Dalitz Plot Parameters for $K \to 3\pi$ Decays

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The Dalitz plot distribution for $K^{\pm} \to \pi^{\pm}\pi^{\pm}\pi^{\mp}$, $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$, and $K^0_L \to \pi^+\pi^-\pi^0$ can be parameterized by a series expansion such as that introduced by Weinberg [1]. We use the form

$$\begin{aligned}\left|M\right|^2 \propto 1 &+ g\frac{(s_3 - s_0)}{m^2_{\pi^+}} + h\left[\frac{s_3 - s_0}{m^2_{\pi^+}}\right]^2 \\ &+ j\ \frac{(s_2 - s_1)}{m^2_{\pi^+}} + k\left[\frac{s_2 - s_1}{m^2_{\pi^+}}\right]^2 \\ &+ f\ \frac{(s_2 - s_1)}{m^2_{\pi^+}}\frac{(s_3 - s_0)}{m^2_{\pi^+}} + \cdots \ ,\end{aligned} \tag{1}$$

where $m^2_{\pi^+}$ has been introduced to make the coefficients g, h, j, and k dimensionless, and

$$\begin{aligned}s_i &= (P_K - P_i)^2 = (m_K - m_i)^2 - 2m_K T_i \ , \ i = 1, 2, 3, \\ s_0 &= \frac{1}{3}\sum_i s_i = \frac{1}{3}(m^2_K + m^2_1 + m^2_2 + m^2_3) \quad .\end{aligned}$$

Here the P_i are four-vectors, m_i and T_i are the mass and kinetic energy of the i^{th} pion, and the index 3 is used for the odd pion.

The coefficient g is a measure of the slope in the variable s_3 (or T_3) of the Dalitz plot, while h and k measure the quadratic dependence on s_3 and $(s_2 - s_1)$, respectively. The coefficient j is related to the asymmetry of the plot and must be zero if CP invariance holds. Note also that if CP is good, g, h, and k must be the same for $K^+ \to \pi^+\pi^+\pi^-$ as for $K^- \to \pi^-\pi^-\pi^+$.

Since different experiments use different forms for $\left|M\right|^2$, in order to compare the experiments we have converted to g, h, j, and k whatever coefficients have been measured. Where such conversions have been done, the measured coefficient a_y, a_t, a_u, or a_v is given in the comment at the right. For definitions of these coefficients, details of this conversion, and discussion of the data, see the April 1982 version of this note [2].

References:

1. S. Weinberg, Phys. Rev. Lett. **4**, 87 (1960).
2. Particle Data Group, Phys. Lett. **111B**, 69 (1982).