LHC Optics Determination with Proton Tracks Measured in the Roman Pot Detectors of the TOTEM Experiment

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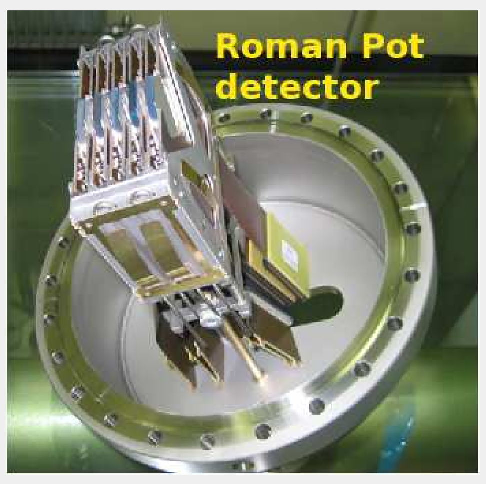
1. The Roman Pots of the TOTEM experiment

Proton-proton elastic scattering has been measured by the TOTEM experiment at the CERN Large Hadron Collider at $\sqrt{s}$ = 7 TeV in dedicated runs. The small scattering angles (down to a few μradians) are detected with the movable near-beam insertions (Roman Pots) equipped with stacks of silicon microstrip detectors, installed on the outgoing LHC beams.

The Roman Pots of TOTEM

- 4 stations at s $\approx \pm 147$ m and s $\approx \pm 220$ m
- 6 Pots per station (4 vertical + 2 horizontal devices)
- a total of 24 Roman Pots

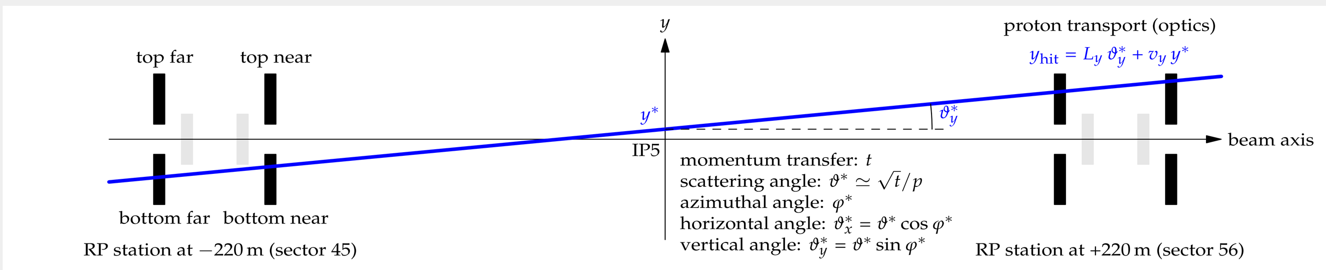


High angular and spatial resolution of track reconstruction:

- $\sigma(\Theta_x) \approx \sigma(\Theta_y) \approx 1.7\mu$rad

Precise detector alignment:

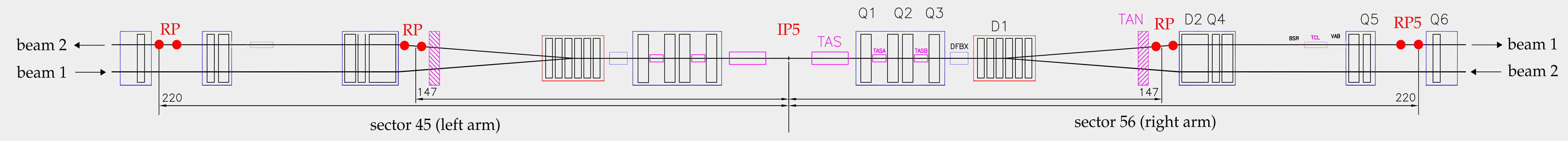
- Beam touching alignment around 50μm
- Track based alignment $\delta x < 10$ μm, $\delta y = 10$ μm



Side view of an elastic event at IP5

2. How to transport protons from the interaction point to the detectors ?

Scattered protons are detected after having moved through a segment of the LHC lattice containing 29 magnets per beam.



At IP5 they are characterized by transverse vertex position $(x, y)_{IP}$ and scattering angle $(\Theta_x, \Theta_y)_{IP}$ and they are observed with transverse positions $(x, y)_{RP}$ and angles $(\Theta_x, \Theta_y)_{RP}$. Proton trajectories are described by a transport matrix:

$$\begin{pmatrix} x \\ \Theta_x \\ y \\ \Theta_y \\ \Delta p/p \end{pmatrix}_{RP} = \begin{pmatrix} v_x & L_x & m_{13} & m_{14} & D_x \\ v'_x & L'_x & m_{23} & m_{24} & D'_x \\ m_{31} & m_{32} & v_y & L_y & D_y \\ m_{41} & m_{42} & v'_y & L'_y & D'_y \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ \Theta_x \\ y \\ \Theta_y \\ \Delta p/p \end{pmatrix}_{IP}$$

with $v_{x,y}$ *magnification* and L_y *effective length* being particularly important, which can be expressed in terms of *betatron amplitude* β:

$$L_{x,y}(s) = \sqrt{\beta_{x,y}(s)\beta_{IP}}\sin\Delta\mu_{x,y}(s) \qquad \Delta\mu_{x,y}(s) = \int_{s_0}^{s}\beta_{x,y}(s')^{-1}ds'$$

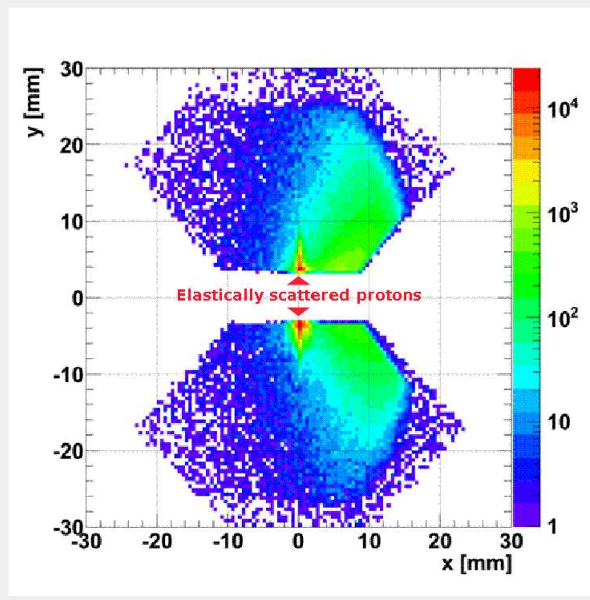
In case of a perfect machine all $m_{i,j}$ would be 0. In reality, they are close to zero and approximately

$$\Theta_{y,IP} \approx \frac{y_{RP}}{L_{y,RP}} \qquad \Theta_{x,IP} \approx \frac{1}{\frac{dL_{x,RP}}{ds}}\left(\Theta_{x,RP} - \frac{dv_{x,RP}}{ds}x_{IP}\right)$$

In reality, the vertex contributions are cancelled due to the anti-symmetry between the beams. For elastically scattered protons the four momentum transfer is given by

$$t = -p^2 \cdot \left(\Theta^2_{x,IP} + \Theta^2_{y,IP}\right)$$

As the values of the reconstructed angles are directly depend on the optical functions, the accuracy of optics defines the systematic errors of the final physics results.



Reconstructed proton tracks.

3. Optics errors induced by LHC imperfections

Proton transport matrix

The proton transport matrix $T(s; \mathcal{M})$ is defined by the machine settings $\mathcal{M}$. Therefore it is calculated with the MAD-X code for each group of runs with identical optics based on several data sources:

- TIMBER: actual currents of the magnets
- FIDEL + LSA: current to strength conversion curves
- WISE: measured imperfections (harmonics, displacements, rotations)

The lattice is subject to additional $\Delta\mathcal{M}$ *imperfections*, not measured well enough so far, which alter the transport matrix

$$T(s; \mathcal{M}) \rightarrow T(s; \mathcal{M} + \Delta\mathcal{M}) = T(s; \mathcal{M}) + \Delta T$$

$\Delta\beta/\beta$ beating measurement with 5–10% accuracy is not enough for the TOTEM physics program.

The effect of machine imperfections: ΔT

ΔT is a function of machine imperfections of which the most important are:

- Magnet strength conversion error $I \rightarrow B$, $\sigma(B)/B \approx 10^{-3}$, FIDEL
- Beam momentum offset, $\sigma(p)/p \approx 10^{-3}$

Other imperfections are of lower importance

- Magnet rotations $\sigma(\phi) \approx 1$ mrad, WISE database
- Beam harmonics, $\sigma(B)/B \approx 10^{-4}$, WISE database
- Power converter errors $\sigma(I)/I \approx 10^{-4}$
- Magnet positions $\Delta x, \Delta y \approx 100$ μm, WISE database

The allowed magnitude of $|\Delta T|$ is determined by the nominal optics settings together with the above quoted tolerances. In addition ΔT can be approximately determined from proton tracks in the Roman Pots.

Optical function sensitivity to LHC imperfections

Sensitivity of the vertical effective length L_y to magnet strengths and beam momentum imperfections with the relative error of 1‰ for low and large β_{IP} settings.

$\beta_{IP} = 3.5$ m	
Perturbed	$\delta L_y/L_y$ [%]
MQXA.1R5	0.98
MQXB.A2R5	-2.24
MQXB.B2R5	-2.42
MQXA.3R5	1.45
MQY.4R5.B1	-0.10
MQML.5R5.B1	0.05
Δp/p	-2.19

$\beta_{IP} = 90$ m	
Perturbed	$\delta L_y/L_y$ [%]
MQXA.1R5	0.14
MQXB.A2R5	-0.23
MQXB.B2R5	-0.25
MQXA.3R5	0.20
MQY.4R5.B1	-0.01
MQML.5R5.B1	0.04
Δp/p	0.01

$\beta_{IP} = 90$ m optics is robust against perturbations.

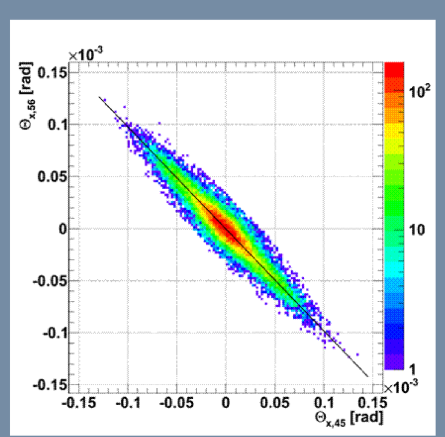
4. Constraints for optics estimation from distributions measured with Roman Pots for $\beta_{IP} = 3.5\,m$

Elastically scattered proton collinearity constraints

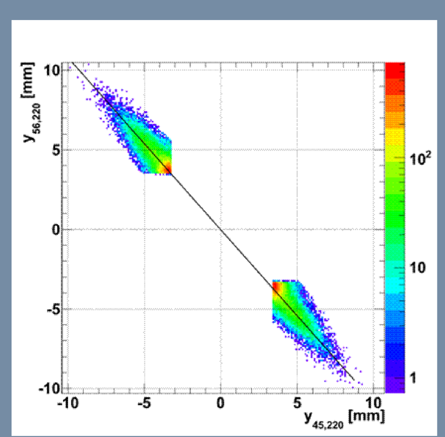
Elastic scattering relates the optical functions of beam 1 and 2

$$R_1 \equiv \frac{\Theta_{x,b_1,RP}}{\Theta_{x,b_2,RP}} \approx \frac{\frac{dL_{x,b_1,RP}}{ds}}{\frac{dL_{x,b_2,RP}}{ds}} \quad \text{and} \quad R_2 \equiv \frac{y_{b_1,RP}}{y_{b_2,RP}} \approx \frac{L_{y,b_1,RP}}{L_{y,b_2,RP}}$$

$b_{1,2} \equiv$beam 1 and 2. $R_{1,2}$ can be estimated with a 0.5% precision.



Horizontal angle beam 1 vs. beam 2 @RP



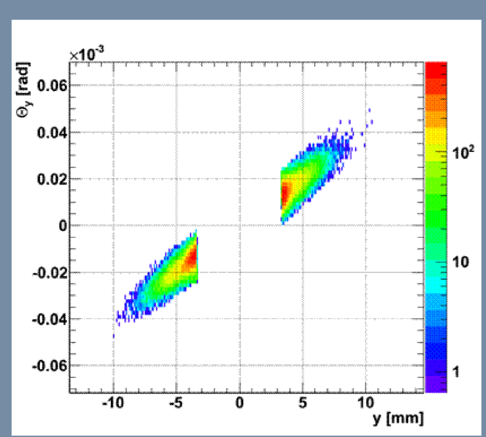
Horizontal position beam 1 vs. beam 2 @RP

Vertical and horizontal angle vs. position constraints

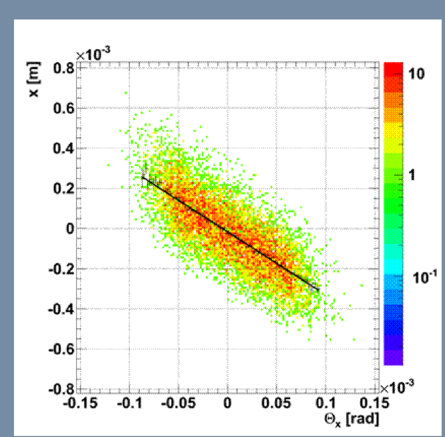
$\frac{dL_y}{ds}/L_y$ and $\frac{dL_x}{ds}/L_x$ can be expressed with measurables:

$$R_3 \equiv \frac{\Theta_{y,b_1,RP}}{y_{b_1,RP}} \approx \frac{\frac{dL_{y,b_1,RP}}{ds}}{L_{y,b_1,RP}} \quad \text{and} \quad R_5 \equiv \frac{x_{b_1,RP}}{\Theta_{x,b_1,RP}} \approx \frac{L_{x,b_1,RP}}{\frac{dL_{x,b_1,RP}}{ds}}$$

and $R_{4,6}$ follow this pattern for beam 2. Their precision is 0.5 %.



Vertical position vs. vertical angle @RP



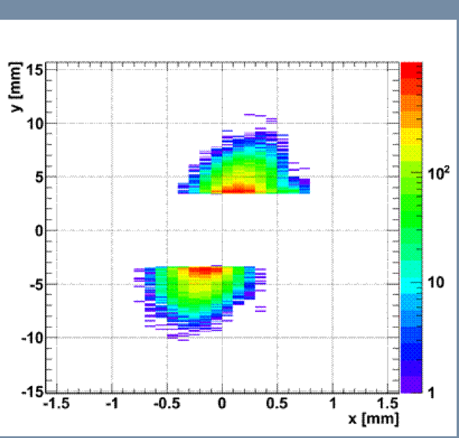
Horizontal angle vs. horizontal position @RP

X,Y coupling estimation and constraints

The coupling components of the transport matrix can be estimated:

$$R_7 \equiv \frac{x_{b_1,near\ pots}}{y_{b_1,near\ pots}} \approx \frac{m_{14,b_1,near\ pots}}{L_{y,b_1,near\ pots}} \quad \text{and} \quad R_8 \equiv \frac{x_{b_1,far\ pots}}{y_{b_1,far\ pots}} \approx \frac{m_{14,b_1,far\ pots}}{L_{y,b_1,far\ pots}}$$

with a precision of 3%. $R_{9,10}$ is similarly defined with beam 2.



Vertical vs. horizontal position @RP

5. Real optics estimation: matching the machine parameters for $\beta_{IP} = 3.5\,m$

From detailed sensitivity studies it is known that there are 6 relevant magnets per beam segments between IP5 and Roman Pots

- the inner triplet (2 MQXA and 2 MQXB magnets)
- the MQML, MQY magnets (less important, but not negligible)

In total 36 constraints were applied

- 10 constraints R_i from measurements
- $2_{beams} \times 6_{magnets} \times 2_{constraints(strength,rotation)}$ = 24 magnet design constraints
- 2 beam momentum constraints

26 parameters optimized, magnet strengths, rotations, beam momenta.

The phase space is $\mathcal{T} = \mathbb{R}^{26}$, and the $\chi^2 : \mathcal{T} \rightarrow \mathbb{R}$ function is minimized, where

$$\chi^2 = \chi^2_{measured} + \chi^2_{design}, \qquad \chi^2_{measured} = \sum_{i=1}^{10}\left(\frac{R_i - R_{i,MADX}}{\sigma(R_i)}\right)^2$$

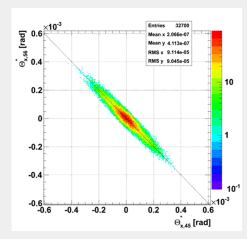
$$\chi^2_{design} = \sum_{i=1}^{12}\left(\frac{k_i - k_{i,MADX}}{\sigma(k_i)}\right)^2 + \sum_{i=1}^{12}\left(\frac{\phi_i - \phi_{i,MADX}}{\sigma(\phi_i)}\right)^2 + \sum_{i=1}^{2}\left(\frac{p_i - p_{i,MADX}}{\sigma(p_i)}\right)^2$$

The design part defines the nominal machine as an attractor $\mathcal{A} \in \mathcal{T}$, while the measured part "pushes" the place of minimum from $\mathcal{A}$ to a nearby P to meet with the measured constraints.

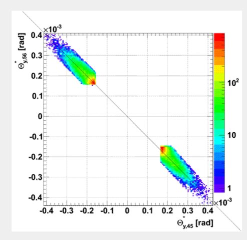
In practice both the CERN MINUIT package and Windows Solve were used to estimate P. Finally, dL_x/ds and L_y were used for data analysis.

6. Matching results

- Before matching
  - Beam 1
    - L_y=22.4 m dL_x/ds=-3.21·10^{-1}
  - Beam 2
    - L_y=18.4 m dL_x/ds=-3.29·10^{-1}
- After matching
  - Beam 1
    - L_y=22.6 m, dL_x/ds=-3.12·10^{-1}
  - Beam 2
    - L_y=20.7 m, dL_x/ds=-3.15·10^{-1}



Hor. angle beam1&2 @IP



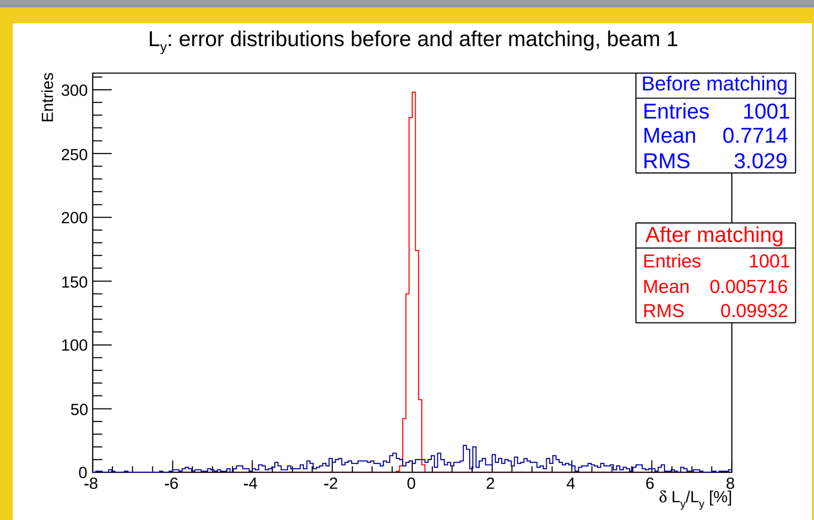
Ver. angle beam1&2 @IP

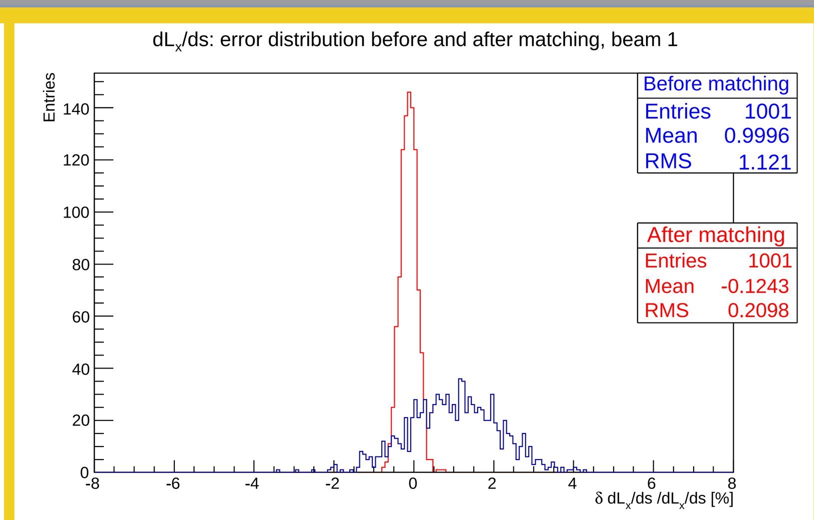
7. Monte-Carlo validation of the method for $\beta_{IP} = 3.5\,m$

In the study the following LHC parameters were perturbed to simulate the effect of $\Delta\mathcal{M}$:

- The strength of relevant magnets
- Beam momenta
- Magnet displacements, rotations
- Kickers, harmonics
- Scattered protons angular-distribution

- Distribution of optical function errors resulting from imperfections:
  - Bias$\left(\frac{\delta dL_x/ds}{dL_x/ds}\right)$=1.0 %, RMS$\left(\frac{\delta dL_x/ds}{dL_x/ds}\right)$=1.1 %
  - Bias$(\delta L_y/L_y)$=0.77 %, RMS$(\delta L_y/L_y)$=3.0 %
- Optics imperfections after matching with Roman Pot data:
  - Bias$\left(\frac{\delta dL_x/ds}{dL_x/ds}\right)$=-0.12 %, RMS$\left(\frac{\delta dL_x/ds}{dL_x/ds}\right)$=0.21 %
  - Bias$(\delta L_y/L_y)$=0.0057 %, RMS$(\delta L_y/L_y)$=0.10 %





Conclusion: for β_{IP}=3.5 m TOTEM can measure the transfer matrix between IP5 and Roman Pots with a precision 0.2 %.

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