EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH

CERN-EP/2004-XXX
25 March 2004

Measurement of the running of the QED coupling in small angle Bhabha scattering at LEP

OPAL Collaboration

Abstract

The running of the effective QED coupling $\alpha(t)$ is measured for spacelike momentum transfer $1.8 \leq -t \leq 6.0$ GeV^2 from the angular dependence of small angle Bhabha scattering by the OPAL detector at LEP. In an almost ideal QED framework, with very favourable experimental conditions and a precise experimental setup, we obtain the strongest direct evidence up to date, with a significance above 5 σ. We also report the first clear experimental evidence for the hadronic contribution to the running, with a significance of about 3 σ.

Draft 1.0

Authors:
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Summary of Comments on paper.1.0.dvi

Page: 1

Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 14.4.2004 7:33:18 PM

replace the abstract:
The running of the effective QED coupling alpha(t) is measured for spacelike momentum transfer 1.8 < -t < 6.0 GeV2 from the angular dependence of small angle Bhabha scattering by the OPAL detector at LEP. In an almost ideal QED framework, with very favourable experimental conditions and a precise experimental setup, we obtain the strongest direct evidence up to date, with a significance above 5 sigma. We also report the first clear experimental evidence for the hadronic contribution to the running, with a significance of about 3 sigma.

with:
Using the OPAL detector at LEP, the running of the effective QED coupling alpha(t) is measured for space-like momentum transfer 1.8 < -t < 6.0 GeV^2 through its effect on the angular spectrum of small angle Bhabha scattering . In an almost ideal QED framework, with very favourable experimental conditions, we obtain the strongest direct evidence ever presented that the running of alpha is consistent with Standard Model expectations. The null hypothesis that alpha remains constant within the above interval of $-t$ is excluded with a significance above 5 sigma. Similarly, our results are inconsistent at the level of about 3 sigma with the hypothesis that only leptonic loops contribute to the running, and therefore provide the first clear experimental evidence that hadronic loops also contribute.

comment:
We have to be clearer on two points:
1) our results are consistent with the SM
2) the quoted significances are wrt to the null hypothesis of no running.

1 Introduction

The electromagnetic coupling constant is a basic parameter of the Standard Model, known to 4×10^{-9} [1]. In QED the coupling becomes effective or *running* with the scale of momentum transfer due to vacuum polarization. This is due to virtual lepton or quark loops. Their contribution increases the effective electric charge at increasing momentum transfer. This can be understood as an effect of screening of a bare electric charge which is probed at smaller and smaller distance. The effective QED coupling is generally expressed as:

$$\alpha(Q^2) = \frac{\alpha_0}{1 - \Delta\alpha(Q^2)} \tag{1}$$

where $\alpha_0 = \alpha(Q^2 = 0) \simeq 1/137.036$ is the fine structure constant. Whereas the leptonic contributions are calculable to very high accuracy, the hadronic ones are more problematic as they involve quark masses and hadronic physics at low momentum scales. The hadronic contribution is traditionally determined from a dispersion integral over a parameterization of the measured annihilation cross section of $e^+e^- \rightarrow$ *hadrons*, supplemented with perturbative QCD above resonances [2, 3]. The main difficulty of this approach comes from the integration of experimental data in the region of hadronic resonances, which in turn gives the dominant uncertainty on $\Delta\alpha$ for positive (*timelike*) Q^2. The effective QED coupling $\alpha(Q^2)$ is an essential ingredient for many precision physics predictions. Its uncertainty is still one of the dominant ones in the electroweak fits constraining the Higgs mass [4]. There are also many evaluations which are more theory-driven, extending the application of perturbative QCD down to 2 GeV or so (see for example the references in [4]). An alternative approach was put forward to use the Adler function [5] and perturbative QCD in the negative Q^2 (*spacelike*) region [6], where $\Delta\alpha$ is a smooth function.

Until now there have been only a few direct observations of the running of the QED coupling [7, 8, 9, 10]. Most of these analyses involve measurements of cross sections and their ratios and obtain values of $\alpha(Q^2)$ which are found to deviate from α_0 or from the assumed value of the coupling at some initial scale. Theoretical uncertainties on the predicted absolute cross sections as well as experimental scale errors can hurt such determinations or reduce their significance. The s-channel results from the TOPAZ [7] and the OPAL [8] experiments were based on e^+e^- annihilations to leptonic final states. Far enough from the Z resonance these processes are dominated by single photon exchange, although they substantially involve the full electroweak theory. Large angle Bhabha scattering has been studied by the VENUS [9] and L3 [10] experiments to measure the running in the spacelike region. In this case both s- and t-channel γ-exchange diagrams are important and the effective QED coupling appears as a function of s or t respectively. Moreover weak contributions of Z-exchange interference are also sizeable.

In this paper we measure the running of α in the spacelike region, by studying the angular dependence of the small angle Bhabha scattering. The spectrum is modified by the running coupling which appears as $\alpha^2(t)$ and the square momentum transfer t is simply related to the polar scattering angle. We use the small angular region accepted for the luminosity measurement, which approximately corresponds to $2 \leq -t \leq 6$ GeV^2 at centre-of-mass energy about the Z resonance peak. At such t scale the average $\Delta\alpha$ is about 2%. The counting rate of small angle Bhabha events is used to determine the integrated luminosity, so that we cannot do an absolute measurement of $\alpha(t)$, rather we will look only at the shape. This is affected by the expected variation of the coupling throughout the acceptance, which is about 0.5%, leading

Page: 2

Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 12.4.2004 11:32:47 PM

replace:
to 4 × 10−9 [1]

with:
with a relative precision of 4×10^{-4}[1] at zero momentum transfer.

Sequence number: 2
Author: Richard Kellogg
Subject: Highlight
Date: 12.4.2004 11:35:12 PM

replace:
the coupling becomes effective or running

with:
the effective coupling changes, or {\em runs}

Sequence number: 3
Author: Richard Kellogg
Subject: Highlight
Date: 12.4.2004 11:42:37 PM

replace:
loops
with:
loop corrections to the photon propagator

Sequence number: 4
Author: Richard Kellogg
Subject: Highlight
Date: 12.4.2004 11:47:55 PM

Their
contribution increases the effective electric charge at increasing momentum transfer. This can be understood as an effect of screening of a bare electric charge which is probed ...

with:
This effect can also be understood as an increasing penetration of the polarized cloud of virtual particles which screen the bare electric charge of a particle as it is probed ....

Sequence number: 5
Author: Richard Kellogg
Subject: Highlight
Date: 12.4.2004 11:53:54 PM

Comments from page 2 continued on next page

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start new paragraph before:
Whereas the leptonic contributions

and insert:
The value of $\Delta \alpha$ can be calculated in field theory.

comment:
a clearer distinction between what is measureed and what is calculated is needed, especially due to the role that ee--> hadrons measurements play in the theoretical application of the dispersion relation to determine D(alpha).

Sequence number: 6
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:42:18 AM
replace:
from

with:
through its relation to

comment:
the trick of using the dispersion relation is just a little too indirect to say "from"

Sequence number: 7
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:39:42 AM
replace:
is traditionally

with:
can be most precisely

comment:
we should avoid giving the impression that our measurement is a viable alternative for dermining the value of dalpha

---

Sequence number: 8
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:44:07 AM
hyphenate everywhere:
timelike (and spacelike)

Sequence number: 9
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:47:38 AM
replace:
Its uncertainty is still one of the dominant ones

Comments from page 2 continued on next page

1 Introduction

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In this paper we measure the running of α in the spacelike region, by studying the angular dependence 12 the small angle Bhabha scattering. The spectrum is modified by the running coupling which appears as $\alpha^2(t)$ and the square momentum transfer t is simply related to the polar scattering angle. We use the small angular region accepted for the luminosity measurement, which approximately corresponds to $2 \leq -t \leq 6$ GeV^2 at centre-of-mass energy 13 out the Z resonance peak. At such t scale the average $\Delta\alpha$ is about 2%. The counting rate of small angle Bhabha events is used to determine the integrated luminosity, so that 14 cannot do an absolute measurement of $\alpha(t)$, rather we will look only at the shape. This is affected by the expected variation of the coupling throughout the acceptance, which is about 0.5%, leading

with:
It contributes one of the dominant uncertainties

Sequence number: 10
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:48:32 AM
replace:
hurt

with:
influence

Sequence number: 11
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:50:46 AM
replace:
weak contributions of Z-exchange interference

with:
interference contributions due to Z exchange

Sequence number: 12
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:55:11 AM
replace:
of the small angle Bhabha scattering. The spectrum is modified by the running coupling which appears as alpha^2(t) and the square momentum transfer t is simply related to the polar scattering angle.

with:
of small angle Bhabha scattering. The square of the momentum transfer t is simply related to the polar scattering angle, and the scattering spectrum is modified by the running coupling which appears as alpha^2 (t) .

Sequence number: 13
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:57:07 AM
replace:
about the Z resonance peak. At such t scale

with:
near the Z resonance peak. At this t scale

Sequence number: 14
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:58:42 AM

Comments from page 2 continued on next page

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replace:
we cannot do

with:
we will not make

comment:
not so clear that such a measurement is infeasible

---

to an observable effect of about 1%. An interesting property of such low $|t|$ region is that, although the absolute $\Delta\alpha$ value is dominated by the leptonic contributions, the variation of the hadronic contribution is predicted to be three times faster, resulting in similar contributions to the variation of the coupling. There has been only one previous attempt to test directly the momentum transfer dependence of α in a way free of normalization errors by the L3 collaboration [10]. To date there exists no direct experimental evidence for the hadronic contribution to the running.

The small angle Bhabha scattering appears as an ideal place for a direct measurement of the running of $\alpha(Q^2)$ in a single experiment. This has been pointed out also recently [11]. Among the advantages are the very high available statistics and the purity of the data sample. In this work a crucial part has the very high precision in measuring the scattering angle which was possible by the OPAL Silicon-Tungsten (SiW) luminometer [12]. Not less important is the cleanliness of the measurement from a theoretical point of view. Small angle Bhabha scattering is strongly dominated by single-photon t-channel exchange, while s-channel photon exchange is practically negligible. It is currently exactly calculable up to the leading $\mathcal{O}(\alpha^2)$ terms in the QED photonic corrections (herein indicated as $\mathcal{O}(\alpha^2 L^2)$, where $L = ln(|t|/m_e^2) - 1$ is the big logarithm). Many existing calculations were described in [13] and there were also widely cross-checked mainly to reduce the theoretical error on the determination of the luminosity at LEP1. Higher order terms are partially accounted through exponentiation. Many of these calculations are available in the convenient form of Monte Carlo programs and have been also extensively checked by the LEP experiments. There exists also a calculation accurate to the subleading $\mathcal{O}(\alpha^2)$ terms [14]. Corrections for Z interference are very small and well known, so that small angle Bhabha scattering is basically a pure QED process. A comparison of data with such precise calculations can determine the value of the effective QED coupling in the most precise way without relying on the correctness of the SU(2)×U(1) electroweak model.

The paper is organized in this way: in section 2 we explain the analysis method, the detector and its Monte Carlo simulation is briefly described in section 3 and the event selection in section 4. The procedure to correct the data distributions is explained in section 5. The fit results including only statistical errors are given in section 6, while the systematic errors are described in detail in section 7. The theoretical uncertainties are discussed in section 8. The results are finally given in section 9, and a conclusive summary in section 10.

2 Analysis method

The Bhabha differential cross section can be written in the following form for small scattering angle:

$$\frac{d\sigma}{dt} = \frac{d\sigma^{(0)}}{dt}\left(\frac{\alpha(t)}{\alpha_0}\right)^2 (1+\epsilon)\ (1+\delta_\gamma) + \delta_Z \tag{2}$$

where:

$$\frac{d\sigma^{(0)}}{dt} = \frac{4\pi\alpha_0^2}{t^2} \tag{3}$$

is the Born term for t-channel single photon exchange, α_0 is the fine structure constant, $\alpha(t)$ is the effective coupling at the momentum transfer scale t. Here ϵ represents the radiative corrections to the Born cross section, δ_γ the contribution of s-channel photon exchange and

Page: 3

Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 7:59:44 AM
replace:
property of such low

with:
property of this low

Sequence number: 2
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 8:00:36 AM
replace:
three times faster

with:
three times larger

Sequence number: 3
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 8:04:38 AM
replace:
one previous attempt to test directly the momentum transfer dependence of α in a way free of normalization errors by the L3 collaboration [10]

with:
one similar attempt [10] to test the momentum transfer dependence of α directly in a way free of normalization errors.

Sequence number: 4
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 8:06:22 AM
replace:
The small angle Bhabha scattering appears as an ideal place

with:
Small angle Bhabha scattering appears to be an ideal process

Sequence number: 5
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 8:07:24 AM
replace:

Comments from page 3 continued on next page

to an observable effect of about 1%. An interesting property of such low $|t|$ region is that, although the absolute $\Delta\alpha$ value is dominated by the leptonic contributions, the variation of the hadronic contribution is predicted to be three times faster, resulting in similar contributions to the variation of the coupling. There has been only one previous attempt to test directly the momentum transfer dependence of α in a way free of normalization errors by the L3 collaboration [10]. To date there exists no direct experimental evidence for the hadronic contribution to the running.

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This has been pointed out also recently

with:
This has also been pointed out recently

Sequence number: 6
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 8:08:10 AM
replace:
a crucial part has

with:
a crucial element has been

Sequence number: 7
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 8:10:31 AM
replace:
which was possible by

with:
provided by

Sequence number: 8
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 8:12:03 AM
replace:
calculations were described in [13] and there were also widely cross-checked mainly

with:
calculations are described in [13] and were also widely cross-checked, mainly

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Sequence number: 9
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 8:21:46 AM
replace:
accounted

with:
accounted for

ccomment:
can be applied globally. I do not believe "accounted" can ever be used without "for"

Sequence number: 10
Author: Richard Kellogg
Subject: Highlight

Comments from page 3 continued on next page

to an observable effect of about 1%. An interesting property of such low $|t|$ region is that, although the absolute $\Delta\alpha$ value is dominated by the leptonic contributions, the variation of the hadronic contribution is predicted to be three times faster, resulting in similar contributions to the variation of the coupling. There has been only one previous attempt to test directly the momentum transfer dependence of α in a way free of normalization errors by the L3 collaboration [10]. To date there exists no direct experimental evidence for the hadronic contribution to the running.

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Date: 13.4.2004 9:32:07 AM
T replace:
Monte Carlo programs and have

with:
Monte Carlo programs which have

Sequence number: 11
Author: Richard Kellogg
Subject: Highlight
Date: 14.4.2004 6:54:11 PM
T replace:
There exists also a calculation accurate to the subleading O(\alpha^2) terms [14]

with:
A calculation accurate to the subleading O(\alpha^2) terms [14] also exists.

Sequence number: 12
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:38:02 AM
T replace:
Bhabha scattering is basically a

with:
Bhabha scattering near the Z pole can be considered an essentially

---

Sequence number: 13
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:39:32 AM
T replace:
The paper is organized in this way: in section 2 we explain the analysis method,

with:
The paper is organized as follows: We explain the analysis method in section 2,

---

δ_Z the contribution of Z exchange. The contributions of δ_γ and δ_Z are much smaller than those of ϵ and the vacuum polarization. Therefore with a precise knowledge of the radiative corrections (ϵ term) one can determine the effective coupling $\alpha(t)$ by measuring the differential cross section. Actually the form of equation (2) is an approximation as the δ_γ term is not really factorized with the effective coupling $\alpha^2(t)$. In fact the s-channel amplitude has coupling $\alpha(s)$. The practical validity is a consequence of the smallness of the δ_γ term, which could even be neglected.

The counting rate of Bhabha events in the SiW luminometers is used to determine the integrated luminosity, so that we cannot do an absolute measurement of $\alpha(t)$, unless an independent determination of the luminosity were available. Instead, the structure of the cross section as written in (2) easily allows to determine the variation of $\alpha(t)$ over the accessible t range. In fact the vacuum polarization gives the term $(\alpha(t)/\alpha_0)^2$, factorized with the dominant piece of the cross section. At the leading order the variable t is simply related to the scattering angle:

$$t = -s\,\frac{1-cos\,\theta}{2} \approx -\frac{s\theta^2}{4} \tag{4}$$

Photon radiation (in particolar Initial State Radiation) smears this correspondence. The event selection that we will be using, described in section 4, has been carefully studied to reduce the impact of radiative events. In particular the energy cuts and the acollinearity cut are very effective. As a result the event sample is strongly dominated by two cluster configurations, with almost full energy back-to-back scattered e^+ and e^-. For such selection the relation (4) is well approximated. The polar scattering angle θ is measured from the radial position R of the scattered e^+ and e^- at reference planes located within the SiW luminometers, at a distance z from the interaction point:

$$\theta = atan(R/z) \tag{5}$$

We use the BHLUMI [15] Monte Carlo generator for small angle Bhabha scattering. It is a multiphoton exponentiated generator accurate up to the leading logarithmic $\mathcal{O}(\alpha^2)$ terms. Higher order photonic contributions are partially included by virtue of the exponentiation. The generated events contain always the scattered electron and positron plus an arbitrary number of (non-collinear) photons. Small contributions from s-channel photon exchange and Z interference are also included. Corrections due to vacuum polarization are implemented with a few choices for the parameterization of the hadronic term [2, 16]. We used the option to generate weighted events, such that we could access all the available intermediate weights which compose the final complete cross section event by event. In particular we could also modify the parameterization of the vacuum polarization or set $\alpha(t) \equiv \alpha_0$ to assume a fixed coupling α_0.

We will compare the radial distribution of the data (and hence the t-spectrum) with the predictions of the BHLUMI Monte Carlo, to determine the running of α within the accepted region. We followed two equivalent methods:

- We calculated the ratios of data and Monte Carlo events in each bin. The Monte Carlo was modified by setting the coupling to the constant value $\alpha(t) \equiv \alpha_0$. Then:

$$R(t) = \frac{N_{data}}{N_{MC}} \propto \left(\frac{1}{1-\Delta\alpha(t)}\right)^2 \tag{6}$$

Page: 4

Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:41:53 AM
Z exchange.

question:
does this include interference?

Sequence number: 2
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:44:03 AM
replace:
Actually the form of equation (2) is an approximation as the δ_γ term is not really factorized

with:
The form of equation (2) is an approximation since the δ_γ term is not really factorizable

---

Sequence number: 3
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:44:59 AM
replace:
has coupling

with:
couples as

Sequence number: 4
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:46:27 AM
replace:
The practical validity

with:
The practical validity of Equation (2)

Sequence number: 5
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:48:54 AM
replace:
we cannot do an absolute

Comments from page 4 continued on next page

δ_Z the contribution of Z exchange. The contributions of δ_γ and δ_Z are much smaller than those of ϵ and the vacuum polarization. Therefore with a precise knowledge of the radiative corrections (ϵ term) one can determine the effective coupling $\alpha(t)$ by measuring the differential cross section. Actually the form of equation (2) is an approximation as the δ_γ term is not really factorized with the effective coupling $\alpha^2(t)$. In fact the s-channel amplitude has coupling $\alpha(s)$. The practical validity is a consequence of the smallness of the δ_γ term, which could even be neglected.

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with:
we cannot make an absolute

Sequence number: 6
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:51:23 AM
replace:
unless an independent determination of the luminosity were available.

with:
without an independent determination of the luminosity.

Sequence number: 7
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:53:26 AM
replace:
allows to determine the variation of $\alpha(t)$ over the accessible t range.

with:
allows the variation of $\alpha(t)$ over the accessible t range to be determined.

Sequence number: 8
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:54:35 AM
replace:
In fact the vacuum polarization

with:
The vacuum polarization

Sequence number: 9
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:55:27 AM
replace:
At the leading order

with:
At leading order

Sequence number: 10
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:56:49 AM
make roman:
cos

use:

Comments from page 4 continued on next page

δ_Z the contribution of Z exchange. The contributions of δ_γ and δ_Z are much smaller than those of ϵ and the vacuum polarization. Therefore with a precise knowledge of the radiative corrections (ϵ term) one can determine the effective coupling $\alpha(t)$ by measuring the differential cross section. Actually the form of equation (2) is an approximation as the δ_γ term is not really factorized with the effective coupling $\alpha^2(t)$. In fact the s-channel amplitude has coupling $\alpha(s)$. The practical validity is a consequence of the smallness of the δ_γ term, which could even be neglected.

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$$R(t) = \frac{N_{data}}{N_{MC}} \propto \left(\frac{1}{1-\Delta\alpha(t)}\right)^2 \tag{6}$$

\cos \sin \atan, etc

Sequence number: 11
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:57:41 AM
replace:
particolar

with:
particular

Sequence number: 12
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 9:58:45 AM
replace:
we will be using,

with:
we use,

Sequence number: 13
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:01:24 AM
replace:
For such selection the relation (4) is well approximated

with:
For such a selection the relation (4) represents a good approximation

---

Sequence number: 14
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:03:36 AM
replace:
generator for small angle Bhabha scattering

with:
generator for all calculations of small angle Bhabha scattering

---

Sequence number: 15
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:04:43 AM
replace:
contain always

with:
always contain

Comments from page 4 continued on next page

δ_Z the contribution of Z exchange. The contributions of δ_γ and δ_Z are much smaller than those of ϵ and the vacuum polarization. Therefore with a precise knowledge of the radiative corrections (ϵ term) one can determine the effective coupling $\alpha(t)$ by measuring the differential cross section. Actually the form of equation (2) is an approximation as the δ_γ term is not really factorized with the effective coupling $\alpha^2(t)$. In fact the s-channel amplitude has coupling $\alpha(s)$. The practical validity is a consequence of the smallness of the δ_γ term, which could even be neglected.

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$$R(t) = \frac{N_{data}}{N_{MC}} \propto \left(\frac{1}{1 - \Delta\alpha(t)}\right)^2 \tag{6}$$

Sequence number: 16
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:07:43 AM

insert after:
to assume a fixed
coupling α 0.

We also used OLDBIS, which is included in the BHLUMI package, to determine the effect of considering only $\Order (\alpha)$ contributions.

Sequence number: 17
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:10:34 AM

replace:
followed two equivalent methods:

with:
followed two equivalent methods, which respectively parameterize either the total running, or its hadronic component:

comment:
but do we actually publish the two methods?

The dominant dependence of $\Delta\alpha(t)$ expected from theory is logarithmic. However within the kinematic region of this analysis it may be approximated with a straight line. There is no statistical sensitivity to deviations from a linear behaviour of the running. So we fitted the ratios as:

$$R(t) = a + b|t| \tag{7}$$

The b slope represents the full observable effect of the running of $\alpha(t)$, both the leptonic and the hadronic component. It is related to the variation of the coupling by:

$$b = 2\frac{\Delta\alpha(t_2) - \Delta\alpha(t_1)}{|t_2| - |t_1|} = \frac{2}{|t_2| - |t_1|}\frac{\alpha^{-1}(t_2) - \alpha^{-1}(t_1)}{\alpha_0^{-1}} \tag{8}$$

where t_1 and t_2 correspond to the acceptance limits.

- The hadronic contribution to the vacuum polarization is included in the Monte Carlo with the parameterization [16] of the form :

  $$\Delta\alpha_{had} = A + B\ ln\ (1 + C\ |t|) \tag{9}$$

  The coefficients A, B and C have different values in intervals of $|t|$ which depend on the detailed method of extraction of the parameterization. We fixed A and C to their values at the average $|t|$ of our data sample, leaving B as a free parameter. In this case the leptonic contribution to the vacuum polarization $\Delta\alpha_{lep}$ was kept at the calculated value.

The effective slope defined in (7) is slightly variable for the different data samples, as their average centre-of-mass energy varies. To combine the results we can practically redefine b in (7) as:

$$b = b^* \frac{\Delta t^*}{\Delta t} \tag{10}$$

where Δt is the actual energy-dependent t range, Δt^* corresponds to a reference centre-of-mass energy $\sqrt{s} = 91.1$ GeV, and then fit for b^*. With the acceptance cuts specified in section 4 the reference t range is: $t_1^* = -1.78$ GeV^2, $t_2^* = -5.96$ GeV^2, $\Delta t^* = |t_2^*| - |t_1^*| = 4.18$ GeV^2.

3 Detector, data samples and Monte Carlo simulation

The OPAL detector and trigger have been described in detail elsewhere [17]. In particular this analysis is based on the silicon-tungsten luminometer (SiW), which was used to determine the luminosity from the counting rate of accepted Bhabha events, starting from 1993. The SiW was designed to improve the precision of the luminosity measurement to better than 1 per mille. In fact it achieved a fractional experimental systematic error of 3.4×10^{-4}. The detector and the luminosity measurement are extensively described in [12]. Here we only review briefly the detector aspects relevant for this analysis.

The OPAL SiW luminometer consisted of 2 identical cylindrical calorimeters, encircling the beam pipe simmetrically at about ± 2.5 m from the interaction point. Each calorimeter is a stack of 19 silicon layers interleaved with 18 tungsten plates, with a sensitive depth of 14 cm, representing 22 radiation lenghts (X_0). The first 14 tungsten plates are each 1 X_0 thick, while

Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:17:04 AM
replace:
it may be approximated with a straight line. There is no statistical sensitivity to deviations from a linear behaviour of the running. So we fitted

with:
, and the statistical sensitivity of the data, the expected dependence may be approximated with a straight line. We fitted

---

Sequence number: 2
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:18:01 AM
replace:
and the hadronic component.

with:
and hadronic components.

Sequence number: 3
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:19:28 AM
The hadronic contribution to the vacuum polarization is included in the Monte Carlo

comment:
as far as I can see this alternative is not actually used in the paper. If this is the case, drop the description.

Sequence number: 4
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:23:13 AM
replace:
To combine the results we can practically redefine

with:
To take this into account when combining the results we define

comment:
If I understand correctly, this does not change the meaning of b, but merely adds the new quantity b*.

Sequence number: 5
Author: Richard Kellogg

Comments from page 5 continued on next page

The dominant dependence of $\Delta\alpha(t)$ expected from theory is logarithmic. However within the kinematic region of this analysis it may be approximated with a straight line. There is no statistical sensitivity to deviations from a linear behaviour of the running. So we fitted the ratios as:

$$R(t) = a + b|t| \tag{7}$$

The b slope represents the full observable effect of the running of $\alpha(t)$, both the leptonic and the hadronic component. It is related to the variation of the coupling by:

$$b = 2\frac{\Delta\alpha(t_2) - \Delta\alpha(t_1)}{|t_2| - |t_1|} = \frac{2}{|t_2| - |t_1|}\frac{\alpha^{-1}(t_2) - \alpha^{-1}(t_1)}{\alpha_0^{-1}} \tag{8}$$

where t_1 and t_2 correspond to the acceptance limits.

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Subject: Highlight
Date: 13.4.2004 10:24:12 AM
replace:
starting from 1993.

with:
from 1993 until the end of LEP running.

Sequence number: 6
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:24:55 AM
replace:
luminometer consisted of

with:
luminometer consists of

Sequence number: 7
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:26:06 AM
replace:
simmetrically

with:
symmetrically

---

the last 4 are each 2 X_0 thick. The sensitive area fully covers radii between 6.2 and 14.2 cm from the beam axis. Each detector layer is segmented with $R - \phi$ geometry in a 32×32 pad array. The pad size is 2.5 mm radially and 11.25 degrees in azimuth. In total the whole luminometer had 38,912 readout channels corresponding to the individual silicon pads. The calibration was studied with electrical pulses generated both on the readout chips and on the front-end boards, as well as with ionization signals generated in the silicon using test beams and laboratory sources. Overall pad-to-pad gain variations were within 1%.

We use the data samples collected in 1993-95 at energies close to the Z resonance peak. In total they amount to 101 pb^{-1} of OPAL data.

For the LEP2 data-taking started in 1996 the detector configuration changed, with the installation of tungsten shields designed to protect the inner tracking detectors from synchrotron radiation. These introduced about 50 radiation lengths of material in front of the calorimeters between 26 and 33 mrad from the beam axis, thus reducing the useful acceptance of the detector at the lower polar angle limit. Moreover the new fiducial acceptance cut fell right in the middle of the previous acceptance, where the preshowering material was maximum. For these reasons we have limited this analysis to the LEP1 data samples.

The OPAL SiW detector simulation does not rely on a detailed physical simulation of electromagnetic showers in the detector. Instead it is based on a parameterization of the detector response obtained from the data [12]. This approach gives a much more reliable description of the tails of the detector response functions, which are primarily due to extreme fluctuations in shower development, than we could obtain using any existing program which attempts to simulate the basic interactions of electrons and photons in matter. The measured LEP beam size and divergence, as well as the measured offset and tilt of the beam with respect to the calorimeters are also incorporated in this simulation. The Monte Carlo simulation is used to correct the acceptance for the effects of the detector energy response, the coordinate resolution and LEP beam parameters. The data are divided in 9 subsamples according to the average centre-of-mass energy and the values of the beam parameters, which slightly varied. For each subsample we generated an independent sample of BHLUMI events subjected to detector simulation with corresponding setting of the parameters. The statistics were always at least 10 times those of the corresponding data set.

There are other acceptance corrections which are not accounted by the Monte Carlo simulation, but rather applied directly to data. These include the trigger efficiency, accidental background, detector metrology and most importantly biases in the reconstructed radial coordinate. The latter is crucial for this analysis and will be discussed in section 5.

4 Event selection

The event selection criteria can be classified into *isolation* cuts, which isolate a sample of pure Bhabha scattering events from the off-momentum background, and acceptance defining, or *definition* cuts. The isolation cuts are used to define a fiducial set of events which lie within the good acceptance of both calorimeters and are essentially background free. The definition cuts then select subsets of events from within the fiducial sample. Showers generated by incident electrons and photons are recognized as clusters in the calorimeters and their energies and

Page: 6

Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:27:14 AM
replace:
had 38,912 readout channels

with:
has 38,912 readout channels

Sequence number: 2
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:29:25 AM
replace:
101 pb−1 of OPAL data.

with:
101 pb−1 of OPAL data, corresponding to $X.X \times 10^6$ accepted small angle Bhabha events.

Sequence number: 3
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:30:34 AM
replace:
For the LEP2 data-taking

with:
LEP2 data-taking

Sequence number: 4
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:31:14 AM
replace:
which slightly varied.

with:
which varied slightly.

Sequence number: 5
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:32:42 AM
replace:
subjected to detector simulation with corresponding setting of the parameters

Comments from page 6 continued on next page

the last 4 are each 2 X_0 thick. The sensitive area fully covers radii between 6.2 and 14.2 cm from the beam axis. Each detector layer is segmented with $R - \phi$ geometry in a 32×32 pad array. The pad size is 2.5 mm radially and 11.25 degrees in azimuth. In total the whole luminometer had 38,912 readout channels corresponding to the individual silicon pads. The calibration was studied with electrical pulses generated both on the readout chips and on the front-end boards, as well as with ionization signals generated in the silicon using test beams and laboratory sources. Overall pad-to-pad gain variations were within 1%.

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There are other acceptance corrections which are not accounted by the Monte Carlo simulation, but rather applied directly to data. These include the trigger efficiency, accidental background, detector metrology and most importantly biases in the reconstructed radial coordinate. The latter is crucial for this analysis and will be discussed in section 5.

4 Event selection

The event selection criteria can be classified into *isolation* cuts, which isolate a sample of pure Bhabha scattering events from the off-momentum background, and acceptance defining, or *definition* cuts. The isolation cuts are used to define a fiducial set of events which lie within the good acceptance of both calorimeters and are essentially background free. The definition cuts then select subsets of events from within the fiducial sample. Showers generated by incident electrons and photons are recognized as clusters in the calorimeters and their energies and

with:
reflecting its individual beam and energy response parameters

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Sequence number: 6
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:40:53 AM
replace:
not accounted by

with:
not accounted for by

Sequence number: 7
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:33:24 AM
replace:
applied directly to data.

with:
applied directly to the data.

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coordinates determined. The fine segmentation of the detectors allows incident particles with separations greater than 1 cm to be individually reconstructed with good efficiency.

The coordinate system used throughout this paper is cylindrical, with the z-axis pointing along the direction of the electron beam, passing through the centers of the two calorimeter bores. The origin of the azimuthal coordinate, ϕ, is in the horizontal plane, towards the inside of the LEP ring. All radial coordinate measurements are projected to reference planes at a distance of ± 246.0225 cm from the nominal intersection point. These reference planes correspond to the nominal position of the silicon layers 7 X_0 deep in the two calorimeters.

The *isolation* cuts consist of the following requirements, imposed on (R_R,ϕ_R) and (R_L,ϕ_L), the radial and azimuthal coordinates of the highest energy cluster associated with the Bhabha event, in each of the right and left calorimeters, and on E_R and E_L, the total fiducial energy deposited by the Bhabha event in each of the two calorimeters, explicitly including the energy of radiated photons:

- Loose radial cut, right (left) $\quad 6.7 \text{ cm} < R_R < 13.7 \text{ cm}$ $\quad (6.7 \text{ cm} < R_L < 13.7 \text{ cm})$
- Acoplanarity cut $\quad ||\phi_R - \phi_L| - \pi| < 200 \text{ mrad}$
- Acollinearity cut $\quad |R_R - R_L| < 2.5 \text{ cm}$
- Minimum energy cut, right (left) $\quad E_R > 0.5 \cdot E_{beam}$ $\quad (E_L > 0.5 \cdot E_{beam})$
- Average energy cut $\quad (E_R + E_L)/2 > 0.75 \cdot E_{beam}$

Note that by defining the energy cuts relative to the beam energy, E_{beam}, the selection efficiency is largely independent of $\sqrt{s}$.

The acollinearity cut (which corresponds to approximately 10.4 mrad) is introduced in order to ensure that the acceptance for single radiative events is effectively determined geometrically and not by the explicit energy cuts.

The isolation cuts accept events in which the radial coordinate, in both the Right and the Left side, is more than two pad width (0.5 cm) away from the edge of the sensitive area of the detector. The *definition* cuts, based solely on the reconstructed radial positions (R_R, R_L) of the two highest energy clusters, then require the radial position on either side to be within two extra pads towards the inside of the acceptance. For the correction procedure explained in section 5 we refer to one specified silicon layer, which can be varied with some freedom. The Right and Left *definition* cuts are chosen so as to correspond closely to radial pad boundaries in the same detector layer. When the chosen layer is the reference one at 7 X_0, the definition cuts are:

- Right side $\quad 7.2 \text{ cm} < R_R < 13.2 \text{ cm}$
- Left side $\quad 7.2 \text{ cm} < R_L < 13.2 \text{ cm}$

Expressed in terms of polar angles, these cuts correspond to 29.257 and 53.602 mrad. When alternative layers are chosen the acceptance cuts are projected to the layer at 7 X_0. For

Page: 7

Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:41:53 AM
replace:
including the energy

with:
including any detected energy

Sequence number: 2
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:42:40 AM
replace:
more than two pad width

with:
more than two pad widths

Sequence number: 3
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:43:52 AM
7.2 cm < RR < 13.2 cm

comment:
add note and justification for using an expanded acceptance wrt [12]

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example when using the layer at 4 X_0, $R_{min} = 7.25842$ cm and $R_{max} = 13.30710$ cm. The radial distributions after the isolation cuts are shown in Fig. 1 for the complete LEP1 statistics and compared to Monte Carlo distributions normalized to the same number of events. The agreement is very good, except in the central part where effects of the preshowering material are expected. Correction of the radial distributions is one of the main issues and is described in the next section.

5 Radial coordinate correction

The reconstruction of the radial coordinate is the key element to control the systematic errors to the desired accuracy. It is explained in detail in [12]. The last step is the so-called *anchoring*. This procedure was meant to correct any residual bias which could remain on the final reconstructed radius R on either side of the calorimeter at the pad boundaries defining the edges of the acceptance. For the luminosity measurement the correction affected only the total acceptance. Here we want to bin the radial distribution to determine its shape, so we need to correct for radial biases occuring at any bin edge.

The reconstruction method respects the simmetry condition that a shower which deposits equal energies on two adjacent pads in the reference layer at 7 X_0 has to be reconstructed in the mean exactly at the boundary between the pads. In reality, due to the $R - \phi$ geometry of the pads, the true position of such showers is at a smaller radius than the pad boundary. This is termed the *pad boundary bias* and depends on the lateral shower spread. The pad boundary bias has been measured in a test beam.

The test beam employed a SiW calorimeter module of 3 azimuthal wedges fully equipped in depth, and a four-plane, double-sided Si micro-strip telescope with a resolution of better than 3 μm for individual tracks. The geometry of the calorimeter pads with respect to the telescope was determined using a beam of 100 GeV muons. The muon beam was alternated with one of 45 GeV electrons.

As the radial position of the incoming particles is scanned across a radial pad boundary in a single layer, the probability for observing the largest pad signal above or below this boundary shifts rapidly, giving an image of the pad boundary as shown in Fig. 2. These plots are obtained from OPAL data taken in 1993-94 and refer to three radial pad boundaries in layer 4 X_0 of the Right calorimeter. The pad boundary images are modelled with an error function (a gaussian convoluted with a step function), where the gaussian width σ[1] is related to the resolution at the boundary and the difference between the nominal and the apparent boundary position, defined by the half-height of the step, is called the radial offset R_{off}.

At the test beam the difference in R_{off} obtained by changing from electron to muon beam was the measured pad boundary bias, which was found to follow a linear dependence increasing with σ. The fitted linear parameterization was assigned an error of 2 μm. During the OPAL running the radial position is determined by the SiW calorimeter alone, so that the width observed from the pad boundary images is an apparent one. The relation between the true σ and the apparent σ_a was also measured at the test beam and is used to convert the observed σ_a

[1] In [12] the variable w was defined differing by a numerical factor from the width: $w = \sqrt{2}\,\sigma$.

Page: 8

Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:45:15 AM
replace:
one of the main issues and

with:
one of the main issues of this analysis and

Sequence number: 2
Author: Richard Kellogg
Subject: Highlight
Date: 14.4.2004 6:50:59 PM
replace these three paragraphs with the file radial_coord.tex:

The reconstruction of the radial coordinate is the key element to control the systematic errors to the desired accuracy. It is explained in detail in [12]. The last step is the so-called anchoring. This procedure was meant to correct any residual bias which could remain on the final reconstructed radius R on either side of the calorimeter at the pad boundaries defining the edges of the acceptance. For the luminosity measurement the correction affected only the total acceptance. Here we want to bin the radial distribution to determine its shape, so we need to correct for radial biases occuring at any bin edge.
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Sequence number: 3
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:52:25 AM
replace:
the gaussian width \sigma

with:
the gaussian width $\sigma = w_a/\sqrt 2$

and drop footnote

Comments from page 8 continued on next page

example when using the layer at 4 X_0, $R_{min} = 7.25842$ cm and $R_{max} = 13.30710$ cm. The radial distributions after the isolation cuts are shown in Fig. 1 for the complete LEP1 statistics and compared to Monte Carlo distributions normalized to the same number of events. The agreement is very good, except in the central part where effects of the preshowering material are expected. Correction of the radial distributions is one of the main issues and is described in the next section.

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[1] In [12] the variable w was defined differing by a numerical factor from the width: $w = \sqrt{2}\,\sigma$.

Sequence number: 4
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:51:50 AM

replace:
the half-height of the step,

with:
the radius at which the step reaches half its height,

Sequence number: 5
Author: Richard Kellogg
Subject: Highlight
Date: 14.4.2004 6:52:04 PM

drop the entire paragraph:
At the test beam the difference in Roff obtained by changing from electron to muon beam was
the measured pad boundary bias, which was found to follow a linear dependence increasing
with \sigma. The fitted linear parameterization was assigned an error of 2 \mum. During the OPAL
running the radial position is determined by the SiW calorimeter alone, so that the width
observed from the pad boundary images is an apparent one. The relation between the true \sigma
and the apparent \sigma_a was also measured at the test beam and is used to convert the observed
\sigma_a....

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to the true σ. The uncertainty on the conversion factor from σ_a to σ has been estimated from the difference between the test beam with no additional material and with 0.84 X_0 of material in front of the detector.

In OPAL data R_{off} measures the shift of the observed pad boundary image from the nominal position of the pad boundary. Such shifts can be produced by a large number of causes: pad gain fluctuations, metrology shifts, detector malfunctions, resolution effects and preshowering. The pad boundary bias is determined by converting the apparent σ_a to the true σ and then using the test beam results to find the corresponding geometric bias. Fig. 2 shows that a gaussian resolution does not perfectly describe the tails of the distribution. To the extent that the pad boundary image maintains an odd symmetry about the apparent pad boundary, its non-gaussian behaviour does not affect the determination of R_{off} as can be seen from the close agreement of the data points and the fitted curve near the pad boundary. We have also considered a model in which the apparent pad boundary is taken as the median of the observed resolution function. The difference between the two models is assigned as a systematic error of the fit method, when it is larger than the fit statistical error, otherwise the latter is kept as estimate. A further difference of the test beam with respect to the OPAL data is that it was carried out at a radial position close to the inner acceptance cut. The geometrical bias due to $R-\phi$ pads is expected to scale as $1/R$, thus decreasing at a greater radius of pad curvature. Therefore we have scaled in this way the bias estimated by using the test beam results, but assign an additional systematic error equal to 50% of the expected bias to account for possible deviations from this behaviour.

The total net bias (also called *anchor*) δR on the position of a pad boundary is given by:

$$\delta R = R_{off} + \delta R_{R\phi} + \delta R_{res} \tag{11}$$

where R_{off} is the coordinate offset which may have positive or negative sign, $\delta R_{R\phi}$ is the pad boundary bias, always positive and δR_{res} is a small (positive) additional bias due to the resolution flow. The latter results from the steeply falling radial resolution and can be thought as a second-order effect.

From Fig. 2 one can see that the width is similar at the inner and outer radius, while it is considerably greater at the central radius. The offset R_{off} is found very small at the inner edge while it increases to $\approx 10-20$ μm at the central and the outer radius. Among other effects, the observed R_{off} is affected by fluctuations in the pad gain. We have checked these effects directly on data, by studying R_{off} as a function of the 32 azimuthal divisions of the calorimeters. We assign the size of the azimuthal variations, $(R_{off})_{RMS}/\sqrt{32}$, as a systematic error in the anchors, due to pad gain vaiations.

The anchors determined from 1993-94 data for the layers at 4 X_0 and all the pad boundaries used in the analysis are shown in Fig. 3. A similar trend is visible in the two sides, in particular the rise of the anchor from about zero at the inner edge to $20-25$ μm around $R=9$ cm. The inner error bars are the statistical errors in the fit of the pad boundary images. The full error bars include in quadrature the systematic errors from: fit method, pad gain variations, σ_a conversion, test beam parameterization and the assumed $1/R$ scaling of the pad boundary bias. The anchors determined from 1995 data have similar features although with lower statistics.

The anchors have been determined separately for 1993-94 and 1995 data, because the amount of preshowering material was different in the two sub-samples. A clear relation with the amount

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Sequence number: 1
Author: Richard Kellogg
Subject: Highlight
Date: 14.4.2004 6:49:17 PM
drop:
to the true \sigma. The uncertainty on the conversion factor from \sigma_a to \sigma has been estimated from the difference between the test beam with no additional material and with 0.84 X0 of material in front of the detector.

Sequence number: 2
Author: Richard Kellogg
Subject: Highlight
Date: 13.4.2004 10:56:30 AM
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The pad boundary bias is determined by converting the apparent sigma_a to the true sigma and then using the test beam results to find the corresponding geometric bias.
