# Evaluation of the phase of the CP violation parameter $\eta_{+-}$ and the $\mathrm{K_L} - \mathrm{K_S}$ mass difference from a correlation analysis of different experiments


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The best estimation of $\varphi_{+-}$ (the phase of the CP violation parameter $\eta_{+-}$) and of $\Delta m$ (the $\mathrm{K_L} - \mathrm{K_S}$ mass difference) is obtained by averaging the results of different experiments, taking into account the different correlation, existing for most of the experiments, between the measurement of $\varphi_{+-}$ and $\Delta m$. Including the recent measurements, we obtain the average values $<\Delta m> = (530.7 \pm 1.3) \times 10^7 \hbar/\mathrm{s}$ and $<\varphi_{+-}> = 43.82^\circ \pm 0.63^\circ$. This value of $\varphi_{+-}$ is in good agreement with the superweak phase $\varphi_{SW} = 43.49^\circ \pm 0.08^\circ$.

## 1 Introduction

The world's best limits of CPT violation [1,2] are obtained by comparing the phase $\varphi_{+-}$ of the CP violation parameter $\eta_{+-}$ with the superweak phase $\varphi_{SW} \equiv \tan^{-1}(2\Delta m/\Delta\Gamma)$. Here $\Delta m(\Delta\Gamma)$ is the mass (total decay width) difference between the $\mathrm{K_L}$ and $\mathrm{K_S}$. Since the present experimental results [3-8,10-13] are of comparable precision, the best estimation of the value of $\varphi_{+-}$ is obtained by averaging the results of all these experiments. Given the different strong correlation of the measurement of $\varphi_{+-}$ and $\Delta m$ for most of the experiments, averaging the measurements of $\varphi_{+-}$ and $\Delta m$ independently [1] seems not to be the adequate method. A more precise method consists in using all the available experimental information to construct a global likelihood distribution $\mathcal{L}$ depending on the parameters $\Delta m$, $\varphi_{+-}$ (and $\tau_S$, the $\mathrm{K_S}$ mean life), as the product of individual likelihood distributions of each experiment. The best estimations for the values of $\Delta m$ and $\varphi_{+-}$ are then obtained by maximizing

Table 1
Measurements of $\varphi_{+-}$ considered in our fit. For the experiments [6,7] we assume a common systematic error of $\pm 0.3°$ due to regeneration uncertainties.

| Experiment | $\varphi_{+-}$ | stat. errors $\varphi_{+-}$, $\Delta m$ | $\rho$ |
|---|---|---|---|
| | [°] | [°], [$10^7\hbar$/s] | |
| Geweniger [3] | $49.4 \pm 1.0 + 0.565\,(\Delta m - 540.0)$ | | $> 0.99$ |
| Carithers [4] | $45.5 \pm 2.8 + 0.224\,(\Delta m - 534.8)$ | | |
| Carosi [5] | $46.9 \pm 1.6 + 0.579\,(\Delta m - 535.1) + 303\,(\tau_S - 0.8922)$ | | |
| † E731 [6] | $42.2 \pm 0.9 + 0.189\,(\Delta m - 525.7) - 460\,(\tau_S - 0.8922)$ | 0.75, 4.4 | 0.74 |
| E773 [7] | $43.53 \pm 0.76 + 0.173\,(\Delta m - 528.2) - 275\,(\tau_S - 0.8926)$ | 0.58, 3.0 | 0.67 |
| CPLEAR [8] | $42.7 \pm 1.1 + 0.316\,(\Delta m - 527.4) + 30\,(\tau_S - 0.8926)$ | 0.9, 6.7 | 0.92 |

† The 1994 PDG [1] quotation has an error in the central value for the $\Delta m$ dependence

$\mathcal{L}$. The values of the experiments [3–6, 10-12] adopted by PDG [1], as well as the values of three recent experiments [7, 8, 13] are presented in Table 1 (for $\varphi_{+-}$), together with their quoted $\Delta m$ and $\tau_S$ dependence, and in Table 2 (for $\Delta m$).

A gaussian likelihood distribution taking into account the correlation between $\varphi_{+-}$ and $\Delta m$ can be defined with the published information for the experiments [6–8]. The authors of the other experiments [3–5] simply state that their value of $\varphi_{+-}$ is strongly correlated with $\Delta m$ and give a linear dependence. In order to define a gaussian likelihood distribution also for these experiments, we have to make some assumptions about the correlation between $\varphi_{+-}$ and $\Delta m$ and about the central value of $\Delta m$. For experiment [3], which gives the most precise value of $\varphi_{+-}$ among these three experiments, we have been able to deduce that the correlation is larger than 99% [9]. We assume this correlation also holds for the other two experiments [4, 5]. For such a strong correlation between $\varphi_{+-}$ and $\Delta m$, the error ellipse of the individual experiment is degenerated to a band (see Figure 1) within the limit of $\Delta m$ defined by the other experiments [6–8, 10–13]. Consequently the experiments [3–5] contribute to the fit only as one degree of freedom. The systematic uncertainty for the average values due to our assumptions is estimated by varying the correlation between $\varphi_{+-}$ and $\Delta m$ and the central value of $\Delta m$.

The dependence of $\varphi_{+-}$ on the value of $\tau_S$ is less important, since $\tau_S$ is known to a better precision than $\Delta m$. For the same reason, the change in the experimental value of $\Delta m$ is negligible, when varying the value of $\tau_S$ within its error (Table 2). The world average value of $\tau_S$ [1] is dominated by the exper-

Table 2
Measurements of $\Delta m$ considered in our fit. The experiments [10–13] measure $\Delta m$ independently of $\varphi_{+-}$, whereas the experiments [6–8] obtain $\Delta m$ from a fit with floating $\Delta m$ and $\varphi_{+-}$. The experiments [11] and [12] have a common systematic error of $\pm 1.5 \times 10^7 \hbar/\mathrm{s}$.

| Experiment | $\Delta m [10^7 \hbar/\mathrm{s}]$ |
|---|---|
| E731 [6] | $525.7 \pm 4.9$ |
| E773 [7] | $529.7 \pm 3.7$ |
| CPLEAR [8] | $529.5 \pm 6.7$ |
| Cullen [10] | $542.0 \pm 6.0$ |
| Gjesdal [11] | $533.4 \pm 4.0$ |
| Geweniger [12] | $534.0 \pm 3.0 + 12\,(\tau_S - 0.8994)$ |
| CPLEAR [13] | $527.4 \pm 2.9 \pm 83\,(\tau_S - 0.8926)$ |

iment of Ref. [6]. The authors assume in their fit $\varphi_{+-} = \varphi_{SW}$ and note that a difference of $1^\circ$ between $\varphi_{+-}$ and $\varphi_{SW}$ would shift the value of $\tau_S$ by only $0.0008 \times 10^{-10}\,\mathrm{s}$. In our fitting procedure we only take into account the linear dependence of $\varphi_{+-}$ with $\tau_S$ when it is given (Table 1).

## 2 Description of the fits

We assume that the measured values $X_i = ({\varphi_{+-}}^i, \Delta m^i)^T$ are distributed around the true value $X$ according to a gaussian likelihood distribution:

$$L_i(X) = k_i e^{-\frac{1}{2}(X_i - X)^T C_i^{-1}(X_i - X)}, \tag{1}$$

where $C_i$ is the corresponding covariance matrix:

$$C_i = \begin{pmatrix} (\sigma_\varphi^i)^2 & c_{\varphi\Delta m}^i \\ c_{\varphi\Delta m}^i & (\sigma_{\Delta m}^i)^2 \end{pmatrix}, \tag{2}$$

and $k_i$ is a normalization factor which generally depends on the elements of $C_i$. In the case of experiments [10-13], $X_i = \Delta m^i$. The combined likelihood distribution of all measurements is then given by:

$$\mathcal{L}(X) = \prod_i L_i(X)\ . \tag{3}$$

The best estimate of $X$ and its error is given by maximizing $\mathcal{L}(X)$, i.e. minimizing $\chi^2(X) = \sum(X_i - X)^T C_i^{-1}(X_i - X)$ with respect to $X$. Errors common to different experiments can be taken into account in this procedure by expanding $X_i$ and $C_i$ as follows:

$$X_i \rightarrow X_i = \begin{pmatrix} X_k \\ X_l \end{pmatrix} \qquad C_i \rightarrow C_i = \begin{pmatrix} C_k & \sigma^2 \\ \sigma^2 & C_l \end{pmatrix}, \tag{4}$$

where $\sigma$ is the error common to experiment $k$ and $l$.

Most of the experiments quote the value of $\varphi_{+-}$ for a fixed value $\Delta m = \Delta\tilde{m}$ together with the linear dependence on $\Delta m$. We recall that for a fixed value of $\Delta m$, $\Delta\tilde{m}$, the value of $\varphi_{+-}$ and its error, $\tilde{\varphi}_{+-}$ and $\tilde{\sigma}_\varphi$ respectively, are given by:

$$\tilde{\varphi}_{+-} = \varphi_{+-} + \rho\frac{\sigma_\varphi}{\sigma_{\Delta m}}(\Delta\tilde{m} - \Delta m) \tag{5}$$

$$\tilde{\sigma}_\varphi = \sigma_\varphi\sqrt{1-\rho^2} \tag{6}$$

where $\varphi_{+-}$, $\sigma_\varphi$ and $\Delta m$, $\sigma_{\Delta m}$ are the central values and errors of a two parameter fit and $\rho$ is the correlation coefficient between $\varphi_{+-}$ and $\Delta m$. Therefore we are able to reconstruct the full covariance matrix by using in addition to the value of $\varphi_{+-}$ and its statistical error, the value of $\Delta m$ and its error and the linear dependence (Table 1 and 2). The systematic errors of $\varphi_{+-}$ and $\Delta m$ are added in quadrature to the diagonal elements of the covariance matrix.

### *2.1 Average values for $\varphi_{+-}$ and $\Delta m$*

The data used as input to our fits is summarized in Table 1 and Table 2 and also presented together with the final result in Figure 1. Since the values of $\varphi_{+-}$ depend on the value of $\tau_S$, we leave $\tau_S$ as an additional free parameter in the fit. However, its value is constrained by the precision of the world average, $<\tau_S> = (0.8926 \pm 0.0012) \times 10^{-10}$ s [1].

Using the experimental data available up to 1994 as input to our fit , we find similar results as PDG [1]. However our error for $<\Delta m>$ is smaller compared to the PDG result due to the additional information used. For $\Delta m$ we find $<\Delta m>$ $= (532.1 \pm 1.8 \pm 0.1) \times 10^7 \hbar/\mathrm{s}$ and for $\varphi_{+-}$ the value $<\varphi_{+-}> = 44.3° \pm 1.0° \pm 0.1°$, where the second error reflects the uncertainty of the correlation between $\varphi_{+-}$ and $\Delta m$ for the experiments [3–5]. Two experiments [6,10] give a large contribution to the $\chi^2$, the first giving a low value and the second a high value of $\Delta m$ compared to the average value. The $\chi^2$/degree of freedom (dof) for the

Table 3
Results from Fit A and Fit B. Fit A is made using all the experiments whereas Fit B is based only on the experiments [6–8, 10–13] which give the full covariance matrix of their measurements. The error for Fit A includes an additional error of $\pm 0.1 \times 10^7 \hbar/\mathrm{s}$ for $\Delta m$ and $\pm 0.1^\circ$ for $\varphi_{+-}$, obtained by varying the correlation between $\varphi_{+-}$ and $\Delta m$ for the older experiments [3–5].

| Parameter | Fit A | Fit B |
|---|---|---|
| $\Delta m [10^7 \hbar/\mathrm{s}]$ | $530.7 \pm 1.3$ | $530.9 \pm 1.5$ |
| $\varphi_{+-}$ | $43.82^\circ \pm 0.63^\circ$ | $43.71^\circ \pm 0.66^\circ$ |
| $\tau_S$ $[10^{-10}\,\mathrm{s}]$ | $0.8922 \pm 0.0010$ | $0.8923 \pm 0.0011$ |
| $\chi^2/\mathrm{dof}$ | 0.89 | 1.02 |
| $\varphi_{SW}^{\dagger}$ | $43.49^\circ \pm 0.08^\circ$ | $43.50^\circ \pm 0.08^\circ$ |

$^\dagger$ For the $\mathrm{K_L}$ mean life we used $\tau_L = (5.15 \pm 0.04) \times 10^{-8}\,\mathrm{s}$ [1].

combined average of $\varphi_{+-}$ and $\Delta m$ is 1.0, and in contrast to PDG we do not need to scale the error of $\Delta m$ by 1.2.

If we include the three recently published measurements [7,8,13] of $\Delta m$ and $\varphi_{+-}$ we find the results shown in Table 3 (Fit A). The value of $\Delta m$ is lower by $1\,\sigma$ and only one experiment [10] now gives a large contribution to the $\chi^2$. Since this experiment deviates only by $2\sigma$ and the total $\chi^2/\mathrm{dof}$ is 0.8, we have retained it in our fit. By using only the experiments [6–8, 10–13], which give the full covariance matrix of their measurements, we obtain the results shown in Table 3 (Fit B), which are in good agreement with Fit A.

The quoted systematic error of [6,7] concerning their phase measurement which is mainly determined by the knowledge of the regeneration amplitudes has been subject of discussion [14–16]. Increasing the common systematic error of these two experiments to $1^\circ$ ($3^\circ$) yields $\varphi_{+-} = 43.94^\circ \pm 0.75^\circ$ ($44.06^\circ \pm 0.87^\circ$) and $\Delta m = [530.8 \pm 1.4\ (530.9 \pm 1.4)] \times 10^7 \hbar/\mathrm{s}$. We conclude that the results of our correlated analysis do not change significantly, even with enlarged systematic errors for the measurements reported by [6] and [7], although the precision on $\varphi_{+-}$ deteriorates.

## 3 Conclusion

In order to determine the best values of $\Delta m$ and $\varphi_{+-}$ using the data available from different experiments, we performed a correlated fit to the data, using when available the individual correlations of this two parameters. Our final result using the experiments [3–8,10–13] is:

$$<\varphi_{+-}> = 43.82^\circ \pm 0.63^\circ \tag{7}$$
$$<\Delta m> = (530.7 \pm 1.3) \times 10^7 \hbar/\mathrm{s} \ , \tag{8}$$

with a correlation coefficient of 0.70 between $\varphi_{+-}$ and $\Delta m$, and $-0.40$ between $\varphi_{+-}$ and $\tau_S$, i.e.:

$$\varphi_{+-} = 43.82^\circ \pm 0.41^\circ + 0.339\,(\Delta m - 530.7)^\circ - 252\,(\tau_S - 0.8922)^\circ \ . \tag{9}$$

The value of $\varphi_{+-}$ is in good agreement with the superweak phase $\varphi_{SW} = 43.49^\circ \pm 0.08^\circ$ as expected from CPT invariance.

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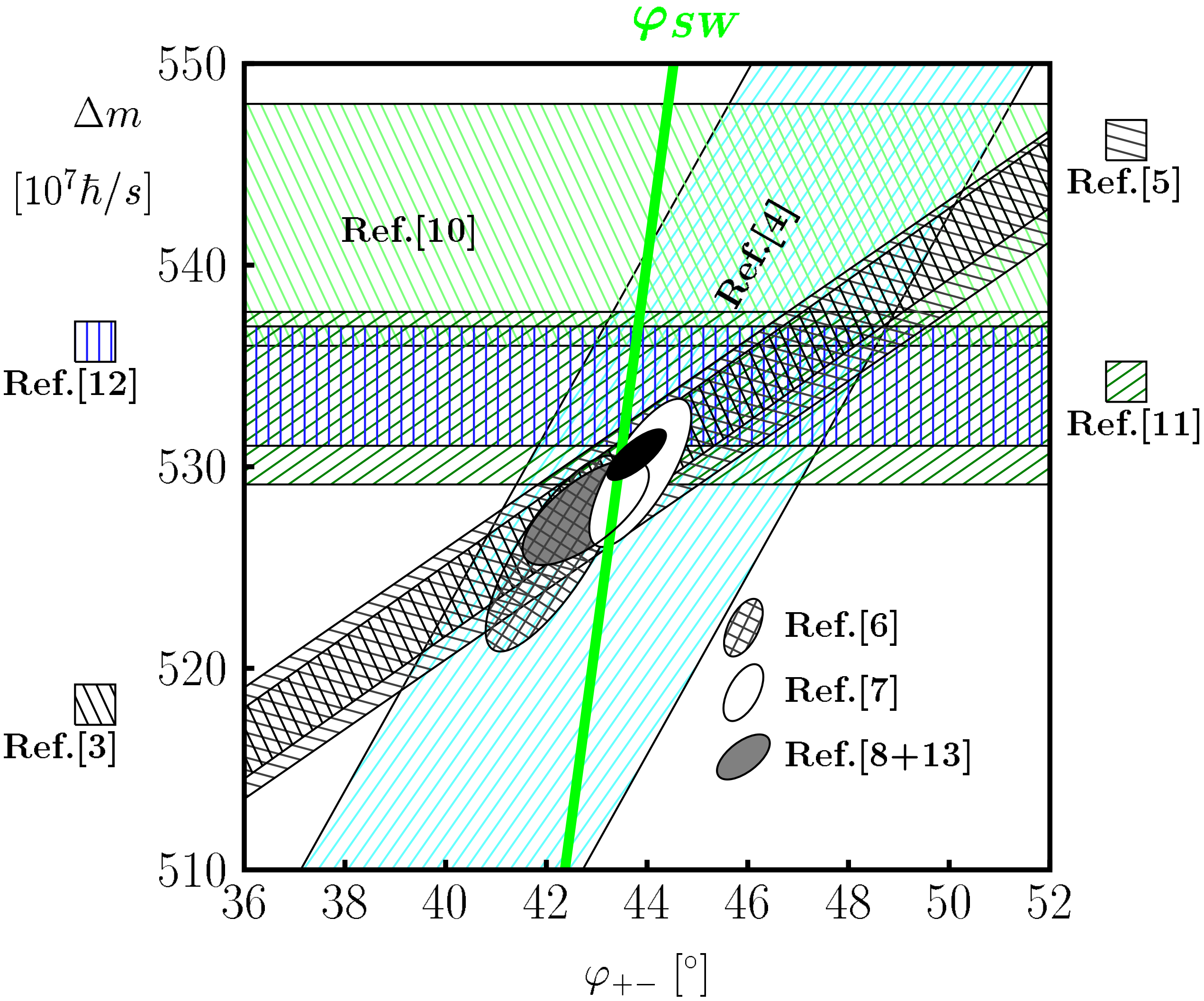


Fig. 1. The plot shows the $1\sigma$ contour plots of all measurements listed in Table 1 and 2. The black ellipse in the center represents the result of our fit. The expected region for the value of $\varphi_{SW}$ is also shown.