Z-pole Data Constraints on Standard Model Parameters

How various components of the Z-pole data and the direct measurement of $m_{\rm W}$ constrain $m_{\rm t}$ and $m_{\rm H}$ can be clearly illustrated on a plot of $m_{\rm t}$ vs $\log_{10}(m_{\rm H}/{\rm GeV})$.

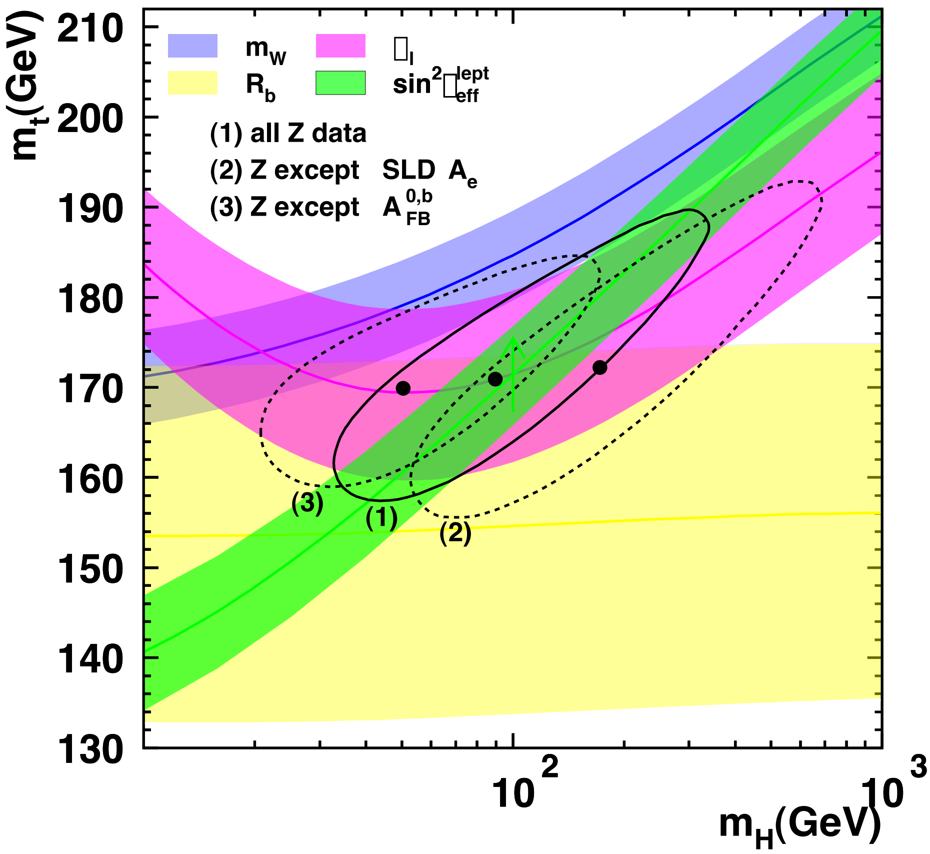

Figure 1: $m_{\rm t}$ and $m_{\rm H}$ constraints from Z-pole measurements and $m_{\rm W}$.

- The green diagonal band indicates the 1σ constraint from the $\sin^2\theta_{\rm eff}^{\rm lept}$ measurements. Higher-order electroweak corrections do not significantly affect the linear relationship expected from the lowest-order terms, and the slope of this band is approximated well by the ratio of coefficients of $m_{\rm t}^2$ and $ln(m_{\rm H})$ in the expression for $\Delta\kappa$

$$\Delta\kappa = \frac{3G_{\rm F}m_{\rm t}^2}{8\pi^2\sqrt{2}}\frac{\cos^2\theta_{\rm W}}{\sin^2\theta_{\rm W}} - \frac{11}{3}\frac{G_{\rm F}m_{\rm W}^2}{8\pi^2\sqrt{2}}\left(\ln\frac{m_{\rm H}^2}{m_{\rm W}^2} - \frac{5}{6}\right) + \cdots, \quad (1)$$

$$\frac{dm_{\rm t}}{dln(m_{\rm H})} = \frac{11}{9}\frac{m_{\rm Z}^2\sin^2\theta_{\rm W}}{m_{\rm t}} \quad (2)$$

- The purple banana-shaped band shows the 1σ constraint from the $\Gamma_{\ell\ell}$ measurement. The slope of this band at large $log(m_{\rm H})$ agrees well with the linear relation expected from the ratio of coefficients in the lowest-order expression for $\Delta\rho$ in this region,

$$\Delta\rho = \frac{3G_{\rm F}m_{\rm t}^2}{8\pi^2\sqrt{2}} - \frac{3G_{\rm F}m_{\rm W}^2}{8\pi^2\sqrt{2}}\frac{\sin^2\theta_{\rm W}}{\cos^2\theta_{\rm W}}\left(\ln\frac{m_{\rm H}^2}{m_{\rm W}^2} - \frac{5}{6}\right) + \cdots, \quad (3)$$

$$\frac{dm_{\rm t}}{dln(m_{\rm H})} = \frac{m_{\rm Z}^2\sin^2\theta_{\rm W}}{m_{\rm t}}. \quad (4)$$

But the behavior at low $log(m_{\rm H})$ is different, even changing the sign of the first derivative.

- Due to the fact that $R_{\rm b}$ is controlled by vertex corrections determined by the $t-b$ mass-splitting, its measurement provides a constraint on $m_{\rm t}$ almost independent of $log(m_{\rm H})$, as shown by the horizontal yellow band (also 1σ).

- The central ellipse shows the 68% CL contour for the Standard Model fit to all Z-pole measurements. The dominant role played by $\sin^2\theta_{\rm eff}^{\rm lept}$ in determining the minor axis is evident, and the turn-over of the $\Gamma_{\ell\ell}$ banana provides the lower bound of the major axis. The $R_{\rm b}$ measurement provides the upper bind. If the $R_{\rm b}$ constraint is removed, the fit in fact no longer yields any upper limit for $m_{\rm H}$.

- The other two ellipses show similar fit contours when either $A_{\rm LR}$ or the $A_{\rm FB}^{0,\,\rm b}$ measurements are excluded from the fit. The noticeable shrinkage of the major axis in the case when the $A_{\rm FB}^{0,\,\rm b}$ measurement is dropped is due to the fact that the $\sin^2\theta_{\rm eff}^{\rm lept}$ constraint then begins to move around the corner of the $\Gamma_{\ell\ell}$ banana. If the $\sin^2\theta_{\rm eff}^{\rm lept}$ measurement moved even lower, the $\sin^2\theta_{\rm eff}^{\rm lept}$ and $\Gamma_{\ell\ell}$ constraints would become almost perpendicular, eliminating the usual $m_{\rm t} - log(m_{\rm H})$ error correlation.

- The agreement of the indirect measurement of $m_{\rm t}$ from the Z-pole measurements alone with the direct measurement made in pp collisions is an important experimental confirmation of the validity of electroweak corrections. The remarkable stability of the indirect measurement's central value under shifts in $\sin^2\theta_{\rm eff}^{\rm lept}$ can be seen to result from a complex interplay between the relatively weak constraint from $R_{\rm b}$ and the exact relation between the $\sin^2\theta_{\rm eff}^{\rm lept}$ measurement band and the position of the $\Gamma_{\ell\ell}$ corner.

- The arrows show how the measurement bands would shift under one-sigma changes in the $\Delta\alpha_{\rm had}^{(5)}(m_{\rm Z}^2)$ determination. Only the effect on $\sin^2\theta_{\rm eff}^{\rm lept}$ is seen to be significant with respect to the current measurement errors.

- The diagonal purple band shows the constraint from the direct measurement of $m_{\rm W}$. It is clearly more compatible with the lower $m_{\rm H}$ values favored by $A_{\rm LR}$ than the higher ones favored by $A_{\rm FB}^{0,\,\rm b}$.

- The effect of applying the constraint of the direct $m_{\rm t}$ measurement can easily be visualized by imagining a horizontal band at $m_{\rm t} = 175 \pm 5$GeV. Notice that at the operating point of the Z-pole fit, the direct $m_{\rm t}$ measurement essentially surplants the constraints provided by $\Gamma_{\ell\ell}$.

- It is perhaps interesting to remark on the fact that all measurements are compatible with the broad $\Gamma_{\ell\ell}$ extremum in $log(m_{\rm H})$. Only the failure to find direct production of the Higgs at LEP II indicates nature's choice to lie on the $\Gamma_{\ell\ell}$ upper branch.