# Are the LEP and SLD heavy flavour electroweak data combined correctly?

$R_{\rm b}$–$R_{\rm c}$ mini workshop
XXXIst Rencontre de Moriond
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Many thanks to: LEP electroweak working group
LEP and SLD heavy flavour groups

## Is combination of data correct?

# YES

## BUT...

# Back to Basics

Start with a single measurement: $a_1 \pm \sigma_1$, Gaussian p.d.f.

$$\chi^2(x) = (x - a_1)^2/\sigma_1^2$$

$\chi^2(a_1) = 0$, $\chi^2(a_1 \pm \sigma_1) = 1$.

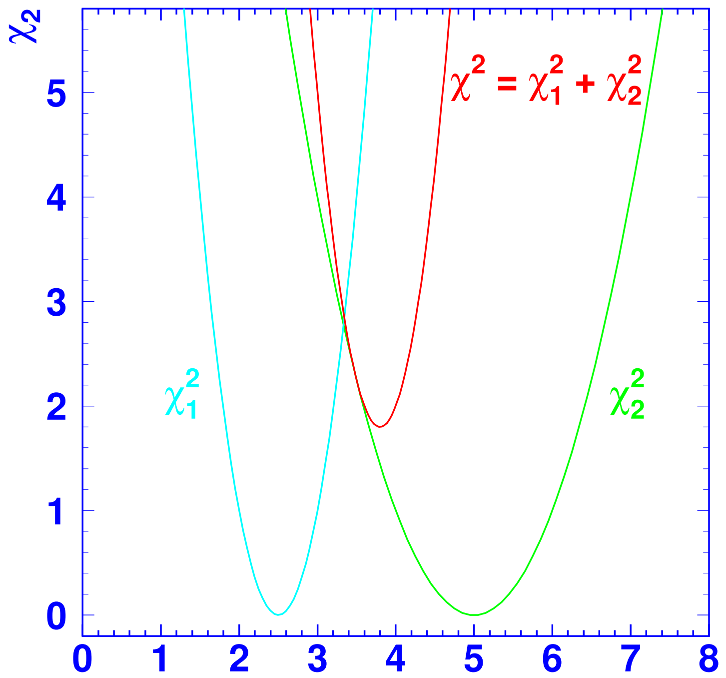
Now combine two measurements: $a_1 \pm \sigma_1$, $a_2 \pm \sigma_2$

Best estimate is linear combination: $a_0 = wa_1 + (1 - w)a_2$

$$\chi^2(x) = (x - a_1)^2/\sigma_1^2 + (x - a_2)^2/\sigma_2^2$$

Minimum $\chi^2$ at $x = a_0$,

1 standard deviation error from $\chi^2(a_0 \pm \sigma_0) - \chi^2(a_0) = 1$



# Correlations

Take into account correlations with covariance matrix, $\mathcal{C}$:

$$\chi^2(x) = \sum_{ij=1,2} (x - a_i)\mathcal{C}^{-1}_{ij}(x - a_j)$$

Example, for uncorrelated errors $u_i$ and 100% correlated $c_i$

i.e. $a_1 \pm u_1 \pm c_1$ , $a_2 \pm u_2 \pm c_2$, covariance matrix given by

$$\mathcal{C} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} u_1^2 + c_1^2 & c_1 c_2 \\ c_1 c_2 & u_2^2 + c_2^2 \end{pmatrix}$$

Minimum $\chi^2$ at $x = a_0$

1 standard deviation error from $\chi^2(a_0 \pm \sigma_0) - \chi^2(a_0) = 1$

If there are several sources of correlated errors, $c_i^p$,

off-diagonal terms in covariance matrix $\Longrightarrow \sum_p c_1^p c_2^p$

Can be extended to many measurements of the same parameter

(Known as Best Linear Unbiassed Estimate – BLUE)

# More than one parameter

Want the best estimates of 10 heavy flavour quantities denoted $x^\mu$ (index $\mu$ corresponds to the parameter)

$$\begin{aligned} x^\mu \quad = \quad & R_{\mathrm{b}},\ R_{\mathrm{c}}, \\ & A_{\mathrm{FB}}^{\mathrm{b\bar{b}}},\ A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}, \\ & \mathcal{A}_{\mathrm{b}},\ \mathcal{A}_{\mathrm{c}} \\ & \mathrm{BR}(\mathrm{b} \to \ell^-),\ \mathrm{BR}(\mathrm{b} \to \mathrm{c} \to \ell^+),\ \overline{\chi}, \\ & \mathrm{BR}(\mathrm{c} \to \mathrm{D}^\star \to \pi \mathrm{D}^0). \end{aligned}$$

Individual measurements are denoted $r_i^\mu$

- index $i$ runs over the 76 measurements
- index $\mu$ $\Longrightarrow$ which of the 10 parameters this measures

Minimise $\chi^2$ w.r.t. best estimates $x^\mu$

$$\chi^2 = \sum_{ij} (r_i^\mu - x^\mu)\, \mathcal{C}_{ij}^{-1} (r_j^\nu - x^\nu)$$

where $\mathcal{C}$ is the covariance matrix for all the heavy flavour measurements, in this case a 76$\times$76 matrix.

1 standard deviation error for 1 parameter defined by $\Delta\chi^2$ = 1, minimising $\chi^2$ w.r.t. all other parameters.

# Full covariance matrix

Calculation of covariance matrix for heavy flavour results is an essential part of the averaging procedure

- $\mathcal{C} = \mathcal{C}^{\mathrm{stat}} + \mathcal{C}^{\mathrm{syst}}$
- $\mathcal{C}^{\mathrm{stat}}$ statistical covariance matrix – includes off-diagonal terms from multiparameter fits

$$\begin{pmatrix} a & 0 & 0 & 0 & 0 & 0 & .. \\ 0 & b & 0 & 0 & 0 & 0 & .. \\ 0 & 0 & c & x & y & 0 & .. \\ 0 & 0 & x & d & z & 0 & .. \\ 0 & 0 & y & z & e & 0 & .. \\ 0 & 0 & 0 & 0 & 0 & f & .. \\ : & : & : & : & : & : & g \end{pmatrix}$$

- Square of total systematic errors on the diagonal $\mathcal{C}^{\mathrm{syst}}_{ii}$
- Off-diagonal terms of $\mathcal{C}^{\mathrm{syst}}$ calculated from detailed breakdown of systematic errors. (N.B. relative signs of systematic errors important)

$$\mathcal{C}^{\mathrm{syst}}_{ij} = \sum_p c_i^p c_j^p$$

where the sum is over all sources of uncertainty, $p$, which affect both results $i$ and $j$.

# Interdependence

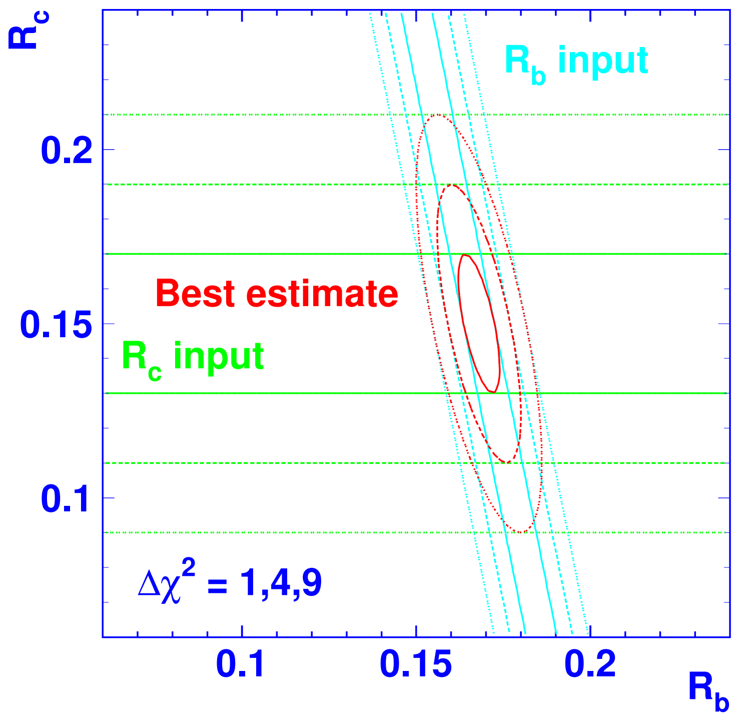
Measurements $r_i^\mu$ of parameter $x^\mu$ may depend on value of some other parameter $x^\nu, \nu \neq \mu$.

$\Longrightarrow$ Take this into account explicitly in $\chi^2$.

Example, $k$th measurement is a precise double-tag value of $R_\mathrm{b}$. Result depends on $R_\mathrm{c}$:

$$r_k^{R_\mathrm{b}} = R_\mathrm{b}^\mathrm{meas} + a_k^{R_\mathrm{c}} \frac{(x^{R_\mathrm{c}} - R_\mathrm{c}^\mathrm{used})}{x^{R_\mathrm{c}}},$$

Combine this with a measurement of $R_\mathrm{c}$ with b–c separation of D mesons $\Longrightarrow$ independent of $R_\mathrm{b}$:



# Cross checks

Many cross checks to verify that combination procedure is correctly implemented

- Code is driven by input tables with detailed breakdown of results, and uses Minuit to minimise $\chi^2$ $\Longrightarrow$ flexible
- Check that $\chi^2$ is well behaved with a unique minimum for each parameter
- Compare results of independently written code
- Verify that input tables for complicated multiparameter fits reproduce the fit results
- Verify that averages made from subsets of results are reasonable, eg. consistent with simpler methods.

$\Longrightarrow$ Confident that averages are technically correct

# Common systematic errors - I

Representative table of significant contributions to total error.

"Value and range" correspond to PDG '92, "New value" is either PDG '95 or unofficial LEP average.

"Pull" is (Average – Standard Model) / (Error from this source).

| Error source | Value and range | New value | $R_b$ pull | $R_c$ pull |
|---|---|---|---|---|
| Decay model $\mathrm{b} \to \ell$ | ACCMM$^{+\mathrm{ISGW}}_{-\mathrm{ISGW**}}$ | | –47 | –34 |
| Decay model $\mathrm{c} \to \ell$ | CL1 $^{+\mathrm{CL2}}_{-\mathrm{CL3}}$ | | –88 | $\infty$ |
| BR($\mathrm{c} \to \ell$) | $(9.8 \pm 0.5)$% | | +21 | +5 |
| BR($\mathrm{b} \to \bar{\mathrm{c}} \to \ell^-$) | $(1.3 \pm 0.5)$% | | –161 | +168 |
| $\langle x_E(\mathrm{b})\rangle$ (if not in fit) | $0.70 \pm 0.02$ | $0.695 \pm 0.010$ LEP | –129 | +16 |
| $\langle x_E(\mathrm{c})\rangle$ (if not in fit) | $0.51 \pm 0.02$ | $0.515 \pm 0.013$ LEP | –100 | +13 |
| b fragmentation model | Peterson, Kartvelishvelli, Collins–Spiller | | $\infty$ | +67 |
| c fragmentation model | " | | –135 | –8 |
| BR($\mathrm{D}^0 \to \mathrm{K}^-\pi^+$) | $(3.84 \pm 0.13)$% PDG'95 | | +61 | +9 |
| BR($\mathrm{D}^+ \to \mathrm{K}^-\pi^+\pi^+$) | $(9.1 \pm 0.6)$% PDG'95 | | +168 | +25 |
| BR($\mathrm{D_s}^+ \to \phi\pi^+$) | $(3.5 \pm 0.4)$% PDG'95 | | +178 | +34 |
| BR($\Lambda_\mathrm{c} \to \mathrm{pK}^-\pi^+$) | $(4.4 \pm 0.6)$% PDG'95 | | +198 | +30 |
| c baryon/$\Lambda_\mathrm{c}$ | $1.15 \pm 0.05$ | | $\infty$ | –94 |

# Common systematic errors - II

| Error source | Value and range | New value | $R_b$ pull | $R_c$ pull |
|---|---|---|---|---|
| $D^0$ fraction in $c\bar{c}$ events | $0.557 \pm 0.053$ | $0.583 \pm 0.035$ LEP | –132 | $\infty$ |
| $D^+$ fraction | $0.248 \pm 0.037$ | $0.233 \pm 0.024$ LEP | –11 | +55 |
| $(D^0 + D^+)$ fraction | $0.80 \pm 0.07$ | | –19 | +123 |
| $D_s$ fraction | $0.12 \pm 0.05$ | $0.097 \pm 0.022$ LEP | –18 | +119 |
| $\Lambda_c$ fraction | $0.08 \pm 0.05$ | $0.076 \pm 0.033$ LEP | –198 | $\infty$ |
| $D^0$ lifetime (ps) | $0.420 \pm 0.008$ | $0.415 \pm 0.004$ PDG'95 | –35 | –76 |
| $D^+$ lifetime (ps) | $1.066 \pm 0.023$ | $1.057 \pm 0.015$ PDG'95 | –31 | –52 |
| $D_s$ lifetime (ps) | $0.450^{+0.030}_{-0.026}$ | $0.467 \pm 0.017$ PDG'95 | –44 | –96 |
| $\Lambda_c$ lifetime (ps) | $0.191^{+0.015}_{-0.012}$ | $0.206 \pm 0.012$ PDG'95 | –323 | –30 |
| B decay multiplicity | $5.5 \pm 0.5$ OPAL | $5.7 \pm 0.3$ OPAL, DELPHI | –263 | +52 |
| D decay multiplicity | $2.53 \pm 0.06$ | | –15 | +101 |
| $BR(D \to K_s{}^0 X)$ | $0.46 \pm 0.06$ | | +17 | –109 |
| $N(g \to b\bar{b})$ | $(0.16 \pm 0.08)\%$ | | –41 | $\infty$ |
| $N(g \to c\bar{c})$ | $(1.6 \pm 0.8)\%$ theory | $(2.38 \pm 0.48)\%$ OPAL | –28 | –24 |
| Rate long-lived light h | JETSET$\pm 10\%$ | | –26 | +118 |
| Light quark frag. | JETSET $\pm$ small | | +20 | –72 |

# Inputs from lower energy

Are results from lower energy experiments valid at LEP?

Semileptonic branching ratios: BR$(\mathrm{b} \to \ell^-)$ and BR$(\mathrm{b} \to \mathrm{c} \to \ell^+)$ now sufficiently well measured at LEP to be able to drop CLEO measurement (where only $\mathrm{B}^0$ and $\mathrm{B}^+$ are produced). BR($\mathrm{c} \to \ell$) taken from lower energy data.

Charm background for $R_\mathrm{b}$: Assume that lower energy measurements of D fractions produced in $\mathrm{c\overline{c}}$ events are valid.

- Expectation is that above threshold, fragmentation to different charm hadrons independent of energy.
- Consistent results with those observed at LEP.

Measurements of $R_\mathrm{c}$: assumptions on rates of $\mathrm{D}^0$, $\mathrm{D}^+$ not needed, but still gain precision from using lower energy BR($\mathrm{c} \to \mathrm{D}^\star \to \pi\mathrm{D}^0$):

- BR $= 0.178 \pm 0.013$ (CLEO,ARGUS,PEP/PETRA)
- BR $= 0.164 \pm 0.008$ (LEP)

# Hemisphere correlations

$$
\begin{aligned}
N_t/2N_{\mathrm{had}} &= \varepsilon_{\mathrm{b}} R_{\mathrm{b}} + \varepsilon_{\mathrm{c}} R_{\mathrm{c}} + \varepsilon_{\mathrm{uds}}(1 - R_{\mathrm{b}} - R_{\mathrm{c}}) \\
N_{tt}/N_{\mathrm{had}} &= \mathcal{C}_{\mathrm{b}} \varepsilon_{\mathrm{b}}^2 R_{\mathrm{b}} + \varepsilon_{\mathrm{c}}^2 R_{\mathrm{c}} + \varepsilon_{\mathrm{uds}}^2 (1 - R_{\mathrm{b}} - R_{\mathrm{c}}) \\
\mathcal{C}_{\mathrm{b}} &= \varepsilon_{\mathrm{b}}^{\mathrm{d}} / \varepsilon_{\mathrm{b}}^2 \\
\lambda_{\mathrm{b}} &= (\varepsilon_{\mathrm{b}}^{\mathrm{d}} - \varepsilon_{\mathrm{b}}^2)/(\varepsilon_{\mathrm{b}} - \varepsilon_{\mathrm{b}}^2)
\end{aligned}
$$

where $\varepsilon_{\mathrm{b}}^{\mathrm{d}} \equiv$ double-tagging efficiency for $\mathrm{b\overline{b}}$ event

$$R_{\mathrm{b}} \approx \mathcal{C}_{\mathrm{b}} N_t^2 / 4 N_{tt} N_{\mathrm{had}}$$

| | $\mathcal{C}_{\mathrm{b}}$ | $\lambda_{\mathrm{b}}$ (%) | $\varepsilon_{\mathrm{b}}$ (%) | $\mathcal{C}_{\mathrm{b}} - 1$ (%) |
|---|---|---|---|---|
| OPAL 92–94 | $1.006 \pm 0.003$ | | 23 | $+0.6 \pm 0.3$ |
| ALEPH 92 | | $-2.0 \pm 0.3$ | 26 | $-5.7 \pm 0.8$ |
| DELPHI 91 | | $-0.1 \pm 0.4$ | 24 | $-0.4 \pm 1.3$ |
| DELPHI 92 | | $-0.3 \pm 0.2$ | 20 | $-1.0 \pm 0.7$ |
| DELPHI 93 | | $-1.3 \pm 0.2$ | 21 | $-4.8 \pm 0.6$ |
| SLD 93–95 | | $+0.5 \pm 0.2$ | 36 | $-0.9 \pm 0.4$ |

Estimates of error due to $\mathcal{C}_{\mathrm{b}}$ dominated by statistics (Monte Carlo or data) $\Longrightarrow$ assume uncertainty not correlated between measurements Varying assumptions on correlated uncertainty due to $\mathcal{C}_{\mathrm{b}}$ has tiny effect on $R_{\mathrm{b}}$ value or error

Hemisphere correlations area of intense study

# ... and finally

## Summary of combination procedure

- Define and propagate a consistent set of external inputs.
- Form the full covariance matrix relating the measurements.
- Take into account any explicit dependence of a measurement on the other electroweak parameters.
- Perform a $\chi^2$ minimisation with respect to the combined electroweak parameters.

Small corrections to convert physical observables to parameters for pure $\mathrm{Z}$ exchange

Photon exchange contributions for $R_\mathrm{b}$, $R_\mathrm{c}$:

$$
\begin{aligned}
R_\mathrm{b} &\equiv \sigma(\mathrm{e^+e^-} \to \mathrm{b\bar{b}})/\sigma(\mathrm{e^+e^-} \to \mathrm{hadrons}) \\
R_\mathrm{b}^0 &\equiv \Gamma(\mathrm{Z} \to \mathrm{b\bar{b}})/\Gamma(\mathrm{Z} \to \mathrm{hadrons}) \\
R_\mathrm{b}^0 &= R_\mathrm{b} + 0.0003 \\
R_\mathrm{c}^0 &= R_\mathrm{c} - 0.0003
\end{aligned}
$$

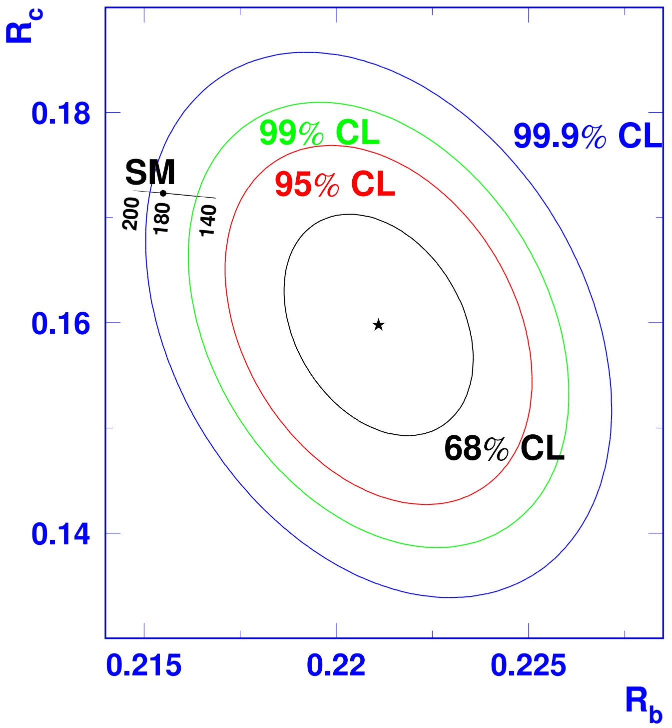
(Measured asymmetries converted to “pole” asymmetries, with corrections for $\sqrt{s}$, QED, QCD, $\gamma$ exchange, $\gamma\mathrm{Z}$ interference.)

# Electroweak Results

$$
\begin{aligned}
R_{\mathrm{b}}^{0} &= 0.2211 \pm 0.0016 \\
R_{\mathrm{c}}^{0} &= 0.1598 \pm 0.0070 \\
A_{\mathrm{FB}}^{0,\,\mathrm{b}} &= 0.1002 \pm 0.0028 \\
A_{\mathrm{FB}}^{0,\,\mathrm{c}} &= 0.0759 \pm 0.0051 \\
\mathcal{A}_{\mathrm{b}} &= 0.842 \pm 0.052 \\
\mathcal{A}_{\mathrm{c}} &= 0.618 \pm 0.091
\end{aligned}
$$

with $\chi^2$/d.o.f. = 41/(76–10) and error correlation matrix:

$$
\begin{pmatrix}
1.000 & -0.30 & 0.01 & -0.03 & -0.06 & 0.05 \\
-0.30 & 1.000 & 0.05 & 0.02 & 0.07 & -0.06 \\
0.01 & 0.05 & 1.00 & 0.10 & 0.04 & 0.00 \\
-0.03 & 0.02 & 0.10 & 1.00 & -0.01 & 0.06 \\
-0.06 & 0.07 & 0.04 & -0.01 & 1.00 & 0.07 \\
0.05 & -0.06 & 0.00 & 0.06 & 0.07 & 1.00
\end{pmatrix}
$$



# Is combination of data correct?

# YES

Averaging procedure does what is expected of it

# BUT...

Combined results are only as correct as individual results

- All dependences are assumed to be linear, all errors are assumed to be Gaussian. Keep this in mind when discussing 99.99% C.L.
- Input values and ranges for systematic errors will be updated, but no big deviations expected
- Unknown sources of error are not taken into account, mistakes/incorrect assumptions are still present.