# Quantitative analysis of two-neutron correlations in the ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right){ }^{14} \mathrm{C}$ reaction 

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The ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right){ }^{14} \mathrm{C}$ and ${ }^{12,13} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right){ }^{13,14} \mathrm{C}$ reactions are studied at 84 MeV . Mass distributions and energy spectra of the ejectiles are measured, indicating the selectivity of these reactions to populate two- and one-neutron configurations in the states of the residual nucleus, respectively. The measured absolute cross-section angular distributions are analyzed by exact finite range coupled reaction channel calculations based on a parameter free double-folding optical potential. The form factors for the $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reaction are extracted within an extreme cluster and independent particles scheme with shell-model-derived coupling strengths. The results show that the measured cross sections are accurately described for the first time without the need for any arbitrary scaling factor. The $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reaction is thus found to be a powerful tool for quantitative spectroscopic studies of pair configurations in nuclear states.

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## I. INTRODUCTION

Direct transfer reactions have the fundamental property to select specific degrees of freedom in the complicated many body nuclear system. For this reason, they have always played a crucial role in the exploration of the nuclear structure. In particular, the relation between transfer probabilities in twoneutron transfer reactions and pairing correlations in nuclei is a subject of great interest [1]. The strength of the pairing force is indeed directly connected to microscopically derived transition densities for pair addition and removal modes [2,3]. Compared to other methods where the pairing force is studied by the exploration of ground state properties, transfer reactions are particularly interesting because a large variety of excited states are accessed. In this way, the effects of the pairing as a function of the occupation of different single-particle orbitals and linear and angular momentum transfer, including the rise of collective modes, can be determined. A renewed attention in this field was recently triggered by the possibility to use radioactive ion beams to study the role of pairing in nuclei far from stability [4-7].

A key issue in the study of transfer reactions is the concept of spectroscopic factor. This is experimentally accessible for almost pure single-particle and $\alpha$ cluster states but less evident for two-neutron configurations, since the two-neutron system is unbound in the vacuum. However, the pairing force, which acts in the mean field of a nuclear system, binds the two neutrons together and makes the pair-pluscore configurations relevant for some particular states. A detailed study of such states gives quantitative constraints to the pairing force, basically contained in the two-particle spectroscopic factors. In contrast to the simpler one-nucleon transfer processes, it is not straightforward to extract structure information from the two-neutron transfer cross sections. One
of the difficulties consists of the coupling of the initial and final states with inelastic excitations, which opens alternative routes for the reaction. Coupled channel corrections are necessary to explain, for example, the anomalous behavior of the angular distributions observed in the transitions to the $2^{+}$vibrational states in medium mass nuclei [8,9]. Other important couplings are those with states of intermediate partitions populated by the sequential transfer of the two nucleons. These second order processes are important especially when dealing with heavy ions [10-12].

In the past, spectroscopic investigations of pairing correlations above the Fermi level were mainly conducted by $(t, p)$ reactions [13], but nowadays the use of triton beams is very limited, mainly due to restrictive radioprotection rules. However, the potentiality of heavy-ion two-neutron transfer reactions has not been fully exploited [8,14], partly due to the experimental difficulties in producing high-resolution spectra covering a wide energy and angular range [15]. Nevertheless, studies with heavy projectiles demonstrate their reliability for quantitative analyses, as long as the multistep processes remain comparatively weak [16].

From a theoretical point of view, it is known that, in heavyion two-neutron transfer reactions, the standard technique to extract the optical potential from fits of elastic scattering angular distributions fails in reproducing the differential cross sections $[17,18]$. Attempts to use folded-density calculations of the real part were made without success [19]. Another feature that should be taken into account is the finite size of the nuclei and the recoil effects [20,21]. Computational problems have historically limited the calculation of finite range form factors in multinucleon transfer [22]. In addition, in a single cross-section analysis, only the product of projectile and target spectroscopic factors is accessible, thus making the extraction of spectroscopic information not so straightforward.

Consequently, even the most successful calculations based on the coupled channel Born approximation required the use of scaling or "unhappiness" factors sensibly larger than one and even as high as hundreds to reproduce the absolute cross sections, whose origin has not been clarified so far [9,14,22-24]. In such conditions, the extraction of spectroscopic factors is not reliable.

Here we show that the above-mentioned difficulties can be largely overcome. In particular, we have studied the ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right){ }^{14} \mathrm{C}$ reaction above the Coulomb barrier. The $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reactions are good candidates to show the role of pairing interaction thanks to the existence of a correlated pair of neutrons in the ${ }^{18} \mathrm{O}$ ground state wave function and the very low polarizability of the ${ }^{16} \mathrm{O}$ core. The study is complemented by the analysis of the ${ }^{12,13} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right){ }^{13,14} \mathrm{C}$ reaction, in order to ascertain the selectivity of the two processes in exciting two-particle and single-particle configurations, respectively.

## II. EXPERIMENT AND RESULTS

The experiment was performed at the Istituto Nazionale di Fisica Nucleare Laboratori Nazionali del Sud in Catania, Italy. A beam of ${ }^{18} \mathrm{O}^{6+}$ ions was accelerated at 84 MeV incident energy on $50 \mu \mathrm{~g} / \mathrm{cm}^{2}$ pure ${ }^{12} \mathrm{C}$ and $99 \%$ enriched ${ }^{13} \mathrm{C}$ targets. The ejectiles produced in the collisions were momentum analyzed by the MAGNEX spectrometer [25], working in full acceptance mode ( $\Omega \sim 50$ Millisteradian solid angle and $\Delta p / p=\Delta B \rho / B \rho \sim 24 \%$ momentum acceptance), and detected by its focal plane detector [26]. The experiment was performed at three angular settings, with the spectrometer optical axis centered at $\theta_{\text {opt }}=8^{\circ}, 12^{\circ}$, and $18^{\circ}$. The large angular acceptance guarantees wide overlaps between contiguous settings. The data reduction technique, based on a differential algebraic method [27-29], and the performances of the whole


FIG. 1. Energy spectrum obtained from the ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right){ }^{13} \mathrm{C}$ oneneutron transfer reactions at 84 MeV in the angular region $8^{\circ}<\theta_{\text {lab }}$ $<10^{\circ}$. The peaks marked with an asterisk and a triangle represent the transition to the excited states of the ${ }^{17} \mathrm{O}$ ejectiles at 0.87 and 3.05 MeV , respectively.


FIG. 2. ${ }^{14} \mathrm{C}$ energy spectra for the (top) one-neutron and (bottom) two-neutron transfer reactions. In the top panel the peaks marked with an asterisk, a triangle, a circle, and a diamond represent the transition to the excited states of the ${ }^{17} \mathrm{O}$ ejectiles at $0.87,3.05,3.84$, and 4.55 MeV , respectively.
system are described in Refs. [30,31]. A mass resolution of $1 / 160$ was measured [32] and an overall resolution of 160 keV (full width at half maximum) in energy and $0.3^{\circ}$ in angle was obtained in the laboratory frame, mainly determined by the multiple scattering in the target and the beam divergence. Examples of the obtained energy spectra and angular distributions are shown in Figs. 1, 2, and 3. In Figs. 1 and 2, $E^{*}$ is defined as $Q-Q_{0}$, where $Q_{0}$ is the $Q$ value for the transfer to the ground state of the residual and ejectile nuclei. An accuracy of $\pm 10 \mathrm{keV}$ was obtained in $E *$ and about $10 \%$ in the absolute cross section, mainly due to the uncertainty on the target thickness. The error bars in the angular distributions include both a statistical contribution and a component due to the solid angle determination (about 3\%) [33].

Ejectiles mass distribution for pure one- and two-neutron removal channels are extracted for the ${ }^{18} \mathrm{O}+{ }^{12} \mathrm{C}$ reaction by selecting the ${ }^{17} \mathrm{O}$ and ${ }^{16} \mathrm{O}$ ions distributed over the $8^{+}$, $7^{+}$, and $6^{+}$charge states. The efficiency for the population of the different charge states is estimated by the INTENSITY code [34]. This procedure introduces uncertainties lower than $1 \%$ since the yields in the spectra are concentrated in the high-kinetic-energy region corresponding to the $8^{+}$. The energy integrated yields in the angular region $7^{\circ}<\theta<18^{\circ}$ are $1.9 \times$ $10^{4}$ for one-neutron removal and $2.3 \times 10^{4}$ for two-neutron


FIG. 3. (Color online) Comparison of the experimental angular distributions with theoretical calculations for the (left) ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right){ }^{14} \mathrm{C}$ and (right) ${ }^{13} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right){ }^{14} \mathrm{C}$ reactions at 84 MeV . No scaling factors are used.
removal. The transfer cross sections for the ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right)^{13} \mathrm{C}$ and ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right){ }^{14} \mathrm{C}$ reactions are also extracted by limiting the integration only to the peaks corresponding to transitions to bound and resonant states. Thanks to the good energy resolution, the accuracy of this procedure can be maintained within $10 \%$. We find strikingly similar results: $73 \pm 4 \mathrm{mb}$ for the oneneutron and $71 \pm 4 \mathrm{mb}$ for the two-neutron transfer channel. Assuming a pure second order process for the two-neutron transfer (independent transfer of two neutrons), the expected probability should be the square of the single-neutron transfer, with a consequent sensible reduction of the cross section. A reduction factor of about 3 is found, for example, in Ref. [35] at sub-barrier energies. Thus, the clear enhancement of the two-neutron transfer cross section observed in our data gives a strong model-independent clue of the relevant role of the direct transfer of a correlated pair of neutrons in this reaction.

Further information on the reaction mechanism comes from the analysis of the energy spectra. Figure 1 shows an example of a spectrum obtained by the ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right){ }^{13} \mathrm{C}$ reaction. One notices that transitions to the well-known single-particle states of ${ }^{13} \mathrm{C}$ products and ${ }^{17} \mathrm{O}$ ejectiles are dominant. Similarly to the results of ${ }^{12} \mathrm{C}(d, p){ }^{13} \mathrm{C}$ reported in Ref. [36], the
weak population of states with more complex configurations indicates that the direct transfer of one neutron is the leading mechanism.

Inspecting the energy spectra measured in the ${ }^{13} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right){ }^{14} \mathrm{C}$ and ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right){ }^{14} \mathrm{C}$ reactions in Fig. 2, a first evidence is that the ${ }^{14} \mathrm{C}$ states are populated with rather different cross sections by the two processes. Angular integrated cross sections confirm the relative yields shown in Fig. 2.

In the one-neutron transfer ${ }^{14} \mathrm{C}$ spectrum, the doublet at $6.73 \mathrm{MeV}\left(3^{-}\right)$and $7.34 \mathrm{MeV}\left(2^{-}\right)$with $\left|\left[\left({ }^{13} C_{\text {g.s. }}\right)^{1 / 2^{-}} \otimes\left(1 d_{5 / 2}\right)_{v}^{5 / 2^{+}}\right]^{2^{-}, 3^{-}}\right\rangle$single-particle configuration has the largest yield, similarly to what found by ${ }^{13} \mathrm{C}(d, p){ }^{14} \mathrm{C}$ reactions [37,38]. However, in the $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reaction, the $7.34-\mathrm{MeV}$ state is scarcely populated. In the weak coupling ( $j-j$ ) model, such states are expected with comparable strength in a two-neutron transfer reaction as a result of a two-step mechanism [39]. In the strong coupling ( $L-S$ ) model the $2^{-}$unnatural parity state cannot be excited in a one-step transfer of a neutron pair (intrinsic spin $S=0$ and isospin $T=1$ ) with respect to the ${ }^{12} \mathrm{C} 0^{+}$core. Our data thus confirm the existence of a clear $S=0$ coupling of the two
neutrons and the direct one-step nature of the transition, giving strong experimental support to the transfer of a neutron pair.

All the states with well-known dominant configuration of two neutrons in the $s d$ shell coupled to a ${ }^{12} \mathrm{C} 0^{+}$core, such as the $2^{+}$at 7.01 and 8.32 MeV and the $4^{+}$at 10.74 MeV [13,37,40,41], are weak or not at all populated in either the $\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right)$ or the $(d, p)$ reaction [37]. In these one-neutron transfer reactions they can be excited only by two-step mechanisms, namely the neutron transfer to the $s d$ shell plus the promotion of the $p_{1 / 2}$ neutron of the ${ }^{13} \mathrm{C}_{\text {g.s. }}$ to the same shell. Our result emphasizes that such two-step mechanisms are significantly suppressed here. Conversely these states are strongly populated in the $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reaction, as expected for the direct transfer of a cluster and observed in $(t, p)$ reactions [13]. Indeed, in the $L-S$ model, these states are described as a main configuration of a cluster of neutrons with $L=2$ and 4 [13,40,42]. The population of all the other states observed in the $\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right)$ and $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reactions is also similar to that found in the $(d, p)$ and $(t, p)$ reactions, respectively.

Summarizing, only states with a well-known structure based on a two-neutron cluster coupled to the ${ }^{12} \mathrm{C} 0^{+}$core are efficiently populated in the $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reaction analyzed in this work. We believe that the suppression of single-particle states, which would require an uncorrelated transfer of two neutrons and the breaking of the initial neutron pair in the ${ }^{18} \mathrm{O}$ ground state, reveals the minor role of the two-step dynamics. This strengthens the conclusions of Ref. [43] for a similar system, where calculations based on the removal of two uncorrelated neutrons from the ${ }^{18} \mathrm{O}$ projectile describe only the continuum of the energy spectrum but not the narrow resonances with two-particle-three-hole configuration, because of the lack of neutron-neutron correlation.

The absolute cross-section angular distributions for some of the transitions induced by the $\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right)$ and $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reactions are shown in Fig. 3. In the ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right){ }^{14} \mathrm{C}$ reaction, the transition to the ${ }^{14} \mathrm{C}$ ground state exhibits a pronounced oscillating pattern, characteristic of the expected $L=0$ angular momentum transfer. The other transitions, characterized by $L \neq 0$, do not show such oscillations, still preserving a certain degree of sensitivity to the transferred angular momentum in the slope. In Ref. [16] the damping of oscillations in heavy-ion reactions was attributed to the different phases of the transfer amplitude components exhibited by the different angular momentum projections $M_{L}$, giving a clear signature of the $L=$ 0 mode. Instead, in the ${ }^{13} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{17} \mathrm{O}\right){ }^{14} \mathrm{C}$ angular distributions, weak oscillations are present perhaps due to the simpler singleparticle configurations excited in this way. All of these features resemble what was found in the ${ }^{26} \mathrm{Mg}\left({ }^{16} \mathrm{O},{ }^{15} \mathrm{~N}\right){ }^{27} \mathrm{Al}$ and ${ }^{26} \mathrm{Mg}\left({ }^{16} \mathrm{O},{ }^{14} \mathrm{C}\right){ }^{28} \mathrm{Si}$ reactions at 128 MeV [44], demonstrated to be direct and selective processes.

## III. THEORETICAL ANALYSIS

We performed exact finite range (EFR) distorted wave Born approximation (DWBA) and coupled reaction channel (CRC) cross-section calculations using the FRESCO code [45]. In all the calculations the São Paulo double folding potential (SPP) was used as a real part in the optical model [46,47]. A matter diffuseness of 0.61 fm was used for the ${ }^{18} \mathrm{O}$ and ${ }^{17} \mathrm{O}$ nuclei,
following the prescriptions of Refs. [48-50]. The imaginary part of the optical potential was built with the SPP shape. According to Ref. [51], a scaling factor of 0.6 was used for the entrance partitions, to account for the missing couplings with the dissipative processes as well as for the coupling to continuum states, which are not explicitly considered. In the outgoing partitions the imaginary part was scaled by a larger factor (0.78). In Ref. [52] it is demonstrated that this prescription is adequate, in the mass and energy region of the present study, when no couplings are introduced. The wave functions used in the form factor calculations were generated by a Woods-Saxon shaped potential, whose depth was adjusted to fit the experimental separation energies for one and two neutrons. The reduced radii and diffuseness are 1.2 and 0.6 fm for ${ }^{16} \mathrm{O}$ and 1.25 and 0.8 fm for ${ }^{12} \mathrm{C}$, respectively. The deformation parameters for the collective excitation in the entrance partition were taken from Ref. [53]. The EFR, prior, full real remnant approximation was used.

The spectroscopic amplitudes were determined by performing a shell-model calculation with ${ }^{12} \mathrm{C}$ treated as closed core and valence protons and neutrons in the orbits $1 p_{1 / 2}, 1 d_{5 / 2}$, and $2 s_{1 / 2}$. This model space, allowing the construction of both positive- and negative-parity states, may be considered a good starting point for the description of the C and O isotopes involved in the present study and makes feasible a complete shell-model calculation with the NUSHELL code [54], which gives us the possibility to compute both one- and two-particle transfer amplitudes. As effective interaction, we used the modified zbm interaction $[55,56]$, which was set for this model space starting from the reaction matrix of the HamadaJohnston potential. In fact, this interaction leads to results that account for many of the observed properties of nuclei around ${ }^{16} \mathrm{O}$ and are substantially similar to those obtained in Ref. [57], where the same model space was adopted with a phenomenological interaction entirely determined from the experimental data. It is worth mentioning, however, that more recently other interactions have been proposed for larger model spaces, $p$-sd space (see Ref. [58] and references therein) and $p-s d-p f$ space [59]. The experimental level schemes of C and O isotopes are satisfactorily reproduced by theory $(600-\mathrm{keV}$ deviations on average) and the calculated spectroscopic amplitudes, listed in Table I, are consistent with those of Refs. $[23,60]$. The adopted coupling schemes are sketched in Fig. 4 and the results of the calculated angular distributions are compared with the experimental ones in Fig. 3.

For the one-neutron transfer reaction, the coupling scheme of the CRC calculations is shown in Fig. 4(a). The CRC cross sections give an accurate estimate of the measured absolute values, together with a remarkable description of the period and amplitude of the observed oscillations. This supports the reliability of our basic theoretical approach.

For the transitions induced by the $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reaction, we used both the extreme cluster model and the independent coordinate scheme to calculate the two-particle state wave functions in CRC [see Fig. 4(b)] [45]. Despite that a twonucleon wave function can be ideally described both in a cluster and in an independent particle basis representation, differences arise when limited model spaces are used, as in the present case. In the cluster approach, the relative

TABLE I. Spectroscopic amplitudes used in the CRC and DWBA calculations. $j$ and $j_{1} j_{2}$ are the spins of the neutron orbitals for one- and two-neutron transfer, respectively; $J_{12}$ is the angular momentum of the two-neutron system. For the states indicated in Fig. 4 and not listed in this table, a spectroscopic amplitude of 1 was used, because only one configuration is present in the model space.

| One-neutron amplitudes |  |  |  |
| :---: | :---: | :---: | :---: |
| Initial state | j | Final state | Spectroscopic Amplitude |
| ${ }^{18} \mathrm{O}_{\mathrm{gss}}\left(0^{+}\right)$ | $\mathrm{d}_{5 / 2}$ | ${ }^{17} \mathrm{O}_{\mathrm{gs} .}\left(5 / 2^{+}\right)$ | +1.305 |
| ${ }^{18} \mathrm{O}_{1.98}\left(2^{+}\right)$ | $\mathrm{d}_{5 / 2}$ | ${ }^{17} \mathrm{O}_{\mathrm{gs} .}\left(5 / 2^{+}\right)$ | -0.929 |
|  | $\mathrm{s}_{1 / 2}$ |  | -0.666 |
| ${ }^{18} \mathrm{O}_{\mathrm{gs},}\left(0^{+}\right)$ | $\mathrm{s}_{1 / 2}$ | ${ }^{17} \mathrm{O}_{0.87}\left(1 / 2^{+}\right)$ | +0.566 |
| ${ }^{18} \mathrm{O}_{\mathrm{gs} .}\left(0^{+}\right)$ | $p_{1 / 2}$ | ${ }^{17} \mathrm{O}_{3.06}\left(1 / 2{ }^{-}\right)$ | -0.929 |
| ${ }^{17} \mathrm{O}_{\mathrm{g} s .}\left(5 / 2^{+}\right)$ | $\mathrm{d}_{5 / 2}$ | ${ }^{16} \mathrm{O}_{\mathrm{gss}}(0+)$ | +0.972 |
| ${ }^{17} \mathrm{O}_{0.87}\left(1 / 2^{+}\right)$ | $\mathrm{s}_{1 / 2}$ | ${ }^{16} \mathrm{O}_{\mathrm{gss}}(0+)$ | +0.975 |
| ${ }^{17} \mathrm{O}_{\mathrm{g} s .}\left(5 / 2^{+}\right)$ | $p_{1 / 2}$ | ${ }^{16} \mathrm{O}_{6.13}(3-)$ | -0.718 |
| ${ }^{13} \mathrm{C}_{\mathrm{g}, \mathrm{s}}\left(1 / 2^{-}\right)$ | $\mathrm{p}_{1 / 2}$ | ${ }^{14} \mathrm{C}_{\mathrm{gs.}}\left(0^{+}\right)$ | +1.291 |
| ${ }^{13} \mathrm{C}_{\mathrm{g} .4}\left(1 / 2^{-}\right)$ | $\mathrm{p}_{1 / 2}$ | ${ }^{14} \mathrm{C}_{6.59}\left(0^{+}\right)$ | -0.412 |
| ${ }^{13} \mathrm{C}_{3.09}\left(1 / 2^{+}\right)$ | $\mathrm{s}_{1 / 2}$ | ${ }^{14} \mathrm{C}_{\mathrm{ss} .}\left(0^{+}\right)$ | -0.296 |
| ${ }^{13} \mathrm{C}_{3.85}\left(5 / 2^{+}\right)$ | $\mathrm{d}_{5 / 2}$ | ${ }^{14} \mathrm{C}_{\mathrm{gss}}\left(0^{(0}\right)$ | -0.496 |


| Two-neutron amplitudes |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Initial state | $\mathrm{j}_{1} \mathrm{j}_{2}$ | $\mathrm{J}_{12}$ | Final state | Spectroscopic Amplitude |
| ${ }^{18} \mathrm{O}_{\mathrm{gs},}\left(0^{+}\right)$ | $\left(\mathrm{p}_{1 / 2}\right)^{2}$ | 0 | ${ }^{16} \mathrm{O}_{\mathrm{gs},}\left(0^{+}\right)$ | +0.241 |
|  | $\left.\left(\mathrm{d}_{5}\right)^{2}\right)^{2}$ |  |  | -0.871 |
|  | $\left(\mathrm{s}_{1 / 2}\right)^{2}$ |  |  | -0.367 |
| ${ }^{18} \mathrm{O}_{1.98}\left(2^{+}\right)$ | $\left(\mathrm{d}_{512}\right)^{2}$ | 2 | ${ }^{16} \mathrm{O}_{\mathrm{g},}\left(0^{+}\right)$ | +0.641 |
|  | $\mathrm{d}_{5 / 2} \mathrm{~s}_{1 / 2}$ |  |  | +0.638 |
| ${ }^{18} \mathrm{O}_{\mathrm{g} .}\left(0^{+}\right)$ | $\mathrm{p}_{1 / 2} \mathrm{~d}_{5 / 2}$ | 3 | ${ }^{16} \mathrm{O}_{6.13}\left(3^{-}\right)$ | +0.801 |
| ${ }^{12} \mathrm{C}_{\mathrm{g} s}\left(0^{+}\right)$ | $\left(p_{1 / 2}\right)^{2}$ | 0 | ${ }^{14} \mathrm{C}_{\mathrm{gs} .}\left(0^{+}\right)$ | +0.913 |
|  | $\left(s_{1 / 2}\right)^{2}$ |  |  | +0.209 |
|  | $\left(\mathrm{d}_{512}\right)^{2}$ |  |  | +0.351 |
| ${ }^{12} \mathrm{C}_{\mathrm{g} s}\left(0^{+}\right)$ | $\left(p_{1 / 2}\right)^{2}$ | 0 | ${ }^{14} \mathrm{C}_{6.59}\left(0^{+}\right)$ | +0.292 |
|  | $\left(\mathrm{s}_{1 / 2}\right)^{2}$ |  |  | -0.935 |
|  | $\left(\mathrm{d}_{552}\right)^{2}$ |  |  | -0.201 |
| ${ }^{12} \mathrm{C}_{\mathrm{g} s}\left(0^{+}\right)$ | $\mathrm{d}_{5 / 2} \mathrm{~s}_{1 / 2}$ | 2 | ${ }^{14} \mathrm{C}_{7.01}\left(2^{+}\right)$ | +0.913 |
|  | $\left(\mathrm{d}_{55}\right)^{2}$ |  |  | +0.408 |
| ${ }^{12} \mathrm{C}_{\mathrm{g} s}\left(0^{+}\right)$ | $\mathrm{d}_{5 / 2} \mathrm{~s}_{1 / 2}$ | 2 | ${ }^{14} \mathrm{C}_{8,32}\left(2^{+}\right)$ | +0.408 |
|  | $\left(\mathrm{d}_{512}\right)^{2}$ |  |  | -0.913 |
| ${ }^{12} \mathrm{C}_{\mathrm{g} s}\left(0^{+}\right)$ | $\left(p_{1 / 2}\right)^{2}$ | 0 | ${ }^{14} \mathrm{C}_{9.75}\left(0^{+}\right)$ | +0.286 |
|  | $\left(s_{1 / 2}\right)^{2}$ |  |  | +0.286 |
|  | $\left(\mathrm{d}_{55}\right)^{2}$ |  |  | -0.915 |

motion of the two-neutron system is frozen and separated from the center of mass. In its extreme model, only the component with the two neutrons coupled antiparallel to a zero intrinsic angular momentum ( $S=0$ ) participates in the transfer. In this case, the relevant parameters for the definition of the wave function of the cluster with respect to the core are the principal quantum number $N$ and the angular momentum $L$. These are inferred from the conservation of total number of quanta in the transformation of the wave functions of the two independent neutrons in orbits $n_{i}, l_{i}$ into a cluster [61]: $\sum_{i=1}^{2} 2\left(n_{i}-1\right)+l_{i}=2(N-1)+L$. In all the present calculations, the $N=3, L=0$ configuration is used for the cluster in the ${ }^{18} \mathrm{O}$ ground state. The spectroscopic amplitude from the shell model is equal to 0.945 . The $N=2$, $L=0$ configuration with amplitude 0.241 gives a negligible contribution.

In the independent coordinates scheme, the transfer of two neutrons is described taking into account single-particle information obtained by shell-model calculations in the available model space (see Table I). In this case the bound-state potential depth was adjusted in order to reproduce half of the two-neutron binding energy, which is a common choice [62].

According to the perturbative formalism of two-particle transfer, the total transition amplitude up to second order should contain three contributions: the simultaneous transfer, the sequential transfer, and the nonorthogonality term [62]. To
our knowledge, such a complete treatment has been developed for ( $p, t$ ) reactions only within the DWBA [63], which is a good approximation when inelastic channels give a negligible contribution. A complete second order treatment including inelastic transitions is still missing. In our approach, we did a separate calculation of the sequential transfer of the two neutrons within the two-step DWBA formalism, introducing the intermediate partition ${ }^{13} \mathrm{C}+{ }^{17} \mathrm{O}$ as sketched in Fig. 4(c). The nonorthogonality terms are included in both CRC onestep and DWBA sequential calculations, despite that small deviations can arise by the different adopted model spaces.

Let us take a closer look at the $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ calculations. The transition to the ${ }^{14} \mathrm{C}_{\text {g.s. }}$ is described, especially at the largest angles, by the cluster model with $N=2$ and $L=0$. Both the shape and the absolute value of the angular distribution are satisfactorily reproduced, even setting to 1 the spectroscopic amplitude for this configuration as shown in Fig. 3. An amplitude of 0.89 is obtained by scaling the CRC calculated cross section to the experimental data. Negligible differences are found by extracting the amplitudes from DWBA results. This value is in good agreement with the 0.913 predicted by shellmodel calculations for the $\left(p_{1 / 2}\right)^{2}$ configuration (see Table I). In another calculation, using the $N=3, L=0$ cluster configuration only, a spectroscopic amplitude of 0.45 is extracted, which is similar to the 0.408 shell-model combined amplitude for $\left(s_{1 / 2}\right)^{2}$ and $\left(d_{5 / 2}\right)^{2}$ configurations (see Table I). These results


FIG. 4. Coupling scheme for (a) one-neutron transfer, (b) direct two-neutron transfer, and (c) sequential two-neutron transfer.
confirm the validity of the reaction model, which allows to determine reliable spectroscopic amplitudes without the need for any arbitrary "unhappiness" factor found in the literature.

A smaller cross section is obtained in the independent coordinates scheme, remarking a relevant contribution of components of the true wave function beyond the adopted model space. The difference in the two results is a consequence of the larger number of pairs of single-particle wave functions that would be necessary to describe the cluster structure. This finding is a strong evidence of the presence of two-neutron pairing correlations in the ${ }^{14} \mathrm{C}$ ground state. One-step DWBA calculations within the cluster model are compared to CRC results. The absence of coupling to intermediate states in the DWBA somewhat deteriorates the agreement with the data found in the CRC. The main effect is a shift of the angular distribution of about $2^{\circ}$ toward backward angles. A slightly larger shift is observed in the two-step sequential transfer calculations. These latter account only for a minor contribution to the absolute cross section, justifying a posteriori our approximated approach to separate the sequential transfer from the CRC calculations. Nevertheless, the coherent sum of the amplitudes of the direct and sequential processes, shown in Fig. 5, indicates the role of the interference in the differential cross section. The sequential mechanism


FIG. 5. (Color online) Comparison of the experimental angular distributions with the coherent sum of direct and sequential calculations for the ${ }^{12} \mathrm{C}\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right){ }^{14} \mathrm{C}_{\text {g.s. }}$. reaction at 84 MeV . The spectroscopic amplitudes of Table I are used.
contributes to sensibly improve the agreement of the cluster model with the experimental data and deteriorate the results of independent coordinates calculations.

The calculated angular distribution of the transition to the $2^{+}$state at 8.32 MeV does confirm the expected $L=2$ transfer. In the cluster model a spectroscopic amplitude of 0.30 is extracted by scaling the CRC calculations with $N=$ $2, L=2$ to the experimental data. This indicates the need for a larger model space for the cluster wave function, which is limited to the $S=0$ component in our approach. Instead, in the independent coordinates calculations, the cross section is accurately reproduced. These results confirm the weak nature of the coupling of the two neutrons in the $s_{1 / 2} d_{5 / 2}$ model space for this state.

For the transition to the $4^{+}$state at 10.74 MeV , calculations assuming arbitrarily wrong angular momenta for the final state were also performed in order to test the sensitivity of the reaction to the angular momentum transfer. The results show that low $L(L \leqslant 3)$ can be safely ruled out by the comparison with the experimental data, while higher $L$ values better represent the data. Using the expected $L=$ 4 , a spectroscopic amplitude equal to 0.55 is extracted for the cluster configuration with $N=1, L=4$. In this case the independent coordinates calculation underestimates the cross section, indicating that a larger space is required in the shell-model calculations. This is not surprising since one expects to find relevant $\left(d_{5 / 2} d_{3 / 2}\right)$ contributions in the $4^{+}$wave function, which are excluded in our model space.

## IV. CONCLUSIONS

The experimental cross sections for a two-neutron transfer reaction induced by ${ }^{18} \mathrm{O}$ projectiles were reproduced for the first time without the need of any "unhappiness" factor. The performed calculations indicate the dominance of a one-step reaction mechanism for this reaction, despite that heavy ions are involved. The strong selectivity of the natural parity
transitions in the measured energy spectra indicates that the transfer of a correlated pair of neutrons $(S=0)$ plays a dominant role. This is also confirmed by the observed enhancement of the two-neutron transfer cross section.

The neutron-neutron pairing correlations, present in the ${ }^{18} \mathrm{O}$ ground state, determine a sizeable amount of cluster configurations in the ground state wave functions of ${ }^{14} \mathrm{C}$ populated in the $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reaction, similarly to what happens in the transfer of very tightly bound systems such as $\alpha$ particles. Our calculations allow also to distinguish states with main $j-j$ coupling configurations from the cluster ones. The latter require a much larger single-particle model space and are more naturally described in an $L-S$ coupling.

On the basis of the discussed results, the spectroscopic factors for two-neutron pair states can be operatively defined.

Here we have shown that a spectroscopic factor close to 1 is found for the ${ }^{14} \mathrm{C}$ ground state. Smaller values are extracted for the $2^{+}$and $4^{+}$states where the $j-j$ coupling plays a role. The $\left({ }^{18} \mathrm{O},{ }^{16} \mathrm{O}\right)$ reaction at 84 MeV is thus found to be a very powerful tool to give a quantitative indication of the effects of the pairing force in the structure of light atomic nuclei. We believe that a systematic exploration of the response of other nuclei to this probe will provide a major source of information on the nature of such a force.

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[1] R. A. Broglia, O. Hansen, and C. Riedel, in Advances in Nuclear Physics, edited by M. Baranger and E. Vogt (Plenum Press, New York, 1973), Vol. 6, pp. 287-457.
[2] W. J. Mulhall, R. J. Liotta, J. A. Evans, and R. P. J. Perazzo, Nucl. Phys. A 93, 261 (1967).
[3] C. H. Dasso, E. Maglione, and G. Pollarolo, Nucl. Phys. A 500, 127 (1989).
[4] I. Tanihata, M. Alcorta, D. Bandyopadhyay, R. Bieri, L. Buchmann, B. Davids, N. Galinski, D. Howell, W. Mills, S. Mythili, R. Openshaw, E. Padilla-Rodal, G. Ruprecht, G. Shiffer, A. C. Shotter, M. Trinczek, P. Walden, H. Savajols, T. Roger, M. Caamano, W. Mittig, P. Roussel-Chomaz, R. Kanungo, A. Gallant, M. Notani, G. Savard, and I. J. Thompson, Phys. Rev. Lett. 100, 192502 (2008).
[5] G. Potel, F. Barranco, E. Vigezzi, and R. A. Broglia, Phys. Rev. Lett. 105, 172502 (2010).
[6] A. Lemasson, A. Navin, M. Rejmund, N. Keeley, V. Zelevinsky, S. Bhattacharyya, A. Shrivastava, D. Bazin, D. Beaumel, Y. Blumenfeld, A. Chatterjee, D. Gupta, G. de France, B. Jacquot, M. Labiche, R. Lemmon, V. Nanal, J. Nyberg, R. G. Pillay, R. Raabe, K. Ramachandran, J. A. Scarpaci, C. Schmitt, C. Simenel, I. Stefan, and C. N. Timis, Phys. Lett. B 697, 454 (2011).
[7] M. Assié, J. A. Scarpaci, D. Lacroix, J. C. Angélique, D. Bazin, D. Beaumel, Y. Blumenfeld, W. N. Catford, M. Chabot, A. Chatterjee, M. Fallot, H. Iwasaki, F. Maréchal, D. Mengoni, C. Monrozeau, J. Nyberg, C. Petrache, F. Skaza, and T. Tuna, Eur. Phys. J. A 42, 441 (2009).
[8] P. D. Bond, H. J. Korner, M. C. Lemaire, D. J. Pisano, and C. E. Thorn, Phys. Rev. C 16, 177 (1977).
[9] M. C. Lemaire and K. S. Low, Phys. Rev. C 16, 183 (1977).
[10] W. von Oertzen and A. Vitturi, Rep. Prog. Phys. 64, 1247 (2001).
[11] C. H. Dasso and A. Vitturi, Collective Aspects in Pair-Transfer Phenomena, Conf. Proc. Vol. 18 (Società Italiana di Fisica, Napoli, 1987).
[12] L. Corradi, G. Pollarolo, and S. Szilner, J. Phys. G: Nucl. Part. Phys. 36, 113101 (2009).
[13] S. Mordechai, H. T. Fortune, G. E. Moore, M. E. Cobern, R. V. Kollarits, and R. Middleton, Nucl. Phys. A 301, 463 (1978).
[14] M. C. Mermaz, A. Greiner, B. T. Kim, M. A. G. Fernandes, N. Lisbona, E. Muller, W. Chung, and B. H. Wildenthal, Phys. Rev. C 20, 2130 (1979).
[15] S. Szilner, F. Haas, Z. Basrak, R. M. Freeman, A. Morsad, and M. P. Nicoli, Nucl. Phys. A 779, 21 (2006).
[16] S. Kahana and A. J. Baltz, in Advances in Nuclear Physics, edited by M. Baranger and E. Vogt (Plenum Press, New York, 1977), Vol. 9, pp. 1-122.
[17] K. S. Low and T. Tamura, Phys. Rev. C 11, 789 (1975).
[18] C. Olmer, M. Mermaz, M. Buenerd, C. K. Gelbke, D. L. Hendrie, J. Mahoney, D. K. Scott, M. H. Macfarlane, and S. C. Pieper, Phys. Rev. C 18, 205 (1978).
[19] J. P. Vary and C. B. Dover, Phys. Rev. Lett. 31, 1510 (1973).
[20] R. M. Devries and K. I. Kubo, Phys. Rev. Lett. 30, 325 (1973).
[21] D. G. Kovar, B. G. Harvey, F. D. Becchetti, J. Mahoney, D. L. Hendrie, H. Homeyer, W. von Oertzen, and M. A. Nagarajan, Phys. Rev. Lett. 30, 1075 (1973).
[22] D. H. Feng, T. Udagawa, and T. Tamura, Nucl. Phys. A 274, 262 (1976).
[23] J. F. Petersen, D. A. Lewis, D. Dehnhard, H. P. Morsch, and B. F. Bayman, Phys. Rev. Lett. 36, 307 (1976).
[24] E. Maglione, G. Pollarolo, A. Vitturi, R. A. Broglia, and A. Winther, Phys. Lett. B 162, 59 (1985).
[25] F. Cappuzzello, D. Carbone, M. Cavallaro, and A. Cunsolo, in Magnets: Types, Uses, and Safety (Nova Science Publishers, New York, 2011), pp. 1-63.
[26] M. Cavallaro, F. Cappuzzello, D. Carbone, A. Cunsolo, A. Foti, A. Khouaja, M. R. D. Rodrigues, J. S. Winfield, and M. Bondì, Eur. Phys. J. A 48, 59 (2012).
[27] A. Cunsolo, F. Cappuzzello, A. Foti, A. Lazzaro, A. L. Melita, C. Nociforo, V. Shchepunov, and J. S. Winfield, Nucl. Instrum. Methods A 484, 56 (2002).
[28] A. Cunsolo, F. Cappuzzello, A. Foti, A. Lazzaro, A. L. Melita, C. Nociforo, V. Shchepunov, and J. S. Winfield, Nucl. Instrum. Methods A 481, 48 (2002).
[29] A. Lazzaro, F. Cappuzzello, A. Cunsolo, M. Cavallaro, A. Foti, S. E. A. Orrigo, M. R. D. Rodrigues, J. S. Winfield, and M. Berz, Nucl. Instrum. Methods A 602, 494 (2009).
[30] F. Cappuzzello, D. Carbone, and M. Cavallaro, Nucl. Instrum. Methods A 638, 74 (2011).
[31] M. Cavallaro, F. Cappuzzello, D. Carbone, A. Cunsolo, A. Foti, R. Linares, D. Pereira, J. R. B. Oliveira, P. R. S. Gomes, J. Lubian, and R. Chen, Nucl. Instrum. Methods A 648, 46 (2011).
[32] F. Cappuzzello, M. Cavallaro, A. Cunsolo, A. Foti, D. Carbone, S. E. A. Orrigo, and M. R. D. Rodrigues, Nucl. Instrum. Methods A 621, 419 (2010).
[33] M. Cavallaro, F. Cappuzzello, D. Carbone, A. Cunsolo, A. Foti, and R. Linares, Nucl. Instrum. Methods A 637, 77 (2011).
[34] J. A. Winger, B. M. Sherrill, and D. J. Morrissey, Nucl. Instrum. Methods B 70, 380 (1992).
[35] L. Corradi, S. Szilner, G. Pollarolo, G. Colo, P. Mason, E. Farnea, E. Fioretto, A. Gadea, F. Haas, D. Jelavic-Malenica, N. Marginean, C. Michelagnoli, G. Montagnoli, D. Montanari, F. Scarlassara, N. Soic, A. M. Stefanini, C. A. Ur, and J. J. Valiente-Dobon, Phys. Rev. C 84, 034603 (2011).
[36] H. Ohnuma, N. Hoshino, O. Mikoshiba, K. Raywood, A. Sakaguchi, G. G. Shute, B. M. Spicer, M. H. Tanaka, M. Tanifuji, T. Terasawa, and M. Yasue, Nucl. Phys. A 448, 205 (1986).
[37] R. J. Peterson, H. C. Bhang, J. J. Hamill, and T. G. Masterson, Nucl. Phys. A 425, 469 (1984).
[38] P. Schumacher, N. Ueta, H. H. Duhm, K.-I. Kubo, and W. J. Klages, Nucl. Phys. A 212, 573 (1973).
[39] N. Anyas-Weiss, J. C. Cornell, P. S. Fisher, P. N. Hudson, A. Menchaca-Rocha, D. J. Millener, A. D. Panagiotou, D. K. Scott, D. Strottman, D. M. Brink, B. Buck, P. J. Ellis, and T. Engeland, Phys. Rep. 12, 201 (1974).
[40] W. von Oertzen, H. G. Bohlen, M. Milin, Tz. Kokalova, S. Thummerer, A. Tumino, R. Kalpakchieva, T. N. Massey, Y. Eisermann, G. Graw, T. Faestermann, R. Hertenberger, and H.-F. Wirth, Eur. Phys. J. A 21, 193 (2004).
[41] H. T. Fortune, M. E. Cobern, S. Mordechai, G. E. Moore, S. Lafrance, and R. Middleton, Phys. Rev. Lett. 40, 1236 (1978).
[42] S. Lie, Nucl. Phys. A 181, 517 (1972).
[43] F. Cappuzzello, C. Rea, A. Bonaccorso, M. Bondì, D. Carbone, M. Cavallaro, A. Cunsolo, A. Foti, S. E. A. Orrigo, M. R. D. Rodrigues, and G. Taranto,, Phys. Lett. B 711, 347 (2012).
[44] D. Sinclair, Phys. Lett. B 53, 54 (1974).
[45] J. I. Thompson, http://www.fresco.org.uk.
[46] L. C. Chamon, D. Pereira, M. S. Hussein, M. A. Cândido Ribeiro, and D. Galetti, Phys. Rev. Lett. 79, 5218 (1997).
[47] L. C. Chamon, B. V. Carlson, L. R. Gasques, D. Pereira, C. DeConti, M. A. G. Alvarez, M. S. Hussein, M. A. Cândido Ribeiro, E. S. Rossi, Jr., and C. P. Silva, Phys. Rev. C 66, 014610 (2002).
[48] J. F. P. Huiza, E. Crema, A. Barioni, D. S. Monteiro, J. M. B. Shorto, R. F. Simoes, and P. R. S. Gomes, Phys. Rev. C 82, 054603 (2010).
[49] E. Crema, D. R. Otomar, R. F. Simoes, A. Barioni, D. S. Monteiro, L. K. Ono, J. M. B. Shorto, J. Lubian, and P. R. S. Gomes, Phys. Rev. C 84, 024601 (2011).
[50] E. S. Rossi, Jr., D. Pereira, L. C. Chamon, C. P. Silva, M. A. G. Alvarez, L. R. Gasques, J. Lubian, B. V. Carlson, and C. De Conti, Nucl. Phys. A 707, 325 (2002).
[51] D. Pereira, J. Lubian, J. R. B. Oliveira, D. P. de Sousa, and L. C. Chamon, Phys. Lett. B 670, 330 (2009).
[52] L. R. Gasques, L. C. Chamon, P. R. S. Gomes, and J. Lubian, Nucl. Phys. A 764, 135 (2006).
[53] S. Raman, C. W. Nester Jr., and P. Tikkanen, At. Data Tables 78, 1 (2001).
[54] W. D. M. Rae, http://www.garsington.eclipse.co.uk/.
[55] A. P. Zuker, B. Buck, and J. B. McGrory, Phys. Rev. Lett. 21, 39 (1968).
[56] A. P. Zuker, Phys. Rev. Lett. 23, 983 (1969).
[57] J. B. McGrory and B. H. Wildenthal, Phys. Rev. C 7, 974 (1973).
[58] C. Yuan, T. Suzuki, T. Otsuka, F. Xu, and N. Tsunoda, Phys. Rev. C 85, 064324 (2012).
[59] M. Bouhelal, F. Haas, E. Caurier, F. Nowacki, and A. Bouldjedri, Nucl. Phys. A 864, 113 (2011).
[60] H. T. Fortune and G. S. Stephans, Phys. Rev. C 25, 1 (1982).
[61] M. Moshinsky, Nucl. Phys. 13, 104 (1959).
[62] G. R. Satchler, Direct Nuclear Reactions (Oxford University Press, New York, 1983).
[63] G. Potel, F. Barranco, F. Marini, A. Idini, E. Vigezzi, and R. A. Broglia, Phys. Rev. Lett. 107, 092501 (2011).

