

DIRAC NOTE 2007-11 (updated)

DIPGEN (DIRAC Pairs Generator)

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Abstract

DIPGEN is a kinematics generator to simulate the physics events which are of interest for the DIRAC experiment. This note describes the version which was used by the most recent MC production (MC200606). Up-to-date description can be found at “<http://www.cern.ch/jabitski/dirac/dipgen>”.

Introduction

The DIRAC analyzes $\pi^+\pi^-$ -pairs with small relative momenta Q in their center of mass system in order to find out signal from the ponium ionization in target. One of approaches in analysis does rely on the correct MC simulation of this process. In addition simulation of other $\pi^+\pi^-$ -pairs inclusively produced in pNi-collisions, which are background to pairs from ponium ionization, allows to estimate the number of produced poniums. Thus the probability of ponium ionization (break-up) in target, which is the ratio of the number of observed ionized pairs over the number of produced atoms, can be deduced from experimental data.

The DIPGEN generator was written as a MC generator which is in-phase with the direct calculation of break-up probability [1]. It is also constructed to eliminate drawbacks of previously used “DIRAC events generator” [2].

The DIPGEN simulates kinematics of ionized $\pi^+\pi^-$ pairs from ponium break-up at the point of ionization and kinematics of background $\pi^+\pi^-$ pairs at the point of their production. The DIPGEN can provide the kinematical input for the GEANT-DIRAC simulation program [10] for further transportation of the generated $\pi^+\pi^-$ pairs through the DIRAC setup.

1 Physics motivations

Pairs from pionium break-up (so-called “atomic” pairs) are characterized by small relative momentum $Q < 3\text{MeV}/c$ in their center of mass system. On break-up pairs acquire rather small momentum transfers which are nearly perpendicular to the initial pionium momentum. So the value of Q on break-up depends mainly on the pionium Bohr momentum which is about $0.5\text{MeV}/c$ for $1S$ state and scales as $\approx 0.5/n\text{MeV}/c$ for nS states.

Due to multiple scattering [4] and finite setup resolution on relative momentum (σ_{Q_X} , σ_{Q_Y} and σ_{Q_L} are about $0.5\text{MeV}/c$ [5])¹ the Ω_Q space, where the search for atomic signal is performed, is typically extended up to $Q_t = \sqrt{Q_X^2 + Q_Y^2} < 6\text{MeV}/c$. Component parallel to the pionium total momentum is up to $|Q_L| < 22\text{MeV}/c$ in order to detect $\pi^+\pi^-$ pairs without interaction in the final state. Within the defined Ω_Q space pairs effective mass

$M = 2m_\pi \sqrt{1 + \left(\frac{Q}{2m_\pi}\right)^2} < 1.004 \times 2m_\pi$, therefore it is reasonable to parametrize pairs

kinematics as a product of pairs distribution on c.m.s. relative momentum $\frac{dN}{d\vec{Q}}$ times their

distribution on total momentum $\frac{dN}{d\vec{P}_\Sigma} = \int_{\Omega_Q} \frac{dN}{d\vec{P}_\Sigma d\vec{Q}} d\vec{Q} \propto \frac{dN}{d\vec{P}_\Sigma d\vec{Q}} \Big|_{Q \rightarrow 0}$.

DIRAC uses very thin targets, for example nuclear efficiency of $100\ \mu\text{m}$ Ni foil is less than 10^{-3} . For simplicity vertices of inelastic pNi-interactions are generated uniformly over the target thickness.

1.1 Atomic pairs

The position of the break-up point and the atomic state $|nlm\rangle$, from which ionisation occurred, are obtained by numerically solving the transport equations with the cross sections for excitation and break-up [1]. User can specify the maximal principal quantum n_{max} used for calculations and the particular scheme for the evolution of higher excited states with $n > n_{\text{max}}$. The momentum and angular distributions of pions from break-up from different initial states of pionium have been derived in [3] in the pionium centre-of-mass system and tabulated as for [2]. The laboratory momentum of atoms is the same as for Coulomb-correlated pion pairs (see later).

1.2 Other inclusive pairs

Main background for atomic pairs are pion pairs inclusively produced in p-Ni collisions. These pairs in their turn are subdivided into pairs with Coulomb interaction in the final state (CC — *Coulomb-correlated pairs*) and without interaction (NC — *non-correlated pairs*). The latter are supposed to have the uniform distribution of pairs relative momenta in their center-of-mass system, while for Coulomb-correlated pairs the uniform distribution is modified by Gamov-Sommerfeld factor $A_C(Q)$

$$\frac{dN_{CC}}{dQ} \propto \frac{dN_{NC}}{dQ} A_C(Q), \quad A_C(Q) = \frac{2\pi m_\pi \alpha / Q}{1 - \exp(-2\pi m_\pi \alpha / Q)}. \quad (1)$$

¹We define Q_L as the component of the c.m.s. relative momentum collinear to the c.m.s. flight direction (see Appendix A for details).

At the moment only the shape of the total inclusive differential cross section is known [7]:

$$\frac{dN_{\text{incl}}}{dP_{\Sigma}d\Theta} = \frac{dN_{CC}}{dP_{\Sigma}d\Theta} + \frac{dN_{NC}}{dP_{\Sigma}d\Theta}. \quad (2)$$

This experimental inclusive distribution of pairs with small Q is split between Coulomb-correlated and non-correlated pairs by the approximation of the $w_l^0(P_{\Sigma})$ function defined in FRITIOF-6:

$$w_l^0(P_{\Sigma}) \doteq \frac{dN_{NC}}{dP_{\Sigma}} \bigg/ \left(\frac{dN_{CC}^0}{dP_{\Sigma}} + \frac{dN_{NC}}{dP_{\Sigma}} \right) = \exp(-0.4066 - 0.2864 \cdot P_{\Sigma}), \quad P_{\Sigma} \text{ is in GeV}/c, \quad (3)$$

where $\frac{dN_{CC}^0}{dP_{\Sigma}}$ is the distribution of Coulomb-correlated pairs without interaction in the final state.

$$\frac{dN_{CC}}{dP_{\Sigma}d\Theta} = \int_{\Omega_Q} \frac{dN_{CC}^0}{d\vec{P}d\vec{Q}} A_C(Q) d\vec{Q} d\varphi. \quad (4)$$

As $\frac{dN_{CC}^0}{d\vec{P}d\vec{Q}}$ is expected to be a smooth function of Q near $Q = 0$:

$$\frac{dN_{CC}}{dP_{\Sigma}d\Theta} = \frac{dN_{CC}^0}{dP_{\Sigma}d\Theta} \frac{\int_{\Omega_Q} A_C(Q) Q^2 dQ}{\int_{\Omega_Q} Q^2 dQ} = \frac{dN_{CC}^0}{dP_{\Sigma}d\Theta} \mathbf{A}_C(\Omega_Q), \quad (5)$$

$$w_l(P_{\Sigma}, \Omega_Q) \doteq \frac{dN_{NC}}{dP_{\Sigma}} \bigg/ \left(\frac{dN_{CC}}{dP_{\Sigma}} + \frac{dN_{NC}}{dP_{\Sigma}} \right) = \frac{w_l^0(P_{\Sigma})}{\mathbf{A}_C(\Omega_Q) (1 - w_l^0(P_{\Sigma})) + w_l^0(P_{\Sigma})}. \quad (6)$$

For $Q < 8 \text{ MeV}/c$ $\mathbf{A}_C(\Omega_{Q<8}) \approx 1.74$. Shape of the $w_l(P_{\Sigma})$ function (fig. 1) and the modification of the expected shape for Coulomb-correlated pairs by the Coulomb enhancement (fig. 2) are presented below.

One has to take into account that formula (3) is coming from the model, therefore the shape of CC/NC pairs distribution is the subject of future improvements/modifications,

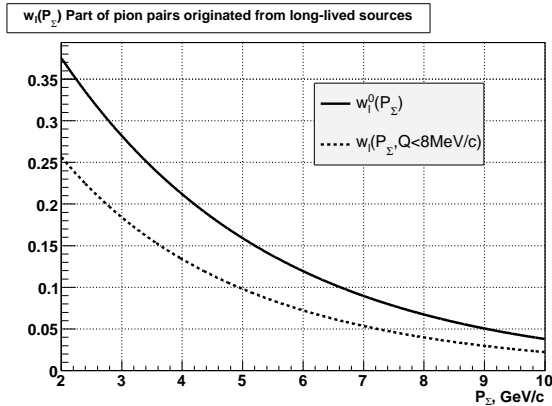


Figure 1: $w_l^0(P_{\Sigma})$ vs. $w_l(P_{\Sigma}, \Omega_{Q<8})$.

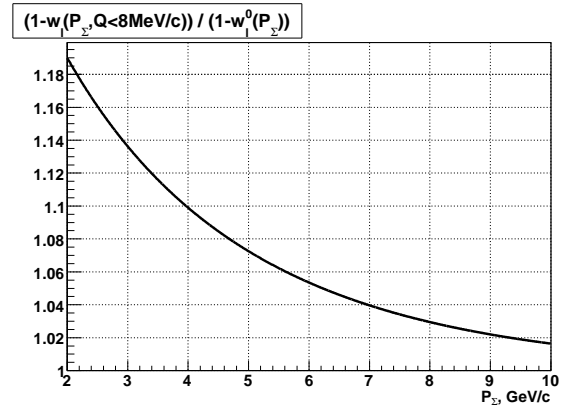


Figure 2: $\frac{1 - w_l(P_{\Sigma}, \Omega_{Q<8})}{1 - w_l^0(P_{\Sigma})}$.

perhaps their shape can be measured by DIRAC itself. To facilitate analysis we suggest to replace the shape of the total inclusive differential cross section by the simple analytical function

$$\frac{d^2 N_{\text{MC}}}{dP_{\Sigma} d\Theta} \propto P_{\Sigma}^2 \exp(-1.1P_{\Sigma}). \quad (7)$$

The latter can be easily modified by weights in analysis. The generation of laboratory momenta covers the full range of the DIRAC acceptance.

1.3 Accidentals

These are pion pairs which originate from two different proton-nuclei collisions, therefore they are generated by a simple combination of inclusively produced positive and negative pions [6]:

$$\frac{dN}{d\vec{p}_+ d\vec{p}_-} \propto \frac{dN_{\pi^+ X}}{d\vec{p}_+} \frac{dN_{\pi^- X}}{d\vec{p}_-}. \quad (8)$$

Comparison of MC accidentals to experimental accidentals can be used to verify data analysis procedures. This comparison is especially useful in the case of analysis without forward detectors where distributions of inclusively produced hadrons ($h^{\pm} X$) are well measured [6].

Input distributions for different types of generated pairs are summarized in Tab. 1.

Table 1: Input distributions for different types of pion pairs

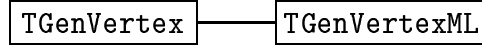
	Laboratory (total) momentum	c.m.s. relative momentum
$A_{2\pi}$	$\frac{dN_{CC}}{dP_{\Sigma} d\Theta_{\Sigma}}$	\vec{Q} from break-up
CC	$\frac{dN_{CC}}{dP_{\Sigma} d\Theta_{\Sigma}} = (1 - w_l^0(P_{\Sigma})) \frac{dN_{\text{incl}}}{dP_{\Sigma} d\Theta_{\Sigma}}$	$\frac{dN_{CC}}{dQ} = Q^2 A_C(Q)$
NC	$\frac{dN_{NC}}{dP_{\Sigma} d\Theta_{\Sigma}} = w_l^0(P_{\Sigma}) \frac{dN_{\text{incl}}}{dP_{\Sigma} d\Theta_{\Sigma}}$	$\frac{dN_{NC}}{dQ} = Q^2$
Acc	$\frac{dN_{\pi^- X}}{d\vec{p}_+} \cdot \frac{dN_{\pi^+ X}}{d\vec{p}_-}$	

Generator keeps the number of produced pairs (atoms) and the number of pairs inside of the predefined Q volume, which are required for the proper normalisation (so-called k -factor).

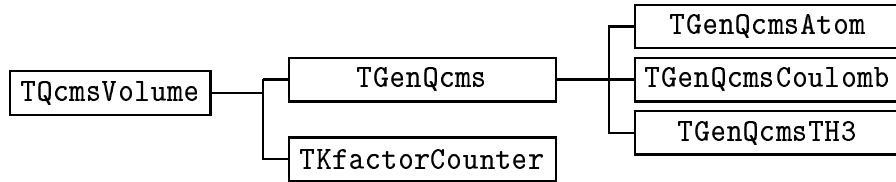
2 Software implementation

DIPGEN is a library written as a set of C++ *classes*. The hierarchy of its classes does rely on *inheritance* and *polymorphism*, which also allow an easy extension of the generator functionality by user. Below we will show only the scheme of the class hierarchy. Interested reader should read HTML documentation at <http://www.cern.ch/jabitski/dirac/dipgen> for the detailed description.

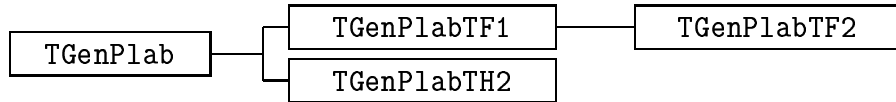
- Vertex can be generated either for a single layer target (TGenVertex) or for a multi-layer target (TGenVertexML).



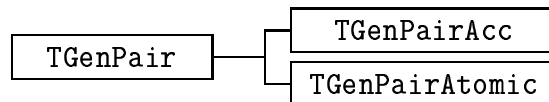
- TQcmsVolume is the base class for c.m.s. relative momentum generation. Type of the coordinate system (rectangular, cylindrical or spherical) and appropriate Q limits can be set. It can check whether a given pair (\vec{p}_+, \vec{p}_-) belongs to the predefined Q volume. TGenQcms generates pairs c.m.s. momenta according to the uniform distribution. TGenQcmsCoulomb generates momenta according to the uniform distribution modified by Gamov-Sommerfeld factor. TGenQcmsAtom provides the complete MC simulation of the atom interaction with the target. TKfactorCounter is a simple counter which provides the total number of generated pairs and the number of pairs within the predefined Q volume.



- TGenPlab is an abstract class which provides the common interface for derived classes. TGenPlabTH2 generates (P_Σ, Θ) according to a 2d-histogram. Other classes generate laboratory momenta according to analytic functions of P_Σ (TGenPlabTF1) or (P_Σ, Θ) (TGenPlabTF2). All other components of the laboratory momentum are uniformly distributed.



- Based on above classes pairs kinematics can be generated. Inclusive pairs (CC or NC) are generated with TGenPair class which has the following constructor
`TGenPair(TGenVertex* vertex, TGenPlab* Plab, TGenQcms *Qcms)`
 Accidentals and atomic pairs are generated by TGenPairAcc and TGenPairAtomic classes respectively.



2.1 Example

Below is a small example for ROOT interpreter which generates 50000 Coulomb-correlated pairs from 98 μm Ni target with $Q < 8 \text{ MeV}/c$ and P_Σ according to a simple analytic function, then saves their kinematics to the file “kine.dat”. Examples for other types of pairs are available at `examples` sub-directory of `dipgen` sources.

```
// first load libraries
TGenVertex *vertex = new TGenVertex( 98.e-4 );

TF1 *f1 = new TF1( "PlabforCC",
                  "x^2*exp(-1.1*x)*(1-exp(-0.4066-0.2864*x))",
                  2.5, 10.0 );
TGenPlabTF1 *Plab = new TGenPlabTF1(f1);

double mpi = 0.13957018;
TGenQcmsCoulomb *Qcms = new TGenQcmsCoulomb( mpi, mpi );
Qcms->fselect = kselQregion::kQcms;
Qcms->SetQRRange( 0.0, 0.008 );

TGenPair *pair = new TGenPair( vertex, Plab, Qcms );
pair->fcheckQcms = kTRUE;

TTree *t = pair->GenXPRandom( 50000 );
WriteGeantInputTTree( t, "kine.dat" );
```

2.2 Code repository

Sources of the generator codes are located under CERN Central CVS [11] Service (<http://www.cern.ch/cvs>) as the DIPGEN repository. Inside this repository the module “dipgen” stands for the generator codes and the module “crsc” for cross-section tables. For example to extract “dipgen” sources one has to issue following commands (tcsh):

```
setenv CVSROOT :kserver:isscvs.cern.ch:/local/reps/dipgen
cvs co dipgen
```

2.3 External packages

DIPGEN code relies on following external packages:

- ROOT (<http://root.cern.ch>). ROOT classes are used for histograms, analytic functions and pseudo-random number generation. The CINT interpreter, which is a part of the ROOT package, is useful to perform generation (see the example). `root` is supposed to be in your path directories.
- LAPACK (<http://www.netlib.org/lapack>). Used to solve eigenvalue problem. Not needed by MC.
- GSL — GNU Scientific Library (<http://www.gnu.org/software/gsl/gsl.html>). Some special functions used (e.g. Riemann zeta function).

A \vec{Q} components as used by DIRAC

Let pairs total momentum $\vec{P}_\Sigma = \vec{p}_+ + \vec{p}_- = \{P_x, P_y, P_z\}$ be in the secondary horizontal system of reference $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ ². We will define a new system of reference $\{\vec{e}'_x, \vec{e}'_y, \vec{e}'_z\}$, which fulfills following conditions:

1. $\vec{e}'_z \uparrow\uparrow \vec{P}_\Sigma$ (\vec{e}'_z is along \vec{P}_Σ),
2. \vec{e}'_x is in the plane $\{\vec{e}_x, \vec{e}_z\}$.

It is straightforward to express $\{\vec{e}'_x, \vec{e}'_y, \vec{e}'_z\}$ through $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$:

$$\begin{aligned} \vec{e}'_x &= \frac{(P_z \vec{e}_x - P_x \vec{e}_z)}{\sqrt{P_x^2 + P_z^2}}, \\ \vec{e}'_y &= \frac{-P_x P_y \vec{e}_x + (P_x^2 + P_z^2) \vec{e}_y - P_y P_z \vec{e}_z}{|\vec{P}_\Sigma| \sqrt{P_x^2 + P_z^2}}, \\ \vec{e}'_z &= \frac{\vec{P}_\Sigma}{|\vec{P}_\Sigma|} = \frac{1}{|\vec{P}_\Sigma|} (P_x \vec{e}_x + P_y \vec{e}_y + P_z \vec{e}_z). \end{aligned}$$

Then vector components $\{v'_x, v'_y, v'_z\}$ of a vector \vec{v} in the new system of reference will be expressed through its components $\{v_x, v_y, v_z\}$ in the initial system of coordinates as follows

$$\begin{pmatrix} v'_x \\ v'_y \\ v'_z \end{pmatrix} = \begin{pmatrix} \frac{P_z}{\sqrt{P_x^2 + P_z^2}} & 0 & \frac{-P_x}{\sqrt{P_x^2 + P_z^2}} \\ \frac{-P_x P_y}{|\vec{P}_\Sigma| \sqrt{P_x^2 + P_z^2}} & \frac{P_x^2 + P_z^2}{|\vec{P}_\Sigma| \sqrt{P_x^2 + P_z^2}} & \frac{-P_y P_z}{|\vec{P}_\Sigma| \sqrt{P_x^2 + P_z^2}} \\ \frac{P_x}{|\vec{P}_\Sigma|} & \frac{P_y}{|\vec{P}_\Sigma|} & \frac{P_z}{|\vec{P}_\Sigma|} \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}. \quad (9)$$

One can easily check that transformation (9) represents a simple rotation. After it $\vec{p}_+ = \{p'_x, p'_y, p'_{+z}\}$ and $\vec{p}_- = \{-p'_x, -p'_y, p'_{-z}\}$;

$$\vec{\beta}_{\text{cms}} = \frac{\vec{P}_\Sigma}{E_\Sigma} = \frac{\{0, 0, p'_{+z} + p'_{-z}\}}{E_+ + E_-}. \quad (10)$$

Finally for components of pairs relative momentum

$$Q_L^* = \gamma ((p'_{+z} - p'_{-z}) - \beta_{\text{cms}}(E_+ - E_-)) = 2 \frac{p'_{+z} E_- - p'_{-z} E_+}{\sqrt{(E_+ + E_-)^2 - (p'_{+z} + p'_{-z})^2}}; \quad (11)$$

$$Q_x^* = 2p'_x, \quad Q_y^* = 2p'_y, \quad Q_L^* = \frac{2}{M} (p'_{+z} E_- - p'_{-z} E_+), \quad (12)$$

here M — pairs effective mass.

²The secondary horizontal system is the target system of reference rotated by 100 mrad around its x -axis.

References

- [1] M. Zhabitsky, *Direct calculation of probability of the ponium ionization in target*, DIRAC internal note, 2007-10.
- [2] C. Santamarina, *DIRAC event generator*, DIRAC internal note, 2004-02.
- [3] T.A. Heim, K. Hencken, M. Schumann, D. Trautmann and G. Baur, *Distribution of pions from breakup of ponium*, hep-ph/0112293.
- [4] A. Dudarev, V. Kruglov, L. Kruglova, M. Nikitin *Pion multiple Coulomb scattering in the DIRAC experiment*, DIRAC internal note, 2005-02;
A. Benelli, D. Drijard, O. Gorchakov, L. Tauscher, V. Yazkov, *Evaluation of the measurement of multiple scattering for the DIRAC set-up, using standard DIRAC analysis tools*, DIRAC internal note, 2006-02.
- [5] B. Adeva et al. (DIRAC Collaboration), *DIRAC: A high resolution spectrometer for ponium detection*, Nucl. Instr. Meth. A515 (2003) 467.
- [6] M. Zhabitsky, *Parametrization of single particle spectra at the DIRAC kinematic range*, DIRAC internal note, 2006-06.
- [7] M. Zhabitsky, *Parametrization of $\pi^+\pi^-$ pairs spectra at the DIRAC kinematic range*, DIRAC internal note, 2007-01.
- [8] *Criteria for pre-selection*, <http://www.cern.ch/DIRAC/pre-sel.html>
- [9] D. Drijard, M. Hansroul, V. Yazkov, *DIRAC offline reconstruction program Ariane*, <http://www.cern.ch/dirac>.
- [10] P. Zrelov, V. Yazkov, O. Gortchakov, *The GEANT-DIRAC simulation program*, <http://www.cern.ch/dirac>.
- [11] CVS — The Concurrent Versions System, <http://www.nongnu.org/cvs>.