# LATTICE OPTIMIZATION FOR A REALLY LARGE HADRON COLLIDER (RLHC)

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#### Abstract

Long arc cells would lead to major cost savings in a high field high  $T_c$  hadron collider, operating in the regime of significant synchrotron radiation. Two such lattices, with half cell lengths of 110 and 260 m, are compared. Both allow flexible tuning, and have large dynamic apertures when dominated by chromatic sextupoles. Lattices with longer cells are much more sensitive to systematic magnet errors, which are expected to dominate.

# **1 INTRODUCTION**

Contemporary hadron colliders operate in a regime of insignificant synchrotron radiation damping. Conceptual designs of a high field post-LHC "Really Large Hadron Collider" (RLHC), with parameters only slightly beyond those of the SSC, benefit greatly from damping times much less than the storage time. They deliberately incorporate 'short' damping times to desensitize machine operations with respect to sources of phase space dilution, and to increase the integrated luminosity[1, 2]. High field RLHC designs explicitly use "high  $T_c$ " superconducting magnets, to safely absorb the radiation heat load of up to 5 W/m. They implicitly assume that the high  $T_c$  commercial magnets which are beginning to appear will, in due course, be developed into accelerator quality magnets. Magnet R&D based on recent advances in high  $T_c$  materials suggest the possibility of practical tape wound magnets[3].

This paper addresses lattice optimization issues, taking for granted the primary parameters listed in Table 1 that are discussed elsewhere[1, 2]. The advantages of very long arc cells are examined by studying two detailed lattices, one near each end of the spectrum of plausible arc cell length. The short cell lattice has 6 dipoles per half cell of length L = 110 m, while the long cell lattice has 15 dipoles per half cell of length L = 260 m. The optical and dynamical aperture performance of each lattice is discussed.

# 2 HALF CELL LENGTH

The optimum half arc cell length, L, depends on a dynamic balance between two effects pushing for longer cells, and one pushing for shorter cells. Longer cells save money through fewer quadrupoles, fewer correctors, and fewer spool pieces. They also have reduced strength chromaticity sextupoles, and an increased dynamic aperture from this source. However, shorter cells and smaller beams make

Parameter	units	value
Storage energy Injection energy Dipole field (store) Dipole length Dipole coil ID Triplet quad gradient (store) Number of Interaction Regions Transverse rms emittance, $\epsilon$ Longitudinal rms bunch area, S Longitudinal rms emittance, $\epsilon_s$ RF frequency, $f_{RF}$	[TeV] [TeV] [T] [m] [T/m] [τ/m] [eV-s] [m] [MHz]	30.0 1.0 12.5 17.0 50 - 60 300 2 1.0 0.5 .0102 360
Transverse damping time (store)	[hr]	$\approx 2.1$

Table 1: Primary design parameters. Values which vary significantly with time, such as  $\epsilon$ , are quoted at injection.

more modest demands on magnetic field quality.

The SSC half cell length stayed remarkably constant at  $L \approx 100$  m during repeated optimization exercises[4]. This is largely due to the consistent use of an almost invariant set of magnet error statistics, which assumed that random errors would dominate systematics. Recent experience with RHIC magnets shows (scaled) errors that are much smaller than in the SSC model, and shows, to the contrary, that systematics dominate randoms[5]. Measured errors in several real SSC dipoles were significantly smaller than in the standard SSC model used for tracking[6]. This suggests that RLHC half cells might be much longer than 100 meters.

It is not yet possible to postulate a plausible magnet error model for high  $T_c$  dipoles. Instead, suppose that particle motion is stable within a "good field aperture" of  $r_{GF} = 15$  mm, about half of the coil radius, in the arcs. If the phase advance per cell is 90 degrees, the maximum beta function is given by  $\hat{\beta} = 3.41 L$ , generating a maximum transverse rms injection beam size of

$$\widehat{\sigma}_{\beta} = \sqrt{\epsilon \widehat{\beta}/(\beta \gamma)} = \sqrt{\widehat{\beta}/1066} \ [mm]$$
 (1)

The maximum allowable value of L occurs when the beam size fills the good field aperture, when  $r_{GF} = n\hat{\sigma}_{\beta}$ , where a conservative value for n is 15. This yields a maximum

Parameter	units	SHORT	LONG
Half cell length, $L$ Max. cell beta, $\hat{\beta}$ Max. cell dispersion, $\hat{\eta}$ Max. betatron size, $\hat{\sigma}_{\beta}$ Circumference, $C$ Horizontal tune, $Q_x$ Vertical tune, $Q_y$ Number of dipoles	[m] [m] [m] [mm] [km]	110 376 3.85 .594 55.44 65.195 66.185 2888	29.185 2900
Number of sextupoles Slip factor, $\alpha$ Mmtm. width, $\sigma_p/p$ RMS bunch length, $\sigma_s$ Synchrotron tune, $Q_s$ Voltage slope, $dV/ds$ RF voltage, $V$	[10 <sup>-3</sup> ] [10 <sup>-3</sup> ] [m] [10 <sup>-3</sup> ] [MV/m] [MV]	456 .299 .1545 .0619 6.59 103.29 13.69	168 1.813 .0401 .238 2.63 2.78 .37

Table 2: Longitudinal parameters for SHORT and LONG cell lattices, at injection. There are two sextupoles per cell.

allowable half cell length of

$$\widehat{L} = \frac{\beta \gamma}{\epsilon} \frac{r_{GF}^2}{3.41 n^2} = 313 \ [m]$$
 (2)

Long arc cells have a significant impact on longitudinal parameters, as shown in the values in Table 2. The maximum dispersion,  $\hat{\eta} = 2.71 L^2/R$ , increases quadratically with *L*, causing the horizontal beam width contribution from momentum spread to rise much faster than the betatron contribution. The small momentum spreads quoted in Table 2 are determined by setting

$$\sigma_p / p = \hat{\sigma}_\beta / \hat{\eta} \tag{3}$$

to equalize the two contributions to the total beam width. The large slip factor in the LONG lattice ( $\approx 1/Q_{arc}^2$ ) compensates for the small momentum width to make the beam acceptably resistant to collective effects, such as the microwave instability. The small LONG RF voltage (at 360 MHz) yields acceleration times of order 10<sup>4</sup> seconds.

## **3 LATTICE OPTICS**

Figure 1 shows a simple 4 quad telescope matching into empty arc cells. The telescope consists of a close packed triplet starting at  $L^* = 20$  m from the collision point, followed by a 4th quadrupole placed L/2 from the first regular strength arc cell quadrupole [7]. The same optics also match into dispersion suppressor (DS) cells, as shown for the Interaction Regions (IR) cluster in Figure 2. Each DS has four 90 degree cells, with 3/4 the length of regular arc cells, and 2/3 of the number of dipoles. The 2 IRs are separated by back-to-back DSs, in addition to the DSs that match

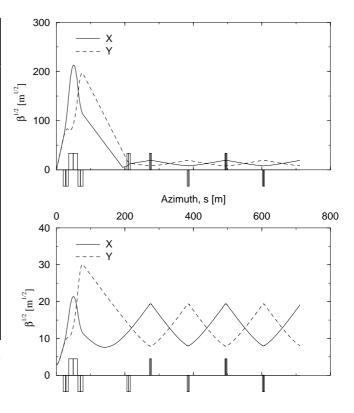


Figure 1: Telescope optics, with  $\beta^* = 0.1$  m (storage) in the top figure, and  $\beta^* = 8.0$  m (injection) in the bottom.

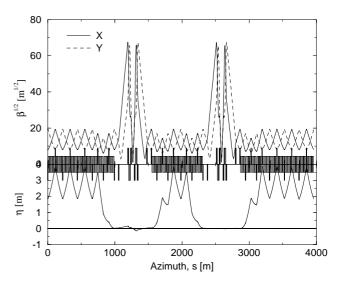


Figure 2: Layout of the IR cluster, with  $\beta^* = 1.0$  [m].

into the regular arcs. On the opposite side of the RLHC circumference are two utility straights of identical geometry, constructed from empty arc cells.

The maximum triplet gradient of 300 T/m causes peak and collision point betas to be related by an "effective length"  $L_e$ , defined by

$$\widehat{\beta} = L_e^2 / \beta^* \tag{4}$$

and given by  $L_e \approx 70$  m for both lattices [1, 8]. When  $\beta^* =$ 

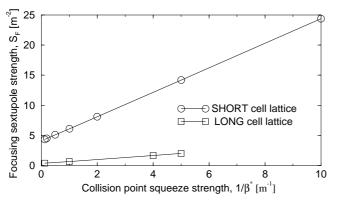


Figure 3: Chromaticity sextupole strength versus squeeze strength at constant net chromaticity.

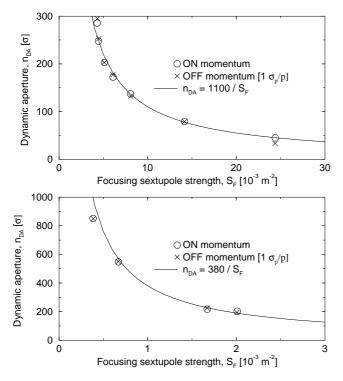


Figure 4: Dynamic aperture (10<sup>4</sup> turns) in the SHORT lattice (top) and the LONG lattice (bottom).

0.1 m at 30 TeV, the triplet beam size is  $\sigma_{trip} \approx 1.2$  mm. Triplet quads can have modest bores.

#### **4 DYNAMIC APERTURE**

When  $\beta^*$  is varied with the net chromaticities held constant  $(\chi_x = \chi_y = 2.0)$ , the total sextupole strength has a contribution proportional to  $1/\beta^*$ , due to the natural chromaticity from the telescopes. Figure 3 illustrates this, with LONG strengths much weaker than the SHORT.

The ON momentum dynamic aperture is inversely proportional to the sextupole strength, as shown by the computed dynamic apertures recorded in Figure 4. Particles are launched with equal betatron motion amplitudes, parametrized by *n* where, if the motion is linear,  $x_{max}/\sigma_x = y_{max}/\sigma_y = \sqrt{0.5} n$ . The OFF momentum dynamic aperture is also plotted, for particles with a synchrotron oscillation amplitude of  $1\sigma_p/p$  (injection). In most cases the OFF and ON momentum dynamic apertures are indistinguishable, owing to the small momentum spread. The exception is the SHORT lattice with  $\beta^* = 0.1 \text{ m} (S_F =$  $24.4 \times 10^{-3} \text{ m}^{-2})$ , where chromatic distortions due to the telescopes are at their strongest.

The huge dynamic apertures with only chromatic sextupoles present  $(n_{DA} \ge 33)$  are rather illusory, since neither a realistic physical aperture  $(n \sim 40)$  nor realistic dipole errors have been included. For example, suppose the dipoles have a systematic sextupole error of  $\Delta B/B = b_2$  at a reference radius of  $r_0 \approx 15$  mm. The integrated sextupole field of a half cell of dipoles equals the injection sextupole strength at a critical value of  $b_2$  given by

$$b_{2,\,cr\,it} \approx r_0^2/\widehat{\eta}^2 \sim L^{-4} \tag{5}$$

The apparent advantages of very long cells may disappear when realistic errors for tape wound high  $T_c$  magnets become known.

### **5** CONCLUSIONS

Lattices with unusually long arc cells have potential advantages, including large cost savings, in a high field high  $T_c$  RLHC. Both LONG and SHORT lattices, with half cell lengths of 110 and 260 m, allow  $\beta^*$  to be squeezed down from 8.0 m at injection to 0.1 m at storage. However, to optimize the cell length it is necessary derive realistic magnet errors. This includes addressing the relative importance of random and systematic harmonics - do systematics dominate, as at RHIC, or do randoms dominate, as assumed for the SSC? It is also necessary to explore the scaling of collective instabilities with respect to arc cell length.

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