70. D_s^+ Branching Fractions

Updated September 2019 by J.L. Rosner (University of Chicago) and C.G. Wohl (LBNL).

Figure 70.1 shows a partial breakdown of the D_s^+ branching fractions. The rest of this note is about how the figure was constructed. The values shown make heavy use of CLEO measurements of inclusive branching fractions [1]. For references to other data cited in the following, see the Listings.

70.1. Modes with leptons

The bottom $(18.0 \pm 1.0)\%$ of Fig. 70.1 shows the fractions for the modes that include leptons. The measured $K^0 e^+ \nu_e$ and $K^{*0} e^+ \nu_e$ fractions have been doubled to take account of the corresponding $\mu^+ \nu_\mu$ fractions. The sum of the exclusive $X e^+ \nu_e$ fractions is $(6.0 \pm 0.3)\%$, consistent with an inclusive semileptonic measurement of $(6.5 \pm 0.4)\%$. There seems to be little missing here.

70.2. Inclusive hadronic $K\overline{K}$ fractions

The Cabibbo-favored $c \to s$ decay in D_s^+ decay produces a final state with both an s and an $\bar{s}$; and thus modes with a $K\overline{K}$ pair or with an η, ω, η', or ϕ predominate (as may already be seen in Fig. 70.1 in the semileptonic fractions). We consider the $K\overline{K}$ modes first. A complete picture of the exclusive $K\overline{K}$ charge modes is not yet possible, because branching fractions for many of those modes have not yet been measured. However, CLEO has measured the inclusive K^+, K^-, K^0_S, K^+K^-, $K^+K^0_S$, $K^-K^0_S$, and $2K^0_S$ fractions (these include modes with leptons) [1]. And each of these inclusive fractions with a K^0_S is equal to the corresponding fraction with a K^0_L: $f(K^+K^0_L) = f(K^+K^0_S)$, $f(2K^0_L) = f(2K^0_S)$, *etc.* Therefore, of all inclusive fractions pairing a K^+, K^0_S, or K^0_L with a K^-, K^0_S, or K^0_L, we know all but $f(K^0_S K^0_L)$.

We can get that fraction. The total K^0_S fraction is

$$f(K^0_S) = f(K^+K^0_S) + f(K^-K^0_S) + 2f(2K^0_S) + f(K^0_S K^0_L) + f(\text{single } K^0_S)\ ,$$

where $f(\text{single } K^0_S)$ is the sum of the branching fractions for modes such as $K^0_S\pi^+2\pi^0$ with a K^0_S and no second K. The $K^0_S\pi^+2\pi^0$ mode is in fact the only unmeasured single-K^0_S mode (throughout, we shall assume that fractions for modes with a K or $K\overline{K}$ and more than three pions are negligible), and we shall take its fraction to be the same as for the $K^0_S 2\pi^+\pi^-$ mode, $(0.30 \pm 0.11)\%$. Any reasonable deviation from this value would be too small to matter much in the following. Adding the several small single-K^0_S branching fractions, including those from semileptonic modes, we get $f(\text{single } K^0_S) = (1.7 \pm 0.2)\%$.

Using this, we have:

$$\begin{aligned} f(K^0_S K^0_L) &= f(K^0_S) - f(K^+K^0_S) - f(K^-K^0_S) \\ &\quad - 2f(2K^0_S) - f(\text{single } K^0_S) \\ &= (19.0 \pm 1.1) - (5.8 \pm 0.5) - (1.9 \pm 0.4) \\ &\quad - 2 \times (1.7 \pm 0.3) - (1.7 \pm 0.2) \\ &= (6.2 \pm 1.4)\%\ . \end{aligned}$$

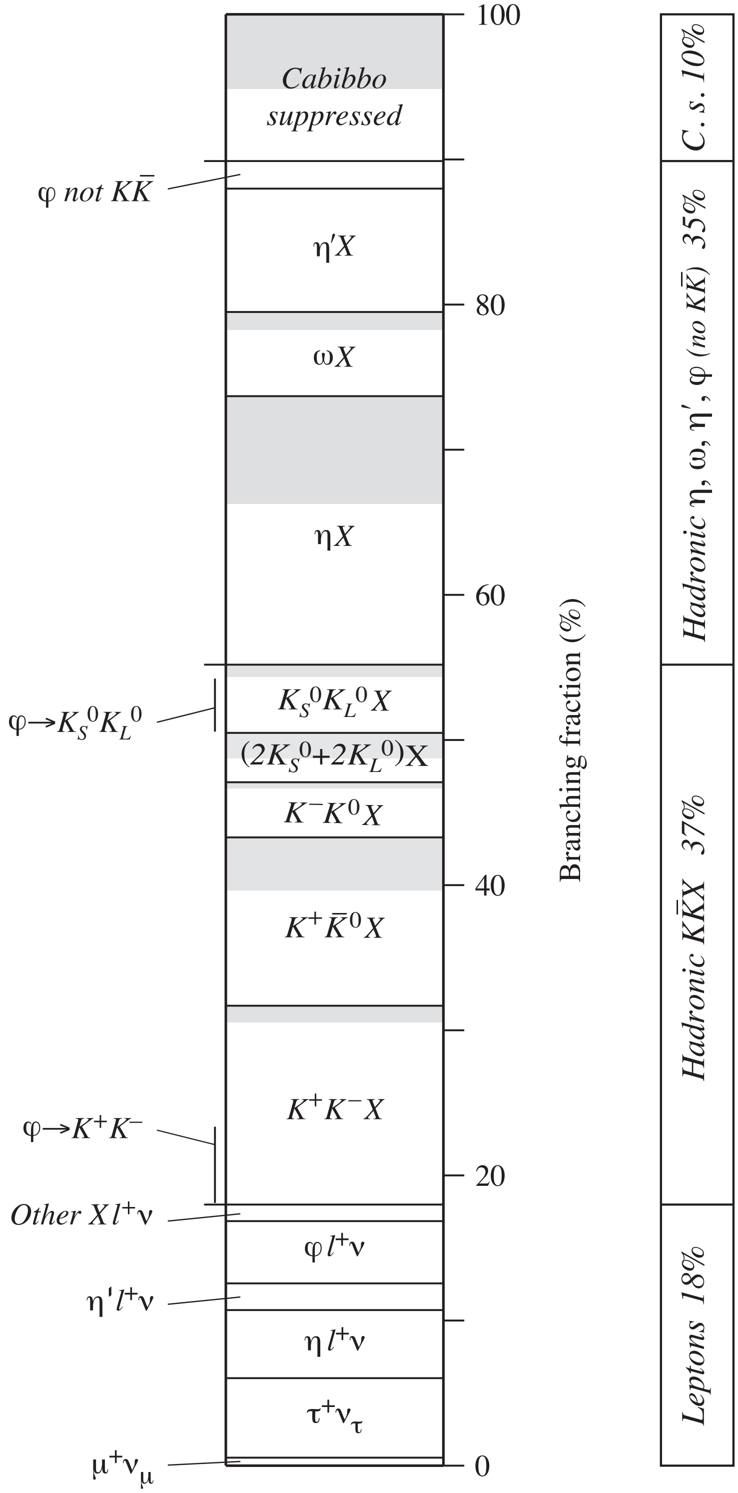

Figure 70.1: A partial breakdown of D_s^+ branching fractions. The hadronic bins in the left column show inclusive fractions. Shading within a bin shows how much of the inclusive fraction is not yet accounted for by adding up all the relevant exclusive fractions. The inclusive hadronic ϕ fraction is spread over three bins, in proportion to its decay fractions into K^+K^-, $K_S^0K_L^0$, and no-$K\bar{K}$ modes.

Here and below we treat the errors as uncorrelated, although often they are not. However, our main aim is to get numbers for Fig. 70.1; errors are secondary.

There is a check on our result: The ϕ inclusive branching fraction is $(15.7 \pm 1.0)\%$, of which 34%, or $(5.34 \pm 0.34)\%$ of D_s^+ decays, produces a $K_S^0 K_L^0$. Our $f(K_S^0 K_L^0) = (6.2 \pm 1.4)\%$ has to be at least this large—and it is, within the sizable error.

We now have all the inclusive $K\overline{K}$ fractions. We use $f(K^+\overline{K}^0) = 2\ f(K^+K_S^0)$, and likewise for $f(K^-K^0)$. For K^+K^- and $K_S^0K_L^0$, we subtract off the contributions from $\phi\ell^+\nu$ decay to get the purely hadronic $K\overline{K}$ inclusive fractions:

$$\begin{aligned}
f(K^+K^-, \text{hadronic}) &= (15.8 \pm 0.7) - (2.1 \pm 0.3) \\
&= (13.7 \pm 0.8)\% \\
f(K^+\overline{K}^0, \text{hadronic}) &= (11.6 \pm 1.0)\% \\
f(K^-K^0, \text{hadronic}) &= (3.8 \pm 0.8)\% \\
f(2K_S^0 + 2K_L^0, \text{hadronic}) &= (3.4 \pm 0.6)\% \\
f(K_S^0K_L^0, \text{hadronic}) &= (6.2 \pm 1.4) - (1.5 \pm 0.2) \\
&= (4.7 \pm 1.4)\% \ .
\end{aligned}$$

The fractions are shown in Fig. 70.1. They total $(37.2 \pm 2.2)\%$ of D_s^+ decays.

We can add more information to the figure by summing up measured branching fractions for exclusive modes within each bin:

K^+K^- *modes*—The sum of measured $K^+K^-\pi^+$, $K^+K^-\pi^+\pi^0$, and $K^+K^-2\pi^+\pi^-$ branching fractions is $(12.6 \pm 0.6)\%$. That leaves $(1.1 \pm 1.0)\%$ for the $K^+K^-\pi^+2\pi^0$ mode, which is the only other K^+K^- mode with three or fewer pions. In Fig. 70.1, this unmeasured part of the K^+K^- bin is shaded.

$K^+\overline{K}^0$ *modes*—Two times the sum of the measured $K^+K_S^0$, $K^+K_S^0\pi^0$, and $K^+K_S^0\pi^+\pi^-$ branching fractions is $(8.0 \pm 0.5)\%$. This leaves $(3.6 \pm 1.1)\%$ for the unmeasured $K^+\overline{K}^0$ modes (there are three such modes with three or fewer pions). This is shaded in the figure.

K^-K^0 *modes*—Twice the $K^-K_S^0 2\pi^+$ fraction is $(3.4 \pm 0.2)\%$, which leaves about $(0.4 \pm 0.8)\%$ for $K^-K^0 2\pi^+\pi^0$, the only other K^-K^0 mode with three or fewer pions.

$2K_S^0 + 2K_L^0$ *modes*—The $2K_S^0\pi^+$ and $2K_S^0 2\pi^+\pi^-$ fractions sum to $(0.86 \pm 0.07)\%$; this times two (for the corresponding $2K_L^0$ modes) is $(1.72 \pm 0.14)\%$. This leaves about $(1.7 \pm 0.7)\%$ for other $2K_S^0 + 2K_L^0$ modes.

$K_S^0K_L^0$ *modes*—Most of the $K_S^0K_L^0$ fraction is accounted for by ϕ decays (see below).

70.3. Inclusive hadronic η, ω, η', and ϕ fractions

These are easier. We start with the inclusive branching fractions, and then, to avoid double counting, subtract: (1) fractions for modes with leptons; (2) η mesons that are included in the inclusive η' fraction; and (3) K^+K^- and $K^0_SK^0_L$ from ϕ decays:

$$\begin{aligned}
f(\eta \text{ hadronic}) &= f(\eta \text{ inclusive}) - 0.65\, f(\eta' \text{ inclusive}) \\
-f(\eta\ell^+\nu) &= (18.5 \pm 3.0)\% \\
f(\omega \text{ hadronic}) &= f(\omega \text{ inclusive}) - 0.026\, f(\eta' \text{ inclusive}) \\
&= (5.8 \pm 1.4)\% \\
f(\eta' \text{ hadronic}) &= f(\eta' \text{ inclusive}) - f(\eta'\ell^+\nu) \\
&= (8.5 \pm 1.5)\% \\
f(\phi \text{ hadronic}, \not\to K\overline{K}) &= 0.17\, \left[f(\phi \text{ inclusive}) - f(\phi\ell^+\nu)\right] \\
&= (1.9 \pm 0.2)\% \; .
\end{aligned}$$

The factors 0.65, 0.026, and 0.17 are the $\eta' \to \eta$, $\eta' \to \omega$, and $\phi \not\to K\overline{K}$ branching fractions. Figure 70.1 shows the results; the sum is $(34.7 \pm 3.6)\%$, which is about equal to the hadronic $K\overline{K}$ total.

Note that the bin marked ϕ near the top of Fig. 70.1 includes neither the $\phi\ell^+\nu$ decays nor the 83% of other ϕ decays that produce a $K\overline{K}$ pair. There is twice as much ϕ in the $K^0_SK^0_L$ bin, and nearly three times as much in the K^+K^- bin. These contributions are indicated in those bins.

Again, we can show how much of each bin is accounted for by measured exclusive branching fractions:

η modes—The sum of $\eta\pi^+$, $\eta\pi^+\pi^0$ (nearly all $\eta\rho^+$), and ηK^+ branching fractions is $(11.1 \pm 1.2)\%$, which leaves a good part of the inclusive hadronic η fraction, $(18.5 \pm 3.0)\%$, to be accounted for. This is shaded in the figure.

ω modes—The sum of $\omega\pi^+$, $\omega\pi^+\pi^0$, and $\omega 2\pi^+\pi^-$ fractions is $(4.6 \pm 0.9)\%$, which is nearly as large as the inclusive hadronic ω fraction, $(5.8 \pm 1.4)\%$.

η' modes—The sum of $\eta'\pi^+$, $\eta'\rho^+$, and $\eta' K^+$ fractions is $(9.9 \pm 1.5)\%$, which is larger than but not in serious disagreement with the inclusive hadronic η' fraction, $(8.5 \pm 1.5)\%$.

70.4. Cabibbo-suppressed modes

The sum of the fractions for modes with a $K\bar{K}$, η, ω, η', or leptons is $(89.9 \pm 4.4)\%$. The remaining $(10.1 \pm 4.4)\%$ is to Cabibbo-suppressed modes, mainly single-$K+$ pions and multiple-pion modes (see below). However, it should be noted that some small parts of the modes already discussed are Cabibbo-suppressed. For example, the $(1.1 \pm 0.2)\%$ of D_s^+ decays to $K^0\ell\nu$ or $K^{*0}\ell\nu$ is already in the $X\ell\nu$ bin in Fig. 70.1. And the inclusive measurements of η, ω, and η' fractions do not distinguish between (and therefore include both) Cabibbo-allowed and -suppressed modes. We shall not try to make a separation here.

K^0 + pions—Above, we found that f(single K_S^0) = $(1.7 \pm 0.2)\%$. Subtracting semileptonic fractions with a K_S^0 leaves $(1.3 \pm 0.2)\%$. The hadronic single-K^0 fraction is twice this, about $(2.6 \pm 0.4)\%$. The sum of measured $K^0\pi^+$, $K^0\pi^+\pi^0$, and $K^0 2\pi^+\pi^-$ fractions is $(1.8 \pm 0.3)\%$, about two-thirds as much.

K^+ + pions—The $K^+\pi^0$ and $K^+\pi^+\pi^-$ fractions sum to $(0.72 \pm 0.05)\%$. The total K^+ fraction wanted here is probably in the 1-to-2% range.

Multi-pions—The $2\pi^+\pi^-$, $\pi^+ 2\pi^0$, and $3\pi^+ 2\pi^-$ fractions total $(2.5 \pm 0.2)\%$. Modes not measured might double this.

The sum of the actually measured fractions is, including the semileptonics, $(4.9 \pm 0.3)\%$. The error on our Cabibbo-suppressed total, $(10.1 \pm 4.4)\%$ is too large to know how much we might be missing.

References:

1. S. Dobbs *et al.*, Phys. Rev. **D79**, 112008 (2009).