# Observation of the CP-conserving $K_s \to \pi^+\pi^-\pi^0$ Decay Amplitude

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The interference between CP-conserving $K_S$ and $K_L \to \pi^+\pi^-\pi^0$ decay amplitudes was observed by studying the decay rate asymmetries between initial $K^0$ and $\overline{K}^0$ separately for the phase space regions $E_{CM}(\pi^+) > E_{CM}(\pi^-)$ and $E_{CM}(\pi^+) < E_{CM}(\pi^-)$. For the parameter $\lambda$ we found $\mathrm{Re}(\lambda) = 0.036 \pm 0.010(\mathrm{stat.})^{+0.002}_{-0.003}(\mathrm{syst.})$ and $\mathrm{Im}(\lambda)$ consistent with zero, leading to a branching ratio $\mathrm{B} = \left[4.1^{+2.5}_{-1.9}(\mathrm{stat.})^{+0.5}_{-0.6}(\mathrm{syst.})\right] \times 10^{-7}$ for the CP-conserving $K_S \to \pi^+\pi^-\pi^0$ decay.

## 1 Introduction

The CPLEAR experiment at the Low Energy Antiproton Ring at CERN uses tagged $K^0$ and $\overline{K}^0$ to study CP, T and CPT violation in neutral kaon decays. In a previous CPLEAR publication [1] the measurement of the CP violation parameter $\eta_{+-0}$ in $K_S \to \pi^+\pi^-\pi^0$ decays was reported. In this paper, the observation of the CP-allowed $K_S \to \pi^+\pi^-\pi^0$ decay amplitude is presented.

While the two-pion final state of the neutral kaon decay always has a definite CP eigenvalue of $+1$, the CP eigenvalue of the $\pi^+\pi^-\pi^0$ final state is dependent on the angular momentum configuration of the three pions. The angular momentum configuration is described in terms of the orbital angular momenta between the $\pi^+$ and $\pi^-$ ($l$), and between the $\pi^0$ and the $\pi^+\pi^-$ pair ($l'$). Since the total angular momentum of the three-pion system must equal the spin of the kaon, we have $l = l'$, and thus the CP eigenvalue for the $\pi^+\pi^-\pi^0$ final state is given by $(-1)^{l+1}$. The sum of the masses of the three pions is close to the kaon mass, hence the pions have low kinetic energy $E_{\mathrm{CM}}(\pi)$ in the kaon rest-frame. For this reason decays into $\pi^+\pi^-\pi^0$ states with high pion angular momenta are suppressed.

Amplitudes of kaon decay to a three-pion final state can be parameterized as a function of the two variables $X$ and $Y$, defined as

$$X = \frac{s_2 - s_1}{\mathrm{m}_{\pi^\pm}^2} \quad \text{and} \quad Y = \frac{s_3 - s_0}{\mathrm{m}_{\pi^\pm}^2} \tag{1}$$

where $\mathrm{m}_{\pi^\pm}$ is the mass of the charged pion. The variable $s_i$ is given by

$$s_i = (p_{\mathrm{K}} - p_i)_\mu (p_{\mathrm{K}} - p_i)^\mu \tag{2}$$

where the pion four-momentum is denoted by $p_i$ ($i$ =1 for $\pi^+$, 2 for $\pi^-$ and 3 for $\pi^0$) and $p_{\mathrm{K}}$ is the four-momentum of the neutral kaon. The variable $s_0$ is defined by

$$s_0 = \frac{s_1 + s_2 + s_3}{3}. \tag{3}$$

Given the above definitions, $X$ is proportional to the difference $E_{\mathrm{CM}}(\pi^+) - E_{\mathrm{CM}}(\pi^-)$.

There are two contributions to the $\mathrm{K_S} \to \pi^+\pi^-\pi^0$ decay amplitude $A_{\mathrm{S}}^{+-0}(X,Y)$: one from the decay into a $\pi^+\pi^-\pi^0$ state with CP = +1, and another from the decay into a CP = −1 state. We introduce the following expression:

$$A_{\mathrm{S}}^{+-0}(X,Y) = A_{\mathrm{S}}^{3\pi(\mathrm{CP}=+1)}(X,Y) + A_{\mathrm{S}}^{3\pi(\mathrm{CP}=-1)}(X,Y), \tag{4}$$

with the property

$$A_{\mathrm{S}}^{+-0}(-X,Y) = -A_{\mathrm{S}}^{3\pi(\mathrm{CP}=+1)}(X,Y) + A_{\mathrm{S}}^{3\pi(\mathrm{CP}=-1)}(X,Y). \tag{5}$$

In the $\mathrm{K_L} \to \pi^+\pi^-\pi^0$ decay, suppression of the decay amplitude for $\mathrm{K_L} \to \pi^+\pi^-\pi^0$ (CP = +1) results from both CP violation and the centrifugal effect. Thus, we have

$$A_{\mathrm{L}}^{+-0}(X,Y) = A_{\mathrm{L}}^{3\pi(\mathrm{CP}=-1)}(X,Y) \tag{6}$$

with

$$A_{\mathrm{L}}^{+-0}(-X,Y) = A_{\mathrm{L}}^{+-0}(X,Y). \tag{7}$$

The CP-conserving $\mathrm{K_L}$ decay amplitude $A_{\mathrm{L}}^{+-0} \equiv A_{\mathrm{L}}$ can interfere with both the CP-conserving $\mathrm{K_S}$ decay amplitude $A_{\mathrm{S}}^{3\pi(\mathrm{CP}=+1)}$ and the CP-violating $\mathrm{K_S}$ decay amplitude $A_{\mathrm{S}}^{3\pi(\mathrm{CP}=-1)}$. The first interference term is antisymmetric in $X$ and vanishes when data are integrated over the whole phase space. In order to observe this interference term, events with $X > 0$ and $X < 0$ must be separated. The second interference term is symmetric in $X$.

Let the eigentime-dependent decay rates for the initial $\mathrm{K}^0$ decaying into $\pi^+\pi^-\pi^0$ be $R_+(\tau)$ and $R_-(\tau)$ for $X > 0$ and $X < 0$, respectively, and similarly for the initial $\overline{\mathrm{K}}^0$ let the rates be $\overline{R}_+(\tau)$ and $\overline{R}_-(\tau)$. The time-dependent asymmetry between $R_+(\tau) + R_-(\tau)$ and $\overline{R}_+(\tau) + \overline{R}_-(\tau)$ was used in our previous publication [1] to obtain the CP violation parameter $\eta_{+-0}$ given by

$$\eta_{+-0} = \frac{\int dX\, dY A_{\mathrm{L}}^*(X,Y)\, A_{\mathrm{S}}^{3\pi(\mathrm{CP}=-1)}(X,Y)}{\int dX\, dY |A_{\mathrm{L}}(X,Y)|^2}. \tag{8}$$

In this paper, two asymmetries are determined, separating the rates for $X > 0$ and $X < 0$:

$$\begin{aligned} A_\pm(\tau) &= \frac{\overline{R}_\pm(\tau) - R_\pm(\tau)}{\overline{R}_\pm(\tau) + R_\pm(\tau)} \\ &= 2\,\mathrm{Re}(\varepsilon) - 2\, e^{-\Delta\Gamma\,\tau/2}\, [\mathrm{Re}\,(\eta_{+-0} \pm \lambda)\cos(\Delta m\, \tau) \\ &\qquad -\mathrm{Im}\,(\eta_{+-0} \pm \lambda)\sin(\Delta m\, \tau)] \end{aligned}$$

where $\Delta m$ is the $\mathrm{K_L}$–$\mathrm{K_S}$ mass difference, $\Delta\Gamma$ is the $\mathrm{K_S}$–$\mathrm{K_L}$ decay width difference and $\varepsilon$ is the CP violation parameter in the $\mathrm{K}^0$–$\overline{\mathrm{K}}^0$ mixing.

The parameter $\lambda$ describes the interference between the CP-conserving $\mathrm{K_L}$ and $\mathrm{K_S}$ decay amplitudes and is defined by

$$\lambda = \frac{\int_{X>0} dX\, dY\, A_{\mathrm{L}}^*(X,Y)\, A_{\mathrm{S}}^{3\pi(\mathrm{CP}=+1)}(X,Y)}{\int_{X>0} dX\, dY |A_{\mathrm{L}}(X,Y)|^2}. \tag{9}$$

The phase-space integration is restricted to $X > 0$. A non-zero value of $\lambda$ is an unambiguous sign of the presence of the CP-conserving $\mathrm{K_S} \to \pi^+\pi^-\pi^0$ decay amplitude. From phenomenological considerations $\lambda$ is expected to be real, i.e. $\mathrm{Im}(\lambda) = 0$ [2].

## 2 Experimental method

For the determination of the parameter $\lambda$, the data used for the measurement of $\eta_{+-0}$ were analyzed. Selection criteria and cuts identical to those described in [1] were used. The only difference in the current analysis is the further splitting of the time-dependent decay rates to $\pi^+\pi^-\pi^0$ into phase-space regions with $X > 0$ and $X < 0$.

The experimental asymmetries for $X > 0$ and $X < 0$ are given by

$$A_{\pm}^{\exp}(\tau) = \frac{\overline{N}_{\pm}(\tau) - N_{\pm}(\tau)}{\overline{N}_{\pm}(\tau) + N_{\pm}(\tau)} \tag{10}$$

where $N$ represents the measured decay rates. When the difference between the efficiencies for identifying the initial $\mathrm{K}^0$ and $\overline{\mathrm{K}}^0$ ($\epsilon(\mathrm{K}^0)$ and $\epsilon(\overline{\mathrm{K}}^0)$) and the background are taken into account, as discussed in [1], we obtain

$$A_{\pm}^{\exp}(\tau) \cong \left(\frac{\xi - 1}{\xi + 1}\right) + \frac{4\,\xi\,[1 - \zeta(\tau)]}{(\xi + 1)^2}\, A_{\pm}(\tau) \tag{11}$$

where $A_{\pm}(\tau)$ is the asymmetry given in Eq. 9. The normalization factor $\xi = \epsilon(\overline{\mathrm{K}}^0)/\epsilon(\mathrm{K}^0)$ is treated as a free parameter in the fit to the data. The background fraction $\zeta(\tau)$ is the ratio of the number of background events estimated by simulation to the total number of events in the data sample.

In determining the branching ratio for the CP-allowed $\mathrm{K_S} \to \pi^+\pi^-\pi^0$ decay from $\lambda$, we use the following parameterizations for the decay amplitudes [3–5]:

$$A_{\mathrm{L}}(X, Y) = a + b\,Y + c\,Y^2 + d\,X^2 \quad \text{and} \quad A_{\mathrm{S}}^{3\pi(\mathrm{CP}=+1)}(X, Y) = e\,X. \tag{12}$$

Terms with higher orders in $X$ and $Y$ are suppressed by kinematics [4]. A term proportional to $X\,Y$ in $A_{\mathrm{S}}^{3\pi(\mathrm{CP}=+1)}$ is neglected since its contribution to our final result is estimated to be less than 1%. For the parameters related to $\mathrm{K_L}$ decays, i.e. $a$, $b$, $c$ and $d$, we use the following values extracted from a global fit to all measured kaon decay rates [5]:

$$\begin{aligned} a &= (84.32 \pm 0.43) \times 10^{-8} \;\; b = (28.31 \pm 0.63) \times 10^{-8} \\ c &= -(1.62 \pm 0.16) \times 10^{-8} \;\; d = (0.29 \pm 0.05) \times 10^{-8}. \end{aligned} \tag{13}$$

The coefficient $e$ for the $\mathrm{K_S}$ decay amplitude is obtained directly from the experimental value of $\lambda$ using Eqs. 9, 12 and 13.

## 3 Decay asymmetries and fits

Figure 1 shows the experimental time-dependent decay rate asymmetries for $X > 0$ and $X < 0$ after applying all the correction procedures described in [1]. In addition, we have taken into account the fact that the cuts which remove the 'combinatorial' background [1] act differently, in the two halves of the Dalitz plot, depending on the charge of the primary kaon. Monte Carlo simulation shows that only 5% of the signal events are removed by these cuts and that the resulting shift on $\mathrm{Re}\,(\lambda)$ is $-4 \times 10^{-3}$ for $\mathrm{Im}\,(\lambda) = 0$. The simulation statistical error on this correction is incorporated in the final statistical error.

A simultaneous six-parameter fit of the functions $A_+^{\mathrm{exp}}(\tau)$ and $A_-^{\mathrm{exp}}(\tau)$ to the relevant data samples gives the following results:

$$
\begin{aligned}
\mathrm{Re}\,(\eta_{+-0}) &= (6 \pm 13) \times 10^{-3} \\
\mathrm{Im}\,(\eta_{+-0}) &= (-2 \pm 18) \times 10^{-3} \\
\mathrm{Re}\,(\lambda) &= (32 \pm 13) \times 10^{-3} \\
\mathrm{Im}\,(\lambda) &= (-6 \pm 18) \times 10^{-3} \\
\xi_{X<0} = \xi_{X>0} &= 1.127 \pm 0.010
\end{aligned}
\tag{14}
$$

The values compiled by the Particle Data Group [6] are used for $\Delta\Gamma$ and $\mathrm{Re}(\varepsilon)$, the value of $\Delta m$ being taken from [7]. Given the current information on $\varepsilon'$ [8] and $\eta_{+-0}$ [1,9], and the accuracy discussed here, we can ignore direct CP-violation and assume that $\eta_{+-0} = \eta_{+-}$. In addition, since the normalizations $\xi_{X<0}$ and $\xi_{X>0}$ are compatible, we can assume $\xi_{X<0} = \xi_{X>0} = \xi$.

Therefore simultaneous fits of the CP asymmetries $A_\pm^{\mathrm{exp}}(\tau)$ are performed, with common $\lambda$ and $\xi$, and $\eta_{+-0} = \eta_{+-}$, using the value from [6] for $\eta_{+-}$. The fit gives

$$
\begin{aligned}
\mathrm{Re}\,(\lambda) &= (32 \pm 13) \times 10^{-3} \\
\mathrm{Im}\,(\lambda) &= (-6 \pm 16) \times 10^{-3}
\end{aligned}
\tag{15}
$$

and $\xi = 1.127 \pm 0.006$. Since this result is consistent with the assumption that $\mathrm{Im}(\lambda) = 0$ a final two-parameter fit may be performed, yielding

$$
\mathrm{Re}\,(\lambda) = (36 \pm 10) \times 10^{-3} \tag{16}
$$

and $\xi = 1.126 \pm 0.006$. The solid lines in Fig. 1 show the result of this fit. The non-zero value obtained for $\lambda$ indicates the presence of CP-allowed $K_S \rightarrow \pi^+\pi^-\pi^0$ decay amplitude.

## 4 Systematic errors

The magnitudes of individual systematic errors are listed in Table 1 for $\mathrm{Re}(\lambda)$ with fixed value of $\mathrm{Im}(\lambda) = 0$. The sources of systematic errors are largely identical to those for the determination of $\eta_{+-0}$ [1]. Because the $\lambda$ terms have opposite signs in $A_+(\tau)$ and $A_-(\tau)$ (see Eq. 9) and the normalization factor $\xi$ is present in first order only as an additive term (Eq. 11), sensitivity to the normalization is weak in the simultaneous fits. Uncertainty on the background normalization also cancels to first order. In addition, we have to consider effects which could be different for the two halves of the Dalitz plot, as discussed below.

- The residual background at short decay-times may have a normalization $\xi^B$ which depends on the variable $X$. An example is the 'combinatorial' background discussed in [1], where the inversion of the primary pion with the secondary pion may introduce a systematic shift of $X$ which is positive for a $\overline{K}^0$-tagged event and negative for a $K^0$ event. To quantify this effect we introduce a background normalization difference $y_B = \xi^B_{X>0} - \xi^B_{X<0}$ and repeat the simultaneous fit of the asymmetries, leaving $y_B$ as a free parameter. Since the residual background decreases very rapidly with decay-time, $y_B$ is almost uncorrelated with $\lambda$. The fit yields $y_B = 0.02 \pm 0.22$, resulting in a systematic error on $\mathrm{Re}(\lambda)$ of $^{+2.0}_{-2.5} \times 10^{-3}$.
- Due to the integration over phase space, the acceptance does not cancel in the expression of $\lambda$ (Eq. 9). Simulated data were used to measure the acceptance as a function of $X$, $Y$ and eigentime $\tau$. The effect of the acceptance in the determination of $\mathrm{Re}(\lambda)$ was less than $10^{-3}$. To test the simulation accuracy of the acceptance, the $K^0$ and $\overline{K}^0 \rightarrow \pi^+\pi^-\pi^0$ event distribution in the Dalitz plot was analysed to extract the $K_L \rightarrow \pi^+\pi^-\pi^0$ Dalitz plot parameters [6]. The values extracted for these parameters are in excellent agreement with the current world averages [6] and have comparable errors.

Similarly, we studied the effect of the same sources of systematic errors on the determination of $\mathrm{Im}(\lambda)$ in the fit where both the real and imaginary parts are left free.

| Source of systematic error | $\mathrm{Re}(\lambda) \times 10^{-3}$ |
|---|---|
| Amount of background | 0.2 |
| Normalization of background | - |
| Decay time dependence of normalization* | 0.3 |
| Decay time resolution * | 0.3 |
| Regeneration | - |
| $\Delta m$ and $\Gamma_S$ | 0.1 |
| Residual background normalization asymmetric in X * | (-2.5, +2.0) |
| Acceptance * | 1.0 |

* Error determination currently limited by statistics.

Table 1
Summary of systematic errors on $\mathrm{Re}(\lambda)$ fixing $\mathrm{Im}(\lambda) = 0$

## 5 Final results and conclusions

Assuming there is no correlation between systematic errors, we obtain

$$\begin{aligned}\mathrm{Re}(\lambda) &= \left(32 \pm 13\,(\mathrm{stat.})\,{}^{+2}_{-3}\,(\mathrm{syst.})\right) \times 10^{-3} \\ \mathrm{Im}(\lambda) &= \left(-6 \pm 16\,(\mathrm{stat.})\,{}^{+1}_{-2}\,(\mathrm{syst.})\right) \times 10^{-3}\end{aligned} \tag{17}$$

where $\mathrm{Im}(\lambda)$ is consistent with zero.

Finally assuming $\mathrm{Im}(\lambda) = 0$, we obtain

$$\mathrm{Re}(\lambda) = \left(36 \pm 10\,(\mathrm{stat.})\,{}^{+2}_{-3}\,(\mathrm{syst.})\right) \times 10^{-3} \tag{18}$$

From Eqs. 12, 13 and 18, the branching ratio for the CP-conserving $\mathrm{K_S} \rightarrow \pi^+\pi^-\pi^0$ decay is calculated as

$$\mathrm{B}_{\mathrm{K_S}\rightarrow\pi^+\pi^-\pi^0(\mathrm{CP}=+1)} = \left(4.1\,{}^{+2.5}_{-1.9}\,(\mathrm{stat.})\,{}^{+0.5}_{-0.6}\,(\mathrm{syst.})\right) \times 10^{-7} \tag{19}$$

where the systematic error also includes a contribution from the uncertainty on the $K_L$ decay parameters (Eq. 13) [5].

Note that the CP-conserving $K_S \to \pi^+\pi^-\pi^0$ decay rate is about 300 times larger than the expected CP-violating decay rate, which can therefore be neglected in this context.

In conclusion, we have observed the CP-conserving $K_S \to \pi^+\pi^-\pi^0$ decay amplitude by measuring its interference with the corresponding $K_L$ amplitude. We have obtained values for the parameter $\lambda$ and the branching ratio B, which are in a good agreement with the values predicted by both a phenomenological global fit to all known kaon decay rates [5,10] and chiral perturbation theory [10,11]. Our results are also in good agreement with those of a recent experiment [12], and were obtained with 30% less statistical error. Our systematical errors are negligible for both $\mathrm{Re}(\lambda)$ and $\mathrm{Im}(\lambda)$.

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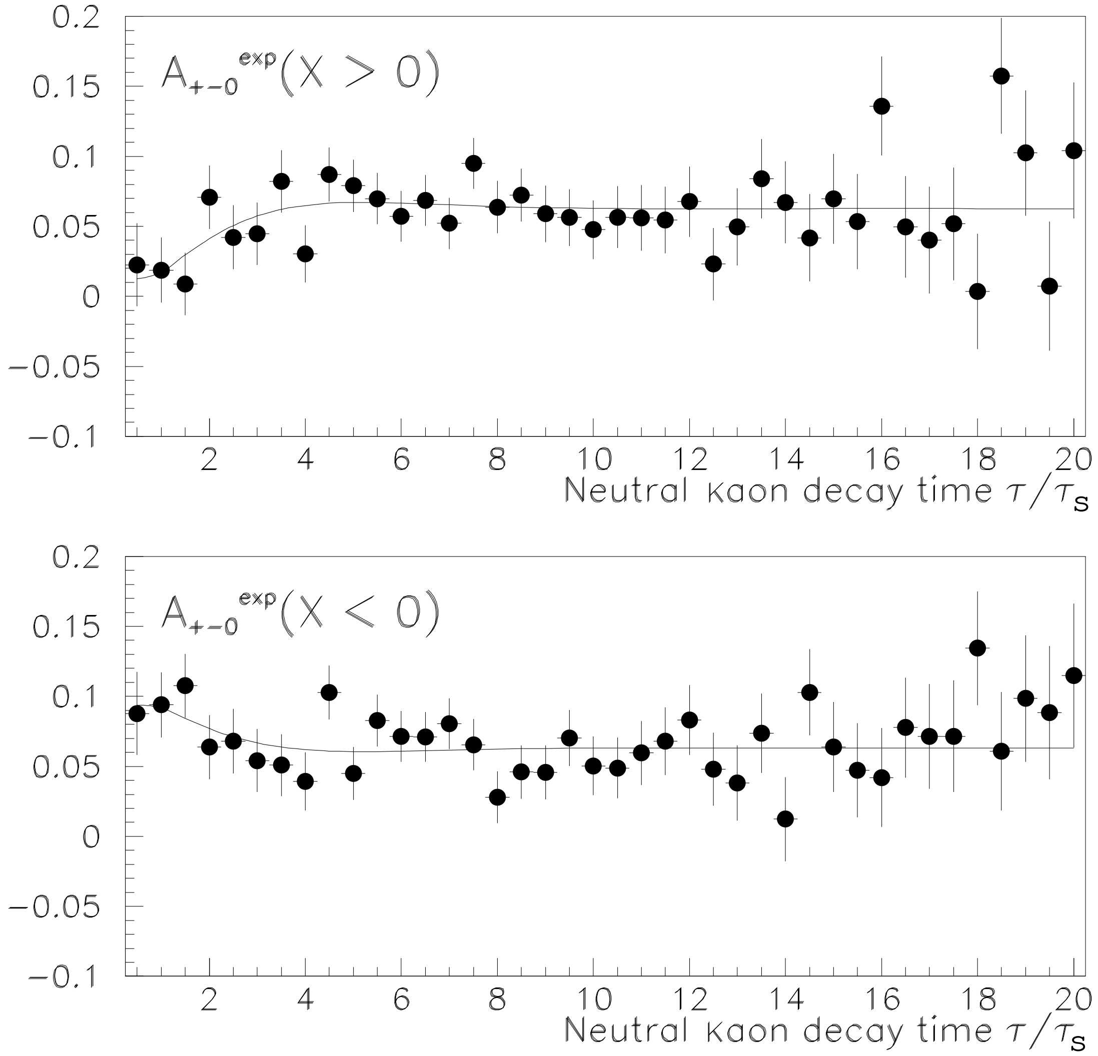


Fig. 1. $\overline{\mathrm{K}}^0$–$\mathrm{K}^0 \to \pi^+\pi^-\pi^0$ measured decay rate asymmetries for $X > 0$ and $X < 0$ as a function of $\tau/\tau_{\mathrm{S}}$, where $\tau_{\mathrm{S}}$ is the $\mathrm{K_S}$ mean life. The solid curves are the result of the simultaneous two-parameter fit of Eq. 11 to the two asymmetries, assuming common $\mathrm{Re}(\lambda)$ and $\xi$.