Department of Physics, Faculty of Science, Chulalongkorn University

Homework - 3 **CL ELECTRODYN II (2304604)**

Due: November 9, 2017 (TUE) **Lecturer: N. Srimanobhas**

Problem 1.

Show explicitly that two successive Lorentz transformations in the same direction are equivalent to a single Lorentz transformation with a velocity

$$v = \frac{v_1 + v_2}{1 + (v_1 v_2/c^2)}.$$

This is an alternative way to derive the parallel-velocity addition law.

Problem 2.

A coordinate system K' moves with a velocity $\mathbf{v}$ relative to another system K. In K' a particle has a velocity $\mathbf{u}'$ and an acceleration $\mathbf{a}'$. Find the Lorentz transformation law for accelerations, and show that in the system K the components of acceleration parallel and perpendicular to $\mathbf{v}$ are

$$\mathbf{a}_{\parallel} = \frac{\left(1 - \frac{v^2}{c^2}\right)^{3/2}}{\left(1 + \frac{\mathbf{v}\cdot\mathbf{u}'}{c^2}\right)^3} \mathbf{a}'_{\parallel}$$

$$\mathbf{a}_{\perp} = \frac{\left(1 - \frac{v^2}{c^2}\right)}{\left(1 + \frac{\mathbf{v}\cdot\mathbf{u}'}{c^2}\right)^3} \left(\mathbf{a}'_{\perp} + \frac{\mathbf{v}}{c^2} \times (\mathbf{a}' \times \mathbf{u}')\right)$$

Problem 3.

The electric field of a laser beam is linearly polarized vertically, is axially symmetric, and has a magnitude which depends upon distance from the laser's axis as follows:

$$\mathbf{E} = \mathbf{E_0} e^{-s^2/s_0^2} = \mathbf{E_0} e^{-(x'^2+y'^2)/s_0^2}$$

The laser beam is pointed perpendicular to a screen which lies a very long distance r ($\gg s_0$) away from the laser. What is the electric field on this screen, as a function of distance $q = \sqrt{x^2 + y^2}$ from the point on the screen at which the laser is aimed? Hint: work in Cartesian coordinates initially, and complete the square in the exponent of the integrand, to carry out the integral.