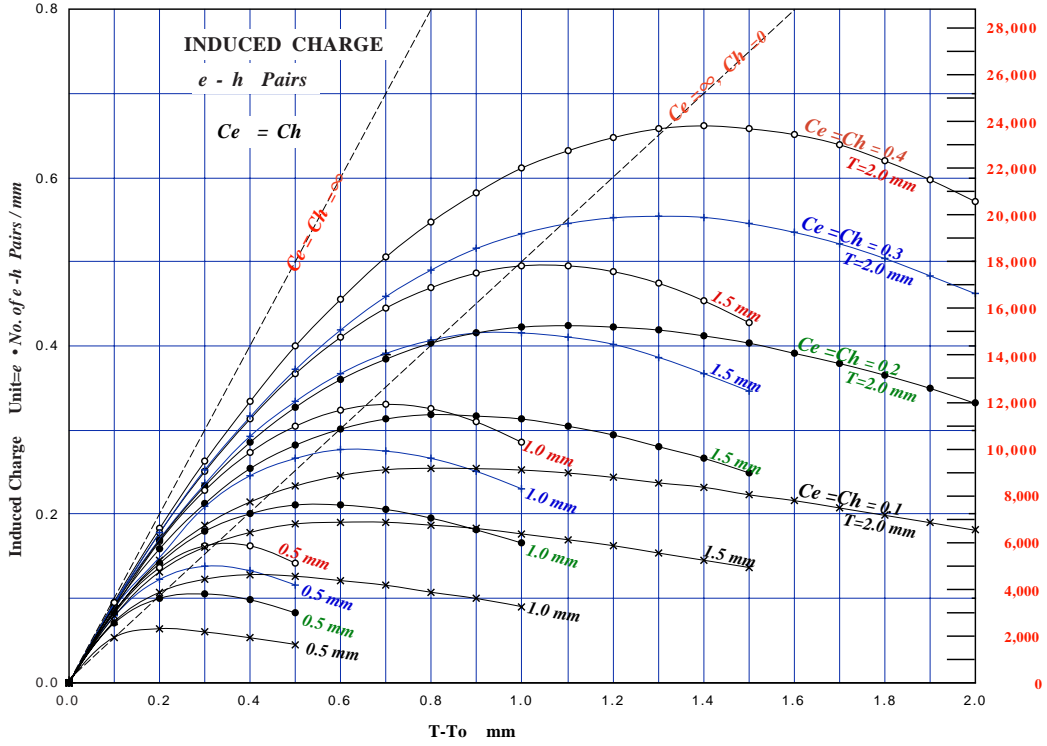


Fig.6. Induced charge for various values of c assuming $c_e = c_h$.

Table 2. Examples of parameter sets for 400 μ m thick diamond layer to achieve $\langle Q_{ind} \rangle \approx 10,000$ electrons.

$c_e = c_h$	T	$e \cdot Ne-h$ pairs/mm	$\langle Q_{ind} \rangle$ electrons ⁵
0.2	1.5 mm	0.254	9,140
	2.0 mm	0.285	10,260
0.3	1.5 mm	0.292	10,510
	2.0 mm	0.317	11,410
0.4	1.0 mm	0.274	9,860
	1.5 mm	0.314	11,300
	2.0 mm	0.335	12,060

This is the range of the value of c we have already been observing, as discussed later.

In Fig. 6, it should be noted that the end points of the curves corresponding to $T_0 = 0$ lie on a straight line

$$Q_{ind}(T_0=0) = \frac{Q_0}{1 + \frac{1}{c}} T \quad (24)$$

⁵ A caution needs to be taken for the ratio between average and the most probable peak of the signal. The ratio depends on the uniformity of the diamond layer and if it is not ideal, the ratio needs to be taken much larger and hence larger number for the average signal.

as can be calculated from formula (23) for $T_0 = 0$. This is because the shape factor $f(c, \frac{T_0}{T})$ becomes 0.5 for $T_0 = 0$ as shown in (13).

No data points are supposed to lie below these lines if the growth condition is the same. Such measurement at the end point, i.e. the as-grown point, is convenient to find the coefficient c .

D) Practical application

One of the features of diamond is that the mobility of the holes is large, namely 67 % of that of electrons. Therefore we need to take account of the contribution from hole motion even in the case of short shaping times of amplifier.

As discussed in later sections for the case of real samples, we are definitely observing the contribution from holes. The observed charge at $T - T_0 = 0.4 \text{ mm}$ (0.2 mm) is about the same ($\sim 140\%$) as the maximum limit ($c = \infty$) for electrons alone. It means that there is a significant contribution from holes.

In principle, a special measurement, in which finely collimated beam of minimum ionizing particles is injected into the diamond layer parallel to the surface, so that ionization localized in the direction perpendicular to the layer takes place, can resolve two components.

Otherwise, two components are indistinguishably additive to each other in the ordinary measurement of the charge against thickness as can be easily understood from **Figs. 3** and **4**. The dependence of the induced charge on the film thickness is a smooth function and mildly dependent on the value of the coefficient c within the range we are observing in the latest samples⁶.

From these figures, it is imaginable that the sum of two components with two different values of c , one large and another small, is similar to the sum of two components both with middle range value of c . **Figs. 7a** and **b** demonstrate such cases. In **Fig. 7a (b)**, the sum of two components, one with $c_e = 0.2$ (0.4) and another with $c_h = 0.1$ (0.2) is compared with the sum of two components with the same value that is the mean of the former two values, $c_e = c_h = 0.15$ (0.3). A further fine tuning of the effective coefficients will diminish the differences. Therefore, a practical application of the formula we have derived is to assume an effective common coefficient for both of the components and simply multiply the formula (11) for electrons by a factor of 2.

⁶ DBDS#39: $c_e = c_h = 0.12$ equivalent .
 DBDS#73: $c_e = c_h = 0.24$ equivalent .
 DBDS#55: $c_e = c_h = 0.30$ equivalent .
 DBDS#86: $c_e = c_h = 0.48$ equivalent .

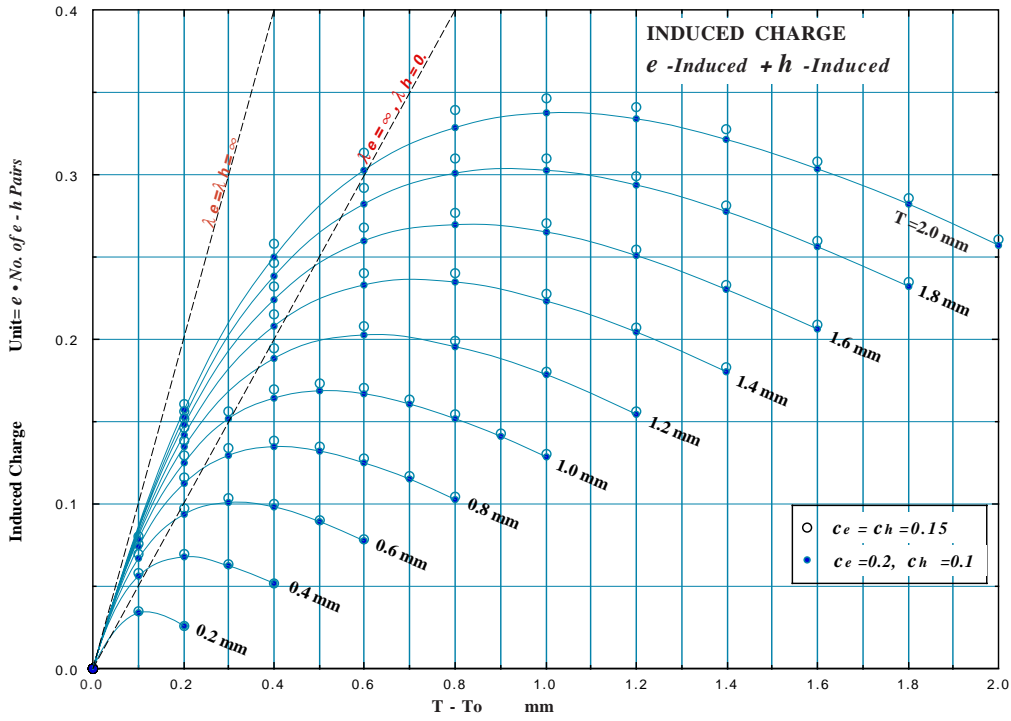


Fig. 7a. Comparison between the induced charge for $c_e = c_h = 0.15$ and the charge for $c_e = 0.2$ and $c_h = 0.1$.

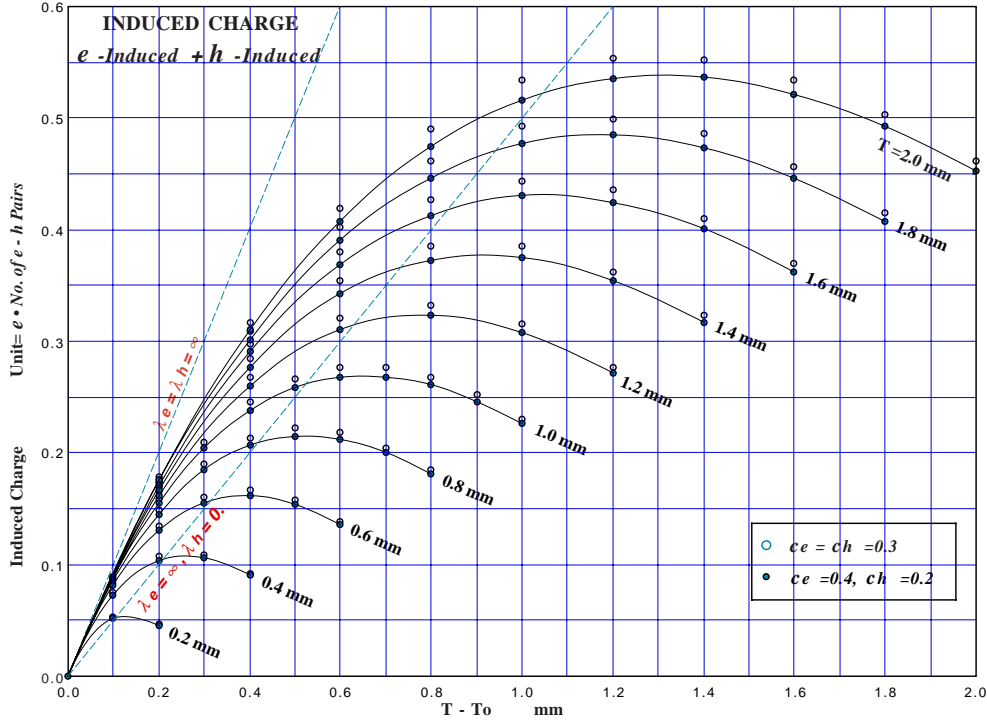


Fig. 7b. Comparison between the induced charge for $c_e = c_h = 0.3$ and the charges for $c_e = 0.4$ and $c_h = 0.2$.

EXTREME CASES

It is useful to consider extreme cases to know the bound of the reality. We consider two cases:

- A) infinite absorption length,
- B) finite constant absorption length throughout the diamond thickness.

A) Infinite absorption length.

$$\lambda(x) = \infty . \quad (25)$$

The induced charge is the product of the geometrical mean free path and the charge of the electrons and holes created uniformly over the diamond thickness, divided by the total thickness. Since the geometrical mean free path is half of the total thickness, the induced charge is

$$\begin{aligned} Q_{ind} &= \frac{1}{T-T_0} \frac{1}{2} (T-T_0) \left(|Q_0^{electron}| + |Q_0^{hole}| \right) (T-T_0). \\ &= \frac{1}{2} (T-T_0) \left(|Q_0^{electron}| + |Q_0^{hole}| \right) . \end{aligned}$$

Here $Q_0^{electron}$ and Q_0^{hole} are the electron charge and hole charge created by the traversing minimum ionizing particle per unit length in the diamond layer. Since they are equal,

$$Q_{ind} = (T-T_0)Q_0 \quad (26)$$

with $Q_0 = e \times \text{Number of electron-hole pairs in diamond per unit length}$
 $= \sim 3600 e / 100 \mu\text{m}.$

It is obvious that (22) can be also derived by putting $c = \infty$ into (20).

This is the **absolute limit** of the induced charge that can be observed from a diamond detector. Naturally the thicker the diamond film the greater the observed signal is.

B) Finite constant absorption length.

$$\lambda(x) = \lambda_0 : \text{constant.} \quad (27)$$

Replacing the $\lambda(x)$ in equation (3) by the above expression, we obtain

$$\begin{aligned} d Q_{ind}(x',x) &= d Q_0(x') \exp \left\{ -\frac{1}{\lambda_0} \int_{\ell=x'}^x d\ell \right\} \\ &= d Q_0(x') \exp \left\{ -\frac{x-x'}{\lambda_0} \right\} . \end{aligned} \quad (28)$$

To simplify the expression, we take $T_0 = 0$ without losing generality. Therefore, equation (7) becomes

$$d Q_{ind}(x') = \frac{1}{T} \int_{x=x'}^T d Q_0(x') \exp \left(-\frac{x-x'}{\lambda_0} \right)$$

$$= \frac{\lambda_0}{T} dQ_0(x') \left\{ 1 - \exp\left(-\frac{T-x'}{\lambda_0}\right) \right\}. \quad (29)$$

$$\begin{aligned} \therefore Q_{ind}^{er} &= \frac{\lambda_0 Q_0}{T} \int_{x'=T_0}^T \left\{ 1 - \exp\left(-\frac{T-x'}{\lambda_0}\right) \right\} dx' \\ &= \lambda_0 Q_0 \left[1 - \frac{\lambda_0}{T} \left\{ 1 - \exp\left(-\frac{T}{\lambda_0}\right) \right\} \right]. \end{aligned} \quad (30)$$

$$Q_{ind}^{e-h \text{ pair}} = 2 \lambda_0 Q_0 \left[1 - \frac{\lambda_0}{T} \left\{ 1 - \exp\left(-\frac{T}{\lambda_0}\right) \right\} \right]. \quad (31)$$

Fig. 8 is a plot of the induced charge derived from the above formula in a universal form as a function of T/λ_0 .

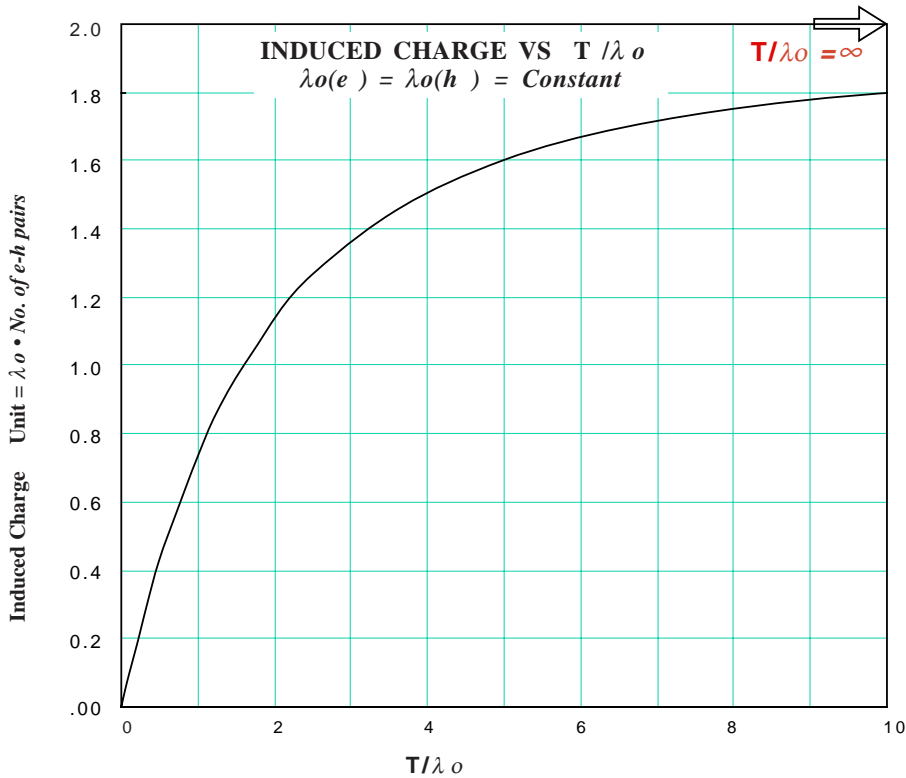


Fig. 8. Universal curve for the induced charge for constant mean free path against dimensionless variable T/λ_0 .

Intuitively, when T is much smaller than λ_0 , the charge grows linearly as $\sim Q_0 T$ with T because the mean free path is half of T as discussed for the case of $\lambda = \infty$ (formula (26)). When T becomes much greater than λ_0 , it is expected that the mean free path is approximately λ_0 irrespective of where the charge is created. Therefore the induced

charge asymptotically approaches constant value $2\lambda_0 Q_0$ while the factor T for the initial minimum ionizing track length and $1/T$ factor for the charge induction cancels out.

Such intuitive picture for the extreme end of the range of the values of T/λ_0 is consistent with what can be calculated from formula (31). For small value of T/λ_0 , the first and the second term of the series expansion of the exponential term are canceled and the remaining third term results in $Q_0 T$ as intuitively expected. For the large value of T/λ_0 the result is obvious.

In **Fig. 9**, formula (31) is plotted against T with various values of λ_0 .

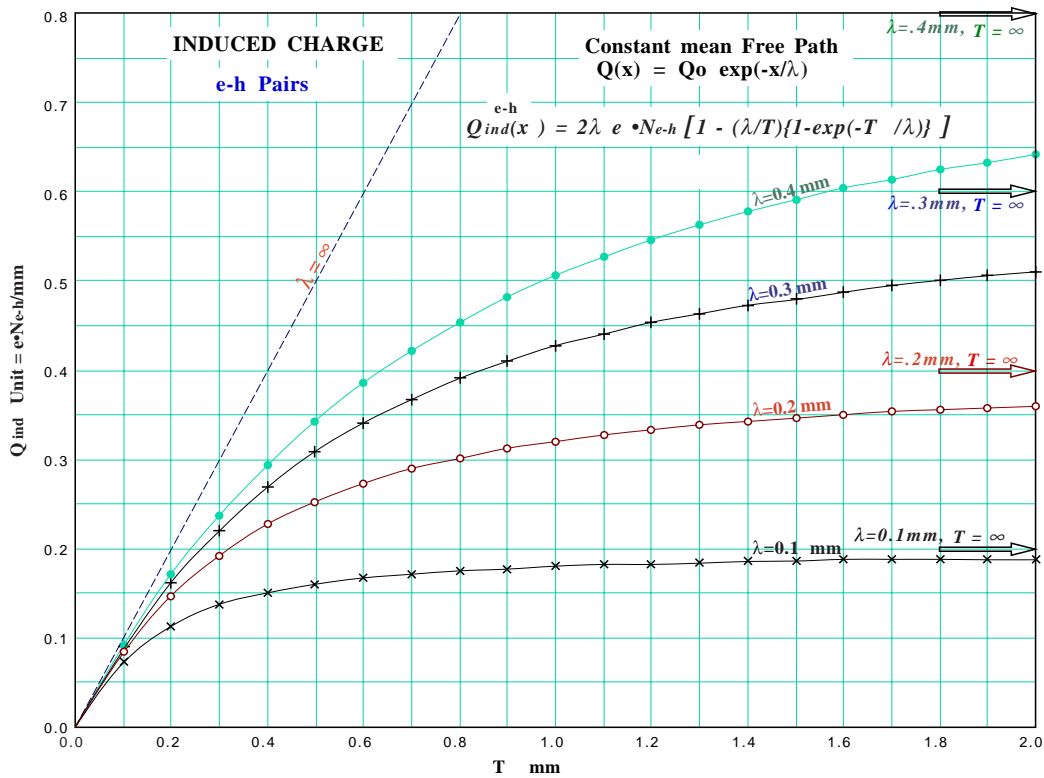


Fig.9. Induced charge for constant mean free path.

APPLICATION TO REAL SAMPLES

As discussed before, a single effective charge absorption coefficient common to both the electrons and holes, assuming that the contributions are equal, appeared to be sufficient to reproduce a true behavior for wide range of the relative proportion of the two components and also the value of c .

The procedure to find the coefficient c is to equate the measured charge to the formula (11) times 2 for the assumption of $c_e = c_h = c$:

$$2Q_0 T f\left(c, \frac{T_0}{T}\right) = Q_{Measured} \quad . \quad (32)$$

Note that $Q_{Measured}$ can be expressed as a product of Q_0 and customarily used *charge collection distance* d_c :

$$Q_{Measured} = Q_0 \cdot d_c \quad . \quad (33)$$

Therefore the above equation becomes :

$$2 f\left(c, \frac{T_0}{T}\right) = \frac{d_c}{T} \quad . \quad (34)$$

In the following we examine two latest samples DBDS #39, #73, and rather old #55. Data are taken from “Report on DeBeers Samples” series.

We take the data for the *saturated* points since we do not quite know the real mechanism of “pumping/unpumping” nor the dependence of the signal on the applied voltage. Also it is safer to take the saturated point which is less sensitive to minor differences in the conditions, either of pumping/unpumping or the voltages, from one measurement to another.

Let’s take DBDS#39 which has been tested at two different thickness.

Table 1. Measured parameters for DBDS #39.

From “Report” 1/97			
D	d_c	OSU t_∞	
1105 μm	200 μm	$T_0=425 \mu\text{m}$, $T=1530 \mu\text{m}$	DBDS#39*
From table in J. Conway’s report in Toronto meeting			
675 μm	7840 electrons => $d_c = 217.8 \mu\text{m}$		

We make two extreme assumptions: one is to assume that the observed signal is only from electrons and the second is to assume that the electrons and holes have the same coefficient $c_e = c_h = c_{effective}$.

Equating formula (11) and (23), respectively, to 0.2 Q_0 as measured for the first point of the thickness of 1530 μm , we obtain

$$c_e = 0.279 \quad \text{and} \quad c_{effective} = 0.117, \quad \text{respectively.}$$

The dependence of the charge on the thickness is plotted in **Fig. 10** using the above values of $c_{effective}$ for various thickness conditions. If we draw the curves for the above two values of the coefficient $c_{effective}$ through the first point, the second point is much closer to the curve for the case in which both electrons and holes are assumed to have the same coefficient indicating that there is a significant contribution from hole motion.

If we take an identical procedure for DBDS#73 the value of the coefficient is

$$c_{effective} = 0.24.$$

DBDS#55 was measured at two points in the thickness and it is consistent with

$$c_{effective} = 0.295$$

2.5 time greater value than that of #39.

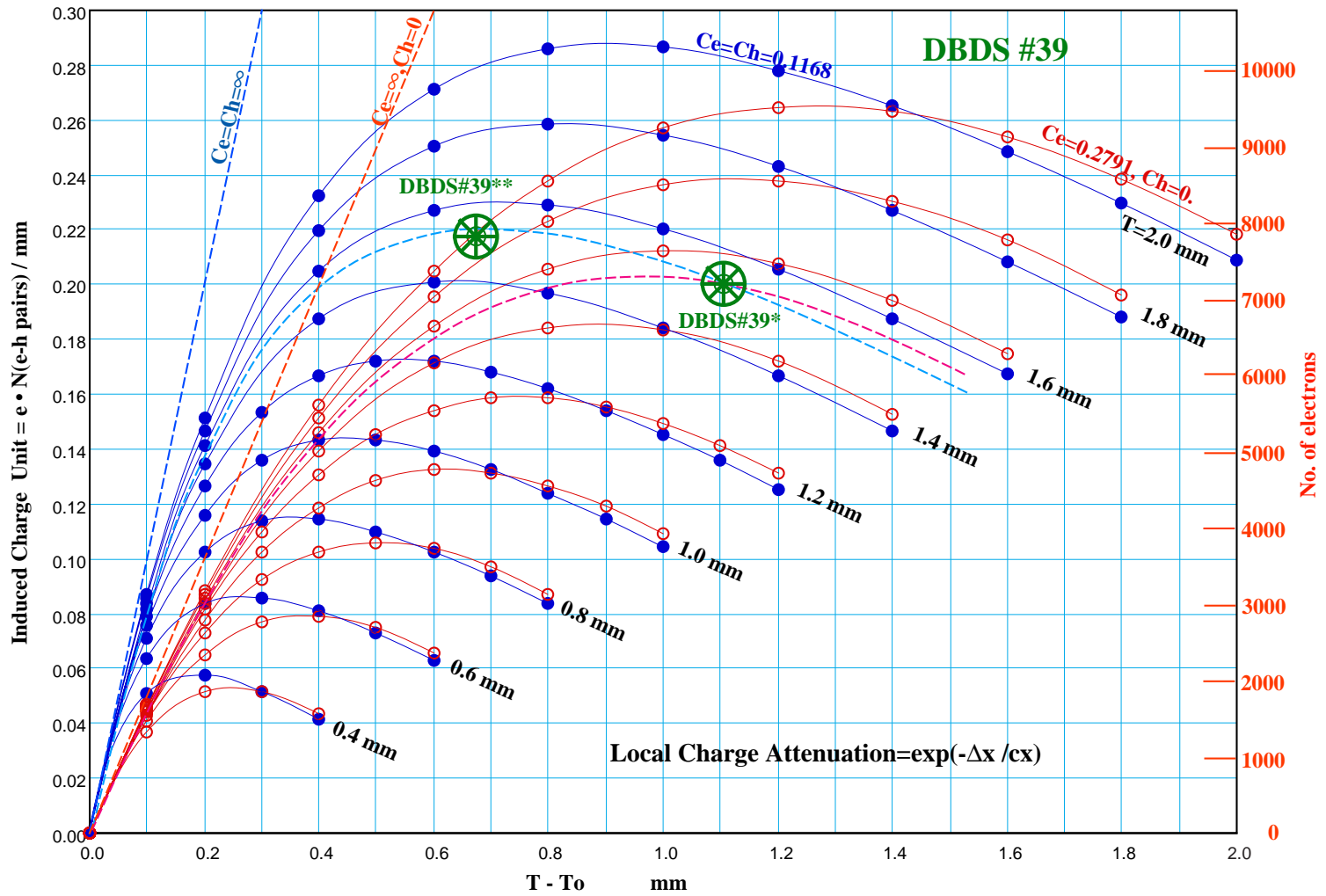


Fig. 10. Induced charge observed on DBDS#39.

Universal template for evaluating the coefficient c .

A possible practical mean of evaluating the linear coefficient c of the samples is to plot the measured charge on a universal template in a normalized scale. Equation (34) derived before is a convenient form for such purpose.

$$2f\left(c, \frac{T_0}{T}\right) = \frac{d_c}{T}. \quad (34)$$

In **Fig. 11** we drew curves of $2f\left(c, \frac{T_0}{T}\right)$ as a function of $\frac{T-T_0}{T}$ and superimposed plots of $\frac{d_c}{T}$ of the DeBeers samples listed in the “Report on DeBeers Samples” series from No. 3/96 through 1/98.

Immediately noticed is the high values of c , which is obvious from the raw number of the collection distance, of DBDS#86 and #83. Since #83* (690 μm) was the result of lapping #83 (782 μm) on growth side, according to the Report, the thickness was treated as as-grown thickness T .

Though #86 was cracked, according to the Report, when it was lapped to 482 μm from the original 920 μm , it is an intriguing exercise to trace the curve of the value of c , ~ 4.8 , found at the original thickness down to 400 μm point, $(T_0-T)/T \approx 0.43$. We find d_c/T to be ~ 0.29 corresponding to 266 μm as the collection distance. If it were grown to more than 1 mm, the signal would have been large enough at 400 μm for any purposes. This map is an indication that some of the CVD diamond growth recipes in hand might be already excellent candidates for mass production for real detector.

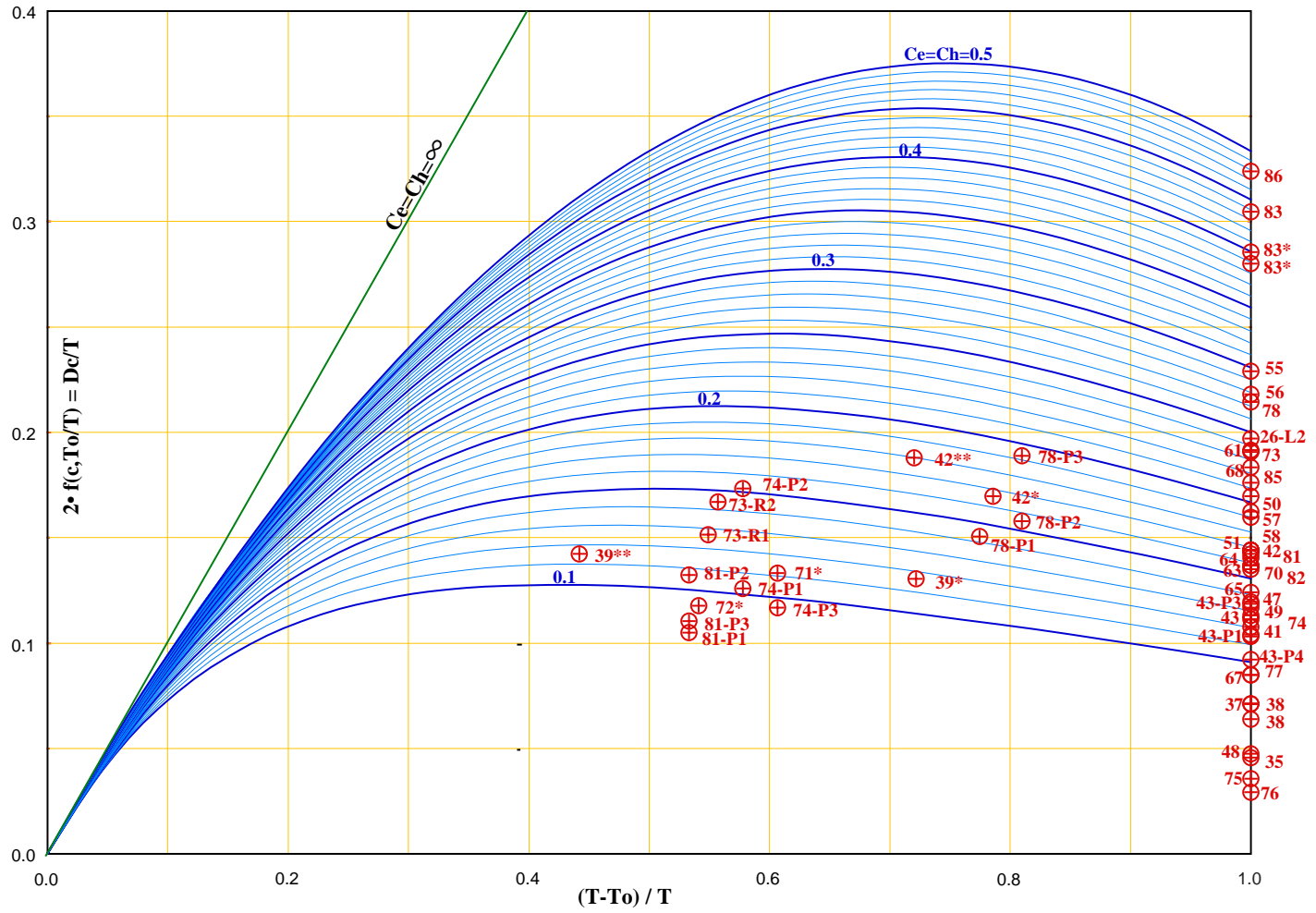


Fig. 11. Universal template for evaluating linear coefficient c . $2 \cdot f(c, T_0/T)$ is plotted against $(T - T_0) / T$. $d c / T$ of DeBeers samples are superimposed.

CONCLUSIONS

- A) A calculation has been performed for the induced charge signal generated in diamond detectors based on linear model in which the local distance of charge transmission was assumed to increase linearly with the diamond growth thickness. As a result an explicit analytical formula was derived for the dependence of the induced charge on the remaining thickness after removing the substrate side.
- B) It was shown that the induced charge is independent of the relative orientation of the electric field.
- C) The measured values of the induced charge on DBDS #39 lapped into different thickness is consistent with the general trend of the dependence indicating the validity of the linear model although a further study is required for confirmation.
- D) The example of DBDS#39 indicates that there is a significant contribution from hole motion.
- E) A universal template for evaluating the linear coefficient c of the samples was demonstrated. Superimposing the measured data of the collection distances of the samples listed in the reports, we find some samples with exceedingly high values of c .

There are some caveats and comments that need to be stated.

- a) A large question remains. Do we have any reason to believe that the linear model is the right model which must have to do with the CVD diamond growth process that is totally covered under proprietary secrets, at least to the author ? Though there are some indications that linear model as phenomenology is close to the reality in some of the samples (DBDS#39, #83), it depends on whether a single sample is grown at a constant condition represented by a constant value of the coefficient c .
- b) It is by no means the intention of the author that the linear coefficient c is the most important parameter to watch. What we need for the detector is a 400 μm thick piece with good transmission and if there is a way to make such pieces economically and reproducibly, whether it is represented by a single parameter c is of no importance.
- c) However, even if we are not able to persuade the manufacturer to provided some hints on such questions, it is still an interesting exercise to pursue this model as a “working” hypothesis for describing the characteristics of the samples *as given*, until we find otherwise.
- d) As has been discussed in the recent meetings, uniformity, or the lack thereof to some degree, is an important issue in addition to the “signal size” which is one of the aspects averaged over some region of the sample. It may be a useful practice if the initial measurement of the collection distance is made at least at several points over the sample surface, especially for the samples larger than ordinary 1 cm x 1 cm area, so that we have some feeling on the uniformity.

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