55. Muon Anomalous Magnetic Moment

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The Dirac equation predicts a muon magnetic moment, $\vec{M} = g_\mu \frac{e}{2m_\mu}\vec{S}$, with gyromagnetic ratio $g_\mu = 2$. Quantum loop effects lead to a small calculable deviation from $g_\mu = 2$, parameterized by the anomalous magnetic moment

$$a_\mu \equiv \frac{g_\mu - 2}{2} \quad . \tag{55.1}$$

That quantity can be accurately measured and, within the Standard Model (SM) framework, precisely predicted. Hence, comparison of experiment and theory tests the SM at its quantum loop level. A deviation in $a_\mu^{\rm exp}$ from the SM expectation would signal effects of new physics, with current sensitivity reaching up to mass scales of $\mathcal{O}$(TeV) [1,2]. For recent thorough muon $g-2$ reviews, see e.g. Refs. [3–5]. .

The E821 experiment at Brookhaven National Lab (BNL) studied the precession of μ^+ and μ^- in a constant external magnetic field as they circulated in a confining storage ring. It found[1] [6]

$$\begin{aligned} a_{\mu^+}^{\rm exp} &= 11\,659\,204(6)(5) \times 10^{-10}\,, \\ a_{\mu^-}^{\rm exp} &= 11\,659\,215(8)(3) \times 10^{-10}\,, \end{aligned} \tag{55.2}$$

where the first errors are statistical and the second systematic. Assuming CPT invariance and taking into account correlations between systematic uncertainties, one finds for their average [6,7]

$$a_\mu^{\rm exp} = 11\,659\,209.1(5.4)(3.3) \times 10^{-10} \quad . \tag{55.3}$$

These results represent about a factor of 14 improvement over the classic CERN experiments of the 1970's [8]. Improvement of the measurement by a factor of four by setting up the E821 storage ring at Fermilab, and utilizing a cleaner and more intense muon beam and improved detectors [9] is in progress with the commissioning of the experiment having started in 2017. First results are expected in 2019. Another muon $g-2$ experiment with similar sensitivity but using an alternative zero-electric-field technique with a low-emittance and low-momentum muon beam is currently under construction at J-PARC in Japan [10].

The SM prediction for $a_\mu^{\rm SM}$ is generally divided into three parts (see Fig. 55.1 for representative Feynman diagrams)

$$a_\mu^{\rm SM} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm Had} \quad . \tag{55.4}$$

[1] The results reported by the experiment have been updated in Eqs. (55.2) and (55.3) to the newest value for the absolute muon-to-proton magnetic ratio $\lambda = 3.183\,345\,107(84)$ [7]. The change induced in $a_\mu^{\rm exp}$ with respect to the value of $\lambda = 3.183\,345\,39(10)$ used in Ref. 6 amounts to $+1.12 \times 10^{-10}$.

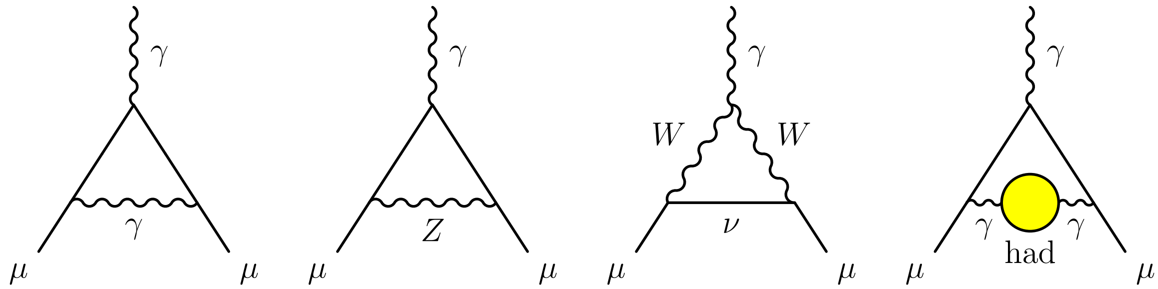

Figure 55.1: Representative diagrams contributing to $a_\mu^{\rm SM}$. From left to right: first order QED (Schwinger term), lowest-order weak, lowest-order hadronic.

The QED part includes all photonic and leptonic (e, μ, τ) loops starting with the classic $\alpha/2\pi$ Schwinger contribution. It has been computed through 5 loops [11]

$$\begin{aligned} a_\mu^{\rm QED} =& \frac{\alpha}{2\pi} + 0.765\,857\,425(17)\left(\frac{\alpha}{\pi}\right)^2 + 24.050\,509\,96(32)\left(\frac{\alpha}{\pi}\right)^3 \\ &+ 130.879\,6(6\,3)\left(\frac{\alpha}{\pi}\right)^4 + 752.2(1.0)\left(\frac{\alpha}{\pi}\right)^5 + \cdots \end{aligned} \quad (55.5)$$

with little change in the coefficients since our last update of this review. Employing $\alpha^{-1} = 137.035\,999\,046(27)$, obtained from the precise measurements of $h/m_{\rm Cs}$ [12], the Rydberg constant, and $m_{\rm Cs}/m_e$ leads to [11]

$$a_\mu^{\rm QED} = 116\,584\,718.92(0.03) \times 10^{-11}\,, \quad (55.6)$$

where the small error results mainly from the uncertainty in α.

Loop contributions involving heavy $W^\pm, Z$ or Higgs particles are collectively labeled as $a_\mu^{\rm EW}$. They are suppressed by at least a factor of $(\alpha/\pi)\cdot(m_\mu^2/m_W^2) \simeq 4\times10^{-9}$. At 1-loop order [13]

$$\begin{aligned} a_\mu^{\rm EW}[\text{1-loop}] &= \frac{G_\mu m_\mu^2}{8\sqrt{2}\pi^2}\left[\frac{5}{3} + \frac{1}{3}\left(1 - 4\sin^2\theta_{\rm W}\right)^2 + \mathcal{O}\left(\frac{m_\mu^2}{M_W^2}\right) + \mathcal{O}\left(\frac{m_\mu^2}{m_H^2}\right)\right] \\ &= 194.8 \times 10^{-11}\,, \end{aligned} \quad (55.7)$$

for $\sin^2\theta_{\rm W} \equiv 1 - M_W^2/M_Z^2 \simeq 0.223$, and where $G_\mu \simeq 1.166\times10^{-5}$ GeV^{-2} is the Fermi coupling constant. Two-loop corrections are relatively large and negative [14]. For a Higgs boson mass of 125 GeV it amounts to $a_\mu^{\rm EW}[\text{2-loop}] = -41.2(1.0)\times10^{-11}$ [14], where the uncertainty stems from quark triangle loops. The 3-loop leading logarithms are negligible, $\mathcal{O}(10^{-12})$ [14,15]. A recent full 2-loop numerical evaluation of the electroweak correction [16] reproduces the total 1+2-loop contribution when adjusted for appropriate light quark masses

$$a_\mu^{\rm EW} = 153.6(1.0)\times10^{-11}\ . \quad (55.8)$$

Hadronic (quark and gluon) loop contributions to $a_\mu^{\rm SM}$ give rise to its main theoretical uncertainties. At present, those effects are not precisely calculable from first principles, but such an approach, at least partially, may become possible as lattice QCD matures [17]. Instead, one currently relies on a dispersion relation approach to evaluate the lowest-order $\mathcal{O}(\alpha^2)$ hadronic vacuum polarization contribution $a_\mu^{\rm Had}[{\rm LO}]$ from corresponding cross section measurements [18]

$$a_\mu^{\rm Had}[{\rm LO}] = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds\, \frac{K(s)}{s} R^{(0)}(s)\,, \tag{55.9}$$

where $K(s)$ is a QED kernel function [19], and where $R^{(0)}(s)$ denotes the ratio of the bare[2] cross section for e^+e^- annihilation into hadrons to the pointlike muon-pair cross section at center-of-mass energy $\sqrt{s}$. The function $K(s) \sim 1/s$ in Eq. (55.9) emphasizes the low-energy part of the integral so that $a_\mu^{\rm Had}[{\rm LO}]$ is dominated by the $\rho(770) \to \pi^+\pi^-$ resonance.

The analysis of Eq. (55.9) results in the representative value [20]

$$a_\mu^{\rm Had}[{\rm LO}] = 6\,939(39)(7) \times 10^{-11}\,, \tag{55.10}$$

where the first error is experimental, dominated by systematic uncertainties in the $e^+e^- \to$ hadrons cross-section data, and the second due to perturbative QCD, which is used at intermediate and large energies in the dispersion integral to predict the contribution from the quark-antiquark continuum. The experimental precision is currently limited by a discrepancy between the most precise $\pi^+\pi^-$ data from the BABAR and KLOE experiments [20]. Other recent evaluations [31,32] of $a_\mu^{\rm Had}[{\rm LO}]$ find consistent results with Eq. (55.10).

Alternatively, one can use precise vector spectral functions from $\tau \to \nu_\tau +$ hadrons decays [21] that can be related to isovector $e^+e^- \to$ hadrons cross sections by isospin symmetry. Analyses replaced e^+e^- data in the two-pion and four-pion channels by the corresponding isospin-transformed τ data, and applied isospin-violating corrections [22]. Owing to the progress in the precision of the e^+e^- data, the τ data are now less precise and less reliable due to additional theoretical uncertainties, so that recent $a_\mu^{\rm Had}[{\rm LO}]$ evaluations ignored them.

Higher order hadronic contributions are obtained from dispersion relations using the same $e^+e^- \to$ hadrons data [23], giving $a_\mu^{\rm Had,Disp}[{\rm NLO}] = (-98.7 \pm 0.9) \times 10^{-11}$ and $a_\mu^{\rm Had,Disp}[{\rm NNLO}] = (12.4 \pm 0.1) \times 10^{-11}$ [24], along with model-dependent estimates of the hadronic light-by-light scattering contribution, $a_\mu^{\rm Had,LBL}[{\rm NLO}]$, motivated by

[2] The bare cross section is defined as the measured cross section corrected for initial-state radiation, electron-vertex loop contributions and vacuum-polarization effects in the photon propagator. However, QED effects in the hadron vertex and final state, as photon radiation, are included.

large-N_C QCD [25–30]. .[3] Following [29], one finds for the sum of the three terms

$$a_\mu^{\text{Had}}[\text{N(N)LO}] = 19(26) \times 10^{-11}\,, \tag{55.11}$$

where the error is dominated by hadronic light-by-light uncertainty.

Adding Eqs. (55.6), (55.8), (55.10) and (55.11) gives the representative SM prediction

$$a_\mu^{\text{SM}} = 116\,591\,830(1)(40)(26) \times 10^{-11}\,, \tag{55.12}$$

where the errors are due to the electroweak, lowest-order hadronic, and higher-order hadronic contributions, respectively. The difference between experiment and theory

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 261(63)(48) \times 10^{-11}\,, \tag{55.13}$$

where the errors are from experiment and theory prediction (with all errors combined in quadrature), respectively, represents an interesting but not conclusive discrepancy of 3.3 times the combined 1σ error. All the recent estimates for the hadronic contribution compiled in Fig. 55.2 exhibit similar discrepancies.

An exciting interpretation is that Δa_μ may be a new physics signal with supersymmetric particle loops as the leading candidate explanation. Such a scenario is quite natural, since generically, supersymmetric models predict [1] an additional contribution to a_μ^{SM}

$$a_\mu^{\text{SUSY}} \simeq \pm\, 130 \times 10^{-11} \cdot \left(\frac{100 \text{ GeV}}{m_{\text{SUSY}}}\right)^2 \tan\beta\,, \tag{55.14}$$

where m_{SUSY} is a representative supersymmetric mass scale, $\tan\beta \simeq 3$–40 a potential enhancement factor, and ± 1 corresponds to the sign of the μ term in the supersymmetric Lagrangian. Supersymmetric particles in the mass range 100–500 GeV could be the source of the deviation Δa_μ. If so, those particles should be directly observable at the Large Hadron Collider at CERN. So far, there is however no direct evidence in support of the supersymmetry interpretation.

New physics effects [1] other than supersymmetry could also explain a non-vanishing Δa_μ. A popular scenario involves the "dark photon", a relatively light hypothetical vector boson from the dark matter sector that couples to our world of particle physics through mixing with the ordinary photon [33–35]. . As a result, it couples to ordinary charged particles with strength $\varepsilon \cdot e$ and gives rise to an additional muon anomalous magnetic moment contribution

$$a_\mu^{\text{dark photon}} = \frac{\alpha}{2\pi}\varepsilon^2 F(m_V/m_\mu)\,, \tag{55.15}$$

[3] Some representative recent estimates of the hadronic light-by-light scattering contribution, $a_\mu^{\text{Had,LBL}}$[NLO] are: $105(26)\times 10^{-11}$ [29], $110(40)\times 10^{-11}$ [25], $136(25)\times 10^{-11}$ [26]. An approach based on dispersion relations is proposed in [28].

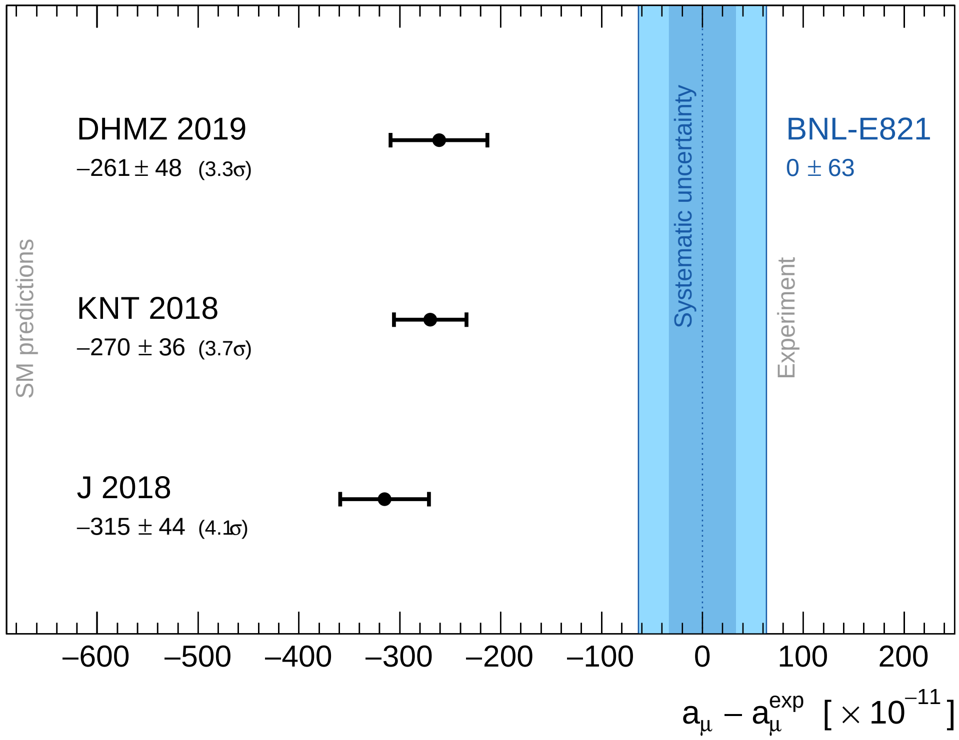

Figure 55.2: Compilation of recent results for a_μ (in units of 10^{-11}), subtracted by the central value of the experimental average (55.3). The shaded (dark shaded) vertical band indicates the total (systematic) experimental uncertainty. The SM predictions are taken from: DHMZ 2019 [20], KNT 2018 [31], and J 2017 [32]. Note that the quoted errors in the figure do not include the uncertainty on the subtracted experimental value. To obtain for each theory calculation a result equivalent to Eq. (55.13), the errors from theory and experiment must be added in quadrature.

where $F(x) = \int_0^1 2z(1-z)^2/[(1-z)^2 + x^2 z]\,dz$. For values of $\varepsilon \sim 1\text{–}2 \times 10^{-3}$ and $m_V \sim 10\text{–}100\,\mathrm{MeV}$, the dark photon, which was originally motivated by cosmology, can provide a viable solution to the muon $g-2$ discrepancy. However, recent experimental constraints disfavor such a scenario [36] under the assumption that the dark photon decays primarily into charged lepton pairs. Direct searches for the dark photon continue to be well motivated [37], but with primary guidance coming from phenomena outside the muon anomalous magnetic moment discrepancy. More recent popular solutions to the muon anomaly discrepancy have focused on loop contributions coming from relatively light new scalar or pseudoscalar particle appendages from physics beyond the SM.

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