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Presentation of LEP Electroweak Heavy Flavour Results for Summer 1996 Conferences

The LEP Electroweak Working Group *

Abstract

The recommendations for the presentation of electroweak heavy flavour results are updated. The changed are motivated by new measurements of the required quantities inside and outside LEP as well as by new developments in the electroweak measurements themselves.

The suggestions in this document apply to results starting at the conferences in the Summer of 1996.

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1 Introduction

In 1994 the LEP heavy flavour group has made suggestions how to treat the electroweak measurements with quarks in a consistent way in order to ease the combination of the data in the LEP collaborations [1, 2]. This procdure was successfully used to average LEP and later on also SLD data [3, 4].

The recommendations at that time have been made on the basis of the analyses available then and mainly based on the 1992 edition of the PDG [5]. In the meantime the analyses at LEP/SLD have evolved and a lot of new measurements became available that sometimes significantly reduce the error on the needed input parameters. For world avarages the suggestions presented here are generally based on the 1996 edition of the PDG [6].

In this note we describe the updated format for presenting heavy flavour electroweak results. We also discuss the changes in the procedure for making average numbers, and the implication of this for existing measurements.

2 Definition of parameters and fitting procedure

The principle of the fitting procedure remains unchanged. The most up to date description can be found in [3]. The electroweak parameters of interest are R_b, R_c, $A_{FB}^{b\bar{b}}$, $A_{FB}^{c\bar{c}}$, $\mathcal{A}_b$and $\mathcal{A}_c$. The b and c forward-backward asymmetries are either split into three different centre of mass energies or all transported to the Z peak before fitting. The fitted forward-backward asymmetries will in future already be corrected for QCD effects. With the given precision this is more appropriate since the exact size of the QCD correction depends on the given analysis.

Some of the analyses like the lepton fits measure other parameters simultaneously with the electroweak parameters. Other measurements depend on the assumed value of those parameters. For that reason some more auxiliary parameters are treated by the fit together with the electroweak parameters of interest. These parameters currently are:

- the semileptonic branching ratio of the b-quark ($\mathrm{Br}(b \to \ell)$),
- the semileptonic cascade branching ratio of the b-quark ($\mathrm{Br}(b \to c \to \ell)$),
- the average $b\bar{b}$-mixing parameter (χ_B),
- the probability for a c quark to fragment into a D^{*+} which subsequently decays into π^+D^0 ($P(c \to D^{*+}) \times BR(D^{*+} \to \pi^+D^0)$),
- the probability for a c quark to fragment into a D^+, D_s or a charmed baryon ($f(D^+)$, $f(D_s)$, $f(c_{baryon})$). The probability that a c quark fragments into a D^0 is calculated from the constraint that all probabilities add up to one.

The list of auxiliary parameters of course may evolve with time depending on new analyses appearing.

Charmed hadron production cross sections are proportional to the product of R_c and the probability that a c quarks fragments into this hadron. To allow the use of single charmed hadron cross section measurements the parameters $R_c \cdot P(c \to D^{*+}) \times BR(D^{*+} \to \pi^+D^0)$, $R_c \cdot f(D^+)$, $R_c \cdot f(D_s)$, $R_c \cdot f(c_{baryon})$ and $R_c \cdot f(D^0)$ are accepted by the fit as input parameters. The last parameter is interpreted as $R_c \cdot (1 - f(D^+) - f(D_s) - f(c_{baryon}))$.

3 Charm production and decay

3.1 The charmed hadron production rates

Given the difference in lifetime between the weakly–decaying charmed hadrons, their tag efficiencies are substantially different, and therefore their relative abundances in $\mathrm{Z} \rightarrow \mathrm{c\bar{c}}$ events strongly affect the overall charm efficiency. Up to now the values recommended by the LEP EWWG [3] were based on ARGUS and CLEO data, assuming that they are valid at $\sqrt{s} = M_\mathrm{Z}$

New precise measurements of the charmed hadron multiplicities in $\mathrm{Z} \rightarrow \mathrm{c\bar{c}}$ events are now available from LEP data. In order to estimate the systematic uncertainty, the $\mathrm{D^0}$ fraction is written as

$$f(\mathrm{D^0}) = 1 - f(\mathrm{D^+}) - f(\mathrm{D_s}) - f(\mathrm{c_{baryon}})$$

The errors on the last three parameters and their correlation matrix are used to evaluate the uncertainty on the charm efficiency. When varying the fractions in the Monte Carlo, the variation of each of the three channels is always compensated by the $\mathrm{D^0}$ fraction. Some details about the averaging procedure along with the final correlation matrix are given below.

3.1.1 The new LEP average

The method is based on the OPAL analysis described in Ref. [8]. The measurements used are:

Experiment	Parameter	Value
	$\mathrm{R_c f(D^0) BR(D^0 \rightarrow K^-\pi^+)}$	0.00389 ± 0.00027
OPAL [8]	$\mathrm{R_c f(D^+) BR(D^+ \rightarrow K^-\pi^+\pi^+)}$	0.00358 ± 0.00046
	$\mathrm{R_c f(D_s) BR(D_s^+ \rightarrow \phi\pi^+)}$	0.00056 ± 0.00015
	$\mathrm{R_c f(\Lambda_c) BR(\Lambda_c^+ \rightarrow pK^-\pi^+)}$	0.00041 ± 0.00019
DELPHI [9]	$\mathrm{R_c f(D^0) BR(D^0 \rightarrow K^-\pi^+)}$	0.00403 ± 0.00089
	$\mathrm{R_c f(D^+) BR(D^+ \rightarrow K^-\pi^+\pi^+)}$	0.00367 ± 0.00078
ALEPH [10]	$\mathrm{R_c f(D_s) BR(D_s^+ \rightarrow \phi\pi^+)}$	0.00077 ± 0.00013

These measured values are combined using the errors (or the covariance matrix) provided in the references, and the values of the charmed hadron branching ratios listed below. They are the new PDG world averages, except for the case of the $\mathrm{BR(D_s^+ \rightarrow \phi\pi^+)}$. Instead, the latest model–independent CLEO analysis [11] which gives a value of the ratio $\mathrm{BR(D_s^+ \rightarrow \phi\pi^+)/BR(D^0 \rightarrow K^-\pi^+)}$ virtually free from theoretical assumptions, is used directly to account for the correlation between $\mathrm{BR(D_s^+ \rightarrow \phi\pi^+)}$ and $\mathrm{BR(D^0 \rightarrow K^-\pi^+)}$.

$\mathcal{B}_{\mathrm{D^0}} = \mathrm{BR(D^0 \rightarrow K^-\pi^+)}$	0.0383	$\pm$ 0.0012
$\mathcal{B}_{\mathrm{D^+}} = \mathrm{BR(D^+ \rightarrow K^-\pi^+\pi^+)}$	0.091	$\pm$ 0.006
$\mathcal{B}_{\mathrm{D_s}} = \mathrm{BR(D_s^+ \rightarrow \phi\pi^+)/BR(D^0 \rightarrow K^-\pi^+)}$	0.91	$\pm$ 0.23
$\mathcal{B}_{\Lambda_\mathrm{c}} = \mathrm{BR(\Lambda_c^+ \rightarrow pK^-\pi^+)}$	0.044	$\pm$ 0.006

	$f(D^0)$	$f(D^+)$	$f(D_s)$	$f(c_{baryon})$	R_c	$\mathcal{R}_{baryon}$	$\mathcal{B}_{D^0}$	$\mathcal{B}_{D^+}$	$\mathcal{B}_{D_s}$	$\mathcal{B}_{\Lambda_c}$
Value	0.600	0.233	0.102	0.065	0.172	1.150	0.038	0.091	0.038	0.044
Error		0.028	0.037	0.029	0.017	0.050	0.001	0.006	0.010	0.006
$f(D^+)$		1.00	−0.36	−0.24	−0.19	−0.02	0.21	−0.42	0.34	0.07
$f(D_s)$		−0.36	1.00	−0.14	0.39	−0.01	−0.10	0.04	−0.91	0.02
$f(c_{baryon})$		−0.24	−0.14	1.00	0.29	0.09	0.05	0.03	0.06	−0.28
R_c		−0.19	0.39	0.29	1.00	0.03	−0.27	−0.16	−0.51	−0.09
$\mathcal{R}_{baryon}$		−0.02	−0.01	0.09	0.03	1.00	0.00	0.00	0.00	0.00
$\mathcal{B}_{D^0}$		0.21	−0.10	0.05	−0.27	0.00	1.00	0.00	0.16	0.00
$\mathcal{B}_{D^+}$		−0.42	0.04	0.03	−0.16	0.00	0.00	1.00	0.00	0.00
$\mathcal{B}_{D_s}$		0.34	−0.91	0.06	−0.51	0.00	0.16	0.00	1.00	0.00
$\mathcal{B}_{\Lambda_c}$		0.07	0.02	−0.28	−0.09	0.00	0.00	0.00	0.00	1.00

Table 1: Details of the fit results: fitted values, errors and correlation matrix

On top of that, an additional constraint is added on the heavy baryon production. It is assumed that

$$\mathcal{R}_{baryon} = f(c_{baryon})/f(\Lambda_c) = (1.15 \pm 0.05)$$

This information is merged with χ^2 minimisation, leaving as free parameters $f(D^+)$, $f(D_s)$, $f(c_{baryon})$, R_c, $\mathcal{R}_{baryon}$, and the four branching ratios listed above. Results, errors and the correlation matrix are listed in Table 1. It should be noted that the measured production rates and R_c are correlated. This influences the explicit dependence of R_b on R_c.

3.2 The charmed hadron decay modes: topological channels

Different decay modes of a given charmed hadron can have different tag efficiencies. Unfortunately, the complete set of measurements of the exclusive branching ratios does not exist for any hadron type. Since the tags basically extract the information from the charged tracks, decay modes can be classified into topological channels, according to the number of charged products. This classification should account for the most part of the difference in efficiency.

Inclusive topological branching ratios have been measured [12] for D^0, D^+ and D_s. In order to calculate the resulting error on the charm efficiency, each channel is varied by its uncertainty except for the largest one, which is used to balance the various shifts. The errors extracted are combined using the correlation coefficients listed below. The assumption of negligible correlation between the 0–prong and the 6–prong channels in the D^0 decay has been made, since for this channel the information given in the reference is incomplete.

Values, errors and correlation coefficients are:

D^0	$f_0 = 0.054$	$f_2 = 0.634$	$f_4 = 0.293$	$f_6 = 0.019$
	$\sigma_0 = 0.011$		$\sigma_4 = 0.023$	$\sigma_6 = 0.009$
	$C_{04} = 0.07$	$C_{46} = -0.46$	$C_{06} = 0$	
D^+	$f_1 = 0.384$	$f_3 = 0.541$	$f_5 = 0.075$	
	$\sigma_1 = 0.023$		$\sigma_5 = 0.015$	
	$C_{15} = -0.33$			

$\mathrm{D_s}$	$f_1 = 0.37$	$f_3 = 0.42$	$f_5 = 0.21$
	$\sigma_1 = 0.10$		$\sigma_5 = 0.11$
	$C_{15} = -0.02$		

where f_i is the probability that a D state decays into i charged prongs, σ_i its error and C_{ij} the correlation between f_i and f_j.

3.3 Charm lifetimes and branching ratios

All charm lifetimes and exclusive branching ratios are taken from the PDG [6]. Some of the results on $\mathrm{BR}(\mathrm{D}^+ \to \mathrm{K}^-\pi^+\pi^+)$ have been obtained by measuring the ratio $\frac{\mathrm{BR}(\mathrm{D}^+\to\mathrm{K}^-\pi^+\pi^+)}{\mathrm{BR}(\mathrm{D}^0\to\mathrm{K}^-\pi^+)}$. However it has been checked that the correlation between the two branching ratios introduced by these measurements is negligible.

For the inclusive semileptonic branching ratio $\mathrm{c} \to \ell$ no usable numbers exist. However as a cross check the individual semileptonic branching ratios of the different charmed hadrons have been folded with the production ratios described above. For the D^0 semileptonic branching ratio the PDG number has been averaged with two recent measurements from ARGUS [13] and CLEO [14] to be $(6.74 \pm 0.27)\%$ and the semileptonic widths of all D mesons have assumed to be equal. This procedure gives $\mathrm{Br}(\mathrm{c} \to \ell) = (9.1 \pm 0.5)\%$ well consistent with the used number of $(9.8 \pm 0.5)\%$. However to avoid correlation of $\mathrm{Br}(\mathrm{c} \to \ell)$ with the D production ratios and $\mathrm{R_c}$ this number will not be used further.

4 b Decay Multiplicities

There are two results in publication, first from OPAL then from DELPHI.

$$\begin{array}{rlllll} \langle N_\mathrm{b} \rangle = & 5.51 & \pm & 0.05 & \pm & 0.51 \quad \text{OPAL}, \\ \langle N_\mathrm{b} \rangle = & 5.84 & \pm & 0.04 & \pm & 0.38 \quad \text{DELPHI} \end{array}$$

where the first error is statistical and the second systematic.

Both results are corrected to include the decay tracks from **all** K^0s and Λs. A more recent paper from OPAL on b-multiplicities did not update the decay multiplicity analysis.

Access was granted to a detailed breakdown of the systematic errors in the OPAL case. In the DELPHI paper, several systematic errors are given without breaking down the exact sub-contributions. It is assumed that the OPAL breakdown is representative for common errors. The OPAL result, being earlier also takes considerably larger uncertainties in R_b and τ_b. Therefore the OPAL errors have been scaled from these sources down to the DELPHI levels, giving a reduction in the overall OPAL systematic error from 0.51 to 0.49.

The situation with fragmentation errors is confused, as rather different approaches have been taken. OPAL calculated the systematic error in the case of b-fragmentation by varying $\langle x_E \rangle_\mathrm{b}$ in the range 0.70 ± 0.02 using the Peterson fragmentation function. DELPHI, on the other hand, have used two JETSET versions, ARIADNE, and two different fragmentation functions in JETSET 7.4 to estimate the different fragmentation effects. The $\langle x_E \rangle_{\mathrm{b}}$ is available for each model. The systematic error estimation procedure adopted seems to be to find the result with each of the MC models, and use the RMS of the results obtained as the systematic. If one calculates the mean and RMS of the $\langle x_E \rangle_{\mathrm{b}}$ for the different Monte Carlo sets, the result is 0.70 ± 0.02, and so similar to the range considered by OPAL. The fragmentation systematic is therefore treated as common between the two. The charm fragmentation systematics are less obviously common between the two, but are treated as such; they contribute a relatively small amount to the error, so this approach is at worst conservative.

OPAL also varied the b decay multiplicity and the B species composition. It is not clear what, if any systematic error contribution is assumed from these sources by DELPHI, and so they are treated as unique to the OPAL measurement. The multiplicity variation is much the more significant of the two, and only contributes $\approx 20\%$ of the total systematic error, and so they will have little effect on the average and overall error.

The total correlated error is assumed to be 0.22. Performing a BLUE average, one obtains:

$$\langle N_{\mathrm{b}} \rangle = \quad 5.73 \quad \pm \quad 0.03 \quad \pm \quad 0.34 \quad (0.34)$$

where the first error is statistical, the second systematic and the third total, *i.e.* the weights are one-third OPAL two-thirds DELPHI.

Treating the DELPHI systematic as completely common to the two results, then the OPAL result only has a 1.5% weight, and we essentially recover the DELPHI result. Going to the other extreme and treating the two results as completely uncorrelated gives:

$$\langle N_{\mathrm{b}} \rangle = \quad 5.72 \quad \pm \quad 0.03 \quad \pm \quad 0.31 \quad (0.31)$$

Thus the proposed average is

$$5.73 \pm 0.35$$

It should again be stressed that these numbers **include** the decay tracks from **all** K^0 and Λs produced. (*i.e.* as if they all decay and are observed in the detector). Details of the DELPHI correction to obtain this value are not available, and so we take the numbers given by OPAL (which should be independent of experiment). This would imply that the correction for the b-multiplicity **excluding** all decay tracks from K^0 and Λs is -0.485 ± 0.065, implying a corrected b decay multiplicity of 5.25 ± 0.35.

5 Gluon splitting

The gluon splitting rate $\mathrm{g} \rightarrow \mathrm{c\bar{c}}$ has been measured by OPAL [15] to be $(2.38 \pm 0.48)\%$, consistent with QCD calculations [7]. No published measurement of $\mathrm{g} \rightarrow \mathrm{b\bar{b}}$ exists to date. However theoretically the rate is expected to be [7]

$$\frac{f(\mathrm{g} \rightarrow \mathrm{b\bar{b}})}{f(\mathrm{g} \rightarrow \mathrm{c\bar{c}})} = \frac{m_c^2}{m_b^2} = 0.13 \pm 0.04.$$

Thus it is suggested to vary first $\mathrm{g} \rightarrow \mathrm{c\bar{c}}$ according to the OPAL measurement and keep $f(\mathrm{g} \rightarrow \mathrm{b\bar{b}}) = 0.13\mathrm{f}(\mathrm{g} \rightarrow \mathrm{c\bar{c}})$ and then vary the ratio between $f(\mathrm{g} \rightarrow \mathrm{c\bar{c}})$ and $f(\mathrm{g} \rightarrow \mathrm{b\bar{b}})$.

6 Lepton spectra

No new measurements of the lepton spectra in b- and c-decays have been published . However preliminary measurements from CLEO are well within the range used up to now with smaller errors.

7 $b \to \tau^- \bar{\nu}_\tau X$ Branching Ratio

Input measurements:

ALEPH: $2.75 \pm 0.48\%$ [16].
L3 (i): $2.40 \pm 1.06\%$ [17].
L3 (ii): $1.70 \pm 1.21\%$ [18].
OPAL: $2.66 \pm 0.53\%$ [19].

A detailed error breakdown is given in Table 2 and a listing of input parameters in Table 3.

	ALEPH	L3 (i)	L3 (ii)	OPAL
Statistics	$\pm 0.30\%$	$\pm 0.81\%$	± 0.50	$\pm 0.11\%$
E_{miss} resolution	$\pm 0.13\%$	$\pm 0.30\%$	± 0.30	$\pm 0.17\%$
Energy scale	—	—	± 0.30	—
$e^\pm/\mu^\pm$ identification	$\pm 0.13\%$	$\pm 0.30\%$	—	$\pm 0.11\%$
$b\bar{b}$ tag	$\pm 0.05\%$	$\pm 0.10\%$	± 0.53	$\pm 0.08\%$
$\langle x_b \rangle$	$\mp 0.23\%$	—	∓ 0.30	$\mp 0.33\%$
$b \to l^- \bar{\nu}_l$X	$\mp 0.15\%$	$\mp 0.50\%$	∓ 0.79	$\mp 0.11\%$
$b, \bar{c} \to l^- \bar{\nu}$X	$\mp 0.07\%$	—	(included)	$\mp 0.12\%$
$D_s^- \to \tau^- \bar{\nu}_\tau$	$\mp 0.11\%$	$\mp 0.20\%$	(included)	$\pm 0.28\%$
Other correlated	$\pm 0.09\%$	—	—	$\pm 0.05\%$

Table 2: Error breakdown

	ALEPH	L3 (i)	L3 (ii)	OPAL
$\langle x_b \rangle$	0.714 ± 0.012	—	0.720 ± 0.014	$0.695 \pm 0.007\%$
$b \to l^- \bar{\nu}_l$X	$11.4 \pm 0.5\%$	$11.0 \pm 0.5\%$	10.85 ± 0.49	$11.08 \pm 0.31\%$
$b, \bar{c} \to l^- \bar{\nu}$X	$8.2 \pm 1.2\%$	—	—	$8.22 \pm 0.46\%$
$D_s^- \to \tau^- \bar{\nu}_\tau$	$3.7 \pm 2.3\%$	$3.7 \pm 2.3\%$	—	$5.7 \pm 2.3\%$

Table 3: Input parameters

Notes

$\langle x_b \rangle$	The value used by ALEPH refers to the primary B hadron. It corresponds to ≈ 0.70 for the weakly decaying one. ALEPH makes a larger variation in $\langle x_b \rangle$ than OPAL to allow for uncertainty in the shape of the fragmentation function. L3 (i) quotes no systematic for $\langle x_b \rangle$. A value of $\langle x_b \rangle = 0.702 \pm 0.008$ will be used as discussed in section 8.
$b \to l^- \bar{\nu}_l \mathrm{X}$	A value of $11.2 \pm 0.4\%$ [20] will be used. (N.B. Results from CLEO/ARGUS [20] and L3 [18] suggest a lower value).
$b, \bar{c} \to l^- \bar{\nu} \mathrm{X}$	A value of $8.2 \pm 1.2\%$ [20] will be used.
$D_s^- \to \tau^- \bar{\nu}_\tau$	A value of $5.7 \pm 2.3\%$ [20] will be used.

Corrected Inputs

OPAL will not be used as it is preliminary. Correcting ALEPH and L3 gives:

ALEPH (corrected): $2.69 \pm 0.43\%$.
L3 (i) (corrected): $2.03 \pm 1.02\%$.
L3 (ii) (corrected): $1.52 \pm 1.08\%$.

The errors are unchanged except for:

Table 4: Corrected error breakdown

	ALEPH	L3 (i)	L3 (ii)
$\langle x_b \rangle$	$\mp 0.15\%$	—	∓ 0.17
$b \to l^- \bar{\nu}_l \mathrm{X}$	$\mp 0.12\%$	$\mp 0.40\%$	∓ 0.64

Result

ALEPH + L3 combined: $2.57 \pm 0.42\%$

Multiplying by the τ leptonic branching ratio gives

$$Br(\mathrm{b} \to \tau \to \ell) = (0.452 \pm 0.074)\%$$

8 Determination of the mean energy of bottom and charm hadrons

The mean scaled energy of bottom and charm hadrons has been measured at LEP on the peak of the Z resonance with a number of different methods. In this section, the individual measurements are summarised, the procedure to average the results is introduced, and the results are presented.

8.1 $\langle x_B \rangle$ Measurements

The mean scaled energy of bottom hadrons, $\langle x_B \rangle$ has been measured by ALEPH [21, 22], DELPHI [23] and OPAL [24, 25]. Techniques used may be divided roughly into two classes; those which use leptons as a b tag and as a means to reconstruct the bottom hadron energy, and more inclusive methods. The exact definition of the mean scaled energy, $\langle x_B \rangle = E_{\mathrm{hadron}}/E_{\mathrm{beam}}$ varies between the different collaborations. Older analyses often quote the energy of the B hadron at the primary level, that is, the energy of the first B hadron produced in the fragmentation chain. More recent analyses prefer to give the energy of the weakly decaying bottom hadron. Both are different, because higher excited B states like $\mathbf{B}^*$ or $\mathbf{B}^{**}$, which subsequently decay into B hadrons, tend to soften the energy spectrum of the weakly decaying hadron. To obtain the mean scaled energy of the primary hadron Monte Carlo simulation is used to correct for the influence of these decays, with an appropriate systematic error introduced from this correction. The typical difference between both definitions is -0.015 for the $\mathbf{B}^{**}$ decays, and -0.005 for the $\mathbf{B}^*$ decays.

The mean scaled energy is extracted from the measured spectra by fitting a fragmentation function to the data. The functional form most often used is the one proposed by Peterson et.al. [26], which has one free parameter ϵ_b. This function is defined in the non-observable variable $z = (E+p)_{\|,\mathrm{hadron}}/(E+p)_{\|,\mathrm{quark}}$. Monte Carlo is used to translate z into the experimentally accessible variable x. Because of this the actual value and meaning of the parameter ϵ_b depends on the Monte Carlo used to do this correction, and is therefore a strongly model dependent quantity. The derivation of the mean scaled energy from this function however is much less sensitive to these modelling issues.

For the purpose of calculating a LEP average all measurements which are used have been corrected to the level of the weakly decaying bottom hadron. Only measurements are used where these corrections are more or less explicitly given in the relevant publication. Some older results have been dropped from the average, with however negligible loss of precision of the final result.

The measurements used in calculating the average are listed in table 5, together with a breakdown of the systematic errors. Combining these measurements taken correlations into account gives

$$\langle x_b \rangle = 0.702 \pm 0.008 \tag{1}$$

for the weakly decaying bottom hadron at LEP. The error quoted contains both statistical and systematic uncertainties. The largest uncertainty comes from the modelling of the shape of the fragmentation function, both due to excited states, and due to the fragmentation function used.

8.2 $\langle x_c \rangle$ Measurements

The mean scaled energy of charmed hadrons has been measured at LEP with leptons, D mesons and $\mathbf{D}^{*+}$ mesons. Each of these analyses is sensitive to a slightly different mixture of charm hadrons, and results in a measurement of a mean scaled energy for a particular mixture of charmed hadron.

Most closely related to the actual weakly decaying charmed hadrons are results obtained using a lepton tag, and results from the inclusive reconstruction of $\mathbf{D}^0$ and $\mathbf{D}^{\pm}$ mesons. The main difference between these will be, that no baryons contribute to the inclusive D measurement, while they do contribute to the lepton sample. The $\mathbf{D}^{*+}$ mesons on the other hand do not correspond to the energy of the weakly decaying hadrons.

	ALEPH		DELPHI	OPAL	
	[21]	[22]	[23]	[24]	[25]
	inclusive	lepton	lepton	inclusive	lepton
mean	0.695	0.709	0.7020	0.695	0.692
(pub)	(0.715)	(0.714)	(0.7020)	(0.695)	(0.697)
correction	$\mathbf{B}^{**}, \mathbf{B}^*$	$\mathbf{B}^*$	–	–	$\mathbf{B}^*$
stat	0.007	0.004	0.0044	0.006	0.006
sys					
internal	0.003	0.004	0.0022	0.003	0.002
frag func	0.006		0.0053	0.007	0.009
$\mathbf{B}^{**}$	0.004			0.006	
$\mathbf{D}^{**}$	0.010			0.013	
$\mathbf{b} \rightarrow \mathbf{l}$ dcy mod	n/a	0.010	0.0037	n/a	0.003
c dcy mod	0.001		0.0023	0.002	

Table 5: Table of published measurements of the mean scaled energy of bottom hadrons at LEP. The mean value shown has been corrected to the weakly decaying hadron level. The main result in the original publication is shown in brackets, and the corrections applied are indicated. The systematic error is broken into an internal component, which is assumed to be uncorrelated between analyses, and correlated sources.

A further complication arises from the fact that a non-negligible fraction of the charmed hadron are produced in the gluon jets, by the splitting of a gluon into a pair of charm quarks [15, 27]. In this note such events are explicitly removed from the sample of weakly decaying hadrons, and all analyses are corrected to this level of no charm produced in gluon splitting events.

The proposed mean scaled energy of charmed hadrons at LEP is obtained in a two step procedure. Firstly, the two types of results, the $\mathbf{D}^{*+}$ on the one side, D mesons and leptons on the other side, are individually combined to arrive at two averages. The number from the analysis of the $\mathbf{D}^{*+}$ mesons is then corrected to the weakly decaying level, and combined with the other result. It should be stressed that this later step is somewhat dependent on the Monte Carlo used to calculate the necessary corrections, and might need to be redone for a different Monte Carlo tune.

The non-$\mathbf{D}^{*+}$ results are listed in table 6. A small correction of $+0.002$ has been applied to the $\mathbf{D}^0$ and $\mathbf{D}^\pm$ results, to account for the production of other than $\mathbf{D}^0$ and $\mathbf{D}^\pm$ mesons. Assuming a common modelling error of ± 0.007 for all measurement they result in a mean scaled energy of

$$\langle x_{\mathbf{l},\mathbf{D}^0,\mathbf{D}^\pm} \rangle = 0.487 \pm 0.006 \pm 0.008 \tag{2}$$

The $\mathbf{D}^{*+}$ based results are listed in table 7. Again using a common modelling error of ± 0.007 the following average has been found:

$$\langle x_{\mathbf{D}^{*+}} \rangle = 0.510 \pm 0.005 \pm 0.008. \tag{3}$$

The translation of $\langle x_{\mathbf{D}^{*+}} \rangle$ into a mean scaled energy of weakly decaying charmed hadrons depends on the relative production of $\mathbf{D}^{*+}$ to $\mathbf{D}^0, \mathbf{D}^\pm$ mesons, and the fraction of $\mathbf{D}^{*+}$ mesons produced in $\mathbf{D}^{**}$ decays. Assuming that the later is $30 \pm 10\%$, and that the ratio $N_{\mathbf{D}^{*+}}/(N_{\mathbf{D}^0} + N_{\mathbf{D}^{*+}}) = 0.75$ for mesons not produced in the decays of excited charmed hadrons, the correction amounts to -0.029.

Combining both numbers the mean scaled energy of the weakly decaying charmed hadrons is found to be:

$$\langle x_{\mathbf{c}} \rangle = 0.484 \pm 0.008. \tag{4}$$

	ALEPH	OPAL	
	[22]	[28]	[28]
	lepton	$\mathbf{D^0}$	$\mathbf{D^\pm}$
published	0.487	0.489	0.481
correction	0.000	+0.002	0.002
used	0.487	0.491	0.483
stat	0.008	0.008	0.013
sys	0.008	$^{+0.011}_{-0.009}$	$^{+0.007}_{-0.011}$

Table 6: Table of measurements of the mean scaled energy for charmed hadron other than $\mathbf{D^{*+}}$ mesons. Shown are the experiment, the method, and the result with its statistical and systematic errors.

	ALEPH	DELPHI	OPAL
	[29]	[30]	[27]
published	0.495	0.487	0.515
correction	+0.007	+0.007	0.000
used	0.502	0.494	0.515
stat	0.011	0.015	$^{+0.008}_{-0.005}$
sys			
internal	0.001	0.005	0.007
modelling	0.007	(0.007)	0.007

Table 7: Table of published measurements of the mean scaled energy of $\mathbf{D^{*+}}$ mesons. The means shown have been corrected for the production of charm in gluon jets. A common modelling error of $\pm\mathbf{0.007}$ has been assigned to all measurements. If no such error was included in the publication, it is put into brackets in the table.

8.2.1 $\langle x_{B\to D}\rangle$ for D mesons in B decays

Most charm production measurements are sensitive to the energy spectra of charm hadrons originating from B decays. Using the $\langle x_b\rangle$ given above one obtains $\langle x_{b\to D}\rangle$ from the CLEO measurements [31] of the energy spectrum of D mesons in the B rest frame. Following the Ansatz proposed in [1] a Peterson like function with $\epsilon = 0.42 \pm 0.07$ is used to parameterise the CLEO energy spectra of D^{*+} mesons in the rest frame of B mesons.

9 $\mathrm{R_b}$ hemisphere correlations

There are basically three sources that give rise to hemisphere correlations in the precise $\mathrm{R_b}$ double tag measurements:

- detector inhomogeneities,
- the use of a common primary vertex,
- QCD effects,

where the different sources influence each other. The systematic uncertainties coming from detector inhomogeneities should not be correlated amongst the experiments and are generally well under control. On the contrary QCD effects should be fully correlated between the experiments.

For most experiments the b-tagging efficiency is roughly proportional to the b-momentum. In this case the correlation can be written as

$$c = \frac{< x_1 x_2 >}{< x >^2} - 1$$

where x_1, x_2 is the scaled momentum of the two B-hadrons. Analytic $O(\alpha)$ QCD calculations give effects of about 1.4% [32]. On the parton level all fragmentation models agree to the 0.2% level with this number. On the hadron level all models agree as well apart from Herwig which gives up to 0.8% larger numbers. Clearly the central value for the hemisphere correlation has to be evaluated with a full detector simulation. All experiments have only large samples of the JETSET parton shower available for this task so that this model will be used for the central value. It is suggested that for the error estimate the experiments evaluate the correlation with JETSET and HERWIG in a simplified model and take half the difference as systematic uncertainty.

Motivated by a comparison of matrix element and parton shower models it is proposed to vary in addition the fraction of events with the two B-hadrons in the same hemisphere by 30%.

In the case where analyses are done using a common primary vertex this usually gives rise to the largest sources of correlation. A large part of this is due to the inclusion of tracks from B-decays into the primary vertex making R_b dependent of B production and decay properties. To estimate the uncertainties correlated between experiments it thus seems appropriate to quote the uncertainty in R_b due to

- B-decay multiplicity,
- B fragmentation,
- B lifetimes.

To account for different efficiencies to different B species it was agreed to inflate the error to $\tau_b = 1.55 \pm 0.05 ps$.

10 QCD corrections to the forward-backward asymmetries

Two detailed notes about the QCD corrections to the forward-backward asymmetries can be found in the appendix. The conclusions of these studies are:

The precision of the theoretical calculations and the uncertainty on α_s introduces a relative error of 0.25% on the asymmetries.

The Lund parton shower Monte Carlo reproduces the correction factors exactly for b quarks and to 13% accuracy for c quarks.

The experimental bias has to be studied by the experiments themselves. For this study the Lund Monte Carlo can be used. The remaining common systematic uncertainty on $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ ($A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$) is 0.0003 (0.0005), completely negligible for the current experimental precision.

In recent studies, it appeared that the experimental event selection and signal extraction can introduce important bias to the QCD corrections. In the case of the DELPHI measurement using lepton, the QCD correction to the b (c) quark asymmetry has been estimated to be reduced by 50% (70%) from its theoretical expectation. For the moment not all LEP analysis have estimated the effect of this bias on their measured asymmetry. For this reason, whenever an evaluation of the effect is not available, it is proposed to correct the measurements using the DELPHI estimation with inflated error to take into account possible variation in the experimental bias. In this case the observed asymmetry has to be divided by 0.984 ± 0.008 for the b and 0.99 ± 0.01 for the c asymmetries.

11 Summary

Table 8 summarises the suggested input parameters for the 1996 summer conferences.

Error Source	Proposed Range	Uncertainty
Internal experimental effects	...	...
$\langle x_E(\mathrm{b})\rangle$	Fitted or 0.702 ± 0.008	
$\langle x_E(\mathrm{c})\rangle$	Fitted or 0.484 ± 0.008	
Choice of b fragmentation function	See [3]	
Choice of c fragmentation function	See [3]	
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	$(1.3 \pm 0.5)\%$	
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	$(0.452 \pm 0.074)\%$	
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	$(0.07 \pm 0.02)\%$	
$\mathrm{Br}(\mathrm{c} \to \ell)$	$(9.8 \pm 0.5)\%$	
Semilept. model $\mathrm{b} \to \ell$ [1]	ACCMM $\left({}^{+\mathrm{ISGW}}_{-\mathrm{ISGW**}}\right)$	
Semilept. model $\mathrm{c} \to \ell$ [1]	ACCMM1 $\left({}^{+\mathrm{ACCMM2}}_{-\mathrm{ACCMM3}}\right)$	
$\mathrm{b} \to \mathrm{D}$ model. [1]	Peterson $\epsilon = 0.42 \pm 0.07$	
D^0 lifetime	0.415 ± 0.004 ps	
D^+ lifetime	1.057 ± 0.015 ps	
$\mathrm{D_s}$ lifetime	0.467 ± 0.017 ps	
Λ_c lifetime	0.206 ± 0.012 ps	
B lifetime	1.55 ± 0.05 ps	
B decay multiplicity	$5.73 \pm 0.35^{1)}$	
D decay multiplicity	see text	
$\mathrm{BR}(\mathrm{D} \to \mathrm{K^0_s})$	$0.46 \pm 0.06^{2)}$	
$\mathrm{g} \to \mathrm{c\overline{c}}$ per multi-hadron	$(2.38 \pm 0.48)\%^{3)}$	
$\mathrm{g} \to \mathrm{b\overline{b}}$ per multi-hadron	$(0.13 \pm 0.04) \times (\mathrm{g} \to \mathrm{c\overline{c}})$	
Rate of long-lived light hadrons	Tuned JETSET$\pm$10% (See [3])	
Light quark fragmentation	See [3]	
QCD hemisphere correlations	see text	
D^+ fraction in $\mathrm{c\overline{c}}$ events	0.233 ± 0.028	
$\mathrm{D_s}$ fraction in $\mathrm{c\overline{c}}$ events	0.102 ± 0.037	
c-baryon fraction in $\mathrm{c\overline{c}}$ events	0.065 ± 0.029	
$\mathrm{Br}(\mathrm{c} \to \mathrm{D}^* \to \mathrm{D}^0\pi \to \mathrm{K}\pi\pi)$	$(0.68 \pm 0.05)\%$	

Table 8: Template table of systematic errors. (Quantities that are fitted do not also give rise to a systematic error in the same analysis.)
[1)] This number includes K^0_s and Λ decays.
[2)] The error is the squared sum over the different charmed hadrons.
[3)] The gluon splitting rate into $\mathrm{b\overline{b}}$ is kept to $f(\mathrm{g} \to \mathrm{b\overline{b}}) = 0.13\mathrm{f}(\mathrm{g} \to \mathrm{c\overline{c}})$.
The D production ratios below the line are fitted parameters. The numbers given here are suggestions how to treat the errors in the experimental notes

A QCD corrections to $A_{\rm FB}^{\rm b\overline{b}}$and $A_{\rm FB}^{\rm c\overline{c}}$

In this note we present a few studies on the QCD corrections to the heavy flavour asymmetries.

We will summarised here the different QCD calculations published on the subject and update their results when needed. Then the results of the second order QCD calculations will be compared with what is obtained using the LUND Monte Carlo. Finally a comparison between the LUND results and the first order QCD calculation will be done for the variation of the QCD corrections in function of a cut on the angular acceptance.

In all this note, the direction considered for the QCD corrections is the quark direction and we used $\alpha_s(M_Z) = 0.123 \pm 0.005$.

A.1 First order QCD corrections including mass corrections

We introduce the first order QCD corrections in the form :

$$A_{\rm FB}^{\rm q\overline{q}} = Cor_{q_1} A_{\rm FB,0}^{\rm q\overline{q}} = \left(1 - \tfrac{\alpha_s}{\pi} C1\right) A_{\rm FB,0}^{\rm q\overline{q}}$$

With $C1 = 1 - F1 + F2$

A.1.1 Full theta range

The correct expression for $F1$ is [33]

$$F1 = \mu\left(x_1 + \frac{2}{3}\mu\left(x_2 + \mu\left(-x_3 + \mu x_4\right)\right)\right)$$

with

$$\begin{aligned}
\mu &= 2\tfrac{m_q}{M_Z} \\
x_1 &= \frac{8}{3} \\
x_2 &= \frac{7}{2} + \frac{\pi^2}{12} - x_r + \frac{x_r^2}{2} \\
x_3 &= \frac{457}{72} \\
x_4 &= \frac{\pi^2}{24} - \frac{1}{16} - \frac{11}{8}x_r + \frac{x_r^2}{4} \\
x_r &= \log\left(\frac{\mu}{2}\right)
\end{aligned}$$

The expression for $F2$ is

$$F2 = 3\mu^2 \frac{v_q^2 + a_q^2 \log\left(\frac{4}{\mu^2}\right)}{v_q^2 + a_q^2}$$

μ	$F1$	$F2$	$C1$	$C1$ from [35]
b quark				
$2\frac{4.5}{MZ}$	.3366	.1283	0.79	0.80
$2\frac{3}{MZ}$	.2135	.0641	0.85	0.86
c quark				
$2\frac{1.5}{MZ}$	.0998	.0191	0.92	0.93
$2\frac{.7}{MZ}$	.0442	.0049	0.96	0.96

Table 9: Value of the coefficients used in the first order QCD calculations obtained with the expression given in the text. For each quark, the mass used on the first line correspond to the pole mass of the quark, the second line to the mass of the quark in the $\overline{MS}$ scheme at the scale M_Z . The last column contains the $C1_q$ coefficient given in [35].

$\|\cos(\theta)\| <$	Cor_{b_1}	$\frac{C1_b}{C1_b nocut}$	Cor_{c_1}	$\frac{C1_c}{C1_c nocut}$
1.	0.969	1.	0.965	1.
0.9	0.966	1.11	0.961	1.10
0.8	0.962	1.23	0.958	1.19
0.7	0.959	1.33	0.954	1.29
0.6	0.956	1.43	0.952	1.37

Table 10: QCD corrections and relative change of the QCD corrections for different acceptance cut on the angle of the quark at the end of the fragmentation. For each quark the numbers given correspond to the average of the results obtained with the two mass hypothesis considered in table 9.

These expressions end up to the results summarised table 9 for the b and c quarks. To take the expression of $F1$ from [33] instead of the one quoted in [34] gives a small difference on $F1$, this end up for $\mu = 2\frac{4.5}{MZ}$ to a value of $C1 = 0.7917$ instead of 0.8076. In the rest of the note, the numbers quoted in [35] will be used (they are supposed to use the same $F1$ than in [33]) except for the next section where the analytical expression of $F1$ and $F2$,given here, will be used.

A.1.2 First order QCD corrections with an angular cut

Using [34] and the expression of $F1$ from [33] an expression of $C1$ in function of a cut in the theta acceptance can be computed. The QCD corrections obtained are presented table 10.

It should be noticed that this change of the QCD corrections in function of the acceptance, is coming from the way the asymmetries are in general extracted from the data. In most/all the data analysis the fit to the theta distribution to extract the asymmetry is based on the hypothesis that the shape of the differential production cross section of $e^+e^- \rightarrow Z \rightarrow b\bar{b}$ and $e^+e^- \rightarrow Z \rightarrow c\bar{c}$ is of the form :

$$\frac{d\sigma}{d\cos\theta} = \frac{3}{8}(1+\cos^2\theta)\sigma_u + \frac{3}{4}\sin^2\theta\sigma_l + \frac{3}{4}\cos\theta\sigma_f$$

with

$$\begin{aligned} \sigma_l &= 0 \\ \sigma_{tot} &= \sigma_u + \sigma_l = \sigma_u \\ A_{\mathrm{FB}}^{\mathrm{q\bar{q}}} &= \frac{3}{4}\frac{\sigma_f}{\sigma_{tot}} \end{aligned}$$

But once the QCD corrections are introduced, we cannot consider any more σ_l as negligible. So the dependence in function of θ of the differential cross section is of the form :

$$\frac{d\sigma}{d\cos\theta} = \sigma_{tot}\frac{3}{8}\frac{1}{b_q}\left(1 + a_q\cos^2\theta + \frac{8}{3}b_q A_{\rm FB}^{\rm q\overline{q}}\cos\theta\right)$$

and the dependence in function of θ of the asymmetries is :

$$A_{\rm FB}^{\rm q\overline{q}}(\theta) = \frac{8}{3}\frac{b_q\cos\theta}{1 + a_q\cos^2\theta}A_{\rm FB}^{\rm q\overline{q}}$$

with $a_b \sim a_c \sim 0.905$ and $b_b \sim b_c \sim 0.976$. So these a_q and b_q introduce some extra corrections in function of the acceptance to the one needed for $A_{\rm FB}^{\rm q\overline{q}}$.

Remark :

The expression function of theta calculated from [34], if used for the full theta range, presents a small difference with the expression quoted in the same [34] for the full theta range. Anyway the discrepancy is extremely small, it seems that a term in μ^2 is missing in the expression of $F2$ for the full theta range. More investigation on this point will be done.

A.2 Second order QCD corrections

We introduce the second order QCD corrections in the form :

$$A_{\rm FB}^{\rm q\overline{q}} = Cor_{q_2}A_{\rm FB,0}^{\rm q\overline{q}} = \left(1 - \tfrac{\alpha_s(M_Z)}{\pi}C1 + \tfrac{\alpha_s^2}{\pi}C2\right)A_{\rm FB,0}^{\rm q\overline{q}}$$

The expression for $C2$, the second order QCD corrections, is given in [36] and is the sum of different term. In the following section we will comment the different contributions to the second order QCD corrections and introduce the measured top mass from D0 and CDF when needed.

A.2.1 2 jets

The so called 2-jets corrections has the form

$$F_{QCD}^{2-jet} = 1 + \tfrac{\alpha_s^2}{\pi}0.058 I_2(z) = 1 + \tfrac{\alpha_s^2}{\pi}0.638$$

With $I_2(z) = 7.21 - 1.481z - 1.363z^2 - 6\log(2\sqrt{z})$ and $z = \left(\frac{M_Z}{2m_t}\right)^2$

for $m_t = 175 \pm 9$ GeV and $M_Z = 91.1884$ GeV,

$I_2(z) = 11.0 \pm 0.3$

in [36] $m_t = 150 \pm 50$ was used giving $I_2(z) = 10^{+1.8}_{-2.6}$

remark :

$0.058 = -\frac{1}{6}\frac{v_b^2 - a_b^2}{v_b^2 + a_b^2}$ for $\sin^2\theta_w = 0.23$

A.2.2 3 jets

The so called 3-jets corrections has the form

$$F_{QCD}^{3-jet} = 1 + \frac{\alpha_s^2}{\pi} 0.23$$

There is a small top contribution here also, but it's small and there is no need to change the bound considered in the original paper.

A.2.3 4 Jets

The production of 2 gluons or 2 quarks during the fragmentation enter in this category.

Contribution from $e^+e^- \to b\bar{b}q\bar{q}$ where $b\bar{b}$ are radiated from a virtual gluon

$$F_{QCD}^{B} = 1 - \frac{\alpha_s^2}{\pi}(3.5 \pm 0.7) fcut_b$$

If such contribution doesn't enter directly in the analysis : $fcut_b = 0$

Other so called D contribution

$$\begin{aligned} F_{QCD}^{D} &= 1 - \frac{3}{4} C_F \frac{\alpha_s(M_Z)}{\pi} + \frac{1}{4} C_F \frac{\alpha_s^2}{\pi} \left(\frac{9}{4} C_F + CC_F + NN_C + TT_R \right) \\ &= 1 - \frac{\alpha_s(M_Z)}{\pi} - (2.6 \pm 0.4) \frac{\alpha_s^2}{\pi} \end{aligned}$$

With

C_F	$=$	$\frac{4}{3}$	contribution from $e^+e^- \to b\bar{b}g$
CC_F	$=$	5.8 ± 0.7	contribution from $e^+e^- \to b\bar{b}gg$
NN_C	$=$	-31.0 ± 0.8	contribution from $e^+e^- \to b\bar{b}gg$
TT_R	$=$	14.2 ± 0.1	contribution from $e^+e^- \to b\bar{b}q\bar{q}$ and $e^+e^- \to b\bar{b}b\bar{b}$

So called F contribution

This term has contribution from $e^+e^- \to b\bar{b}q\bar{q}$ and $e^+e^- \to b\bar{b}b\bar{b}$

$$F_{QCD}^{F} = 1 - \frac{\alpha_s^2}{\pi} 0.13$$

A.2.4 Second order QCD corrections

We have :

$$\begin{array}{lllllll} \mathrm{C2} & = & C_{F^{2-jet}_{QCD}} & + C_{F^{3-jet}_{QCD}} & + C_{F^{B}_{QCD}} & + C_{F^{D}_{QCD}} & + C_{F^{F}_{QCD}} \\ & = & 0.638 & + 0.23 & - (3.5 \pm 0.7)\ fcut_b & - (\ 2.6 \pm 0.4) & - 0.13 \end{array}$$

for $fcut_b = 0$ then we are more or less left with the term coming from F^{D}_{QCD} which appear to be small due to cancellation between the corrections from $e^+e^- \rightarrow b\bar{b}gg$ on one side and $e^+e^- \rightarrow b\bar{b}q\bar{q}$, $e^+e^- \rightarrow b\bar{b}b\bar{b}$ on the other.

Remark : for the selected data events the weight of the different kind of contribution could suppress this cancellation. (reduced weight for b bar(b) b bar(b) events ...)

Finally,if we consider $b\bar{b}$ only from $e^+e^- \rightarrow Z \rightarrow b\bar{b}$ ($fcut_b = 0$), we get

$$Cor_{b_2} = 1 - C1_b \frac{\alpha_s(M_Z)}{\pi} + \frac{\alpha_s^2}{\pi}(-(1.9 \pm 0.4))$$

in the same way we get

$$Cor_{c_2} = 1 - C1_c \frac{\alpha_s(M_Z)}{\pi} + \frac{\alpha_s^2}{\pi}(-(3.2 \pm 0.4) - (26 \pm 6)\, fcut_c - 0.123 I_2(z))$$

And here also if we consider only $c\bar{c}$ from Z ($fcut_c = 0$)

$$Cor_{c_2} = 1 - C1_c \frac{\alpha_s(M_Z)}{\pi} + \frac{\alpha_s^2}{\pi}(-(4.6 \pm 0.4))$$

A.3 Final result from theoretical calculations

For

$$A_{\mathrm{FB}}^{\mathrm{q\bar{q}}} = \left(1 - \frac{\alpha_s(M_Z)}{\pi} C1 + \frac{\alpha_s^2}{\pi} C2\right) A_{\mathrm{FB},0}^{\mathrm{q\bar{q}}} = (1 + C_q)\, A_{\mathrm{FB},0}^{\mathrm{q\bar{q}}} = Cor_{q_2} A_{\mathrm{FB},0}^{\mathrm{q\bar{q}}}$$

We obtain

- for the $b\bar{b}$: $C_b = -0.035 \pm 0.002$ and $Cor_{b_2} = 0.965 \pm 0.0021$
- for the $c\bar{c}$: $C_c = -0.044 \pm 0.003$ and $Cor_{c_2} = 0.956 \pm 0.0029$

detail on the different contribution to the error can be found table 11

	C_b	C_c
first order corrections	-0.0325	-0.0370
mass effect on the first order term error from $\alpha_s(M_Z) = 0.123 \pm 0.005$	$\pm$ 0.0012 $\pm$ 0.0013	$\pm$ 0.0006 $\pm$ 0.0015
second order corrections	-0.0029	-0.0071
theoretical error error on the scale $\alpha_s = (.123 \pm 0.02)$	$\pm$ 0.0006 $\pm$ 0.0010	$\pm$ 0.0006 $\pm$ 0.0023
Total	- 0.035 $\pm$ 0.002	- 0.044 $\pm$ 0.003

Table 11: Corrections to the asymmetries and errors from the second order QCD calculations.

A.4 Comparison with LUND

A.4.1 Comparison with the second order QCD calculation

In the LUND parton shower Monte Carlo, the QCD effect on the quark direction can be quite easily factorised, we obtain the following expression on the corrections to the asymmetries :

$$Cor_q^f = \frac{\sum_f \frac{\cos(\theta_q^i)}{1+\cos(\theta_q^i)^2}}{\sum_f \frac{\cos(\theta_q^f)}{1+\cos(\theta_q^f)^2}}$$

Where :

- θ_q^i = initial quark direction
- θ_q^f = final quark direction at the end of the shower development.
- $\sum_f$ = sum over the same sample of events, contained in a given θ_q^f bin.

For technical reason and to have a better estimation on the error coming from the Monte Carlo statistic, we have computed

$$C_q^f = \frac{\sum_f \left(\frac{\cos(\theta_q^i)}{1+\cos(\theta_q^i)^2} - \frac{\cos(\theta_q^f)}{1+\cos(\theta_q^f)^2}\right)}{\sum_f \frac{\cos(\theta_q^f)}{1+\cos(\theta_q^f)^2}}$$

To have such expression, it should be noticed that $\sum_f$ has to be done in such way that the asymmetry can be factorised. This mean that it should be done in the same way than the one corresponding to an asymmetry measurement. For example one can oblige $\cos(\theta_q^f)$ to be always positive , flipping its sign and the sign of $\cos(\theta_q^i)$ when it's not the case. Doing this , $\sum_f$ can be done on any data subsample.

Using 10^6 hadronic Z decays simulated using LUND with the DELPHI tuning, we got for the full theta acceptance :

		LUND	QCD second order	LUND/QCD
C_b	=	-0.0357 $\pm$ 0.0006	- 0.035 $\pm$ 0.002	1.02 $\pm$ 0.06
C_c	=	-0.0496 $\pm$ 0.0008	- 0.044 $\pm$ 0.003	1.13 $\pm$ 0.08
$\frac{C_b}{C_c}$	=	0.720 $\pm$ 0.0168	0.795 $\pm$ 0.009	0.906 $\pm$ 0.024

$\vert\cos(\theta)\vert <$	$\frac{C_b}{C_b nocut}$	$\frac{C_c}{C_c nocut}$
1.	1.	1.
0.9	1.12	1.12
0.8	1.25	1.24
0.7	1.37	1.36
0.6	1.49	1.47

Table 12: Relative change of the QCD corrections for different acceptance cut on the angle of the quark at the end of the fragmentation estimated from the LUND Monte Carlo.

The QCD corrections to the asymmetries obtained for the b and c quark with the LUND Monte Carlo are in good agreement with the QCD second order calculations. Nevertheless the quite lower value of $\frac{C_b}{C_c}$ in LUND compared to the second order QCD corrections is a bit surprising if we remark that the errors on the second order QCD calculations are more or less 100% correlated.

To interpret this discrepancy as a contribution to mass corrections from the second order QCD corrections (neglected here in the theoretical calculations even at the level of the errors), is not justified as the agreement for the b is better than for the c. As the b is in complete agreement between the calculation and the Monte Carlo and the charm shows a 1.5 sigma effect, the ratio being fixed in the second order QCD calculation, we may suspect the charm to be at the origin of this discrepancy.

A.4.2 Angular cut effect in LUND

The variation of the QCD corrections in function of a cut on the angular acceptance has been estimated from LUND. The results obtained are presented table 12 and are in good agreement with the first order calculation (10% level) if we compare to table 10, but here the c behaviour seems to be exactly the same than the b one which is not the case in the first order QCD calculation.

A.5 Results using the full DELPHI simulation

Using DELPHI Monte Carlo sample an estimation of the QCD correction has been done on one of the DELPHI analysis (binned chi2 fit of the b and c quark asymmetries using a lepton tag). The method used to extract the QCD correction is the following : for a given set of events (a bin in $\cos(\theta_{thrust})$)) the θ weight $\sum \frac{\cos(\theta_{thrust})}{1+\cos^2(\theta_{thrust})}$ has been replaced in the fit procedure by $\sum \frac{\cos(\theta_{initial_q})}{1+\cos^2(\theta_{initial_q})}$. The changes on the asymmetries end up to

$$C_b = -0.0158 +/- 0.0015(MCstat) +/- 0.0010(Sys) +/- 0.0013(Theo.)$$

$$C_c = -0.007 +/- 0.004(MCstat + Sys)$$

The statistical error is measured to be below 0.0010 when only μ from b + cascade are taken into account (no weight in function of p,p_t). To have an estimation at the level of the full fit of this error the μ MC sample has been split in 4 . Then to have only one value at 2 sigma and the 3 other bellow 1 sigma an error of 0.003 had to be taken per sample . So we estimate to $\frac{0.003}{\sqrt{4}} = 0.0015$ our statistical error on the full fit. The error from the DELPHI electron sample is estimated to be a factor 2 bigger and doesn't contribute much to improve this number.

The systematic is estimated by

- fitting $A_{\rm FB}^{\rm b\overline{b}}$with $A_{\rm FB}^{\rm c\overline{c}}$or $A_{\rm FB}^{\rm b\overline{b}}$alone (with $A_{\rm FB}^{\rm c\overline{c}}$= λ $A_{\rm FB}^{\rm b\overline{b}}$, with λ taken from Zfitter)

- using theta of the initial quark not only for the lepton from b , b cascade and c but also for any track from b or c (background).

- the theoretical systematic on this number is $\frac{C_b used}{C_b theory}.0025 = 0.0013$, 0.0025 being the theoretical error on $C_b theory$ (see annex B).

It's clear that some more studies are needed but at this point , considering that the theoretical error on this number is close to be dominant we can be satisfy with such accuracy.

Part of the reduction of the QCD correction can be attributed to the cut in momentum on the lepton sample and the small weight in the fit of prompt lepton coming from b or c in the low momentum region. Such effect can be seen on the figures 1and 2 where the QCD correction are computed for μ and e from prompt b decay in function of their momentum. For p greater than 10-12 GeV the QCD correction is found to be compatible with 0 and at 3-4 GeV it's around -.038 in the case of prompt μ from b decay. This momentum dependence of the QCD correction can be attributed to the fact that only events with hard gluon emission have an effect on $A_{\rm FB}$. It can be noticed that only 1-2 % of the events have a sizable change in the quark direction due to gluon emission. More or less only the events with a hard gluon emission which flip the quark direction have a contribution to the QCD correction. But in such event the "flipped" quark is left with a quite low energy and then the probability than the lepton produced by the b decay pass the experimental cut (3 GeV here) is quite reduced. This kind of "threshold effect" is of course dependent of the lepton source . It should be noticed that for lepton coming from b cascade decay the QCD correction will be even further reduced. The previous plots can be compared to figures 3 and 4 where the lepton are not only coming from prompt b decay but also from cascade ($b \rightarrow c/\bar{c}/\tau \rightarrow \mu$). It can be noticed for example that in the case of the μ the QCD correction is then -0.025 instead of -0.038 at 3-4 GeV.

A.6 Conclusion

The QCD corrections to $A_{\rm FB}^{\rm b\overline{b}}$and $A_{\rm FB}^{\rm c\overline{c}}$obtained with the LUND Monte Carlo and the one obtained with the second order QCD calculation are in agreement. The ratio of the C_q obtained with LUND to the one obtained with the second order QCD calculation is equal to 1.02 ± 0.06 for the b and 1.13 $\pm$ 0.08 for the c quark where C_q ($q = b$ or c) is defined by $A_{\rm FB}^{\rm q\overline{q}} = (1 + C_q)A_{\rm FB,0}^{\rm q\overline{q}}$

The dependence of these QCD corrections in function of an angular cut on the acceptance has been compared between LUND and the first order QCD calculation. A relative increase by $\sim$ 11 % of the QCD corrections for a $\cos(\theta)$ cut of 0.9 as been obtained in both case for the b quark.

From this comparisons we can be quite confident that studies at the Monte Carlo level can be used to estimate the QCD corrections on $A_{\rm FB}^{\rm b\overline{b}}$analysis. The effect of acceptance cuts or the dependence of an event weight in function of its QCD "history" can be studied at the Monte Carlo level. From these studies a relative change to the second order QCD corrections can be estimated.

With such study it has been found that in the case of at least one LEP asymmetry measurement the experimental bias can reduce the QCD correction to 50%-70% of their theoretical expectation.

There is few points which need some more investigations :

- If the corrections for the b and the c are in agreement between LUND and the second order QCD corrections, the ratio of the corrections obtained for one quark to one obtain for the other is not.

- Same thing for what concern the θ dependence of the corrections.

with an indication in both cases that the discrepancy may come from the charm.

As the experimental bias seems to be extremely important for the QCD correction , further work on this point is needed.

B A Few Comments about QCD Corrections to the b asymmetry

This short note is mostly intended for internal use of the LEP Heavy Flavour Electroweak Working Group. The procedure which is currently used to correct the heavy flavour forward–backward asymmetries for QCD effects is extremely conservative [3]. The error is coming from varying the renormalization scale between $\mu^2 = M_z^2/4$ and $\mu^2 = M_z^2$ and it is artificially enlarged because the experimental bias 2–jet vs 3–jet events due to analysis cuts is not given by the experiments. We propose here a more rigorous (and less arbitrary) evaluation of the theoretical uncertainty and compute the experimental bias for a realistic case. This is done only for the b asymmetry for the lepton case but can trivially be extended to charm when leptons or D^* are used. The jet charge case is more subtle, most of the QCD effects are compensated by the measurement of the charge separation.

The first order QCD corrections using the thrust axis for the b direction and without any explicit 2–jet selection have been recently completed [35]. The calculation includes mass corrections. The correction has the form

$$A_{FB}^{QCD} = A_{FB}^{no\,QCD}(1 + c_1\frac{\alpha_s}{\pi}) \tag{5}$$

where $c_1 = -0.79 \pm 0.02$, the error being due to the uncertainty on the effective b mass. The use of this correction raises the following issues :

- In reality the thrust is calculated using charged tracks and calorimetric clusters, the neutrino is missed and we have resolution effects. It has been checked [37] that this effect is negligible at least for the Aleph b asymmetry with leptons [38].

- The use α_s at M_z requires the calculation of second order QCD corrections at the same renormalization scale. This has been done in [36]: its implementation is discussed below.

- The experimental bias on the event shape should be calculated.

The second order QCD corrections are given in [36], they allow to solve the ambiguity on the scale and use $\alpha_s(M_z) = 0.118 \pm 0.007$ [39]. This calculation is done using the true b quark direction and mass corrections are not taken into account. Both effects are expected to be small (from an educated guess based on the first order corrections) and work in the same direction, i.e. the tendency is to lower the size of the second order correction. The QCD corrections including second order effects can be written as [36]

$$A_{FB}^{QCD} = A_{FB}^{no\,QCD}(1 + c_1\frac{\alpha_s(M_z)}{\pi} + (\frac{\alpha_s}{\pi})^2[-(1.9 \pm 0.4) - (3.5 \pm 0.7)f_{cut}]) \tag{6}$$

Source	Error
uncertainty on m_b	0.00075
error on $\alpha_s(M_z)$	0.00158
th. error on second order calculations	0.00058
variation of ren. scale for second order	0.00095
missing thrust and mass effects on second order	0.00138
Total	0.00249

Table 13: Size of the individual components for the systematic error on the QCD correction to the b asymmetry. The total error is the sum in quadrature of each component.

where in the second order part the first term is dominated by hard gluon radiation by both b quarks and the second term represents the gluon splitting to b quark pairs. The acceptance of gluon splits depends on the experimental cuts and this is the origin of f_{cut}. In the current measurements the effect of gluon splitting is corrected for at the analysis level, therefore it should be taken away from the second order formula. We are left with the first term. As the calculation is not complete (missing mass corrections and use of the b quark direction) one could take as uncertainty the full range from zero second order correction to the highest value that the first term in the second order correction can take, with the central value taken in the middle. This is very conservative, as in the first order case the mass and thrust effects when combined have roughly a 20 % effect. For the value of α_s in the second order term we take the range $0.10 \div 0.14$. The size of the various systematic errors on the correction factor are given in table 13. The resulting correction including second order is

$$A_{FB}^{QCD} = A_{FB}^{noQCD}(0.9689 \pm 0.00249) \quad (7)$$

The bias 2–jet vs 3–jet events due to experimental cuts has been evaluated for the ALEPH measurement [38] using the first order calculations for 2–jet events given at five y_{cut} values in table 3 of Ref. [35]. The procedure is the following. The N_{jet} distribution has been computed using a $b\bar{b}$ Monte Carlo including full detector simulation for the unbiased case and after all analysis cuts. This has been done at the five y_{cut} of Ref. [35], the five histograms are shown in fig. 5. The (small) bias is clearly seen, it has been proven that it mainly originates from the momentum and transverse momentum cuts which bias the distribution towards 2–jet events. The number of events which migrate from the 2–jet bin to the other bins at each y_{cut} step (a sort of coarse differential two-jet rate) is computed in the unbiased and biased case. This information allows to reweight the five correction coefficients after analysis cuts and in particular yields $c_1 = 0.72$ when no explicit y_{cut} is applied, to be compared with $c_1 = 0.79$ of equation 5, i.e. a reduction of the QCD effect by 9 %. The final correction factor for the ALEPH measurement is

$$A_{FB}^{QCD} = A_{FB}^{noQCD}(0.9717 \pm 0.00227). \quad (8)$$

Conclusions

The QCD correction to the b asymmetry for lepton tagging is

$$A_{FB}^{QCD} = A_{FB}^{noQCD}(0.9689 \pm 0.00249) \quad (9)$$

The effect of gluon splitting on the measurements should be evaluated by the experiments (and in current measurements normally it is). The bias due to analysis cuts is of the order of the theoretical uncertainty itself, therefore should be given by the individual experiments. For ALEPH [38] the bias is equivalent to a 9 % relative reduction of the QCD correction.

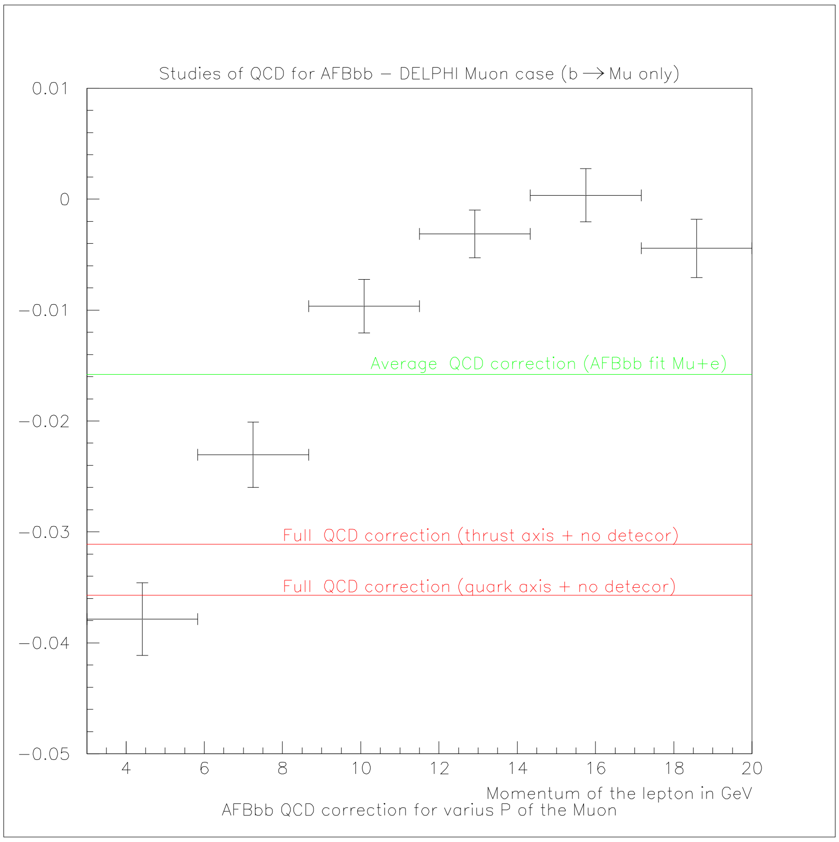

Figure 1: *Value of the QCD correction to* $A_{FB}^{b\bar{b}}$ *(=C_b) in function of the μ momentum. The μ sample contains only mu from $b \rightarrow \mu^-$.*

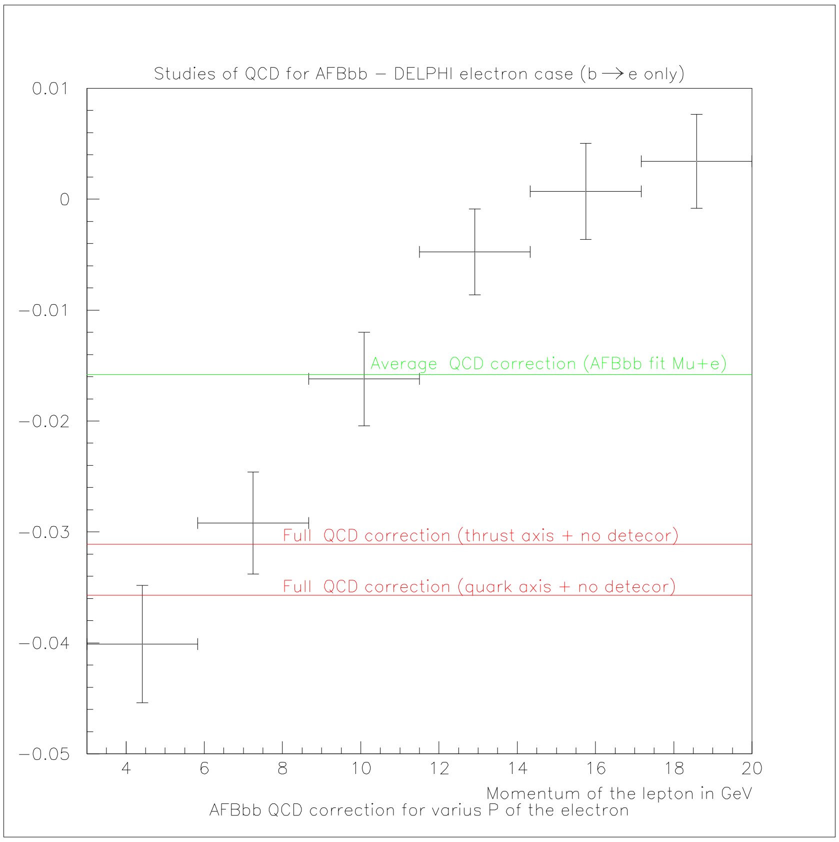

Figure 2: *Value of the QCD correction to* $A_{FB}^{b\bar{b}}$ *(=C_b) in function of the e momentum. The e sample contains only e from $b \rightarrow e^-$.*

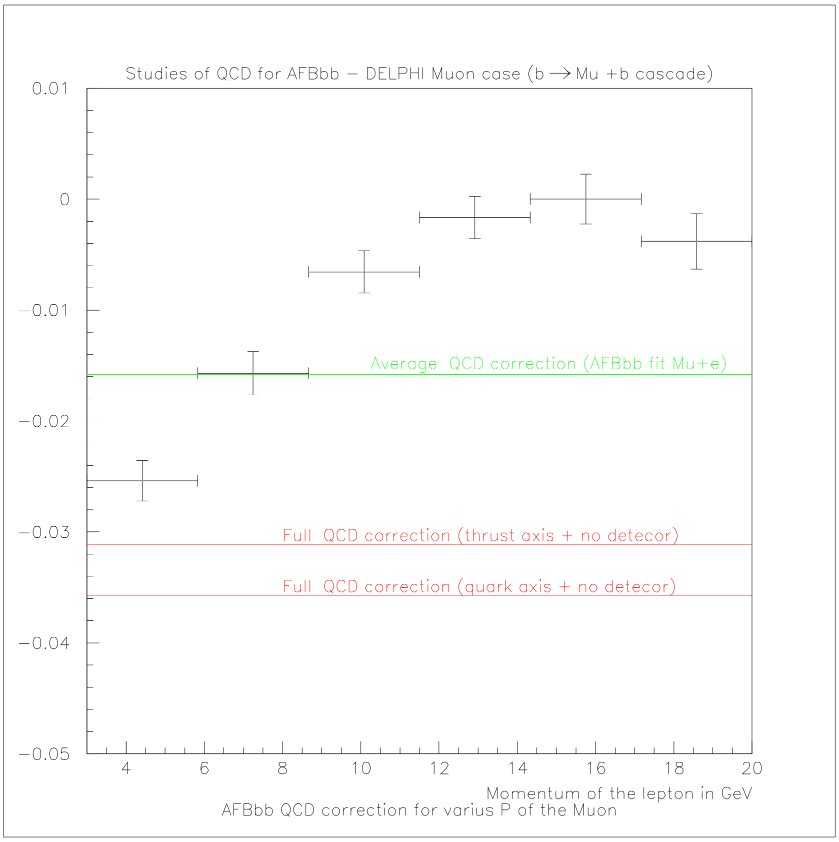

Figure 3: *Value of the QCD correction to* $A_{FB}^{b\bar{b}}$ *(=*C_b*) in function of the* μ *momentum. The* μ *sample contains mu from* $b \to \mu$ *and* $b \to c/\bar{c}/\tau \to \mu$*.*

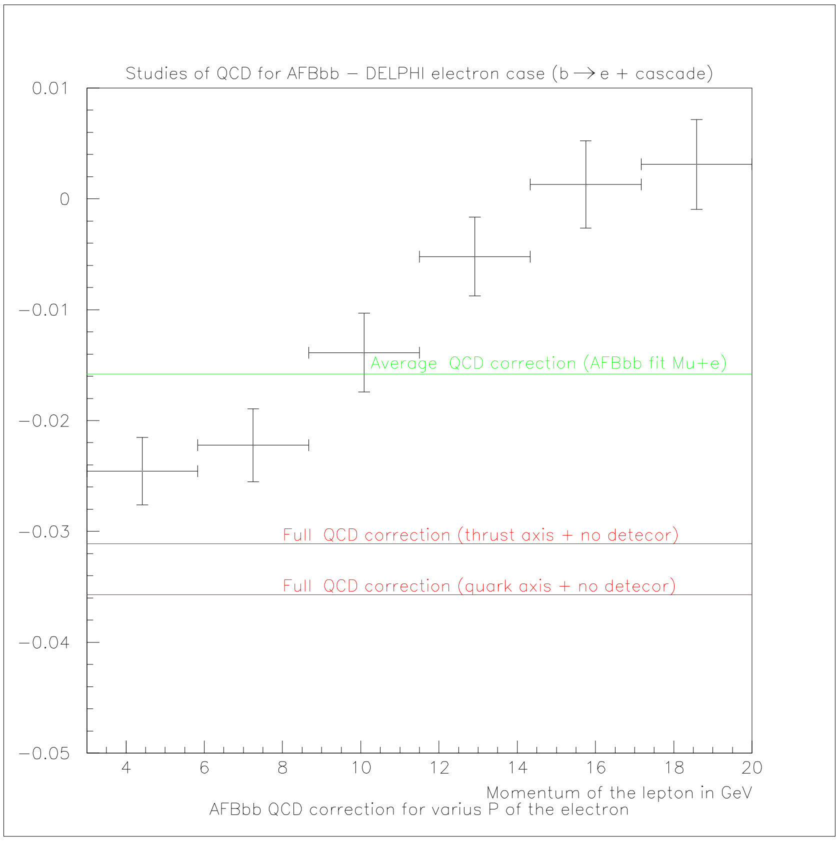

Figure 4: *Value of the QCD correction to* $A_{FB}^{b\bar{b}}$ *(=*C_b*) in function of the e momentum. The e sample contains e from* $b \to e^-$ *and* $b \to c/\bar{c}/\tau \to e$*.*

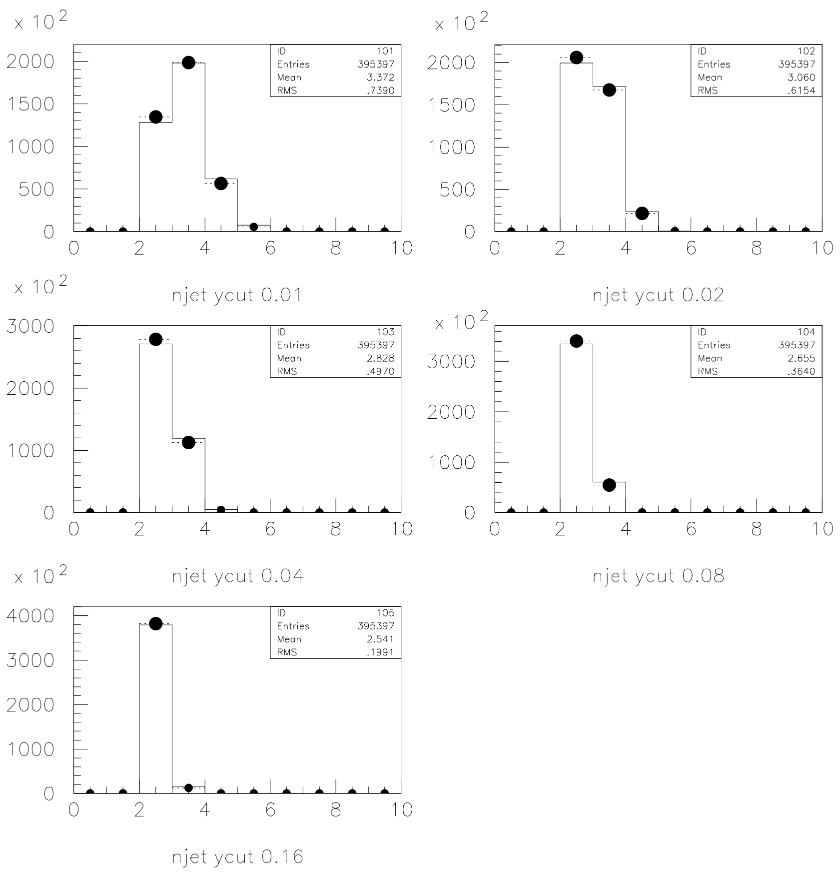

Figure 5: Distribution of the number of jets at five y_{cut} values from the ALEPH $b\bar{b}$ Monte Carlo before (solid histogram) and after (points) the analysis cuts.

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