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Input Parameters for the LEP/SLD Electroweak Heavy Flavour Results for Summer 1998 Conferences

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Abstract

The recommendations for the input parameters used for the electroweak heavy flavour results are updated. New measurements inside and outside LEP allow to reduce the errors and to drop some theoretical assumptions.

The suggestions in this document apply to results used in the LEP/SLD averages starting at the conferences in the Summer of 1998.

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1 Introduction

In 1994 the LEP heavy flavour group has made for the first time suggestions how to treat the electroweak measurements with quarks in a consistent way in order to ease the combination of the data in the LEP collaborations [1, 2]. These parameters have been updated in 1996 to account for the improved input parameters and the new requirements of the experimental analyses [4]. This procedure was successfully used to average LEP and SLD data both for conferences [5] and for the Particle Data Book [6].

With the full LEP1 dataset being available for a few years now several new measurements like the gluon splitting rate into a $\mathrm{b\overline{b}}$-pair have been performed. Also the large statistics from CLEO allows several quantities being measured that have been taken from theory before.

The recommendations in this note are based on published data and, if world averages are used, on the 1998 edition of the PDG [6].

In this note we describe the updated suggestions on the input parameters. For the parameters not described here, the old recommendations stay valid.

2 Definition of parameters and fitting procedure

The fitting procedure remains unchanged. The most up to date description can be found in [4]. The only change is to replace the cross section ratios $\mathrm{R_b}$, $\mathrm{R_c}$ by the ratio of partial width $\mathrm{R_b^0}$, $\mathrm{R_c^0}$. The correction to go from the cross section ratio to the partial width ratio depends slightly on the Z-invariant mass cut used in the cross section definition. Since this cut is not unique it was felt more reasonable to use directly the physically interesting ratios $\mathrm{R_b^0}$ and $\mathrm{R_c^0}$.

The electroweak parameters included in the fit are $\mathrm{R_b^0}$, $\mathrm{R_c^0}$, $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$, $A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$, $\mathcal{A}_\mathrm{b}$and $\mathcal{A}_\mathrm{c}$. The b and c forward-backward asymmetries are either split into three different centre of mass energies or all transported to the Z peak before fitting. The fitted forward-backward asymmetries are already corrected for QCD effects, since the size of the QCD corrections depends strongly on the analysis. A detailed description of the procedure to apply the QCD corrections can be found in [7]. In addition to the electroweak parameters of interest the following auxiliary parameters are left free in the fit:

- the fractions of weakly decaying charmed hadrons $f(\mathrm{D^+})$, $f(\mathrm{D_s})$, $f(\mathrm{c_{baryon}})^*$,
- the probability that a c-quark fragments into a $\mathrm{D^{*+}}$ that decays into $\mathrm{D^0\pi^+}$, $\mathrm{P(c \to D^{*+}) \times BR(D^{*+} \to \pi^+ D^0)}$,
- the $\mathrm{B^0\overline{B^0}}$ effective mixing parameter $\overline{\chi}$,
- the prompt and cascade semileptonic branching ratio of the b-hadrons $\mathrm{Br(b \to \ell)}$ and $\mathrm{Br(b \to c \to \ell)}$.

* $f(\mathrm{D^0})$ is calculated from the constraint $f(\mathrm{D^0}) + f(\mathrm{D^+}) + f(\mathrm{D_s}) + f(\mathrm{c_{baryon}}) = 1$.

3 Charm production and decay

3.1 The charmed hadron production rates

The production ratios of the different charmed hadron species are fit parameters and the dependence on these parameters are quoted e.g. in the $\mathrm{R_b}$ input tables. So the LEP/SLD averages are automatically corrected to the most recent charmed hadron production rates. However, in a publication of an electroweak measurement the charmed hadron production rates should appear as an error rather than an explicitly given dependence. For that reason the most up to date charmed hadron production fractions using published data will be given in this section.

Using only published data a fit has been performed for the electroweak observables with heavy quarks quoted in the PDG [6]. Although not quoted in [6] this fit also produces the charmed hadron fractions. These fractions are the best possible estimate for the given dataset. However, due to the different dependence of the different $\mathrm{R_b}$ analyses these fractions receive a contribution from them. To avoid possible bias of old $\mathrm{R_b}$ analyses in new ones, this fit has been repeated without the $\mathrm{R_b}$ analyses and with $\mathrm{R_b}$ and $\mathrm{R_c}$ fixed to their Standard Model values yielding

$$\begin{aligned} f(\mathrm{D^+}) &= 0.233 \pm 0.027 \\ f(\mathrm{D_s}) &= 0.103 \pm 0.029 \\ f(\mathrm{c_{baryon}}) &= 0.063 \pm 0.028 \end{aligned}$$

with correlation matrix

$$\begin{array}{rrr} 1.00 & -0.28 & -0.26 \\ -0.28 & 1.00 & -0.25 \\ -0.26 & -0.25 & 1.00 \end{array}$$

3.2 The charmed hadron decay modes for the $\mathrm{R_b}$ analyses

A complete set of measured D branching ratios does not exist, so that for the estimation of the c-tagging uncertainty in the $\mathrm{R_b}$ analyses some approximations are needed. The two main features relevant for the c-tagging efficiency are the charged and the neutral decay multiplicity.

For the treatment of charged decay multiplicity a proposal has been described in [4] based on a measurement by MARK III [8] which remains valid.

The influence of neutral multiplicities depends on the detailed properties of the tagging method, especially on how invariant mass cuts are implemented. It is therefore not possible to suggest a common procedure. It was thus agreed to base the method to evaluate those errors on the information given in [6] and to split the error into uncertainties due to the K^0 and due to the π^0 and ν multiplicities. The errors within the two sources will then be fully correlated between the experiments.

3.3 Charm lifetimes and exclusive branching ratios

All charm lifetimes and exclusive branching ratios are taken from the PDG [6]. The average charmed hadron lifetimes from the 1998 PDG update [6] can be found in table 5. The values are basically unchanged compared to the previous update of the input parameters [4].

A list of the new PDG exclusive charm branching ratios [6] relevant for the charm counting and D^* measurements is given in table 1. New results from ALEPH and CLEO on the $BR(\mathrm{D}^0 \to K^-\pi^+)$ yield a 25 % smaller error leaving the mean almost unchanged. New model independent measurements where used to determine $BR(\mathrm{D_s} \to \phi\pi^+)$. Here the average is dominated by the latest CLEO measurement [9] of $BR(\mathrm{D_s} \to \phi\pi^+)/BR(\mathrm{D}^0 \to K^-\pi^+) = 0.92 \pm 0.20 \pm 0.11$. No model independent measurements are available to extract $BR(\Lambda_\mathrm{c} \to pK^-\pi^+)$. In [6] two model dependent estimates where used leading to rather incompatible results, one based on measurements of $BR(\bar{B} \to \Lambda_\mathrm{c}X) * BR(\Lambda_\mathrm{c} \to pK^-\pi^+)$ and other uses the ratio $BR(\Lambda_\mathrm{c} \to pK^-\pi^+)/BR(\Lambda_\mathrm{c} \to \Lambda l^+\nu_l)$. Therefore an average of $BR(\Lambda_\mathrm{c} \to pK^-\pi^+) = 0.050 \pm 0.013$ is quoted, which leads to a factor 2 increase in the error.

Mode	branching ratio
$\mathrm{D}^0 \to K^-\pi^+$	0.0385 ± 0.0009
$\mathrm{D}^+ \to K^-\pi^+\pi^+$	0.090 ± 0.006
$\mathrm{D_s} \to \phi(1020)\pi^+$	0.036 ± 0.009
$\frac{BR(\mathrm{D_s} \to \bar{K}^* K^+)}{BR(\mathrm{D_s} \to \phi\pi^+)}$	0.92 ± 0.09
$\Lambda_\mathrm{c} \to pK^-\pi^+$	0.050 ± 0.013

Table 1: *Charm branching ratios.*

4 b Decay Multiplicities

New measurements have been performed by DELPHI [10] and L3 [11] on the charged particle multiplicity of weakly decaying B hadrons.

The previous value from DELPHI [12] will not be considered anymore, since the method used did not exclude the contribution of the B^{**} decays and possible other excited B states that were unknown at that time. The value from OPAL [13] is still valid because their method excluded them. The values from ARGUS [14] and CLEO [15] cannot be used because of the different B-hadron composition.

The three experiments consider different systematic errors due to the different methods used. Thus in the combination the following assumption were followed. For each experiment *a)* all the error coming form the detector and from the method are combined in a single error; *b)* the errors due to the charm physics are combined in a single error and as well *c)* the errors due to the beauty physics.

For the OPAL result many of the systematic errors coming from charm and beauty physics are scaled down because of the considerably larger uncertainties at that time. The mean b lifetime error is scaled down by a factor of 2 (OPAL was using 10 fs error while now the error is 5 fs) and as well the error due to b fragmentation function, since the measurement is improved in precision from 0.70±0.02 to 0.700 ± 0.008. The uncertainty on R_b was 6% while now it is 0.4%. OPAL gives a global 25% uncertainty on the error due to R_c and the charm sector. Since the main error is $BR(D \to K^0X)$, about 13%, in a conservative way the OPAL error is scaled down by a factor of 2, even if the error on R_c was 20% while now it is 3%.

Finally the three experiments are combined assuming the statistical errors and the errors due to the detector and the method non correlated, while the errors from the charm and the beauty physics fully correlated.

In the following results the K^0 and Λ decay products are excluded. In table 2 the three results with the error treated as described above are listed.

	DELPHI	L3	OPAL	
Mean Multiplicity	4.968	4.90	5.03	
				Corr.
Statistical	0.026	0.040	0.040	0
detector & method	0.060	0.100	0.371	0
charm physics	0.012	0.045	0.061	1
beauty physics	0.019	0.028	0.178	1

Table 2: Measurements and error breakdown used for the B-multiplicity average

The final result is:

$$n_B = 4.955 \pm 0.062.$$

where the total correlated error is 0.028.

The contribution of the K^0 and Λ decay products is given by the difference between the two OPAL results [13] on the mean B charge multiplicity with and without this contribution. It is 0.485 $\pm$ 0.065.

5 Gluon splitting

The gluon splitting rates to heavy flavours are defined as the ratio of the number of events in which a process $g \to Q\bar{Q}$ occurs with respect to the total number of hadronic Z decays.

While measurements for $\mathrm{g} \to \mathrm{c\overline{c}}$ were already presented by OPAL[16, 17], the $\mathrm{g} \to \mathrm{b\overline{b}}$ rate was not measured at the time of the last input parameter recommendations.

It was thus decided to use as input to electroweak heavy flavours measurements the experimental value for $\mathrm{g} \to \mathrm{c\overline{c}}$, and a theoretical estimate for $\mathrm{g} \to \mathrm{b\overline{b}}$ using the ratio $R \equiv \frac{g_{b\bar{b}}}{g_{c\bar{c}}} = \frac{m_c^2}{m_b^2} = 0.13 \pm 0.04$ [18], giving $g_{b\bar{b}} = (3.1 \pm 1.1) \times 10^{-3}$. New measurements from DELPHI[19] and ALEPH[20] allow to replace this theoretical assumption.

To combine the various measurements of $\mathrm{g} \to \mathrm{c\overline{c}}$ and $\mathrm{g} \to \mathrm{b\overline{b}}$ it is necessary to take the correlation between the quantities into account. Especially $\mathrm{g} \to \mathrm{c\overline{c}}$ is an important background in the $\mathrm{g} \to \mathrm{b\overline{b}}$ analyses making the latter dependent on the assumption for the fist.

The choice was to use an iterative method, based on the BLUE technique, to perform the simultaneous combination.

The central value for each analysis can be parametrised as:

$$(g_{c\bar{c}})_i = \frac{A_i}{1 + \frac{g_{b\bar{b}}^{best}}{g_{c\bar{c}}^{best}} B_i} \qquad (g_{b\bar{b}})_j = C_j + D_j \, g_{c\bar{c}}^{best} \tag{1}$$

where A_i, B_i, C_j and D_j are given from the measured efficiencies; the index i runs over $\mathrm{g} \to \mathrm{c\overline{c}}$ measurements, j over $\mathrm{g} \to \mathrm{b\overline{b}}$ ones.

The simultaneous fit can be done using in the first iteration arbitrary values for $g_{c\bar{c}}^{best}$ and $g_{b\bar{b}}^{best}$; new central values for $(g_{c\bar{c}})_i$ and $(g_{b\bar{b}})_j$ are calculated using the relations 1 and the error matrices are scaled to these[†]. A combination based on the BLUE technique is then performed separately for $g \to c\bar{c}$ and $g \to b\bar{b}$, giving first approximation values. These are reused to calculate new values from relations 1. The procedure is repeated until convergence is found.

The method is found stable against variations in the initial assumptions for the rates.

In Table 3 the measurements used for the combinations are reported.

$g \to c\bar{c}$	OPAL e [17]	$(2.18 \pm 0.30 \pm 0.41)\%$
	OPAL μ [17]	$(2.84 \pm 0.58 \pm 1.02)\%$
	OPAL D^* [16]	$(4.4 \pm 1.4 \pm 1.5)\%$
$g \to b\bar{b}$	DELPHI [19]	$(2.1 \pm 1.0 \pm 0.9) \times 10^{-3}$
	ALEPH [20]	$(2.77 \pm 0.42 \pm 0.57) \times 10^{-3}$

Table 3: Experimental measurements used in the fit.

The fit gives the results:

$$g_{c\bar{c}} = (2.33 \pm 0.50)\% \quad (2)$$

$$g_{b\bar{b}} = (0.269 \pm 0.067)\% \quad (3)$$

and the weights from BLUE are presented in Table 4.

$g \to c\bar{c}$	OPAL e [17]	87%
	OPAL μ [17]	11%
	OPAL D^* [16]	2%
$g \to b\bar{b}$	DELPHI [19]	15%
	ALEPH [20]	85%

Table 4: Weights from the BLUE technique.

The shifts given by the cross–correlation between the rates are small with respect to the actual experimental sensitivity, and can be neglected.

6 $\mathbf{Br(b \to \bar{c} \to \ell^-)}$

Until now the input value used for $\mathrm{Br}(b \to \bar{c} \to \ell^-)$ was taken from theoretical calculations. Now several quantities related to this parameter have been measured, and it is therefore possible to make an estimate based upon experimental results.

Different strategies can be adopted to estimate the $\mathrm{Br}(b \to \bar{c} \to \ell^-)$. In the procedure chosen which is described in full details in Appendix A, the inclusive and flavour–specific $B \to D$ and $B \to \Lambda_c^+$ rates measured at CLEO are combined with the $b \to DD(X)$ rates measured in ALEPH to extract the probabilities of producing the different c–hadrons from the *upper vertex* in b decays. These are combined with the c–hadron semileptonic branching ratios to obtain the $\mathrm{Br}(b \to \bar{c} \to \ell^-)$.

The estimate obtained is

$$\mathrm{BR}(b \to \bar{c} \to \ell) = 0.0162\,^{+0.0044}_{-0.0036}\,. \quad (4)$$

[†] All the errors quoted are proportional to central values.

It has been checked that the value obtained is compatible with the latest measurements of charm multiplicity in b decays.

7 Summary

Table 5 summarises the suggested input parameters for the 1998 summer conferences.

Error Source	Proposed Range	Uncertainty
Internal experimental effects	...	...
$\langle x_E(\mathrm{b})\rangle$	Fitted or 0.702 ± 0.008	
$\langle x_E(\mathrm{c})\rangle$	Fitted or 0.484 ± 0.008	
Choice of b fragmentation function	See [3]	
Choice of c fragmentation function	See [3]	
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	$(0.0162\ ^{+0.0044}_{-0.0036})\%$	
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	$(0.452 \pm 0.074)\%$	
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	$(0.07 \pm 0.02)\%$	
$\mathrm{Br}(\mathrm{c} \to \ell)$	$(9.8 \pm 0.5)\%$	
Semilept. model $\mathrm{b} \to \ell$ [3]	ACCMM $\left(^{+\mathrm{ISGW}}_{-\mathrm{ISGW}**}\right)$	
Semilept. model $\mathrm{c} \to \ell$ [3]	ACCMM1 $\left(^{+\mathrm{ACCMM2}}_{-\mathrm{ACCMM3}}\right)$	
$\mathrm{b} \to \mathrm{D}$ model. [3]	Peterson $\epsilon = 0.42 \pm 0.07$	
D^0 lifetime	0.415 ± 0.004 ps	
D^+ lifetime	1.057 ± 0.015 ps	
$\mathrm{D_s}$ lifetime	0.467 ± 0.017 ps	
Λ_c lifetime	0.206 ± 0.012 ps	
B lifetime	1.576 ± 0.016 ps	
B decay multiplicity	4.955 ± 0.062 1)	
D decay multiplicity	See [4]	
$\mathrm{BR}(\mathrm{D} \to \mathrm{K}^0\mathrm{X})$	see text	
$\mathrm{BR}(\mathrm{D} \to \pi^0\mathrm{X})$	see text	
$\mathrm{g} \to \mathrm{c}\overline{\mathrm{c}}$ per multi-hadron	$(2.33 \pm 0.50)\%$	
$\mathrm{g} \to \mathrm{b}\overline{\mathrm{b}}$ per multi-hadron	$(0.269 \pm 0.067)\%$	
Rate of long-lived light hadrons	Tuned JETSET±10% (See [3])	
Light quark fragmentation	See [3]	
QCD hemisphere correlations	see [4]	
D^+ fraction in $\mathrm{c}\overline{\mathrm{c}}$ events	0.233 ± 0.027	
$\mathrm{D_s}$ fraction in $\mathrm{c}\overline{\mathrm{c}}$ events	0.103 ± 0.029	
c-baryon fraction in $\mathrm{c}\overline{\mathrm{c}}$ events	0.063 ± 0.028	
$\mathrm{P}(\mathrm{c} \to \mathrm{D}^{*+}) \times \mathrm{BR}(\mathrm{D}^{*+} \to \pi^+\mathrm{D}^0)$	0.1527 ± 0.0092	

Table 5: Template table of systematic errors. (Quantities that are fitted do not also give rise to a systematic error in the same analysis.)
1) This number does not include K^0_s and Λ decays.
The D production ratios below the line are fitted parameters. The numbers given here are suggestions how to treat the errors in the experimental notes.

A The extraction of BR($b \to \overline{c} \to \ell$)

It is implicitly assumed that the probability of obtaining a D^0, D^+, D_s or Λ_c^+ from the W (so called *upper vertex*) is independent of the nature of the decaying b–hadron, and of the exclusive decay channel.

Measurements used are first presented and discussed, then the result is derived and some consistency checks are performed.

A.1 The CLEO measurements

A.1.1 D^0/D^+ production from upper vertex

In Reference [21] CLEO measures

$$\frac{\Gamma(\mathrm{B} \to \mathrm{D}X)}{\Gamma(\mathrm{B} \to \overline{\mathrm{D}}X)} = 0.100 \pm 0.026 \pm 0.016 \tag{5}$$

where the first error is statistical and the second is systematic, dominated by the knowledge of B mixing. This result is integrated over B^+/B^0, D^+/D^0 states. It can be used to determine the rate of D from the upper vertex once the total D production rate is known. In Reference [22] CLEO reports

$$\frac{\Gamma(\mathrm{B} \to \mathrm{D}^0 X)}{\Gamma(\mathrm{B} \to \mathrm{all})} = 0.636 \pm 0.014 \pm 0.019 \pm 0.0018 \tag{6}$$

$$\frac{\Gamma(\mathrm{B} \to \mathrm{D}^+ X)}{\Gamma(\mathrm{B} \to \mathrm{all})} = 0.235 \pm 0.009 \pm 0.009 \pm 0.0024 \tag{7}$$

where the measurements are done inclusively, without correlating the flavours of the charm and bottom mesons. The first error is statistical, the second is experimental systematic, the third error comes from the branching ratio in the final state used in the analysis, *i.e.* BR($D^0 \to K^-\pi^+$) and BR($D^+ \to K^-\pi^+\pi^+$) respectively. The values used are

$$\mathrm{BR}(\mathrm{D}^0 \to \mathrm{K}^-\pi^+) = 0.0391 \pm 0.0008 \pm 0.0017 \tag{8}$$

$$\mathrm{BR}(\mathrm{D}^+ \to \mathrm{K}^-\pi^+\pi^+) = 0.092 \pm 0.006 \pm 0.008 \tag{9}$$

Measured values given in Equations 5, 6 and 7 need to be rescaled to the latest PDG values for the final state branching ratios given in Section A.3.

The probability of having a D^0 or a D^+ from the upper vertex can then be extracted as

$$P(W \to \mathrm{D}) = \frac{\frac{\Gamma(\mathrm{B}\to\mathrm{D}X)}{\Gamma(\mathrm{B}\to\overline{\mathrm{D}}X)} \times \left(\frac{\Gamma(\mathrm{B}\to\mathrm{D}^0 X)}{\Gamma(\mathrm{B}\to\mathrm{all})} + \frac{\Gamma(\mathrm{B}\to\mathrm{D}^+ X)}{\Gamma(\mathrm{B}\to\mathrm{all})}\right)}{1 + \frac{\Gamma(\mathrm{B}\to\mathrm{D}X)}{\Gamma(\mathrm{B}\to\overline{\mathrm{D}}X)}} \tag{10}$$

A.1.2 D_s production from upper vertex

In Reference [23] the measurement

$$\mathrm{BR}(\mathrm{B} \to \mathrm{D}_s) = (0.1211 \pm 0.0039 \pm 0.0088) \times \frac{0.0035}{\mathrm{BR}(\mathrm{D}_s^+ \to \phi\pi^+)} \tag{11}$$

is quoted, where the first error is statistical, the second is systematic and the dependence on the final state branching ratio is explicitly shown.

The result includes D_s production from lower vertex and has to be corrected for that when extracting the upper vertex rate. Let $P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s}$ be the probability to have a $b \to c$ decay in which a $s\bar{s}$ pair is generated and the c quark hadronises into a D_s. Then the result of Equation 11 can be written as

$$\mathrm{BR}(\mathrm{B} \to \mathrm{D}_s) = P(W \to \mathrm{D}_s) + P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s} \tag{12}$$

A.1.3 c–baryon production from upper vertex

In Reference [24] the measurement

$$\mathrm{BR}(\mathrm{B} \to \Lambda_c^+ X) \times \mathrm{BR}(\Lambda_c^+ \to \mathrm{pK}^-\pi^+) = 0.00273 \pm 0.00051 \pm 0.000039 \tag{13}$$

is quoted, where the first error is statistical and the second is systematic. This can be used, together with the $\mathrm{BR}(\Lambda_c^+ \to \mathrm{pK}^-\pi^+)$ to extract the inclusive production rate of c–baryons from B decays (contributions from baryons heavier than Λ_c^+ are negligible at the level of accuracy needed).

In Reference [25], by studying the charge correlation with a high–$p_\perp$ lepton, CLEO measures the ratio of upper vertex to lower vertex contribution to be

$$R_{\Lambda_c^+} = 0.19 \pm 0.14 \pm 0.04 \tag{14}$$

which can be used to extract the upper vertex contribution, in a way similar to Equation 10:

$$P(W \to \Lambda_c) = \frac{R_{\Lambda_c^+} \times \mathrm{BR}(\mathrm{B} \to \Lambda_c^+ X)}{1 + R_{\Lambda_c^+}} \tag{15}$$

A.2 The ALEPH measurements

In Reference [26] ALEPH presents measurements of exclusive $\mathrm{B} \to \mathrm{DD}(X)$ channels. All these results have to be corrected to account for $b \to u$ and $b \to sg$ transitions, which cannot give two D mesons in the final state.

A.2.1 $\mathrm{D}^0/\mathrm{D}^+$ production from upper vertex

The results

$$\mathrm{BR}(b \to \mathrm{D}^0\mathrm{D}^0(X)) = 0.051\,^{+0.016}_{-0.014}\,^{+0.012}_{-0.011}\,^{+0.003}_{-0.003} \tag{16}$$
$$\mathrm{BR}(b \to \mathrm{D}^0\mathrm{D}^+(X)) = 0.027\,^{+0.015}_{-0.013}\,^{+0.010}_{-0.009}\,^{+0.002}_{-0.002} \tag{17}$$

are quoted, where the first error is statistical, the second is systematic and the third comes from the uncertainty on final state branching ratios. The most important are $\mathrm{BR}(\mathrm{D}^0 \to \mathrm{K}^-\pi^+)$ and $\mathrm{BR}(\mathrm{D}^+ \to \mathrm{K}^-\pi^+\pi^+)$, for which the following values are used:

$$\mathrm{BR}(\mathrm{D}^0 \to \mathrm{K}^-\pi^+) = 0.0383 \pm 0.0012 \tag{18}$$
$$\mathrm{BR}(\mathrm{D}^+ \to \mathrm{K}^-\pi^+\pi^+) = 0.091 \pm 0.006 \tag{19}$$

In order to extract from the result of Equations 16 and 17 the probability of having a D from the upper vertex, one has to allow for a different probability of producing a D in the lower vertex in the case of B_s or b–baryons, and also for processes in which a B meson decays without producing a D meson ($b \to u$ and $b \to sg$ transitions, charmonium production, D_s and baryon production from the lower vertex).

The measurements of Equations 16 and 17 can be added and written as

$$
\begin{aligned}
\mathrm{BR}(b \to \mathrm{DD}) &= (f_{\mathrm{B}^0} + f_{\mathrm{B}^\pm})(1 - P_{b\to u} - P_{b\to sg} - P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s} - P^{\mathrm{low\ V}}_{\mathrm{B}\to\Lambda_b} - P_{\mathrm{B}\to(c\bar{c})})P(W \to \mathrm{D}) \\
&+ f_{\mathrm{B}_s} P^{\mathrm{low\ V}}_{\mathrm{B}_s\to\mathrm{D}} P(W \to \mathrm{D})
\end{aligned} \tag{20}
$$

where $P_{b\to u}$ and $P_{b\to sg}$ are assumed to be independent of the b hadron species, $P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s}$ $P^{\mathrm{low\ V}}_{\mathrm{B}\to\Lambda_b}$ are the probabilities of having a D_s or a baryon from the lower vertex, and similarly and $P^{\mathrm{low\ V}}_{\mathrm{B}_s\to\mathrm{D}}$ is the probability of having a D meson instead of a D_s from the B_s decay.

A possible contribution of D mesons from b baryons has been neglected.

A.2.2 D_s production from upper vertex

The inclusive results

$$
\mathrm{BR}(b \to \mathrm{D}^0\mathrm{D}_s(X)) = 0.091\,^{+0.020}_{-0.018}\,^{+0.013}_{-0.012}\,^{+0.031}_{-0.019} \tag{21}
$$

$$
\mathrm{BR}(b \to \mathrm{D}^+\mathrm{D}_s(X)) = 0.040\,^{+0.017}_{-0.014}\,^{+0.007}_{-0.007}\,^{+0.014}_{-0.009} \tag{22}
$$

are quoted. The first error is statistical, the second is systematic and the third is dominated by the uncertainty in the final state branching ratio. The value used is $\mathrm{BR}(\mathrm{D}_s^+ \to \phi\pi^+) = 0.036 \pm 0.009$ and the values previously quoted for D^0 and D^+.

In order to extract the probability of having a D_s from the upper vertex, these results need to be corrected for $b \to u$ and $b \to sg$ transitions, charmonium production, $B_s \to D$ and $B \to D_s$ transitions in the lower vertex. The formula used is (where again a possible contribution from baryons is neglected):

$$
\begin{aligned}
\mathrm{BR}(b \to \mathrm{DD}_s) &= (f_{\mathrm{B}^0} + f_{\mathrm{B}^\pm}) P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s} P(W \to \mathrm{D}) \\
&+ f_{\mathrm{B}_s}(1 - P_{b\to u} - P_{b\to sg} - P^{\mathrm{low\ V}}_{\mathrm{B}_s\to\mathrm{D}} - P^{\mathrm{low\ V}}_{\mathrm{B}_s\to\Lambda_b} - P_{\mathrm{B}_s\to(c\bar{c})})P(W \to \mathrm{D}) \\
&+ (f_{\mathrm{B}^0} + f_{\mathrm{B}^\pm})(1 - P_{b\to u} - P_{b\to sg} - P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s} - P^{\mathrm{low\ V}}_{\mathrm{B}\to\Lambda_b} - P_{\mathrm{B}\to(c\bar{c})})P(W \to \mathrm{D}_s) \\
&+ f_{\mathrm{B}_s} P^{\mathrm{low\ V}}_{\mathrm{B}_s\to\mathrm{D}} P(W \to \mathrm{D}_s)
\end{aligned} \tag{23}
$$

A.3 Inputs used

A.3.1 Final state branching ratios

The following numbers are used, taken from the new PDG averages [6].

$$
\mathrm{BR}(\mathrm{D}^0 \to \mathrm{K}^-\pi^+) = 0.0385 \pm 0.0009 \tag{24}
$$

$$
\mathrm{BR}(\mathrm{D}^+ \to \mathrm{K}^-\pi^+\pi^+) = 0.090 \pm 0.006 \tag{25}
$$

$$
\mathrm{BR}(\mathrm{D}_s^+ \to \phi\pi^+) = 0.036 \pm 0.009 \tag{26}
$$

$$
\mathrm{BR}(\Lambda_c^+ \to \mathrm{pK}^-\pi^+) = 0.050 \pm 0.013 \tag{27}
$$

A.3.2 $b \to u$ transitions

The inclusive rate of $b \to u$ transitions is taken from the recent ALEPH measurement [27]:

$$P_{b\to u} = 0.0157 \pm 0.0071 \;^{\ddagger} \tag{28}$$

A.3.3 $b \to s$g transitions

In the PDG a 90% CL of 0.068 is reported, which includes also $b \to u$ transitions. Having assumed $P_{b\to u} = 0.0157 \pm 0.0071$, we take for $b \to s$g decays:

$$P_{b\to s\mathrm{g}} = 0.025 \pm 0.025 \tag{29}$$

A.3.4 b–hadron production rates

The following values are used (again from the PDG):

$$f_{\mathrm{B}_s} = 0.105 \,^{+0.018}_{-0.017} \tag{30}$$
$$f_{\Lambda_b} = 0.101 \,^{+0.039}_{-0.031} \tag{31}$$
$$f_{\mathrm{B}^\pm} = f_{\mathrm{B}^0} = \frac{1 - f_{\mathrm{B}_s} - f_{\Lambda_b}}{2} \tag{32}$$

where f_{Λ_b} stays here for the fraction of all kind of b–baryons.

A.3.5 c–hadron semileptonic branching ratios

The following values are used, taken from the PDG [6], and averaging electrons and muons when appropriate:

$$\mathrm{BR}(\mathrm{D}^0 \to \ell^+ X) = 0.0673 \pm 0.0027 \tag{33}$$
$$\mathrm{BR}(\mathrm{D}^+ \to \ell^+ X) = 0.172 \pm 0.019 \tag{34}$$
$$\mathrm{BR}(\Lambda_c^+ \to \ell^+ X) = 0.020 \pm 0.006 \tag{35}$$

The semileptonic branching ratio of D_s is not measured. For that the ratio of D_s to D^0 lifetimes is used together with an estimate for the ratio of semileptonic widths. The values used for the lifetimes, still from the PDG update, are:

$$\tau_{\mathrm{D}^0} = 0.415 \pm 0.004 \;\; \mathrm{ps} \tag{36}$$
$$\tau_{\mathrm{D}_s} = 0.467 \pm 0.017 \;\; \mathrm{ps} \tag{37}$$

The ratio of semileptonic widths is 1 in the spectator model but it is as large as 1.37 in the ISGW2 model [28]. The value

$$\frac{\Gamma(\mathrm{D} \to \ell X \nu)}{\Gamma(\mathrm{D}_s \to \ell X \nu)} = 1.2 \pm 0.2 \tag{38}$$

is used and the B_s SL branching ratio is estimated from:

$$\mathrm{BR}(\mathrm{D}_s^+ \to \ell^+ X) = \frac{\tau_{\mathrm{D}_s}}{\tau_{\mathrm{D}^0}} \frac{\Gamma(\mathrm{D}_s \to \ell X \nu)}{\Gamma(\mathrm{D} \to \ell X \nu)} \mathrm{BR}(\mathrm{D}^0 \to \ell^+ X) \tag{39}$$

‡This is actually calculated as the ratio of charmless to inclusive semileptonic branching ratios

A.3.6 D*/D ratio in upper vertex

The ratio D*/D for the upper vertex contribution is assumed to be the same as for the lower vertex:

$$\frac{\mathrm{BR}(W \to c \to \mathrm{D}^*)}{\mathrm{BR}(W \to c \to \mathrm{D})} = 1.07 \tag{40}$$

No attempt is made to extract this value from existing measurements, and it is conservatively varied between 0 and ∞ to evaluate the error; for each type of particle, equal amounts of neutral and charged states are assumed.

A.3.7 D* branching ratios

These are needed to extract the $\mathrm{D}^+/\mathrm{D}^0$ ratio once the D*/D ratio is assumed for the upper vertex contribution. The values used are

$$\mathrm{BR}(\mathrm{D}^{*0} \to \mathrm{D}^0\pi^0, \gamma) = 1 \tag{41}$$

$$\mathrm{BR}(\mathrm{D}^{*+} \to \mathrm{D}^0\pi^+) = 0.683 \pm 0.014 \tag{42}$$

$$\mathrm{BR}(\mathrm{D}^{*+} \to \mathrm{D}^+\pi^0, \gamma) = 1 - \mathrm{BR}(\mathrm{D}^{*+} \to \mathrm{D}^0\pi^+) \tag{43}$$

A.3.8 B $\to$ D$_s$, B$_s$ $\to$ D and baryon production in the lower vertex, charmonium production

$P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s}$ previously defined is not available experimentally. The probability of hadronising the c quark with a quark coming from the shower (*i.e.* not with the spectator quark) is evaluated to be 35% [29]. The probability that a quark s is picked up, forming a D_s, is assumed to be equal to the probability of having a B_s in the hadronisation of a b quark from the Z decay. Thus we obtain:

$$P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s} = (1 - P_{b\to u} - P_{b\to sg}) \times 0.35 \times 0.105 = 0.035 \tag{44}$$

This is varied between 0.01 and 0.06 to evaluate the systematics.

The ALEPH measurement of Reference [30] gives:

$$\mathrm{BR}(\mathrm{B}_s \to \mathrm{D}_s X) = 0.76 \pm 0.33 \tag{45}$$

This has some correlation with what we want to evaluate due to the final state branching ratio and due to the fact that in this analysis the upper vertex contribution is subtracted. This is ignored here, since this parameter in the end is not very important for the final result. If we consider $1 - \mathrm{BR}(\mathrm{B}_s \to \mathrm{D}_s X)$, this is an upper limit for the probability of having a D from the B_s, since baryon production is neglected. Therefore the value

$$P^{\mathrm{low\ V}}_{\mathrm{B}_s\to\mathrm{D}} = 1 - \mathrm{BR}(\mathrm{B}_s \to \mathrm{D}_s X) = 0.24 \tag{46}$$

varied between 0 and 0.57 to evaluate the systematics is completely conservative.

The branching ratio of B mesons into baryons is given in the PDG as 0.068 ± 0.006, which includes the contribution of the upper vertex. The error is increased by 50% on the low side to allow for the upper vertex contribution, obtaining:

$$P^{\mathrm{low\ V}}_{\mathrm{B}\to\Lambda_b} = 0.068\,^{+0.006}_{-0.009} \tag{47}$$

No measurement is available for the B_s, so the same value is assumed, with the error (arbitrarily) doubled.

$$P^{\text{low V}}_{B_s \to \Lambda_b} = 0.068\,^{+0.012}_{-0.018} \qquad (48)$$

The charmonium production from B mesons is estimated to be (by adding the various channels from the PDG):

$$P_{B \to (c\bar{c})} = 0.0190 \pm 0.0010 \qquad (49)$$

The same is assumed for B_s, with the error doubled.

A.4 Results

The quantities $P(W \to D)$, $P(W \to D_s)$ and $P(W \to \Lambda_c)$ are first extracted from CLEO and ALEPH measurements and averaged, obtaining the figures of Table 6. The values of $P(W \to D_s)$ obtained using ALEPH and CLEO data separately are slightly less than two sigma apart.

	$P(W \to D)$	$P(W \to D_s)$	$P(W \to \Lambda_c)$
ALEPH	0.113 ± 0.037	0.174 ± 0.041	–
CLEO	0.081 ± 0.023	0.083 ± 0.027	0.009 ± 0.001
Average	0.090 ± 0.019	0.110 ± 0.022	0.009 ± 0.001

Table 6: Probability of producing a charm hadron from the upper vertex in b decays estimated from ALEPH and CLEO data, together with the average. Errors shown in this table are the ones used to calculate weights in the average; they are symmetrised.

With the assumptions made on $D^0 - D^+$ relative rates and c-hadron semileptonic branching ratios this translates to:

$$\mathbf{BR}(\boldsymbol{b \to \bar{c} \to \ell}) = \mathbf{0.0162}\,^{+0.0044}_{-0.0036} \qquad (50)$$

The detail of the contributions to the error is given in Table 7.

The error should be solid since the largest contributions come from measured parameters, with the only exception of the D^*/D ratio, for which the assumption is completely safe. The only remaining *caveat* of the estimate is the underlying assumption that the $P(W \to \text{charmed})$ probabilities are the same for all topologies (one may suspect for example that when we have a $B_s \to DK$ transition in the lower vertex the phase space in the upper vertex will be reduced and if a $\bar{c}s$ pair is produced the probability of hadronising into a D_s will be enhanced).

The separate contributions of D^0, D^+ and Λ_c^+ are:

$$\begin{aligned} \mathrm{BR}(b \to W \to D \to \ell) &= 0.0091\,^{+0.0027}_{-0.0026} \\ \mathrm{BR}(b \to W \to D_s \to \ell) &= 0.0071\,^{+0.0035}_{-0.0025} \\ \mathrm{BR}(b \to W \to \Lambda_c^+ \to \ell) &= 0.0002\,^{+0.0001}_{-0.0002} \end{aligned}$$

A.5 Consistency checks

A.5.1 Charm counting

The estimate carried out can be translated to an estimate of the average number of charmed particles produced in b decays, using the following formula:

$$\bar{n}_c = 1 - P_{b\to u} - P_{b\to sg} + P_{b\to(c\bar{c})} + P(W \to \mathrm{D}) + P(W \to \mathrm{D}_s) + P(W \to \Lambda_c) \tag{51}$$

Using the values assumed or derived for the different quantities, the following result is obtained:

$$\bar{n}_c = 1.19 \pm 0.04 \tag{52}$$

which is consistent with the latest LEP and CLEO values.

A.5.2 Total B meson decay rate

A sensitive check is given by the constraint of unitarity for the sum of B meson decay modes. The expression

$$\mathrm{BR}(\mathrm{B} \to \mathrm{D}^0) + \mathrm{BR}(\mathrm{B} \to \mathrm{D}^+) - P(W \to \mathrm{D}) + P^{\mathrm{low\ V}}_{\mathrm{B}\to \mathrm{D}_s} + P_{\mathrm{B}\to(c\bar{c})} + P_{b\to u} + P_{b\to sg} + P^{\mathrm{low\ V}}_{\mathrm{B}\to\Lambda_b} \tag{53}$$

should give one within errors. Using the values assumed or derived the following result is obtained:

$$\sum_i \mathrm{BR}(\mathrm{B} \to i) = 0.944 \pm 0.052 \tag{54}$$

which is one sigma low.

There is no evidence that the estimated error is not conservative enough.

Source	Value	Range	Error on BR($b \to \overline{c} \to \ell$)
CLEO $\frac{\Gamma(\mathrm{B}\to\mathrm{D}X)}{\Gamma(\mathrm{B}\to\overline{\mathrm{D}}X)}$	0.100	0.131 0.069	+0.00161 −0.00170
CLEO $\frac{\Gamma(\mathrm{B}\to\mathrm{D}^0X)}{\Gamma(\mathrm{B}\to\mathrm{all})}$	0.636	0.660 0.612	+0.00016 −0.00016
CLEO $\frac{\Gamma(\mathrm{B}\to\mathrm{D}^+X)}{\Gamma(\mathrm{B}\to\mathrm{all})}$	0.235	0.248 0.222	+0.00009 −0.00009
CLEO $\mathrm{BR}(\mathrm{B}\to\mathrm{D}_s)$	0.1211	0.1307 0.1115	+0.00042 −0.00042
CLEO $\mathrm{BR}(\mathrm{B}\to\Lambda_c^+X)\times\mathrm{BR}(\Lambda_c^+\to\mathrm{pK}^-\pi^+)$	0.00273	0.00337 0.00209	+0.00004 −0.00004
CLEO $R_{\Lambda_c^+}$	0.19	0.34 0.04	+0.00010 −0.00013
ALEPH $\mathrm{BR}(b\to\mathrm{D}^0\mathrm{D}^0)$	0.051	0.071 0.033	+0.00074 −0.00066
ALEPH $\mathrm{BR}(b\to\mathrm{D}^0\mathrm{D}^+)$	0.027	0.045 0.009	+0.00068 −0.00060
ALEPH $\mathrm{BR}(b\to\mathrm{D}^0\mathrm{D}_s)$	0.091	0.115 0.069	+0.00064 −0.00058
ALEPH $\mathrm{BR}(b\to\mathrm{D}^+\mathrm{D}_s)$	0.040	0.058 0.024	+0.00050 −0.00043
$\mathrm{BR}(\mathrm{D}^0\to\mathrm{K}^-\pi^+)$	0.0385	0.0394 0.0376	−0.00026 +0.00028
$\mathrm{BR}(\mathrm{D}^+\to\mathrm{K}^-\pi^+\pi^+)$	0.090	0.096 0.084	−0.00023 +0.00026
$\mathrm{BR}(\mathrm{D}_s^+\to\phi\pi^+)$	0.036	0.045 0.027	−0.00175 +0.00292
$\mathrm{BR}(\Lambda_c^+\to\mathrm{pK}^-\pi^+)$	0.050	0.063 0.037	−0.00004 +0.00006
$P_{b\to u}$	0.0157	0.0228 0.0086	+0.00005 −0.00005
$P_{b\to s\mathrm{g}}$	0.025	0.05 0.	+0.00019 −0.00018
$\mathrm{BR}(\mathrm{D}^{*+}\to\mathrm{D}^0\pi^+)$	0.683	0.697 0.669	−0.00003 +0.00003
$\frac{\mathrm{BR}(W\to c\to\mathrm{D}^*)}{\mathrm{BR}(W\to c\to\mathrm{D})}$	1.07	∞ 0	−0.00154 +0.00165
$\mathrm{BR}(\mathrm{D}^0\to\ell^+X)$	0.0673	0.700 0.646	+0.00044 −0.00044
$\mathrm{BR}(\mathrm{D}^+\to\ell^+X)$	0.172	0.191 0.153	+0.00055 −0.00055
$\mathrm{BR}(\Lambda_c^+\to\ell^+X)$	0.020	0.026 0.014	+0.00005 −0.00005
f_{B_s}	0.105	0.123 0.088	+0.00006 −0.00005
f_{Λ_b}	0.101	0.140 0.070	+0.00031 −0.00023
τ_{D^0}	0.415	0.419 0.411	−0.00007 +0.00007
τ_{D_s}	0.467	0.481 0.450	+0.00025 −0.00025
$\frac{\Gamma(\mathrm{D}\to\ell X\nu)}{\Gamma(\mathrm{D}_s\to\ell X\nu)}$	1.2	1.4 1.0	−0.00099 +0.00138
$P^{\mathrm{low\ V}}_{\mathrm{B}\to\mathrm{D}_s}$	0.035	0.06 0.01	−0.00100 +0.00100
$P^{\mathrm{low\ V}}_{\mathrm{B}\to\Lambda_b}$	0.068	0.074 0.059	+0.00004 −0.00006
$P_{\mathrm{B}\to(c\overline{c})}$	0.019	0.020 0.018	+0.00001 −0.00001
$P^{\mathrm{low\ V}}_{\mathrm{B}_s\to\mathrm{D}}$	0.24	0.57 0	−0.00020 +0.00015
$P^{\mathrm{low\ V}}_{\mathrm{B}_s\to\Lambda_b}$	0.068	0.080 0.050	+0.00001 −0.00001
$P_{\mathrm{B}_s\to(c\overline{c})}$	0.019	0.021 0.017	+0.00001 −0.00001
TOTAL			+0.0044 −0.0036

Table 7: Contributions to the estimated error on $\mathrm{BR}(b\to\overline{c}\to\ell)$

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