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A Consistent Treatment of Systematic Errors for LEP Electroweak Heavy Flavour Analyses

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Abstract

In order to combine measurements of the partial widths and forward-backward asymmetries of Z^0 decays to $b\bar{b}$ and $c\bar{c}$ by the four LEP collaborations, the correlated systematic errors and the interdependencies of the measured quantities must be understood. Here we suggest how the presentation of individual results can be made more complete, so that a consistent average can be formed. In particular we examine the dominant sources of systematic error which lead to correlations between these measurements. This is increasingly important as the precision of the measurements improves, and the systematic errors become relatively more significant.

The suggestions in this document apply to results for conferences in the Winter/Spring of 1994.

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1 Introduction

When forming an average of measurements of electroweak quantities made at LEP for heavy quarks, $\Gamma_{b\overline{b}}/\Gamma_{had}$, $\Gamma_{c\overline{c}}/\Gamma_{had}$, $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$, the increasingly important systematic uncertainties must be taken into account. The measured values of these widths and forward-backward asymmetries depend on each other. This is because any tagging method for a given flavour produces a sample including some contamination from other flavours, and these sample impurities lead to correlations between the measured quantities. Other systematic effects lead to additional correlations between the quantities and between measurements using different methods or by different experiments. The task of making a consistent average can be simplified if all the experiments provide a breakdown of the errors according to the same categories of systematic uncertainties. Certain input parameters are to some extent arbitrary, and we hope to find a set which all the experiments can agree to use. The experiments of course remain free to present their final results in their chosen format. However, if a standard set of input parameters and ranges for their variation can be established, and kept up to date as more information becomes available, this will lead to a more correct LEP average result.

Our aim is to have the complete covariance matrix for these electroweak measurements, derived from both the direct interdependence of the measured values, and the uncertaintes from common sources of systematic errors. If the full breakdown of errors is provided for each of the individual measurements, then constructing the final correlation matrix should be relatively straightforward.

In the following sections we consider the sources of systematic uncertainty, starting with examples of the direct interdependence of measured electroweak parameters, followed by uncertainties in lepton-tag measurements, lifetime-tag measurements and measurements which rely on inferring the primary quark charge from a "jet charge" measure. Finally we comment on the QCD correction necessary to convert from the observed to the pole forward-backward asymmetry

2 Interdependence of electroweak parameters

The measured value of a given electroweak parameter in the heavy flavour sector depends on the values assumed for other electroweak parameters. It is possible to assume that the Standard Model applies and use the predicted values of other parameters. This introduces a model dependence in the measurement, and possibly makes the interpretation more complicated when trying to perform a fit of the Standard Model parameters such as the top quark mass to all the available electroweak measurements. We therefore suggest that where relevant, the dependence of a result on other electroweak parameters should be quoted explicitly, for example:

- $\Gamma_{b\overline{b}}/\Gamma_{had}$ and $\Gamma_{c\overline{c}}/\Gamma_{had}$ – each depends on the other
- $A_{FB}^{b\overline{b}}$, $A_{FB}^{c\overline{c}}$ – depend on each other, and on $\Gamma_{b\overline{b}}/\Gamma_{had}$ and $\Gamma_{c\overline{c}}/\Gamma_{had}$
- mixing (not quite a standard model parameter). $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$ measured with leptons depend on the average mixing in a simple way. We should avoid double counting the error due to the semileptonic decay model in the mixing and the asymmetry (see below). For combining measurements it is useful to know explicitly how this mixing correction was made. Measurements using a vertex tag have a more subtle dependence on mixing, which is one source of correlation between vertex tag and lepton tag results.

This dependence could be provided either as the correlation/covariance matrix from a combined fit, or as an explicit expression from which the former can be calculated.

3 Modelling of prompt leptons

The dominant sources of uncertainty for lepton-tag measurements are:

Choice of semileptonic decay model. Each of the experiments has independently tuned a large set of Monte Carlo parameters to achieve agreement with their data. This procedure includes adjusting the decays of b and c hadrons to agree with the results of lower energy experiments, and tuning the b and c fragmentation parameters. It is not possible simply to suggest a set of parameters for b and c hadron decays for all the experiments to use, as all the other parameters would then need to be retuned, and in addition large samples of events have already been generated. However, for the purposes of results using a lepton tag for $b\overline{b}$ and $c\overline{c}$ events, each experiment can reweight events according to the lepton momentum in the rest frame of the heavy hadron to agree with some model for the lepton spectrum. In this way the results can be provided for a consistent set of models, which should represent the best central value and a reasonable spread of "hardness" of the spectrum to evaluate the systematic uncertainty. The choice of model is discussed in the remainder of section 3.

Semileptonic branching ratios. Semileptonic branching ratios are becoming sufficiently well measured at LEP to avoid having to use results from lower energy experiments. The correspondence between measured branching ratios and semileptonic decay models should be preserved in the measurement of other quantities such as the average mixing and forward-backward asymmetries. The overall error due to modelling can then be quoted explicitly for the electroweak quantities, without double counting.

Fractions of different b and c flavoured hadrons. When average semileptonic branching ratios are measured in the LEP data, then the only remaining uncertainty due to the mixture of heavy hadron states is in whether the range of semileptonic decay model adequately represents the differences in the shape of the lepton spectra for different hadrons. This will usually be the situation for the b flavoured hadrons, once all the experiments measure the $b \to \ell$ branching ratio. However, there are larger uncertainties associated with the mix of c flavoured hadrons produced in b decay, and produced in $c\overline{c}$ events. For example:

- Uncertainty in the relative rates of ℓ^+ and ℓ^- in $b \to c \to \ell$ decay (needed to measure mixing and asymmetries).
- The large change in the $c \to \ell$ branching ratio when the ratio D^0/D^+ is varied. This brings in a correlation with the error coming from the charm sector in lifetime-tag measurements.

b and c fragmentation parameters. The precise measurements of the average energy of b and c hadrons at LEP from lepton and D^* tag measurements mean that heavy flavour fragmentation is often one of the smaller sources of systematic uncertainty. The Peterson model is often the only one considered. This fragmentation function has one free parameter, ϵ, which is varied to reproduce the range in average hadron energy. Other models should also be considered in analyses which are sensitive to the fragmentation uncertainty, and in particular in analyses which fit for the fragmentation.

3.1 Choice of semileptonic decay model – recent history

For existing results the experiments have chosen different models for the lepton momentum in the semileptonic decay of heavy hadrons. However, these models are often motivated by the same low energy data. The models are needed to define the central value of a measurement and to assess the systematic error due to the modelling of semileptonic decay. A brief summary of the choices of models made for previous publications by the LEP experiments are listed in table 1.

$b \to \ell$ decay	
ALEPH	ISGW with 32% D** and ACCMM, quote (average $\pm$ half difference)
DELPHI	ISGW with $32 \pm 5\%$ D**. Also compare with ACCMM and BSW
L3	Correct to CLEO spectrum, vary D** by $\pm 8\%$
OPAL	ACCMM for central values, ISGW with 11% and 32% D** for errors.
$b \to c \to \ell$ decay	
ALEPH	Have been using the spectra provided by CLEO
L3	Used the JETSET $b \to D$ spectrum and the $c \to \ell$ spectrum as below
OPAL	Have been using a spectrum derived following the CLEO prescription
$c \to \ell$ decay	
ALEPH	Reweight to DELCO. Take half difference between DELCO and JETSET as error.
DELPHI	Use the LUND spectrum with K*/K ratio of 0.5.
	Reweight to K*/K ratio of 1 (Mark III) for error.
L3	Reweight to the DELCO spectrum, vary the K*/K ratio for errors.
OPAL	Use the ISGW prediction, and the ACCMM model fitted to DELCO data.

Table 1: An indication of the range of choices of semileptonic decay model made by the LEP experiments in earlier analyses.

The ACCMM model [2] is a free-quark model refined by inclusion of QCD corrections. It has two input parameters, a Fermi momentum parameter, p_{f}, and the mass, m, of the quark produced in the heavy quark decay. This model was used by the CLEO collaboration for their measurement of the $b \to \ell$ branching ratio for b hadrons (B^0 and B^+) produced in in $\Upsilon(4S)$ decays. The parameter values found by CLEO were $p_{\mathrm{f}} = 298$ MeV/c, $m_{\mathrm{c}} = 1673$ MeV/c^2 [1].

The ISGW model [3] is based on a form-factor calculation of an explicit sum of spectra calculated for individual three-body final states. There are no free parameters in the model.

In addition to the ACCMM and ISGW models, the CLEO Collaboration constructed a modified version of the ISGW model, referred to as ISGW**, in which the fraction of semileptonic $b \to \ell$ decays via a D** was allowed to float freely, but the relative rates of the four excited D states with one unit of orbital angular momentum constituting the total D** contribution were unchanged. Although varying the D** content of the ISGW model is a convenient way of changing the hardness of the spectrum, the D** fraction is in fact predicted by the model to be 11%. The modified ISGW** model contains a 32% D** fraction from the result of the fit to CLEO data.

The $b \to c \to \ell$ spectrum must also be modelled. This is more complicated than the $b \to \ell$ spectrum, because both the D spectrum from the $b \to D$ decay, and the lepton spectrum from the $c \to \ell$ (i.e. the $D \to \ell$) decay must be modelled. One should in general remember that the mixture of c hadrons resulting from b decay cannot be assumed to be the same as the mixture of c hadrons in $c\bar{c}$ events.

Some new results for $b \to \ell$ and $b \to c \to \ell$ branching ratios and semileptonic decay models were presented by CLEO at the Summer 1993 conferences. These were consistent with their existing results, for example the D** fraction in the ISGW model, which was previously quoted as $32\pm5\%$, was updated to $21\pm8\%$. We propose that until CLEO update their published paper we should continue to use as input published results constraining semileptonic decay models. This also applies to the D momentum distribution in b$\to$D decays, which is needed to model the $\mathrm{b}\to\mathrm{c}\to\ell$ lepton spectrum.

The effect of uncertainties in the momentum spectrum of leptons from semileptonic c decays must also be considered. From a fit [1] to DELCO data [6], the two free parameters of the ACCMM model for $\mathrm{c}\to\ell$ decays are $p_\mathrm{f} = 282$ MeV/c and $m_\mathrm{s} = 50$ MeV/c^2. As for $\mathrm{b}\to\ell$ decays, the ISGW model makes an exact prediction of the $\mathrm{c}\to\ell$ decay spectrum.

Radiative corrections from low energy photonic virtual loop and bremsstrahlung effects should be made to the predictions of the theoretical model. These have been calculated in [4] and lead to small differences between the distributions for electrons and muons.

In summary, the experiments follow a similar philosophy, in that each chooses models constrained by lower energy data for the central values and ranges of the errors. They use different means to achieve a variation in the model - either changing relative rates of different decay modes or just reweighting to a different model. Feedback from the experiments so far suggests that there is general agreement that these choices are arbitrary, and a simple set of models should be chosen for everyone to use. We therefore decided that the most urgent thing is to provide the LEP experiments with a standard set of semileptonic decay models to integrate into the analysis chain.

Finally, there is the question of whether the $\mathrm{b}\to\ell$, $\mathrm{b}\to\mathrm{c}\to\ell$ and $\mathrm{c}\to\ell$ models should be varied in a correlated way. Because the $\mathrm{b}\to\ell$ and $\mathrm{c}\to\ell$ models are essentially parameterising different data sets (CLEO and DELCO/MARK III) then they should be varied independently. As will be seen below, the proposed $\mathrm{b}\to\mathrm{c}\to\ell$ models are correlated with the $\mathrm{c}\to\ell$ models, and so should be varied simultaneously.

3.2 Proposed model for $\mathrm{b}\to\ell$

The experiments are asked to provide results for three models:

ACCMM with $p_\mathrm{f} = 298$ MeV/c, $m_\mathrm{c} = 1673$ MeV/c^2 [1].

ISGW with the model prediction of 11% D**

ISGW** with the modified 32% D**

The small contribution from $\mathrm{b}\to\mathrm{u}$ decays measured by CLEO for each of these models should be included. These models are illustrated in figure 1, and example data statements (created by OPAL according to this prescription) are given in appendix A.

For use in Z^0 decays, the same models should be assumed to be valid for the $\mathrm{B_s}$ meson and b-baryons (denoted Λ_b for convenience). This would be correct in the simplest spectator model, and is thought more generally to be reasonable for the $\mathrm{B_s}$. The Λ_b constitutes only roughly 10% of b hadrons.

3.3 Proposed model for $\mathrm{c} \to \ell$

The combined measurements of DELCO [6] and MARK III [7] for $\mathrm{c} \to \ell$ decays have been parametrised by the ACCMM model. A full description of the fitting procedure and the choice of models to measure a 1 standard deviation difference from the central values is given in appendix B. The data points used in the fit were inferred by measurement from the figures in the two references, and are listed in appendix C. The results of the fit are:

$$
\begin{aligned}
m_{\mathrm{s}} &= 0.001 \pm 0.152\,\mathrm{GeV}/c^2 \\
p_{\mathrm{f}} &= 0.467^{+0.205}_{-0.114}\,\mathrm{GeV}/c
\end{aligned}
$$

The three distributions proposed are:

ACCMM with $m_{\mathrm{s}} = 0.001, p_{\mathrm{f}} = 0.467$ GeV

ACCMM with $m_{\mathrm{s}} = 0.001, p_{\mathrm{f}} = 0.353$ GeV

ACCMM with $m_{\mathrm{s}} = 0.153, p_{\mathrm{f}} = 0.467$ GeV.

The model with p_{f} shifted from the central value should give the more reliable estimate of a 1σ shift in the result. The other model, with m_{s} shifted from its central value is included for comparison. The resulting distributions of the lepton momentum in the rest frame of the D are shown in figure 2.

3.4 Proposed model for $\mathrm{b} \to \mathrm{c} \to \ell$

The three models proposed for $\mathrm{c} \to \ell$ are combined with the measured b to D spectrum from CLEO [8] to generate three models for the lepton momentum spectrum in the rest frame of the b hadron for $\mathrm{b} \to \mathrm{c} \to \ell$ decays. The resulting distributions of the lepton momentum in the rest frame of the B are shown in figure 3.

An additional, much smaller error comes from the uncertainty in the CLEO $\mathrm{b} \to \mathrm{D}$ spectrum. This spectrum can be well modelled by the Peterson function with $\epsilon = 0.42 \pm 0.07$. The effect of this additional uncertainty on the $\mathrm{b} \to \mathrm{c} \to \ell$ spectrum is illustrated in figure 4.

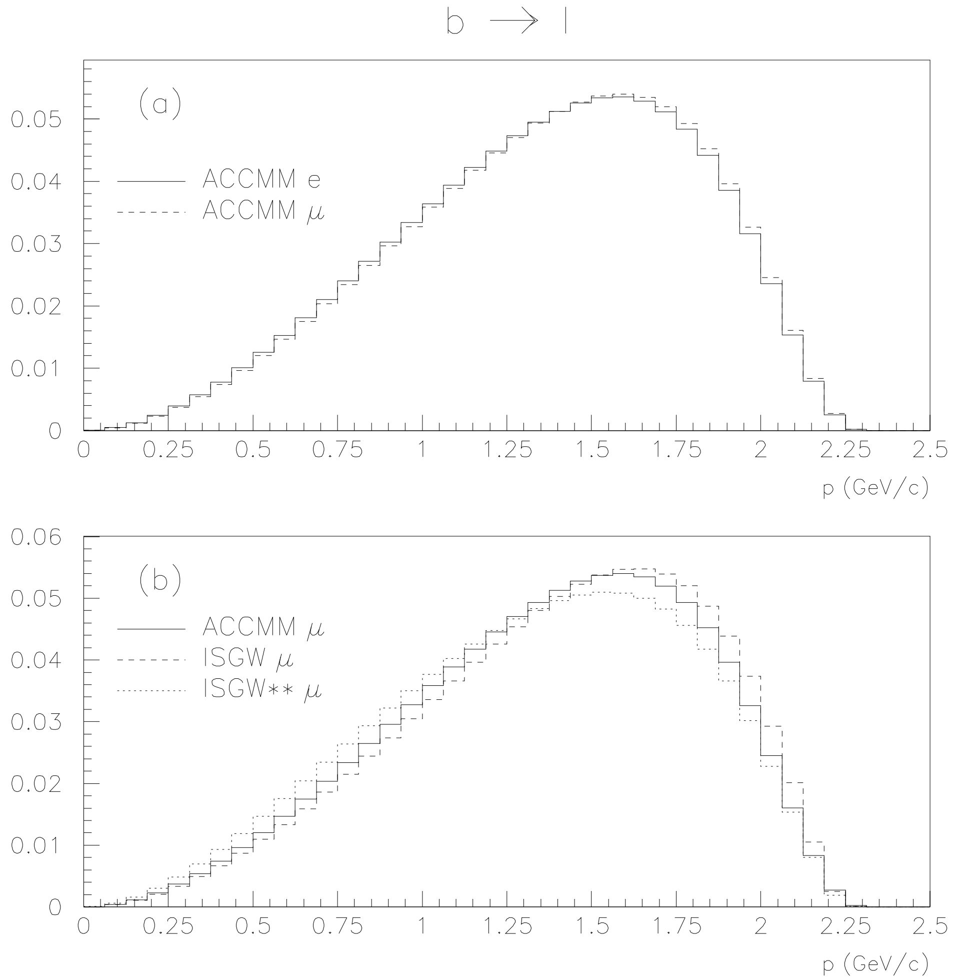

Figure 1: The proposed momentum spectrum for the lepton in the rest frame of the B for $b \to \ell$ decays. (a) Shows the central ACCMM spectrum including radiative corrections for e and μ. (b) Shows the ACCMM, ISGW and ISGW** spectra for μ.

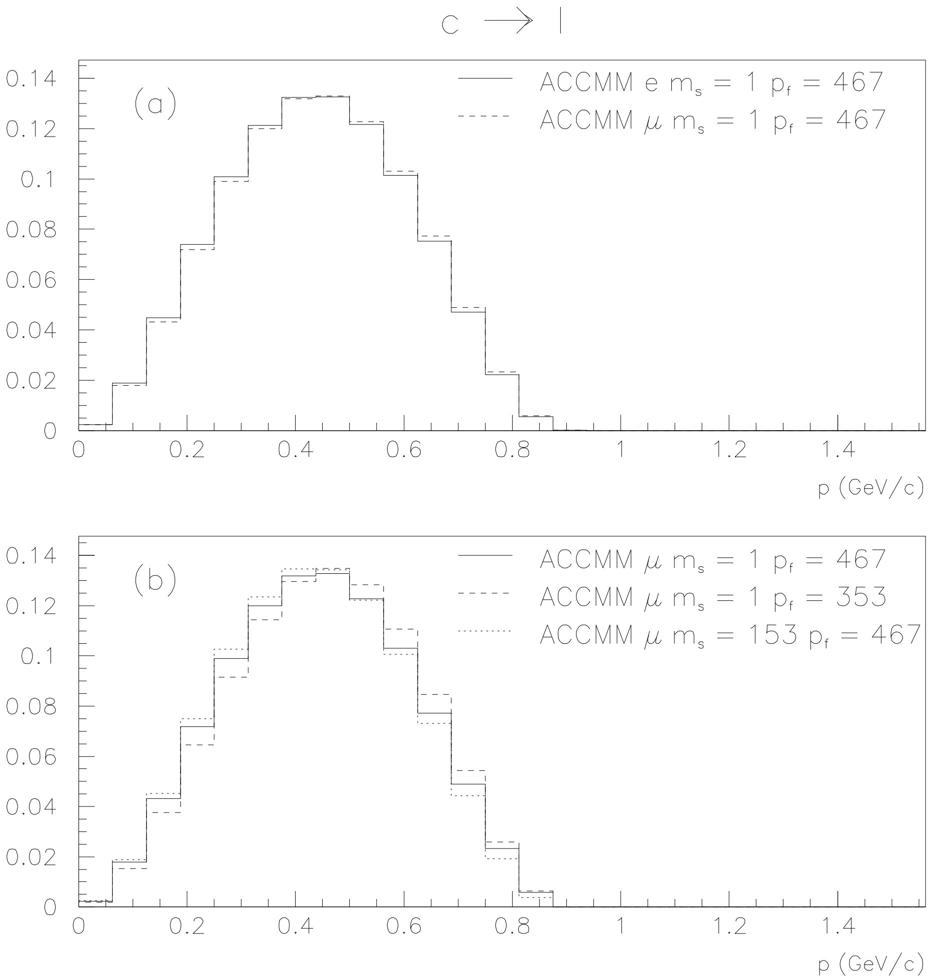

Figure 2: The proposed momentum spectrum for the lepton in the rest frame of the D for $c \to \ell$ decays. (a) Shows the central ACCMM spectrum including radiative corrections for e and μ. (b) Shows the central and one standard deviation ACCMM distributions for μ.

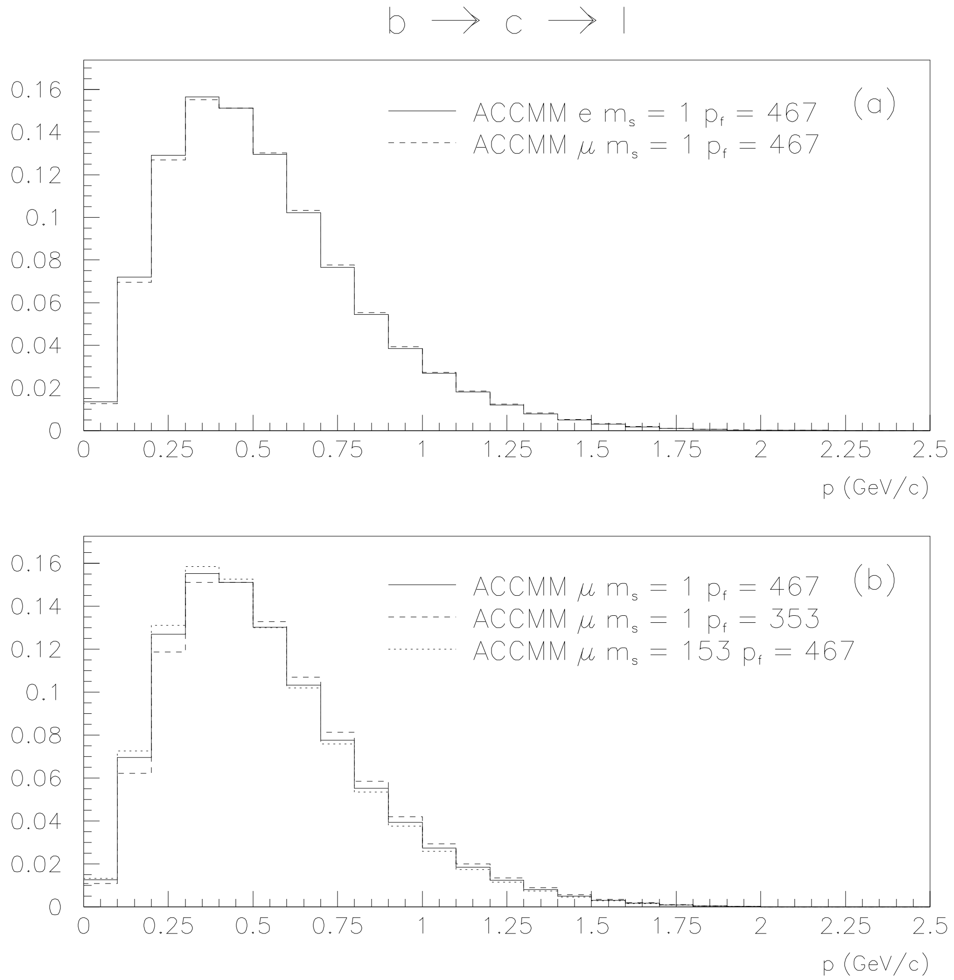

Figure 3: The proposed momentum spectrum for the lepton in the rest frame of the B for $b \to c \to \ell$ decays. (a) Shows the central ACCMM spectrum including radiative corrections for e and μ. (b) Shows the central and one standard deviation ACCMM distributions for μ.

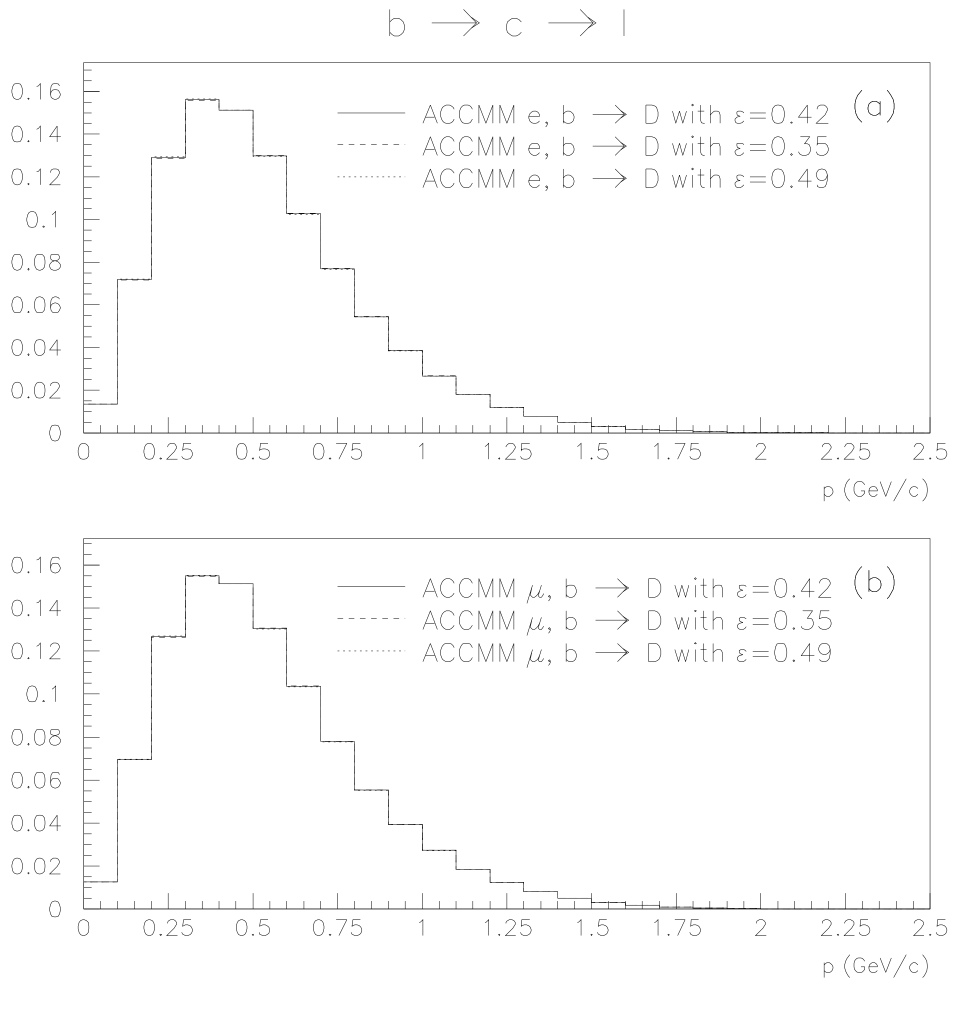

Figure 4: The proposed momentum spectrum for the lepton in the rest frame of the B for $b \to c \to \ell$ decays, illustrating the small variation due to the $b \to D$ uncertainty. (a) is for electrons and (b) for muons.

4 Lifetime tag for $\mathrm{b\overline{b}}$ events

4.1 Hemisphere correlations for $\Gamma_{\mathrm{b\overline{b}}}/\Gamma_{\mathrm{had}}$ from a double tag

In the measurements of $\Gamma_{\mathrm{b\overline{b}}}/\Gamma_{\mathrm{had}}$ using a double tag, the correlation between the tagging efficiencies for b's in the two hemispheres of the event has to be evaluated. There are several sources of error which enter through this hemisphere correlation. For each item, the "yes" or "no" refers to whether this effect is correlated among the LEP experments.

- MC statistics **(no)**
- b fragmentation **(yes)**
- b decay multiplicity **(yes)**
- b hadron composition **(yes)**
- QCD effects - These are usually evaluated from data from the dependence of the result on for example the number of jets **(no, evaluated directly from data)**
- Resolution effects (usually evaluated by varying MC smearing parameters to agree with data) **(no, experiment dependent)**
- b lifetime **(yes)**

4.2 Errors from the charm sector

There are then many effects which lead to an uncertainty in the charm background and which are correlated among the LEP experiments:

- $\Gamma_{\mathrm{c\overline{c}}}/\Gamma_{\mathrm{had}}$ dependence should be provided explicitly
- charm fragmentation (Monte Carlo charm fragmentation parameters should be varied to give a range of $\langle x_E(\mathrm{c})\rangle$ consistent with LEP data. This usually results in a rather small error.)
- the mixture of D hadrons:
  - $\mathrm{D^0/D^+}$ ratio (most sensitive)
  - fraction of $\mathrm{D_s}$
  - fraction of Λ_c

  A prescription for how to vary the D hadron production ratios is given below. The input values are taken from the PDG [5].
- The lifetimes of charm hadrons are also taken from PDG.
- The charm decay multiplicity. So far, the charged multiplicity has been varied in accordance with measurements reported in the PDG. It has also been observed that changing other exclusive branching ratios, so that the number of neutral products changes, effects on the tagging efficiency for charm.

4.2.1 Production rates of $\mathrm{D}^+, \mathrm{D}^0, \mathrm{D_s}$ and charm baryons

Four production fractions are defined:

$$
\begin{aligned}
f_1 &= \text{production fraction of } \mathrm{D}^+ \\
f_2 &= \text{production fraction of } \mathrm{D}^0 \\
f_3 &= \text{production fraction of } \mathrm{D_s} \\
f_4 &= \text{production fraction of charmed baryons}
\end{aligned}
$$

These quantities and their covariance matrix should be specified. The covariance can be constructed from a combination of measurements and constraints by minimizing:

$$\chi^2 = \Delta^T W \Delta + q(1 - f_1 - f_2 - f_3 - f_4)^2 + (\text{other constraint/measurement terms})$$

where Δ is the vector of differences between f_1 to f_4 and the measurements, m_1 to m_4, and W is the weight matrix of the measurements:

$$W = \begin{pmatrix} 1/\sigma_1^2 & 0 & 0 & 0 \\ 0 & 1/\sigma_2^2 & 0 & 0 \\ 0 & 0 & 1/\sigma_3^2 & 0 \\ 0 & 0 & 0 & 1/\sigma_4^2 \end{pmatrix}$$

Note that the diagonal terms can be zero if no measurement information exists for a particular species. The constraint that the f sum to unity is obvious. One can also add to the χ^2 additional terms, e.g.

$$\left(\frac{(f_1/f_2) - r}{\sigma_r}\right)^2$$

where r is, in this case, a measurement of the $\mathrm{D}^+/\mathrm{D}^0$ ratio. A simple example of this is given in a short fortran program in appendix D.

For a measurement of $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$, the error on the charm tagging efficiency is of interest. Given the solution for f_1 to f_4 and the associated covariance V, the systematic error on the charm tagging efficiency due to charm production rates can be calculated in the usual way:

$$Var(\epsilon_c) = \Sigma_{i=1}^4 \Sigma_{j=1}^4 \frac{d\epsilon_c}{df_i} \frac{d\epsilon_c}{df_j} V_{ij}$$

Since $\epsilon_c = f_1\epsilon_1 + f_2\epsilon_2 + f_3\epsilon_3 + f_4\epsilon_4$, the partial derivatives $d\epsilon_c/df_i$ are given by the efficiencies for tagging particular charmed hadrons, ϵ_i. If these 4 numbers are specified along with the derivative of $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$ w.r.t. ϵ_c, the systematic uncertainty on ϵ_c can be recalculated with new f_i and V, allowing one to combine and adjust results from different experiments.

4.2.2 Calculation of D production fractions

The following is a calculation of the $\mathrm{D}^+, \mathrm{D}^0$ and D^{*+} production rates at $\sqrt{s} = 10.55$ GeV. These numbers are expected to be valid within their errors at $\sqrt{s} = M_{\mathrm{Z}}$ as shown in an ALEPH note [9]. (Note that cross-sections for D^0 and D^+ include those coming from D^* decays.)

Measurements of cross-section times branching ratio ($\sigma \times BR$) from ARGUS as quoted in [10] are

$$
\begin{aligned}
\sigma(\mathrm{D}^0 -> K\pi) &= 44 \pm 5.6\text{pb} \\
\sigma(\mathrm{D}^+ -> K\pi\pi) &= 50 \pm 6.9\text{pb} \\
\sigma(\mathrm{D}^{*+} -> \pi K\pi) &= 14 \pm 1.6\text{pb}
\end{aligned}
$$

where the errors include statistical and systematic contributions. Similar measurements from CLEO as quoted in [11] are

$$\begin{aligned}
\sigma(\mathrm{D}^0 -> K\pi) &= 52 \pm 5.0\mathrm{pb} \\
\sigma(\mathrm{D}^+ -> K\pi\pi) &= 47 \pm 7.3\mathrm{pb} \\
\sigma(\mathrm{D}^{*+} -> \pi K\pi) &= 17 \pm 2.1\mathrm{pb} \\
\sigma(\mathrm{D}^{*+} -> \pi K 3\pi) &= 33 \pm 3.5\mathrm{pb}
\end{aligned}$$

where the errors include statistical and systematic contributions. Combining these measurements gives the following $\sigma \times BR$:

$$\begin{aligned}
\sigma(\mathrm{D}^0 -> K\pi) &= 48.5 \pm 3.7\mathrm{pb} \\
\sigma(\mathrm{D}^+ -> K\pi\pi) &= 48.6 \pm 5.0\mathrm{pb} \\
\sigma(\mathrm{D}^{*+} -> \pi K\pi) &= 15.2 \pm 1.3\mathrm{pb} \\
\sigma(\mathrm{D}^{*+} -> \pi K 3\pi) &= 33.0 \pm 3.5\mathrm{pb}
\end{aligned}$$

To turn these into D production cross-sections, the following branching ratios are used:

$$\begin{aligned}
BR(\mathrm{D}^0 -> K\pi) &= 3.54 \pm 0.21\%\,[5] \\
BR(\mathrm{D}^0 -> K3\pi) &= 7.5 \pm 0.5\%\,[5] \\
BR(\mathrm{D}^+ -> K\pi\pi) &= 8.0 \pm 0.8\%\,[5] \\
BR(\mathrm{D}^{*+} -> \mathrm{D}^0\pi) &= 68 \pm 3\%\,[12]
\end{aligned}$$

The cross sections for D^0, D^+, and D^{*+} production are

$$\begin{aligned}
\sigma(\mathrm{D}^0) &= 1.37 \pm 0.13\mathrm{nb} \\
\sigma(\mathrm{D}^+) &= 0.61 \pm 0.09\mathrm{nb} \\
\sigma(\mathrm{D}^{*+}) &= 0.63 \pm 0.06\mathrm{nb}
\end{aligned}$$

where the two D^0 decay modes have been combined in the D^{*+} number.

The total $\mathrm{q}\overline{\mathrm{q}}$ cross section at 10.55 GeV has been measured to be $3.33 \pm 0.05 \pm 0.21$ nb in [13]. The fraction of $\mathrm{c}\overline{\mathrm{c}}$ in the $\mathrm{q}\overline{\mathrm{q}}$ total is expected to be 0.37 ± 0.02 based on a Monte Carlo calculation including ISR as cited in [11]. Thus the charm quark cross section at 10.55 GeV is 2.46 ± 0.21 nb.

Dividing the D cross sections by the charm cross section yields

$$\begin{aligned}
f(\mathrm{D}^0) &= 0.557 \pm 0.053 \pm 0.048 \\
f(\mathrm{D}^+) &= 0.248 \pm 0.037 \pm 0.021 \\
f(\mathrm{D}^{*+}) &= 0.254 \pm 0.023 \pm 0.021
\end{aligned}$$

where the second error comes only from the error on the total charm quark cross section. The sum and ratio of D^0 and D^+ are

$$\begin{aligned}
f(\mathrm{D}^0 + \mathrm{D}^+) &= 0.80 \pm 0.06 \pm 0.07 \\
\mathrm{D}^+/\mathrm{D}^0 &= 0.45 \pm 0.08
\end{aligned}$$

The experimental systematic errors on $\sigma \times BR$ have been treated as independent between D^0 and D^+; the $\mathrm{c}\overline{\mathrm{c}}$ cross section is irrelevant. Thus one expects the sum of $\mathrm{D_s}$ and charm baryons to contribute to $20 \pm 9\%$ of all charm decays. How to split this into the two separate contributions is unclear.

4.2.3 Decay modes of charmed hadrons

For the decay branching ratios, the classification into distinct charged topologies is a useful starting point, as measurements of these have been made, whereas the tables of branching ratios into individual final states is not complete. What additional information should be supplied is not yet clear. Again, for ease of combination and recalculation of $\Gamma_{b\overline{b}}/\Gamma_{had}$ results with different sets of assumptions, the assumptions and the charm tagging efficiency into each charged topology for each charm species should be quoted.

4.3 Further errors from the light quark events

- $b\overline{b}$ and $c\overline{c}$ production from gluon splitting (also in $b\overline{b}$ and $c\overline{c}$ events). OPAL varied the rate by 100% of the JETSET parton shower rate. ALEPH compared JETSET parton shower with JETSET matrix element, which gave roughly a 50% difference for $b\overline{b}$ and 100% for $c\overline{c}$ production. We suggest that this PS vs. ME approach is taken by the other experiments.
- Rate of production of light hadrons with long lifetimes: charged and neutral. ALEPH measured the rate of V0 production. They took the difference between this and the rate in the MC (10%) to apply to charged and neutral hadrons with lifetime, and used this to estimate a systematic error. OPAL also took their measured rates of K0 and Lambda production and used their errors ($\pm 7\%$ and $\pm 20\%$ respectively) in assessing the systematic error. These variations lead to a rather small systematic error, which should be taken as correlated between the experiments.

5 Hemisphere Charge Measurements

Measurements of $B^0\overline{B^0}$ mixing, $A_{FB}^{b\overline{b}}$ and the combined hadronic charge asymmetry have been reported using a "hemisphere charge" technique to infer the charge of the produced quark. In the heavy flavour sector, samples of $b\overline{b}$ enriched events have been selected using a lepton tag for mixing measurements and using a lifetime tag for the $b\overline{b}$ asymmetry. The methods used by the LEP collaborations differ in their use of the mean and b-quark hemisphere charge measurements in data. There are still major areas of common systematics however, which may be broken into the following categories :

- Systematic uncertainties arising from lepton tagging : The source of error on mixing measurements using the hemisphere charge method from the sample composition are generally identical to that of other lepton measurements. These contributions should be stated explicitly and may be treated as fully correlated.
- Systematic uncertainties arising from lifetime tagging : Again, in the case of $A_{FB}^{b\overline{b}}$ measurements, the sources of systematic uncertainty on the b-purity are fully correlated with that of other lifetime tag measurements, eg. $\Gamma_{b\overline{b}}/\Gamma_{had}$ and so should be stated separately.
- Systematic uncertainties from fragmentation : In the case of measurements where the degree of charge retention is estimated from Monte Carlo (typically using JETSET) then the systematic errors obtained from the variation of model parameters may be assumed to be correlated between experiments. In the case of b-quark hemisphere charge measurements in data, either using a lepton or lifetime tag, often similar types of systematic uncertainties are involved for background corrections or hemisphere correlation errors. As a consequence, in all cases the Monte Carlo statistical and model dependent errors should be quoted explicitly.

- Systematic uncertainties arising from χ : These errors fall into two categories depending on whether lepton-signed hemisphere charges are used to constrain the mean b-quark hemisphere charge from data. In both cases the world average χ should be used and the error stated separately from the model parameter variation above. In addition the opposite dependence of $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ on χ relative to lepton asymmetry measurements should be taken into account.

These are the dominant systematic contributions to hemisphere charge measurements to date and so safely contain the bulk of any correlation between experimental systematics. Light quark uncertainties are typically small, suppressed by the ratio of the sum of b- and c-quark purities to that of the light flavours. This generally renders them negligible. However, the c-quark hemisphere charge is typically taken from JETSET predictions and so contributes to the fragmentation uncertainties described above.

6 Event shape measurements

Many of the sources of error already listed also have a direct effect on event shape measures. As for the jet charge measurements, additional uncertainties from the light quark sector may become relevant, depending on the details of the tagging method. These should be quoted separately.

7 Summary of sources of systematic uncertainty

The following tables are intended to remind the reader of all the sources of error which have been discussed, not all of which are relevant for a particular analysis. Authors of papers are encouraged to provide a full breakdown of errors rather than combining errors from similar effects in quadrature, so that a complete treatment of the correlations between measurements is possible.

The first example (tables 2 and 3) is for an analysis which fits several electroweak parameters simultaneously. The correlation matrix for the electroweak parameters and the other fitted parameters is provided. It should be made explicit whether the correlation matrix includes statistical errors only, or also takes into account additional correlations arising from systematic effects. A table listing all the sources of systematic uncertainties and the resulting $\pm 1\sigma$ errors follows. One or two lines have been filled in as a reminder that the sign of the error is also important. N/A indicates a non-applicable source of error.

	$\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$	$\Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}}$	$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$	$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$	χ_M	$\mathrm{Br}(\mathrm{b}\to\ell)$
$\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$	1.	0.1	0.	0.	-0.2	-0.9
$\Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}}$	0.1	1.	0.	0.	0.	0.
$A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$	0.	0.	1.	0.3	0.5	0.
$A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$	0.	0.	0.3	1.	0.	0.
χ_M	-0.2	0.	0.5	0.	1.	0.
$\mathrm{Br}(\mathrm{b}\to\ell)$	-0.9	0.	0.	0.	0.	1.

Table 2: A fictitious correlation matrix for a hypothetical many parameter fit to lepton data.

The second example (tables 4 and 5) is for a measurement of $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$ alone, the correlation matrix indicates the variation of $\Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$ arising from a change in $\Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}}$, and no systematic error due to variations in $\Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}}$ is explicitly given in the second table.

	$\Gamma_{b\overline{b}}/\Gamma_{had}$	$\Gamma_{c\overline{c}}/\Gamma_{had}$	$A_{FB}^{b\overline{b}}$	$A_{FB}^{c\overline{c}}$	χ_M	$Br(b \to \ell)$
b fragmentation	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$
c fragmentation $\langle x_c \rangle \pm 0.2$	∓0.002	±0.003	±0.001	∓0.001	±0.001	$\pm0.04\%$
$Br(b \to \ell)$	N/A	N/A	N/A	N/A	N/A	N/A
$Br(b \to c \to \ell)$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$
$Br(c \to \ell)$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$
Semilept. model $b \to \ell$ ($^{+ISGW}_{-ISGW**}$)	±0.003	∓0.010	±0.002	∓0.005	±0.002	$\mp0.5\%$
Semilept. model $b \to c \to \ell$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$
Semilept. model $c \to \ell$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$	$\pm\sigma$
D^0 fraction in $c\overline{c}$ events	N/A	N/A	N/A	N/A	N/A	N/A
D^+ fraction in $c\overline{c}$ events	N/A	N/A	N/A	N/A	N/A	N/A
D_s fraction in $c\overline{c}$ events	N/A	N/A	N/A	N/A	N/A	N/A
Λ_c fraction in $c\overline{c}$ events	N/A	N/A	N/A	N/A	N/A	N/A
D^0 lifetime	N/A	N/A	N/A	N/A	N/A	N/A
D^+ lifetime	N/A	N/A	N/A	N/A	N/A	N/A
D_s lifetime	N/A	N/A	N/A	N/A	N/A	N/A
Λ_c lifetime	N/A	N/A	N/A	N/A	N/A	N/A
D decay multiplicity	N/A	N/A	N/A	N/A	N/A	N/A
$b\overline{b}$ and $c\overline{c}$ from gluon splitting	N/A	N/A	N/A	N/A	N/A	N/A
Rate of long-lived light hadrons	N/A	N/A	N/A	N/A	N/A	N/A
Light quark fragmentation	N/A	N/A	N/A	N/A	N/A	N/A

Table 3: A fictitious table of systematic errors for a hypothetical many parameter fit to lepton data.

	$\Gamma_{b\overline{b}}/\Gamma_{had}$	$\Gamma_{c\overline{c}}/\Gamma_{had}$
$\Gamma_{b\overline{b}}/\Gamma_{had}$	1.	- 0.1
$\Gamma_{c\overline{c}}/\Gamma_{had}$	- 0.1	1.

Table 4: Correlation matrix between electroweak parameters for a hypothetical $\Gamma_{b\overline{b}}/\Gamma_{had}$ measurement using a lifetime tag.

8 Corrections to asymmetry results

To avoid problems of interpretation we recommend that the experiments provide the measured experimental values for $A_{FB}^{b\overline{b}}$ and $A_{FB}^{c\overline{c}}$, with the thrust axis as the direction indicator, without correcting for centre-of-mass energy or radiative effects. The expression "experimental value" refers to the measured asymmetry after correcting for contamination of other flavours in the sample and for wrong signing of the thrust axis. This wrong signing includes the effects of any mismeasurement, and features inherent to the tagging method, such as the presence of "wrong sign" $b \to c \to \ell$ decays in a lepton tag for $b\overline{b}$ events. It will be useful to be able to form an average of the asymmetry measured at each energy point, especially when the 1993 data are analysed, so that the energy dependence can be studied in a model independent way.

The correction for $B^0\overline{B^0}$ mixing is a special case. The average mixing parameter, χ, and the asymmetry are measured in a simultaneous fit by some experiments. For the others, the value $A_{FB}^{b\overline{b},mix} = A_{FB}^{b\overline{b}} \times (1 - 2\chi)$ is measured, and then corrected to give $A_{FB}^{b\overline{b}}$ according to some value of χ. One has two choices:

- Form the LEP average values, $\langle A_{FB}^{b\overline{b},mix} \rangle$ and $\langle \chi \rangle$, and then calculate the average $b\overline{b}$ mixing as: $\langle A_{FB}^{b\overline{b}} \rangle = \langle A_{FB}^{b\overline{b},mix} \rangle / (1 - 2\langle \chi \rangle)$.

	$\Gamma_{b\overline{b}}/\Gamma_{had}$
b fragmentation	$\pm\sigma$
c fragmentation	$\pm\sigma$
$\mathrm{Br}(\mathrm{b} \to \ell)$	N/A
$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell)$	N/A
$\mathrm{Br}(\mathrm{c} \to \ell)$	N/A
Semilept. model $\mathrm{b} \to \ell$ $(^{+\mathrm{ISGW}}_{-\mathrm{ISGW**}})$	N/A
Semilept. model $\mathrm{b} \to \mathrm{c} \to \ell$	N/A
Semilept. model $\mathrm{c} \to \ell$	N/A
D^0 fraction in $\mathrm{c\overline{c}}$ events	$\pm\sigma$
D^+ fraction in $\mathrm{c\overline{c}}$ events	$\pm\sigma$
$\mathrm{D_s}$ fraction in $\mathrm{c\overline{c}}$ events	$\pm\sigma$
Λ_c fraction in $\mathrm{c\overline{c}}$ events	$\pm\sigma$
D^0 lifetime	$\pm\sigma$
D^+ lifetime	$\pm\sigma$
$\mathrm{D_s}$ lifetime	$\pm\sigma$
Λ_c lifetime	$\pm\sigma$
D decay multiplicity	$\pm\sigma$
$\mathrm{b\overline{b}}$ and $\mathrm{c\overline{c}}$ from gluon splitting	$\pm\sigma$
Rate of long-lived light hadrons	$\pm\sigma$
Light quark fragmentation	$\pm\sigma$

Table 5: A fictitious table of systematic errors for a hypothetical $\Gamma_{b\overline{b}}/\Gamma_{had}$ measurement using a lifetime tag.

- Make the average after correcting the value of $A_{FB}^{b\overline{b},mix}$ from each experiment with the value of χ from the same experiment, to form $\langle A_{FB}^{b\overline{b}} \rangle = \langle A_{FB}^{b\overline{b},mix}/(1-2\chi)\rangle$.

It has been demonstrated that there is a negligible statistical gain from following the first procedure. More important is to treat correctly the correlations between $A_{FB}^{b\overline{b},mix}$ and χ, independent of the order of the averaging. This is because because many systematic effects lead to uncertainties in $A_{FB}^{b\overline{b},mix}$ and χ which partially cancel when calculating $A_{FB}^{b\overline{b}}$.

Events containing a hard radiated gluon can alter the correlation between the directions of the thrust axis and the primary b quark. This leads to a theoretical uncertainty from QCD corrections to $A_{FB}^{b\overline{b}}$ of the form :

$$\left(A_{FB}^{b\overline{b}}\right)^{corrected} = \left(1 - C^{mass}\, C^{thrust}\, \frac{\alpha_s}{\pi}\right) \left(A_{FB}^{b\overline{b}}\right)^{no\,QCD} \quad (1)$$

where C^{mass} and C^{thrust} are corrections to take into account the effects of massive quarks and use of the thrust axis to approximate the b quark direction. To first order in α_s, this gives a relative $A_{FB}^{b\overline{b}}$ correction of 2.7% at the parton level [14]. The applicability of this correction depends on the experiment and the type of measurement, semi-leptonic or hemisphere-charge based.

The lepton tag measurements all quote the asymmetry derived from the event thrust axis, signed by the lepton charge. The effects of experimental resolution of the thrust axis direction can be included as a systematic error, for example from a comparison of the b-quark direction in JETSET with the detector level thrust axis direction. An overall QCD correction can then be applied to the LEP average asymmetry.

In hemisphere-charge measurements where the mean b quark hemisphere-charge is extracted from data, the bulk of the asymmetry dilution of (1) is included in $\langle Q_{FB}^{btag} \rangle$. The remaining components are due to the thrust axis resolution in events close to the $\cos\theta = 0$ boundary and the separations for the lighter quarks. The systematic uncertainty on the mean hemisphere-charge for u,d,s,c quarks reflect this. In both respects, current Monte Carlo generators are likely to be sufficient to estimate the systematic uncertainty. Care must be taken not to effectively apply a QCD correction twice over to these measurements.

A Semileptonic decay models

The following data statements include the semileptonic decay models generated by OPAL using the proposed parameters.

```
*-- b to l spectra. ALT   is Altarelli with CLEO parameters
*--                 ISGW1 is standard ISGW (11% D**)
*--                 ISGW2 is ISGW** (32% D**)
*--                 +E for electrons, +M for muons
*-- 40 bins, pmax = 2.5 GeV/c
       DATA ALTE /
      +.310659E-01,.274147    ,.742908    ,1.41930    ,2.28581    ,
      +3.32519    ,4.52032    ,5.85413    ,7.30951    ,8.86920    ,
      +10.5158    ,12.2316    ,13.9985    ,15.7979    ,17.6107    ,
      +19.4167    ,21.1947    ,22.9221    ,24.5742    ,26.1240    ,
      +27.5410    ,28.7901    ,29.8306    ,30.6135    ,31.0800    ,
      +31.1585    ,30.7598    ,29.7842    ,28.1407    ,25.7055    ,
      +22.4321    ,18.3790    ,13.7356    ,8.93910    ,4.61080    ,
      +1.47989    ,.116056    ,0.0        ,0.0        ,0.0        /
       DATA ALTM /
      +.273760E-01,.248424    ,.682263    ,1.31531    ,2.13318    ,
      +3.12104    ,4.26382    ,5.54615    ,6.95242    ,8.46670    ,
      +10.0728    ,11.7539    ,13.4930    ,15.2724    ,17.0734    ,
      +18.8768    ,20.6617    ,22.4061    ,24.0856    ,25.6734    ,
      +27.1391    ,28.4477    ,29.5580    ,30.4204    ,30.9752    ,
      +31.1487    ,30.8492    ,29.9727    ,28.4226    ,26.0662    ,
      +22.8470    ,18.8122    ,14.1410    ,9.26797    ,4.82453    ,
      +1.56975    ,.126766    ,0.0        ,0.0        ,0.0        /
       DATA ISGW1E /
      +.324692E-01,.286189    ,.774610    ,1.47847    ,2.37966    ,
      +3.46097    ,4.70592    ,6.09869    ,7.62394    ,9.26673    ,
      +11.0123    ,12.8461    ,14.7532    ,16.7182    ,18.7249    ,
      +20.7560    ,22.7920    ,24.8111    ,26.7883    ,28.6943    ,
      +30.4944    ,32.1473    ,33.6039    ,34.8048    ,35.6797    ,
      +36.1447    ,36.1018    ,35.4380    ,34.0275    ,31.7392    ,
      +28.4539    ,24.1048    ,18.7655    ,12.7946    ,6.62022    ,
      +1.55524    ,.569202E-01,0.0        ,0.0        ,0.0        /
       DATA ISGW1M /
      +.285287E-01,.258600    ,.709430    ,1.36654    ,2.21514    ,
      +3.24060    ,4.42859    ,5.76506    ,7.23624    ,8.82855    ,
      +10.5285    ,12.3225    ,14.1968    ,16.1369    ,18.1275    ,
      +20.1519    ,22.1917    ,24.2256    ,26.2295    ,28.1745    ,
      +30.0267    ,31.7449    ,33.2802    ,34.5730    ,35.5523    ,
      +36.1327    ,36.2134    ,35.6775    ,34.3928    ,32.2190    ,
      +29.0238    ,24.7233    ,19.3714    ,13.3162    ,6.97085    ,
      +1.66899    ,.621726E-01,0.0        ,0.0        ,0.0        /
       DATA ISGW2E /
      +.503797E-01,.437272    ,1.16443    ,2.18505    ,3.45527    ,
      +4.93392    ,6.58241    ,8.36467    ,10.2472    ,12.1989    ,
      +14.1912    ,16.1980    ,18.1951    ,20.1608    ,22.0745    ,
```

```
     +23.9174    ,25.6713    ,27.3181    ,28.8388    ,30.2131    ,
     +31.4172    ,32.4233    ,33.1973    ,33.6974    ,33.8719    ,
     +33.6574    ,32.9775    ,31.7427    ,29.8533    ,27.2095    ,
     +23.7369    ,19.4475    ,14.5799    ,9.78819    ,5.06447    ,
     +1.18976    ,.435440E-01,0.0        ,0.0        ,0.0        /
      DATA ISGW2M /
     +.441731E-01,.394304    ,1.06428    ,2.01557    ,3.21003    ,
     +4.61083    ,6.18276    ,7.89249    ,9.70865    ,11.6019    ,
     +13.5451    ,15.5131    ,17.4825    ,19.4319    ,21.3416    ,
     +23.1925    ,24.9666    ,26.6459    ,28.2112    ,29.6419    ,
     +30.9141    ,31.9994    ,32.8633    ,33.4630    ,33.7455    ,
     +33.6457    ,33.0841    ,31.9670    ,30.1882    ,27.6389    ,
     +24.2319    ,19.9632    ,15.0579    ,10.1872    ,5.33270    ,
     +1.27678    ,.475620E-01,0.0        ,0.0        ,0.0        /
*-- b to c to l spectra.
*--       ALT1  has ms = 1 MeV  pf = 467 MeV
*--       ALT2           1           353
*--       ALT3           153         467
*--       +E for electrons, +M for muons
*-- 30 bins, pmax = 3.0 GeV/c
      DATA ALT1E /
     +.665553E-03,.356704E-02,.640844E-02,.776145E-02,.750229E-02,
     +.642592E-02,.507325E-02,.380289E-02,.269718E-02,.191305E-02,
     +.132669E-02,.900415E-03,.598052E-03,.390493E-03,.245066E-03,
     +.149487E-03,.858697E-04,.475059E-04,.242809E-04,.118297E-04,
     +.530431E-05,.216814E-05,.637826E-06,.116892E-06,.110190E-07,
     +.288328E-08,.000000E+00,.000000E+00,.000000E+00,.000000E+00/
      DATA ALT1M /
     +.626047E-03,.343072E-02,.625497E-02,.765371E-02,.745148E-02,
     +.641905E-02,.509169E-02,.383165E-02,.272722E-02,.193989E-02,
     +.134903E-02,.917989E-03,.611628E-03,.400601E-03,.252530E-03,
     +.154365E-03,.890591E-04,.495836E-04,.253846E-04,.124862E-04,
     +.559525E-05,.231568E-05,.677402E-06,.125767E-06,.118628E-07,
     +.288328E-08,.000000E+00,.000000E+00,.000000E+00,.000000E+00/
      DATA ALT2E /
     +.867664E-03,.486224E-02,.915847E-02,.115229E-01,.114382E-01,
     +.999929E-02,.801441E-02,.607723E-02,.435193E-02,.310995E-02,
     +.217159E-02,.148294E-02,.990750E-03,.650077E-03,.410197E-03,
     +.250712E-03,.144488E-03,.798366E-04,.408272E-04,.197791E-04,
     +.887736E-05,.357396E-05,.105660E-05,.187565E-06,.177178E-07,
     +.350057E-08,.000000E+00,.000000E+00,.000000E+00,.000000E+00/
      DATA ALT2M /
     +.816362E-03,.467840E-02,.894488E-02,.113720E-01,.113699E-01,
     +.999634E-02,.804950E-02,.612729E-02,.440298E-02,.315517E-02,
     +.220906E-02,.151235E-02,.101349E-02,.667018E-03,.422719E-03,
     +.258899E-03,.149849E-03,.833168E-04,.426761E-04,.208732E-04,
     +.936257E-05,.381729E-05,.112223E-05,.202069E-06,.190968E-07,
     +.350057E-08,.000000E+00,.000000E+00,.000000E+00,.000000E+00/
      DATA ALT3E /
     +.640677E-03,.340795E-02,.605732E-02,.724556E-02,.692319E-02,
```

```
     +.586359E-02,.457773E-02,.339585E-02,.238350E-02,.167576E-02,
     +.115170E-02,.774569E-03,.508303E-03,.327558E-03,.201459E-03,
     +.121595E-03,.683108E-04,.363076E-04,.183791E-04,.834029E-05,
     +.376279E-05,.137143E-05,.420152E-06,.630992E-07,.599892E-08,
     +.000000E+00,.000000E+00,.000000E+00,.000000E+00,.000000E+00/
      DATA ALT3M /
     +.602625E-03,.327744E-02,.591124E-02,.714259E-02,.687297E-02,
     +.585373E-02,.459083E-02,.341841E-02,.240751E-02,.169736E-02,
     +.116971E-02,.788709E-03,.519161E-03,.335570E-03,.207279E-03,
     +.125368E-03,.707390E-04,.378180E-04,.191767E-04,.878664E-05,
     +.396175E-05,.146350E-05,.445883E-06,.682549E-07,.648908E-08,
     +.000000E+00,.000000E+00,.000000E+00,.000000E+00,.000000E+00/
*--  c to l spectra.
*--       ALT1  has ms = 1 MeV  pf = 467 MeV
*--       ALT2           1           353
*--       ALT3           153         467
*--       +E for electrons, +M for muons
*--  16 bins, pmax = 1.0 GeV/c
      DATA ALT1E /
     +.386162E-02,.302082E-01,.718629E-01,.118654    ,.161916    ,
     +.194763    ,.212600    ,.212735    ,.195181    ,.162632    ,
     +.120762    ,.756482E-01,.355203E-01,.868398E-02,.108848E-03,
     +.000000E+00/
      DATA ALT1M /
     +.363106E-02,.287513E-01,.692039E-01,.115390    ,.158803    ,
     +.192491    ,.211651    ,.213306    ,.197143    ,.165560    ,
     +.124032    ,.785463E-01,.374519E-01,.945793E-02,.108848E-03,
     +.000000E+00/
      DATA ALT2E /
     +.490289E-02,.393273E-01,.960560E-01,.163092    ,.229219    ,
     +.284303    ,.320057    ,.330315    ,.312086    ,.267014    ,
     +.202385    ,.128463    ,.602850E-01,.142183E-01,.132154E-03,
     +.000000E+00/
      DATA ALT2M /
     +.460540E-02,.373837E-01,.923808E-01,.158394    ,.224509    ,
     +.280607    ,.318198    ,.330752    ,.314796    ,.271453    ,
     +.207582    ,.133202    ,.634764E-01,.154643E-01,.132154E-03,
     +.000000E+00/
      DATA ALT3E /
     +.372897E-02,.290699E-01,.688707E-01,.113162    ,.153526    ,
     +.183353    ,.198316    ,.196356    ,.177565    ,.145058    ,
     +.104466    ,.624479E-01,.267350E-01,.503893E-02,.000000E+00,
     +.000000E+00/
      DATA ALT3M /
     +.350866E-02,.276902E-01,.663784E-01,.110144    ,.150704    ,
     +.181372    ,.197604    ,.197056    ,.179508    ,.147800    ,
     +.107389    ,.648973E-01,.282136E-01,.549285E-02,.000000E+00,
     +.000000E+00/
```

B Fitting procedure for the $\mathrm{c} \to \ell$ semileptonic decay model

The LEP EW HF group agreed that the best procedure to follow for extracting the systematic error due to $\mathrm{c} \to \ell$ is to use the DELCO and MARK III data and parametrise their uncertainty using the ACCMM model.

We therefore fit these spectra to extract the 2 parameters of the model, m_{s} and p_{f} using MINUIT. We correct for the boost of the D meson, the final state radiative corrections for the electron and include the detector resolutions. An average momentum resolution of $\sigma_p/p = \sqrt{(0.030)^2 + (0.045p)^2}$ was used when fitting both experiments. The χ^2 of the fit is 45.1 for $(51-2)$ degrees of freedom. As the fitted value of m_{s} is very close to 0, we actually fit to the absolute value of m_{s} to avoid problems at the parameter limits. The data and the fit are shown in Figure 5 and the theory before and after smearing, including the D meson boost, is shown in Figure 6. The fit values for m_{s} and p_{f} are:

$$m_{\mathrm{s}} = 0.001 \pm 0.152\,\mathrm{GeV} \tag{2}$$

$$p_{\mathrm{f}} = 0.467^{+0.205}_{-0.114}\,\mathrm{GeV} \tag{3}$$

We need to extract the uncertainty due to these parameters on a measurement. The procedure we follow is:

- We map the χ^2 distribution of the fit to DELCO and MARK III data in $m_{\mathrm{s}}, p_{\mathrm{f}}$ space (see Figure 7).
- We fit the data in the normal way to extract a measurement (e.g. $B(\mathrm{b} \to \ell\nu X)$) using a number of different values in the $m_{\mathrm{s}}, p_{\mathrm{f}}$ plane (31 in the examples given).
- Using these results we fit a smooth surface to the measurement vs. m_{s} and p_{f} using a polynomial (see Figure 8).
- We pick a large number of points evenly distributed on the $m_{\mathrm{s}}, p_{\mathrm{f}}$ plane, weight the corresponding measurement by $e^{-(\chi^2-\chi^2_{min})/2}$ and plot the distribution of the measurement. This weight corresponds to the probability of obtaining $(\chi^2 - \chi^2_{min})$ greater than this value for 2 degrees of freedom. We have also checked that restricting the points to the part of the plane $\chi^2 < (\chi^2_{min}+1)$, $\chi^2 < (\chi^2_{min}+4)$ and $\chi^2 < (\chi^2_{min}+9)$ gives consistent results.
- From the r.m.s. of this distribution we obtain the systematic error due to $\mathrm{c} \to \ell$. The distributions for $B(\mathrm{b} \to \ell\nu X)$, $\varepsilon_b(z)$ and $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ are shown in Figure 9.

While this procedure is probably the best one to follow it necessitates a relatively large number of fits and needs some work for each experiment to implement and check. We have therefore attempted to see whether we can give a set of points (preferably 2) in the $m_{\mathrm{s}}, p_{\mathrm{f}}$ plane that give a reasonable estimate of the systematic error.

We have used the above procedure to calculate the error on $B(\mathrm{b} \to \ell\nu X)$ fitting the p and p_t distributions of muons, the Peterson fragmentation parameter $\varepsilon_b(z)$, and the b quark asymmetry. We cross-checked the $B(\mathrm{b} \to \ell\nu X)$ result using the counting method and it gives consistent results. **Note that the values given for the parameters and their errors are not official L3 results, they just serve as examples.**

We can then plot the lines in the m_s, p_f plane that correspond to constant values of the measurement and the measurement $\pm 1\sigma$. If these lines are identical for all measurements, or would all intersect at a point, then it would be simple to give m_s, p_f values. The curves are shown in Figure 10. As can be seen the curves do look very similar. We therefore plotted the results vs p_f and overlayed lines corresponding to the 1σ uncertainty in the measurement and in p_f. These are shown in Figure 11. It is clear that the 1σ error on p_f provides a very good estimate on the systematic error on a measurement due to $c \to \ell$. We note that this conclusion applies to a sample with a hard p_t cut (1.5 GeV not including the lepton in the jet), and only 13% charm contamination ($c \to \ell$ and $b \to c \to \ell$). We have not done the same test for a less pure sample.

One worry is that changing the value of p_f does not change the endpoint of the lepton spectrum. We therefore also evaluated the change in the 3 measurements for $p_f = 0.467$ GeVand $m_s = 0.153$ GeV. The measurements are also shown in Figure 11. It can be seen that replacing the $+1\sigma$ point on p_f by $m_s = 0.153$ GeV and $p_f = 0.467$ GeV appears a reasonable choice for $B(b \to \ell\nu X)$ but is not as good for the fragmentation and asymmetry measurements.

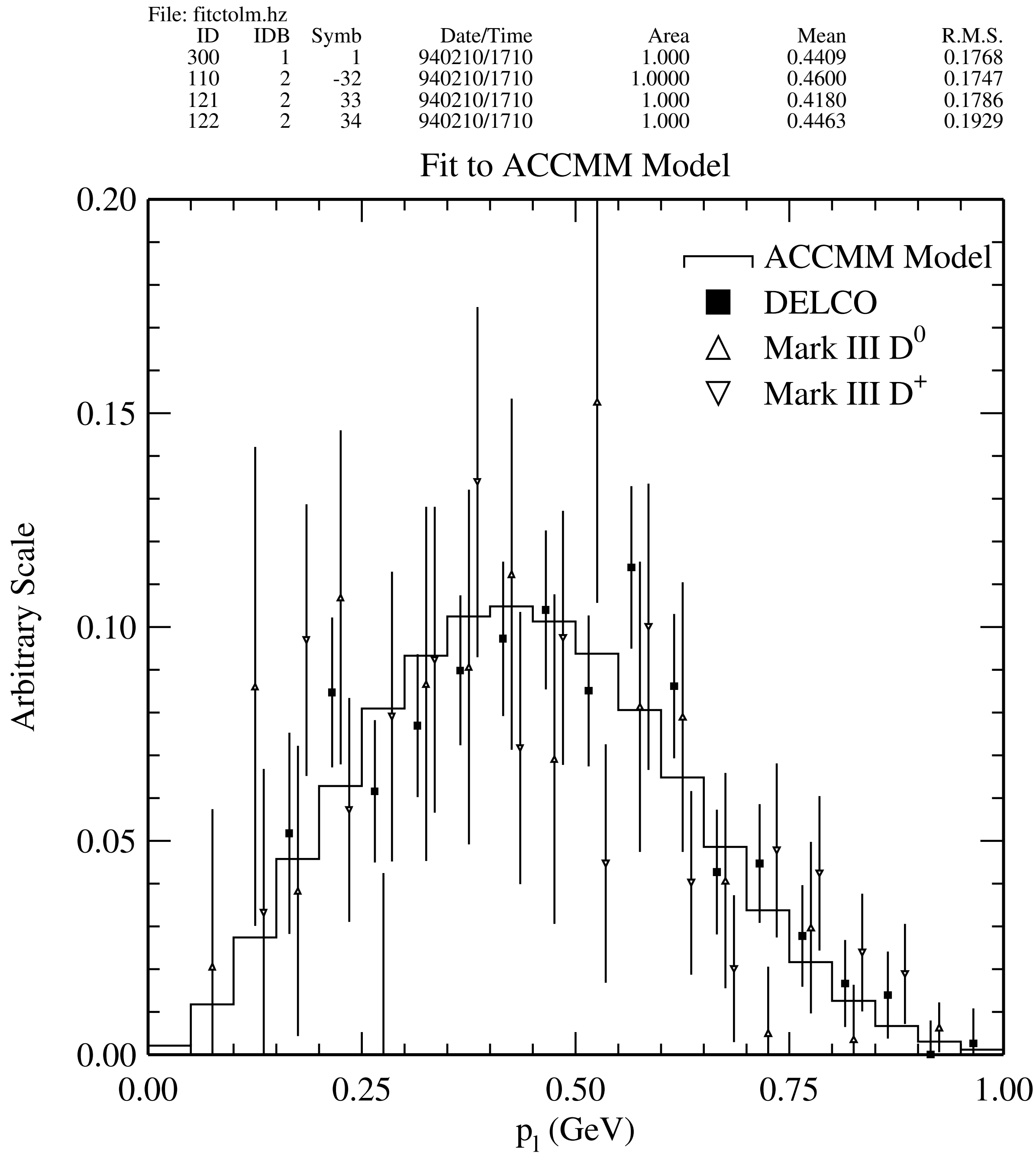

Figure 5: DELCO and MARK III data and the ACCMM spectrum.

File: fitctolm.hz

ID	IDB	Symb	Date/Time	Area	Mean	R.M.S.
300	1	-1	940210/1710	1.000	0.4409	0.1768
310	1	-2	940210/1710	1.0000	0.4416	0.1667

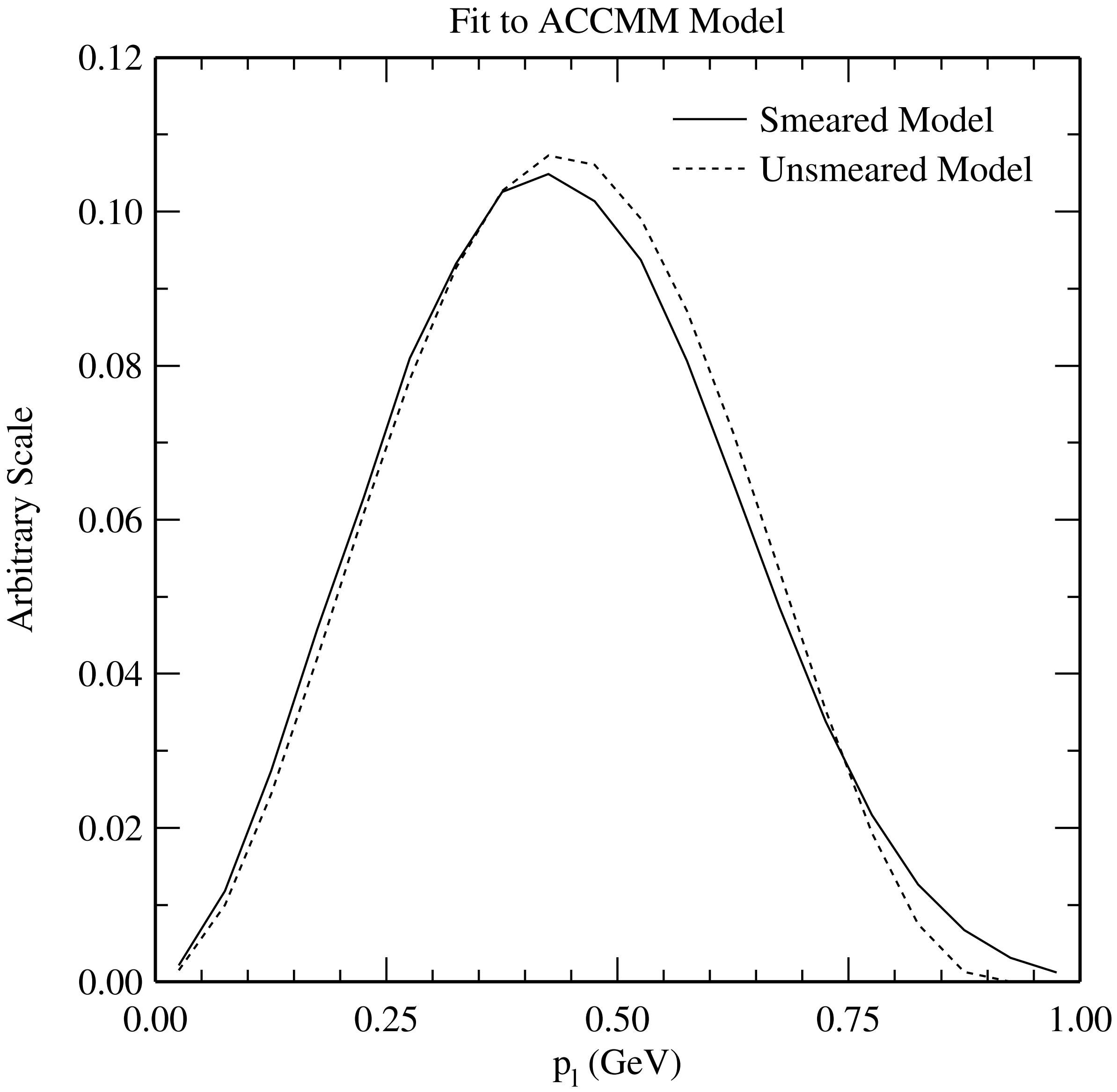

Figure 6: ACCMM Model before and after smearing.

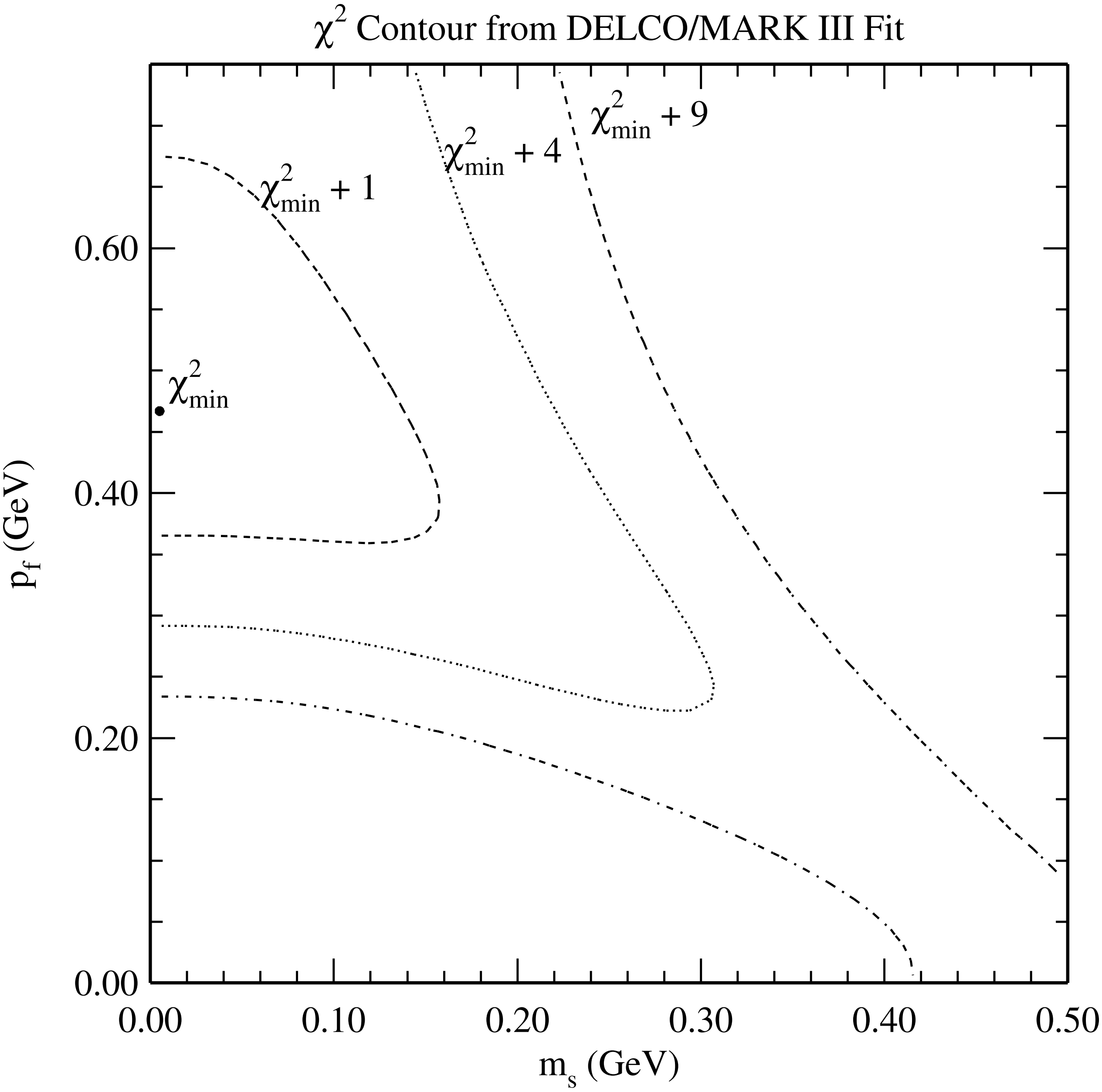

Figure 7: χ^2 contours from the fit to DELCO and MARK III data.

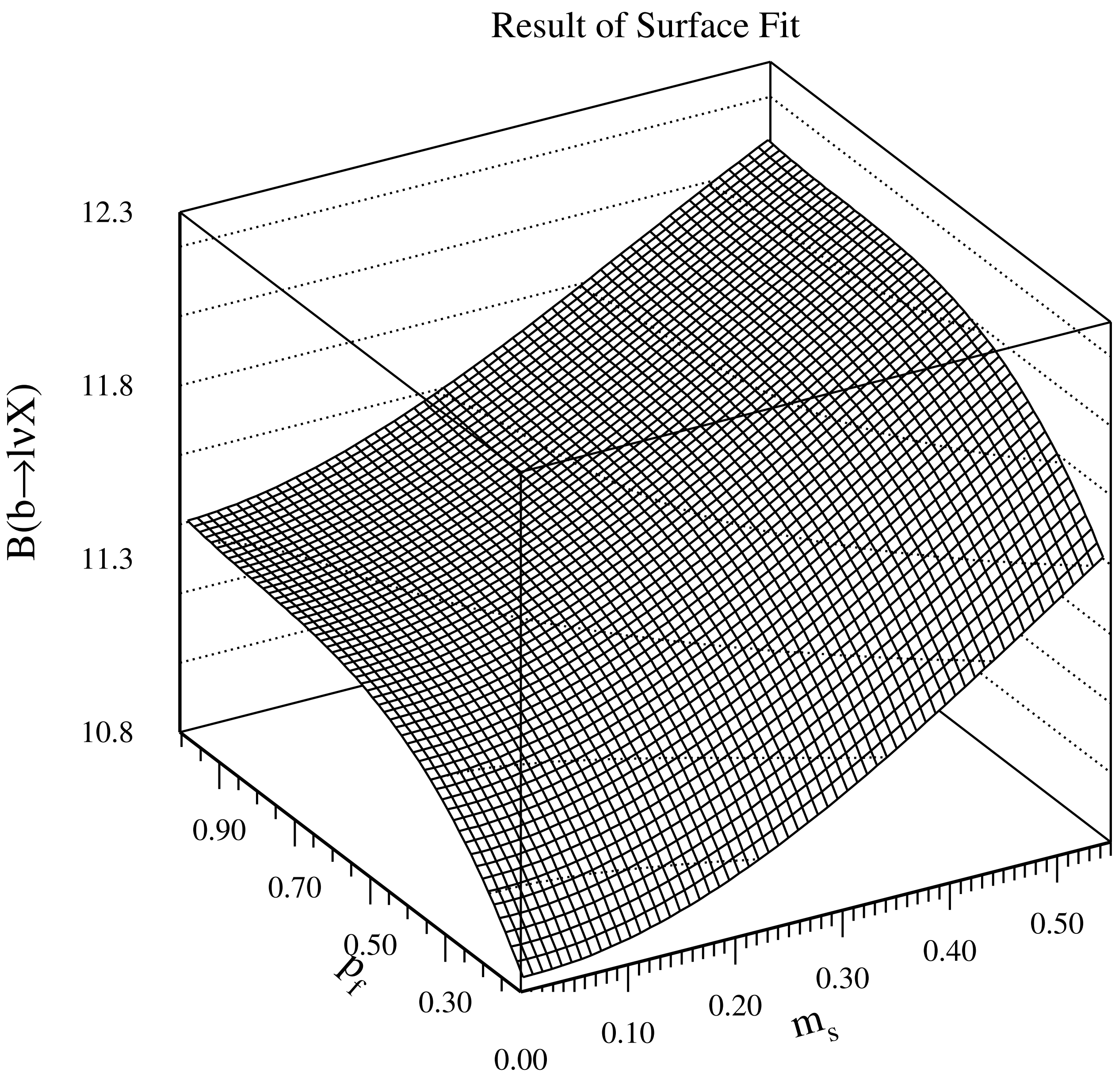

Figure 8: Surface of $B(\mathrm{b} \to \ell\nu X)$ in the $m_{\mathrm{s}}, p_{\mathrm{f}}$ plane.

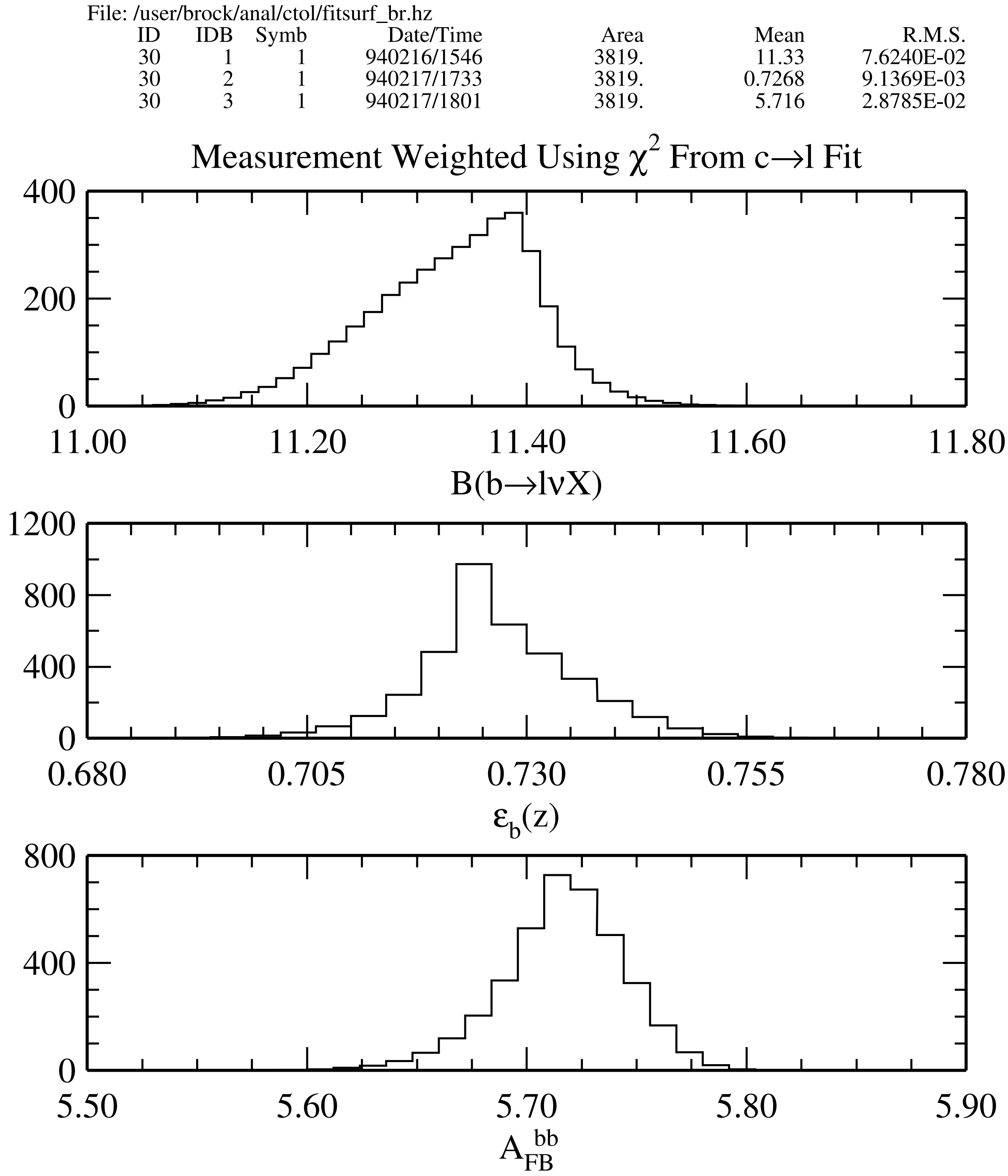

Figure 9: Weighted distribution of a) $B(\mathrm{b} \to \ell\nu X)$, b) $\varepsilon_b(z)$ and c) $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ using the fit to the DELCO and MARKIII data.

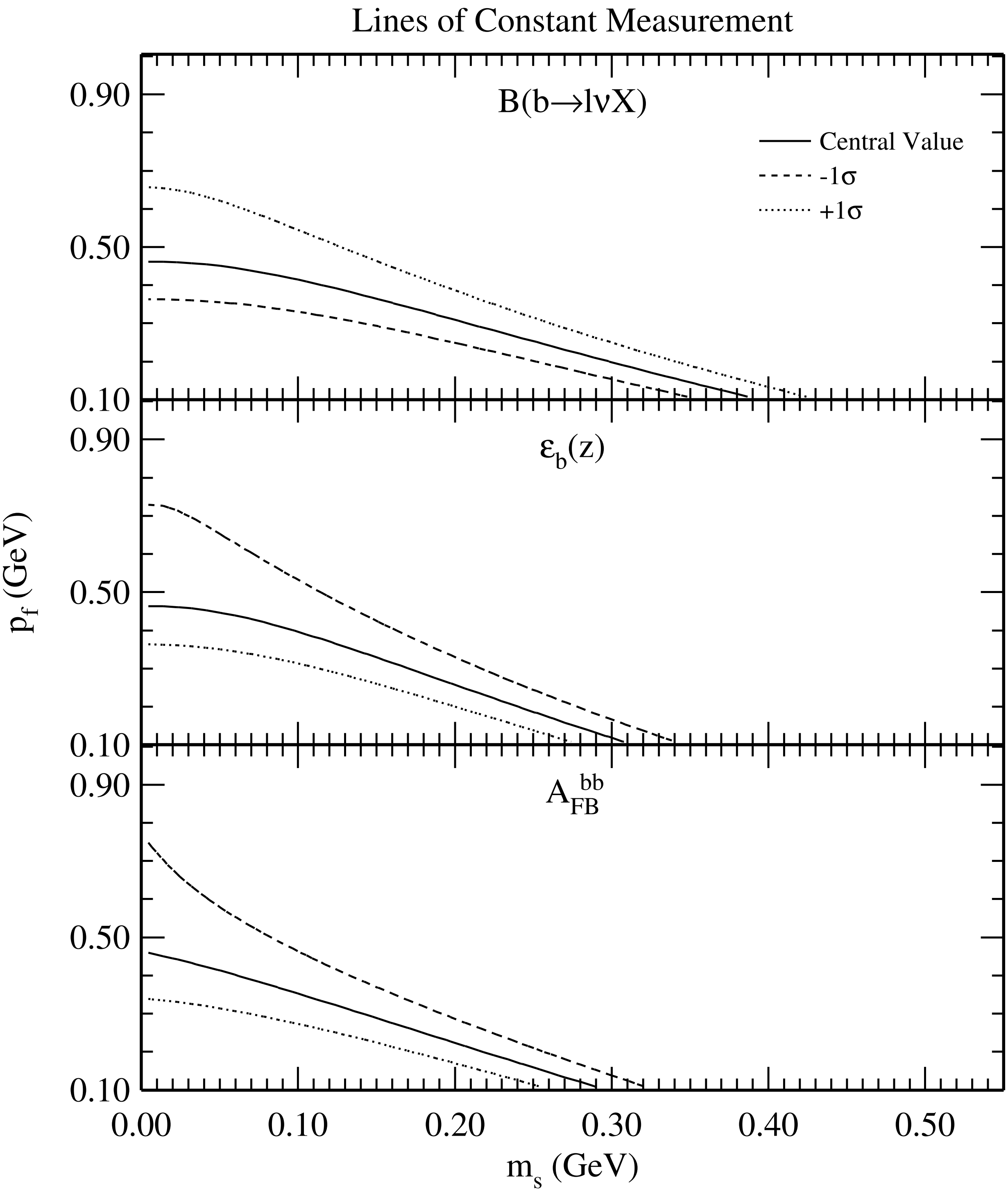

Figure 10: Lines of constant value for a) $B(\mathrm{b} \to \ell\nu X)$, b) $\varepsilon_b(z)$ and c) $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$. The solid line shows the central value, the dashed one $+1\sigma$ and the dotted one -1σ.

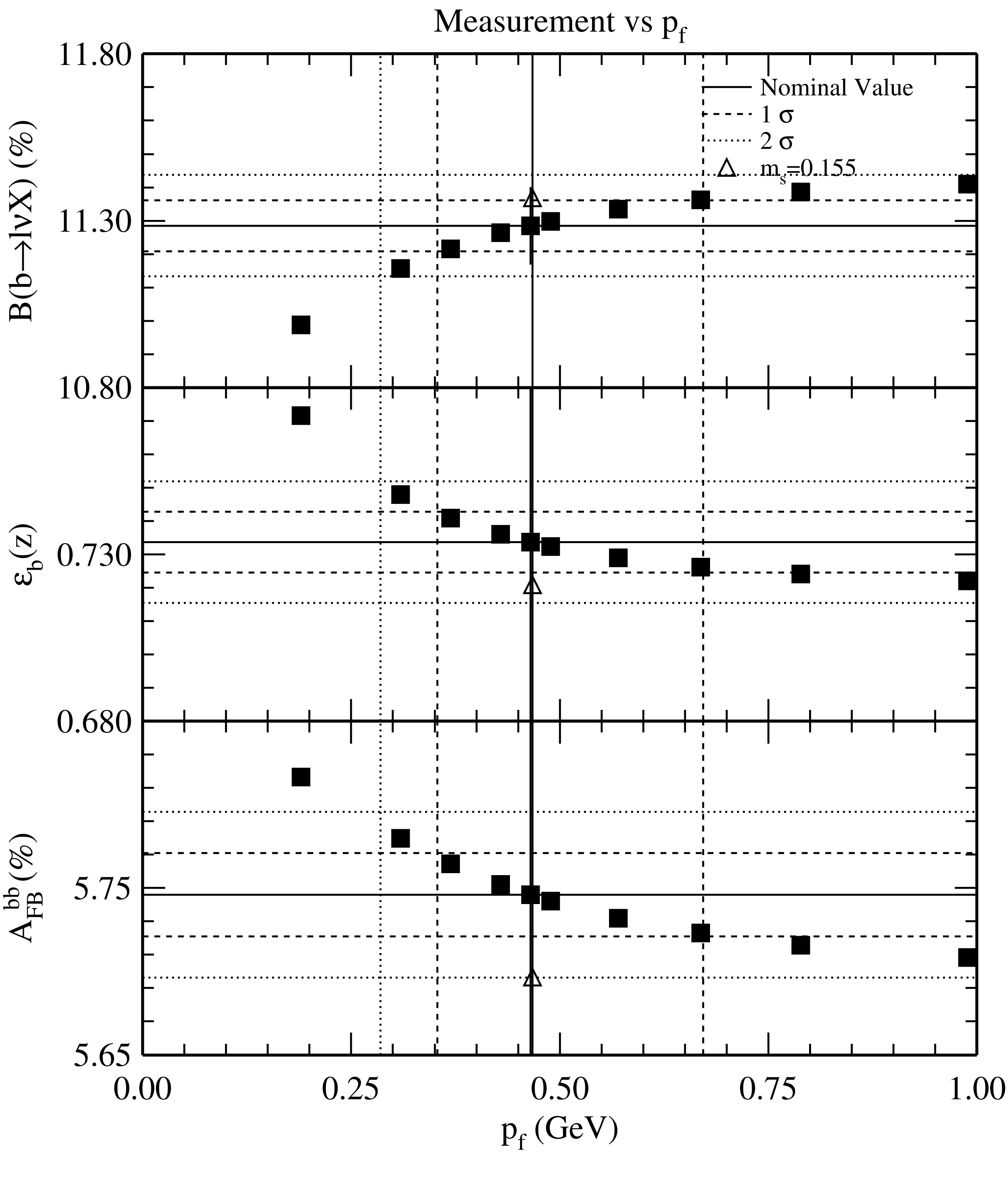

Figure 11: a) $B(\mathrm{b} \to \ell\nu X)$, b) $\varepsilon_b(z)$ and c) $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ vs p_{f}. The solid squares are the measurements for $m_{\mathrm{s}} = 0.001$ GeV and varying p_{f}. The open triangle shows the measurement with $m_{\mathrm{s}} = 0.155$ GeV and $p_{\mathrm{f}} = 0.467$ GeV.

C Input data for the $c \to \ell$ semileptonic decay model

*** ctol.delco.spectrum

.025	.00	.00
.075	.00	.00
.125	.00	.00
.175	19.59	8.89
.225	32.05	6.64
.275	23.30	6.30
.325	29.12	6.30
.375	34.00	6.64
.425	36.81	6.83
.475	39.36	7.02
.525	32.20	6.68
.575	43.11	7.17
.625	32.61	6.38
.675	16.17	5.52
.725	16.92	5.25
.775	10.51	4.50
.825	6.30	3.86
.875	5.29	3.86
.925	.04	3.00
.975	1.01	3.08

*** ctol.mark3.d0spectrum

.025	.00	.00
.075	4.29	7.66
.125	17.90	11.64
.175	7.96	7.05
.225	22.23	8.12
.275	-.30	9.14
.325	18.02	8.61
.375	18.85	8.61
.425	23.36	8.53
.475	14.37	8.00
.525	31.74	9.77
.575	16.92	7.05
.625	16.42	6.56
.675	8.46	5.23
.725	1.06	3.22
.775	6.18	4.17
.825	.76	2.65
.875	-1.63	1.48
.925	1.33	1.21
.975	.00	.00

*** ctol.mark3.dpspectrum

.025	.00	.00
.075	.00	.00
.125	7.34	7.45
.175	21.44	7.03
.225	12.66	5.78
.275	17.49	7.49
.325	20.42	7.91
.375	29.62	9.05
.425	15.86	7.03
.475	21.56	6.58
.525	9.89	6.16
.575	22.13	7.41
.625	8.90	4.75
.675	4.45	3.80
.725	10.57	4.49
.775	9.39	3.99
.825	5.29	3.04
.875	4.18	2.59
.925	.00	.00
.975	.00	.00

D Fortran program for D hadron production rates

```
      program cprod

      implicit none

      integer nvar,ifail
      parameter(nvar=3)
      double precision f0(nvar),f1(nvar),w(nvar,nvar),v(nvar,nvar)
      double precision g(nvar),gg(nvar,nvar),sig0(nvar),r,sigr,sigzz
      double precision del(nvar),chlast,chi2
*

 1    write(6,*) ' Enter D+/D0 ratio and uncertainty '
      read(5,*) r,sigr
* initialize chi**2 to something big
      chi2= 1111.E+21
* sigzz is the uncertainty on the unitarity constraint; this is a convenience
* only, as the exact solution using Lagrange multipliers can be had
      sigzz = 0.0001
* the f0 are measured values
      f0(1) = .463
      f0(2) = .200
      f0(3) = .25
* the sig0 are uncertainties in the measured values
      sig0(1)=.075
      sig0(2)=.038
      sig0(3)=10.
* the f1 are initial estimates of the fitted values
      f1(1) = .46
      f1(2) = .20
      f1(3) = .25
* the initial weight matrix (i.e. inverted covariance) of the f1 follows.
* If the measurements are correlated, one needs to invert the submatrix
* containing the correlated measurements.
      w(1,1)= 1./sig0(1)**2
      w(1,2)= 0.
      w(2,2)= 1./sig0(2)**2
      w(1,3)= 0.
      w(2,3)= 0.
      w(3,3)= 1./sig0(3)**2
      w(2,1)= w(1,2)
      w(3,1)= w(1,3)
      w(3,2)= w(2,3)
*
*  chi**2 = del W del + (f2/f1-r)**2/sigr**2 + (1-f1-f2-f3)**2/sigzz**2
*
 10   del(1) = f1(1)-f0(1)
      del(2) = f1(2)-f0(2)
```

```
      del(3) = f1(3)-f0(3)
* first derivatives of chi**2 w.r.t. f1
      g(1)  = w(1,1)*del(1) + w(1,2)*del(2) + w(1,3)*del(3)
     +      + (f1(2)/f1(1)-r)/sigr**2*(-f1(2)/f1(1)**2)
     +      - (1.-f1(1)-f1(2)-f1(3))/sigzz**2
      g(2)  = w(1,2)*del(1) + w(2,2)*del(2) + w(2,3)*del(3)
     +      + (f1(2)/f1(1)-r)/sigr**2*(1./f1(1))
     +      - (1.-f1(1)-f1(2)-f1(3))/sigzz**2
      g(3)  = w(1,3)*del(1) + w(3,2)*del(2) + w(3,3)*del(3)
     +      - (1.-f1(1)-f1(2)-f1(3))/sigzz**2

* second derivatives of chi**2 w.r.t. f1
      gg(1,1) = w(1,1) + 1./sigzz**2 + 3.*f1(2)**2/f1(1)**4/sigr**2
     +        - 2.*r*f1(2)/f1(1)**3/sigr**2
      gg(1,2) = w(1,2) + 1./sigzz**2 - 2.*f1(2)/f1(1)**3/sigr**2
     +        + r/f1(1)**2/sigr**2
      gg(2,2) = w(2,2) + 1./sigzz**2 + 1./f1(1)**2/sigr**2
      gg(1,3) = w(1,3) + 1./sigzz**2
      gg(2,3) = w(2,3) + 1./sigzz**2
      gg(3,3) = w(3,3) + 1./sigzz**2
      gg(2,1) = gg(1,2)
      gg(3,1) = gg(1,3)
      gg(3,2) = gg(2,3)

* solution to equation  gg*f=g  will be in g after this call
      call dseqn(nvar,gg,nvar,ifail,1,g)
      if(ifail.ne.0) then
        write(6,*) ' DSEQN failed, ',gg
        goto 1
      endif

      write(6,*) ' Initial values f1: ',f1
c      write(6,*) ' Solution for update: ',g

* update f1 by g
      f1(1)  = f1(1) - g(1)
      f1(2)  = f1(2) - g(2)
      f1(3)  = f1(3) - g(3)
      del(1) = f1(1)-f0(1)
      del(2) = f1(2)-f0(2)
      del(3) = f1(3)-f0(3)

* calculate chi**2
      chlast = chi2
      chi2= del(1)*w(1,1)*del(1) + 2.*del(1)*w(1,2)*del(2) +
     +      del(2)*w(2,2)*del(2) + 2.*del(1)*w(1,3)*del(3) +
     +      del(3)*w(3,3)*del(3) + 2.*del(2)*w(2,3)*del(3) +
     +      (f1(2)/f1(1)-r)**2/sigr**2 +
     +      (1.-f1(1)-f1(2)-f1(3))**2/sigzz**2
      write(6,*) ' chi**2 ',chi2
```

```
* check for convergence
      if(abs(chi2-chlast).gt.0.001) goto 10

* invert gg to get covariance matrix of f1
      call dsfinv(nvar,gg,nvar,ifail)
      if(ifail.ne.0) then
        write(6,*) ' DSFINV failed, ',gg
        goto 1
      endif

      write(6,1001) ' start values for f1: ',f0
      write(6,1001) ' final values for f1: ',f1
      write(6,1002) ' Covariance for f1: ',gg(1,1),gg(1,2),gg(2,2)
     +                                   ,gg(1,3),gg(2,3),gg(3,3)
 1001 format(1x,a,3f12.6)
 1002 format(1x,a,/1x,e12.4,/1x,2e12.4,/1x,3e12.4)

      goto 1
      end
```

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