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LEP Electroweak Heavy Flavour Results for Summer 1994 Conferences

The LEP Electroweak Heavy Flavours Working Group*

Abstract

In this document we present the LEP electroweak results from heavy flavour analyses. Average values for the standard model parameters R_b, R_c, $A_\mathrm{FB}^{\mathrm{b\overline{b}}}$ and $A_\mathrm{FB}^{\mathrm{c\overline{c}}}$ are derived, taking into account the correlated errors amongst the different analyses and amongst the experiments. Care has been taken to use a consistent set of input parameters and variations of these parameters.

The document is based on the analyses available on 14 July 1994. Each LEP experiment has been responsible for providing their results in a common format, and for making explicit which older publications are superseded by their new results.

The combined LEP results are:

$$
\begin{aligned}
R_\mathrm{b} &= 0.2202 \pm 0.0020 \\
R_\mathrm{c} &= 0.1583 \pm 0.0098 \\
\mathrm{A}_\mathrm{FB}^{0,\,\mathrm{b}} &= 0.0967 \pm 0.0038 \\
\mathrm{A}_\mathrm{FB}^{0,\,\mathrm{c}} &= 0.0760 \pm 0.0091
\end{aligned}
$$

There is a large correlation coefficient of -0.4 between R_b and R_c. Fixing R_c to the value predicted by the Standard Model yields:

$$R_\mathrm{b} = 0.2192 \pm 0.0018 \text{ with } R_\mathrm{c} = 0.171 \text{ (fixed)}.$$

Internal Note

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1 Introduction

The ability of experiments to tag b- and c-events allows the ratios of partial widths $R_{\mathrm{b}} = \Gamma_{\mathrm{b\overline{b}}}/\Gamma_{\mathrm{had}}$ and $R_{\mathrm{c}} = \Gamma_{\mathrm{c\overline{c}}}/\Gamma_{\mathrm{had}}$ and the forward-backward asymmetries $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ and $A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$ to be determined directly. Such analyses may also determine other quantities, such as the mixing parameter, $\overline{\chi}$, the fragmentation parameters $\langle x_E \rangle_{\mathrm{b}}$ and $\langle x_E \rangle_{\mathrm{c}}$, and the semileptonic branching fractions for b- and c-quarks. Where these quantities are not measured, they often contribute a systematic uncertainty to the Standard Model measurements. Motivated by the lepton fit analyses, seven parameters have been determined: the four parameters of interest in the electroweak sector, R_{b}, R_{c}, $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ and $A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$, and in addition $\overline{\chi}$, $\mathrm{Br}(\mathrm{b} \to \ell)$ and $\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^{+})$, to take into account their correlations with the electroweak parameters. The asymmetries vary rapidly with centre-of-mass energy. The asymmetry measurements on-peak, above peak and below peak have therefore been combined separately.

The results in the following references, which have previously been used in heavy flavour averages, are now superseded by subsequent results: [1–4]. The current results, which are combined in this note, are tabulated in the appendices.

2 Methods of Tagging Heavy Flavours

The tagging of heavy flavour decays by the identification of a lepton in the decay products is well established. The high mass of the heavy quarks and their hard fragmentation functions lead to the leptons having a hard momentum spectrum, and the large mass leads to a large momentum component transverse to the parent jet axis. The transverse momentum is typically smaller for c-decays than for b-decays, allowing the two to be separated. The disadvantage of the lepton tag is the small ($\sim 10\%$) semileptonic branching ratio for b- and c-quarks, compounded by identification inefficiencies for $\mathrm{e}^{\pm}$ and $\mu^{\pm}$ and the inability to identify cleanly $\tau^{\pm}$ in semileptonic decays, leading to b-quark tagging efficiencies of about 10% for 90% purities. Despite this statistical limitation, the lepton tags still provide competitive electroweak measurements.

The relatively long lifetimes of hadrons containing heavy quarks combined with the boost provided in Z^0 decays, give rise to decay lengths of a few millimeters. This lifetime information may be used to select a sample of heavy flavour events. Typically, either several tracks with impact parameters with respect to the primary vertex that are significantly larger than the measurement errors are required, or else a secondary vertex is sought with a significant decay length. The tagging efficiencies achieved for b-quarks are of order 30% for 90% purity. While such techniques were used before, the installation of silicon microvertex detectors in the LEP experiments have allowed this form of tagging to come to the fore.

Neither lepton tagging nor lifetime tagging allow a clean sample of charm events to be isolated. However, while both b- and c-events may give rise to $\mathrm{D}^{*\pm}$, by selecting $\mathrm{D}^{*\pm}$ carrying a large fraction of the beam energy a relatively pure charm sample is obtained. Lifetime, lepton and event-shape information may also be used to separate the b- and c-sources.

Finally, event shape variables have been used to tag heavy flavour events. Unfortunately, heavy flavour events are not very dissimilar to light quark events in these variables. To improve the accuracy of these methods, neural nets based on several shape variables are often used, but the resulting purities still remain low with respect to the other heavy flavour tags.

The charged leptons have the additional advantage of being a good indicator of the parent parton charge, allowing it to be identified as a quark or an antiquark, which is necessary for asymmetry and mixing measurements. The charge of $D^{*\pm}$ mesons produced in the decays of heavy flavours is also a good indicator of the parent parton charge at the time of decay, and is particularly useful in measuring $A_{\rm FB}^{\rm c\bar{c}}$. A less accurate, but statistically more powerful, estimator of the charge of the initial parton contained in a given thrust hemisphere is its hemisphere charge, Q_{hemi}:

$$Q_{hemi} = \frac{\sum_i^N |p_{||i}|^\kappa q_i}{\sum_i^N |p_{||i}|^\kappa}$$

where the sum runs over the N charged tracks of the hemisphere, $p_{||_i}$ is the momentum component of the track i along the thrust axis, q_i is the charge of track i and κ is a parameter which controls the momentum weighting of each particle's charge, and which is tuned in order to optimize the charge determination. Hemisphere charges are particularly useful in the measurement of $A_{\rm FB}^{\rm b\bar{b}}$, after a sample enriched in $\rm b\bar{b}$ events has been selected using a lifetime tag.

3 Heavy Flavour Analysis Methods

3.1 Double-Lifetime and Combined Tagging Measures of $R_{\rm b}$

$R_{\rm b}$ is currently best measured by analyses that compare the number of events with only one hemisphere tagged with that of events with both hemispheres tagged, ("double-tagging methods"). This is because these methods allow the b-tagging efficiency to be derived directly from the data. Results using a lifetime tag dominate these measurements [10, 11, 13]. Additional statistical precision can be gained by allowing either a lifetime or a lepton to tag a hemisphere [12, 13]. If the number of events with a single hemisphere tagged is N_t and with both hemispheres tagged is N_{tt}, then in the absence of background and correlations in the tagging efficiency between the hemispheres,

$$\begin{aligned} \eta_b &= 2N_{tt}/N_t, \\ R_{\rm b} &= N_t^2/(4N_{tt}N_{tot}), \end{aligned}$$

where η_b is the efficiency to tag a hemisphere in a b-event and N_{tot} is the total number of hadronic Z^0 decays in the sample. Backgrounds obviously complicate this picture, and their level must still be inferred by other means. The correlations in tagging efficiencies between the hemispheres are small for the acceptances considered, but are an important source of systematic uncertainty. The value of $R_{\rm c}$ assumed is another important shared systematic uncertainty. The assumptions about fragmentation and heavy flavour production in u, d, s and c-events are also shared; charm production and decay properties are an important source of uncertainty in these analyses.

The use of the double tagging rates has also been used by ALEPH in their event-shapes analysis for $R_{\rm b}$ [14], again allowing the tagging efficiency to be measured. In this case, the purity of the tag is considerably lower, and so the background determination systematics larger.

3.2 Multiparameter Fits to Lepton Data - $R_{\rm b}$, $R_{\rm c}$, $A_{\rm FB}^{\rm b\bar{b}}$, $A_{\rm FB}^{\rm c\bar{c}}$, $\overline{\chi}$, Br($\rm b \to \ell$) and Br($\rm b \to c \to \ell^+$)

High-statistics samples of leptons, able to tag heavy quark charges, provide a means to determine simultaneously $R_{\rm b}$, $R_{\rm c}$, $A_{\rm FB}^{\rm b\bar{b}}$, $A_{\rm FB}^{\rm c\bar{c}}$ and the mixing parameter, $\overline{\chi}$ [7–9,11,12]. The semileptonic branching ratios $\mathrm{Br}(\mathrm{b} \to \ell)$ and $\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+)$ are also often free parameters in the fit. The composition of the sample in terms of the number of single- and double-tagged events, with the double-tags decomposed into like-sign and unlike-sign in the same and in opposite thrust hemispheres, adds important constraints to the fit. The semileptonic decay model assumed is an important and correlated systematic error between these measurements.

The lepton samples are also used to determine $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ and $A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$, either in the same fit or in a separate fit using the output parameters of the fit for the partial widths as constraints [7–9, 18]. The relative rates of $\mathrm{Z}^0 \to \mathrm{b\overline{b}}$ and $\mathrm{Z}^0 \to \mathrm{c\overline{c}}$ are determined from the momentum and transverse momentum of the lepton, while the initial quark direction is estimated by the polar angle of the thrust axis, signed according to the polar angle and charge of the lepton. This measurement must take account of the effects of mixing, as well as the uncertainties arising from the semileptonic decay modelling and branching ratios.

The fitted parameter values from these fits are clearly correlated, and the common systematic uncertainties lead to correlations between the results from the different experiments. It is therefore commonplace to average the results of these fits as a group, rather than simply including them in the individual parameter averages for each experiment.

3.3 Other Measures of R_{c}

Aside from the multiparameter fits, the most common measure of R_{c} comes from the production rates for charmed hadrons [19, 20]. As these can be produced both from direct charm and in the decay chain of b-hadrons, additional information must be used to separate the two components. The $\mathrm{D}^{*\pm}$ energy spectrum, associated lepton production, lifetime information of the charm decay and in the opposite hemisphere to the $\mathrm{D}^{*\pm}$ and also the shape properties of the hemisphere opposite to the $\mathrm{D}^{*\pm}$ have all been used to this end. The b-fragmentation, decay modelling and branching ratios introduce important systematic uncertainties in these measurements.

3.4 Other Measures of $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ and $A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$

In addition to the asymmetry measurements using a lepton tag, ALEPH, DELPHI and OPAL have presented results for $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$ based on samples of high b purity, obtained with lifetime tagging, and the hemisphere containing the quark or antiquark determined using a hemisphere charge measure, Q_{hemi}. The initial quark direction is again estimated by the thrust axis, with the sense determined by the hemisphere charge in the forward and/or backward hemisphere. ALEPH and OPAL have essentially determined the hemisphere charge properties of b-events from the distributions of the charges themselves [16, 17], while the DELPHI study uses lepton information in the opposite hemisphere to measure the hemisphere charge properties [18]. These hemisphere charge analyses are at a similar level of precision to the lepton fits. They have the advantage of being insensitive to the mixing in the b-events, and have generally different systematics to the lepton fits. The main common systematics in these analyses are the assumed frangmentation properties.

Measurements of $A_{\mathrm{FB}}^{\mathrm{c\overline{c}}}$ have been provided by ALEPH, DELPHI and OPAL using samples tagged by $\mathrm{D}^{*\pm}$ mesons. ALEPH and OPAL use the fraction of the beam energy carried by the $\mathrm{D}^{*\pm}$ to separate the b- and c-components [21, 22], while DELPHI have also used decay length information [23], which introduces an uncertainty due to the modelling of the time dependence of mixing. The DELPHI and OPAL analyses also provide a simultaneous measurement of $A_{\mathrm{FB}}^{\mathrm{b\overline{b}}}$. The dominant common systematics uncertainties arise from the b-decay and fragmentation modelling, and from the charm decay schemes used.

4 Corrections to asymmetries

All asymmetries presented in this chapter are given as measured asymmetries at $\sqrt{s} = 91.26$ GeV corrected to full acceptance and using the thrust axis as an estimate of the quark direction before gluon radiation. To relate $\sin^2\theta_{eff}$ to these numbers a few corrections that are summarized in table 1 have to be applied:

- Energy shift to $\sqrt{s} = M_z$ and QED corrections: The slope of the asymmetry around M_Z depends only on the axial coupling and the charge of the initial and final state fermions and is thus independent of the pole asymmetry itself. The corrections due to the energy shift and initial state radiation have been calculated using ZFITTER [24].
- QCD corrections: The QCD corrections using the thrust axis have been calculated recently by Lampe [25] to be $A_{FB}^{QCD} = A_{FB}^{noQCD}(1 + c\frac{\alpha_s}{\pi})$ with with $c = -0.893$ for massless quarks. Using the quark instead of the thrust direction the correction is reduced due to mass effects by 20% (for $m_b = 4.7 GeV$) [26]. It was suggested in [25] that the same mass correction should be applied in case of the thrust direction. Apart from the uncertainty in the mass dependent correction there is an additional uncertainty due to the fact that the present analyses do not examine carefully how far the QCD corrections are modified due to implicit cuts on events with very hard gluons. For these reasons an error of 0.25 is assigned to c.
- γ exchange, γZ interference: These very small corrections have again been calculated using ZFITTER.

source	δA_{FB}^b	δA_{FB}^c
$\sqrt{s} = M_z$	-0.0013	-0.0034
QED corrections	+0.0041	+0.0104
QCD corrections	$+0.0027 \pm 0.0010$	$+0.0022 \pm 0.0005$
γ, γZ	-0.0003	-0.0008
total	$+0.0052 \pm 0.0010$	$+0.0084 \pm 0.0005$

Table 1: Corrections to be applied to the quark asymmetries. The corrections are to be understood as $A_{FB}^0 = A_{FB}^{meas} + \sum_i(\delta A_{FB})_i$

5 Averaging Procedure

The averaging procedure is as follows. We have a number of input results, r_i, and we are trying to find the best average value for a (smaller) number of parameters, x^μ. These are:

$$\begin{aligned} x^\mu &= R_\mathrm{b}, \\ & R_\mathrm{c}, \\ & \mathrm{Br}(\mathrm{b} \to \ell), \\ & \mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+) = \text{“Br(bc}\ell\text{)”} \\ & \overline{\chi}, \\ & A_{FB}^\mathrm{b}(-2), \\ & A_{FB}^\mathrm{c}(-2), \\ & A_{FB}^\mathrm{b}(\mathrm{pk}), \\ & A_{FB}^\mathrm{c}(\mathrm{pk}), \\ & A_{FB}^\mathrm{b}(+2), \\ & A_{FB}^\mathrm{c}(+2) \end{aligned}$$

Each result, r_i, is a measurement of one of the parameters x^μ, and to indicate this we write $r_i = r_i(\mu)$. We minimise the χ^2:

$$\chi^2 = \sum_{ij} (r_i(\mu) - x^\mu)\, C_{ij}^{-1} (r_j(\nu) - x^\nu)$$

where C is the covariance matrix for all the heavy flavour results.

5.1 Corrections to the results $r_i(\mu)$

There are two classes of corrections to the results $r_i(\mu)$: explicit dependences on one of the other parameters $x^\nu, \nu \neq \mu$, and corrections to asymmetries for QCD and energy dependence.

If a result $r_i(\mu)$ depends on the value of some other electroweak parameter $x^\nu, \nu \neq \mu$, this dependence is put in explicitly. No separate error due to the uncertainty in $x^\nu, \nu \neq \mu$ is assigned to $r_i(\mu)$. For example, the value of R_b almost always depends on the value of R_c. If for a particular measurement, $r_k(R_\mathrm{b})$, this dependence was quoted as:

$$R_\mathrm{b} = {R_\mathrm{b}}^0 + a_k \frac{dR_\mathrm{c}}{R_\mathrm{c}}$$

where ${R_\mathrm{b}}^0$ is the value of R_b with $R_\mathrm{c} = 0.171$, then in the χ^2, for this measurement we make the substitution:

$$r_k(R_\mathrm{b}) \to r_k(R_\mathrm{b}) + a_k \frac{x^{R_\mathrm{c}} - 0.171}{0.171}$$

These dependences are given in the tables. (For some measurements the dependence is expressed by quoting the change in the result $r_i(\mu)$ which is derived by varying one of the other electroweak parameters in a specified range.)

The asymmetries were propagated to the following centre-of-mass energies using the predicted dependence from ZFITTER:

$$\begin{aligned} \sqrt{s} &= 89.55\mathrm{GeV}\ (\text{“peak–2”}), \\ \sqrt{s} &= 91.26\mathrm{GeV}\ (\text{“peak”}), \\ \sqrt{s} &= 92.94\mathrm{GeV}\ (\text{“peak+2”}) \end{aligned}$$

These values were chosen as being close to all the $\sqrt{s}$ values at which the measurements were made, and the corrections are in general small. The off-peak points were often only quoted at one average centre-of-mass energy. The ALEPH numbers were provided at each energy point, and each corrected to the below peak, on peak or above peak default quoted above.

The lifetime/jet-charge measurements of asymmetries take into account QCD effects as an inherent part of the analysis, but the measured asymmetries for the analyses using a lepton or $\mathrm{D}^{\star}$ tag need to be corrected for the effects of QCD on the event thrust axis direction. So that the averages could be made at the same level, the jet-charge measurements were adjusted by hand so as to be consistent with the other asymmetry measurements(i.e. 0.0027 was subtracted from each of the three $A_{FB}^{\mathrm{b}}(\mathrm{pk})$ jet-charge measurements before averaging).

After calculating the overall averages, the corrections described in section 4 were made to the average peak asymmetries to derive the pole asymmetries. These overall corrections include a contribution for the effects of QCD.

5.2 Calculation of the covariance matrix $\mathcal{C}$

The covariance matrix can be written as:

$$\mathcal{C} = \mathcal{C}(\mathrm{stat}) + \mathcal{C}(\mathrm{syst})$$

The statistical covariance matrix, $\mathcal{C}(\mathrm{stat})$, has the total statistical error squared for each measurement, i, on the diagonal ($\mathcal{C}(\mathrm{stat})_{ii}$). Some of the measurements are from a multiparameter fit, in which case the statistical correlation matrix for that set of measurements was provided by the experiment.

The systematic covariance matrix, $\mathcal{C}(\mathrm{syst})$ was calculated from the detailed breakdown of systematic errors given by each experiment. The diagonal elements, ($\mathcal{C}(\mathrm{syst})_{ii}$), were the total systematic error for measurement i squared. If the systematic error on result $r_i(\mu)$ due to a source of systematic uncertainty p is written as $s_i(p)$, then an off diagonal element was given by:

$$\mathcal{C}(\mathrm{syst})_{ij} = \sum_{p} s_i(p) s_j(p),$$

where the sum is over all sources of uncertainty p which are correlated between results i and j. Some errors, such as the error from Monte Carlo statistics were uncorrelated for all results. Others, such as those connected with lepton identification, were correlated for any measurements made by the same LEP experiment. The third set of errors were taken as fully correlated for all measurements. These are discussed in more detail in [5, 6].

For some of the multiparameter fits, the full details of the internal experimental systematic errors were not provided, and the full systematic correlation or covariance matrix was given instead. The table giving the breakdown of systematic errors was then only used to calculate the off-diagonal terms in $\mathcal{C}(\mathrm{syst})$, i.e. terms connecting the measurements from the multiparameter fit with other results. The covariance matrix provided by the experiment was used for the block on the diagonal which related the set of results from the same multiparameter fit.

6 Results and cross checks

6.1 Full fit results

The full set of measurements to be averaged were provided in standardised tables by the four LEP experiments. Where necessary, the experiments applied **small corrections to the results quoted in their papers and notes** to use agreed default values and ranges for systematic inputs. These tables are all given in the appendices. Using the averaging procedure defined in the previous section gives the following combined results:

$$
\begin{aligned}
R_{\mathrm{b}} &= 0.2202 \pm 0.0020 \\
R_{\mathrm{c}} &= 0.1583 \pm 0.0098 \\
\mathrm{Br}(\mathrm{b} \to \ell) &= (10.95 \pm 0.19)\% \\
\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+) &= (8.07 \pm 0.44)\% \\
\overline{\chi} &= 0.1149 \pm 0.0069 \\
A^{\mathrm{b}}_{FB}(-2) &= (4.4 \pm 1.5)\% \\
A^{\mathrm{c}}_{FB}(-2) &= (-18.1 \pm 3.9)\% \\
A^{\mathrm{b}}_{FB}(\mathrm{pk}) &= (9.15 \pm 0.37)\% \\
A^{\mathrm{c}}_{FB}(\mathrm{pk}) &= (6.75 \pm 0.91)\% \\
A^{\mathrm{b}}_{FB}(+2) &= (10.5 \pm 1.3)\% \\
A^{\mathrm{c}}_{FB}(+2) &= (9.5 \pm 3.5)\%
\end{aligned}
$$

With the χ^2/dof of 34.3/(58 − 11), and a correlation matrix:

1.000	-.401	-.478	.006	-.102	-.012	-.066	-.047	.082	-.028	.050
-.401	1.000	.325	.059	.023	.009	.075	.083	-.107	.006	-.052
-.478	.325	1.000	.463	.116	.015	.034	.058	-.030	-.005	-.011
.006	.059	.463	1.000	-.529	-.024	-.020	-.056	-.224	-.043	-.104
-.102	.023	.116	-.529	1.000	.031	.031	.258	-.027	.106	.004
-.012	.009	.015	-.024	.031	1.000	.094	.018	.015	-.027	-.054
-.066	.075	.034	-.020	.031	.094	1.000	.012	-.033	-.069	-.034
-.047	.083	.058	-.056	.258	.018	.012	1.000	.084	-.005	.022
.082	-.107	-.030	-.224	-.027	.015	-.033	.084	1.000	.050	.089
-.028	.006	-.005	-.043	.106	-.027	-.069	-.005	.050	1.000	.099
.050	-.052	-.011	-.104	.004	-.054	-.034	.022	.089	.099	1.000

Notice that there is a large correlation of −0.4 between R_{b} and R_{c}. The total corrections for the on peak asymmetries to give the pole asymmetries are $+0.0052 \pm 0.0010$ and $+0.0085 \pm 0.0005$ for b and c respectively. The main results to be used in electroweak fits are then:

$$
\begin{aligned}
R_{\mathrm{b}} &= 0.2202 \pm 0.0020 \\
R_{\mathrm{c}} &= 0.1583 \pm 0.0098 \\
A^{0,\mathrm{b}}_{\mathrm{FB}} &= 0.0967 \pm 0.0038 \\
A^{0,\mathrm{c}}_{\mathrm{FB}} &= 0.0760 \pm 0.0091
\end{aligned}
$$

with their correlation matrix:

1.0	-.401	-.047	.082
-.401	1.0	.083	-.107
-.047	.083	1.0	.084
.082	-.107	.084	1.0

6.2 Cross checks

Many cross checks were performed by several people. The results from making simpler averages of just one or two parameters were compared with the full fit results, and found to be consistent to within a small fraction of the quoted error. Subsets of results combined using exactly the same procedure as for the full fit were also compared with independent averages of the same subset, and again consistency was found.

The parameter whose error has largest contribution from systematic uncertainties is R_{b}. The full fit result for R_{b} with $R_{\mathrm{c}} = 0.171$ (fixed) is

$$R_{\mathrm{b}} = 0.2192 \pm 0.0018 \text{ (Full fit)}$$

This value for R_{b} is dominated by the precise results of the dedicated double-tag analyses for this parameter alone. It is relatively straightforward to combine these single parameter measurements, taking into account the correlated errors. Adding the extra information from the multiparameter lepton fits is much more complicated.

The average of the five single parameter analyses, with fixed $R_{\mathrm{c}} = 0.171$ is

$$R_{\mathrm{b}} = 0.2197 \pm 0.0020 \text{ (Double tags only)}$$

The χ^2/dof is $2.78/(5-1)$. If the multiparameter lepton fits are combined using the full procedure, with all the usual parameters free except for $R_{\mathrm{c}} = 0.171$, the value of R_{b} is:

$$R_{\mathrm{b}} = 0.2173 \pm 0.0048 \text{ (Lepton fits only)}$$

To a good approximation, the double-tag and lepton multiparameter fit measurements of R_{b} are uncorrelated. A simple weighted average of these two measurements gives:

$$R_{\mathrm{b}} = 0.2194 \pm 0.0018 \text{ (Double tag plus lepton fits)}$$

This is in extremely good agreement with the result of the full fit with $R_{\mathrm{c}} = 0.171$.

6.3 Alternative averaging procedure for R_{b}

An independent average for R_{b} with a different formalism was made, where the correlations between the measurements arising from their mutual dependences on any other physical parameters was treated by explicitly including these dependences. In this alternative method a chisquared is constructed with each R_{b} measurement expressed as an explicit function of the parameters P_i on which it depends: $M_i(P_1, P_2, \ldots, P_n)$. The chisquared to be minimised is then given by:

$$\chi^2 = \sum_{\text{results}} \left(\frac{R_{\mathrm{b}} - M_i(P_1, P_2, \ldots, P_n)}{\sigma_{\text{uncorr.}}} \right)^2 + \sum_{\text{parameters}} \left(\frac{\Delta P_i}{\sigma_{P_i}} \right)^2$$

Here $\sigma_{\text{uncorr.}}$ is that part of the error on a given measurement which does not depend on any of the parameters (i.e. the uncorrelated part of the error). The parameters are constrained to their input values by the second term, where ΔP_i is the difference between the fitted parameter and its input value, and σ_{P_i} is the error on the input parameter. This chisquared is minimised w.r.t. R_{b} and the parameters P_i. The dependance of R_{b} on the parameters is assumed to be linear, and is given by the same derivatives used to construct the covariance matrix in the default averaging method.

The average R_{b} obtained with this alternate method agrees well with that obtained using the previous method, as do the errors on the value and the chisquared of the combination.

7 Summary

The combined LEP averages for electroweak parameters in the heavy quark sector are:

$$\begin{aligned}
R_{\mathrm{b}} &= 0.2202 \pm 0.0020 \\
R_{\mathrm{c}} &= 0.1583 \pm 0.0098 \\
\mathrm{Br}(\mathrm{b} \to \ell) &= (10.95 \pm 0.19)\% \\
\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^{+}) &= (8.07 \pm 0.44)\% \\
\overline{\chi} &= 0.1149 \pm 0.0069 \\
A_{FB}^{\mathrm{b}}(-2) &= (4.4 \pm 1.5)\% \\
A_{FB}^{\mathrm{c}}(-2) &= (-18.1 \pm 3.9)\% \\
A_{FB}^{\mathrm{b}}(\mathrm{pk}) &= (9.15 \pm 0.37)\% \\
A_{FB}^{\mathrm{c}}(\mathrm{pk}) &= (6.75 \pm 0.91)\% \\
A_{FB}^{\mathrm{b}}(+2) &= (10.5 \pm 1.3)\% \\
A_{FB}^{\mathrm{c}}(+2) &= (9.5 \pm 3.5)\%
\end{aligned}$$

The correlation matrix, with the parameters in the same order, is:

1.000	-.401	-.478	.006	-.102	-.012	-.066	-.047	.082	-.028	.050
-.401	1.000	.325	.059	.023	.009	.075	.083	-.107	.006	-.052
-.478	.325	1.000	.463	.116	.015	.034	.058	-.030	-.005	-.011
.006	.059	.463	1.000	-.529	-.024	-.020	-.056	-.224	-.043	-.104
-.102	.023	.116	-.529	1.000	.031	.031	.258	-.027	.106	.004
-.012	.009	.015	-.024	.031	1.000	.094	.018	.015	-.027	-.054
-.066	.075	.034	-.020	.031	.094	1.000	.012	-.033	-.069	-.034
-.047	.083	.058	-.056	.258	.018	.012	1.000	.084	-.005	.022
.082	-.107	-.030	-.224	-.027	.015	-.033	.084	1.000	.050	.089
-.028	.006	-.005	-.043	.106	-.027	-.069	-.005	.050	1.000	.099
.050	-.052	-.011	-.104	.004	-.054	-.034	.022	.089	.099	1.000

Converting the peak asymmetries to pole asymmetries for electroweak fits gives:

$$\begin{aligned}
R_{\mathrm{b}} &= 0.2202 \pm 0.0020 \\
R_{\mathrm{c}} &= 0.1583 \pm 0.0098 \\
\mathrm{A}_{\mathrm{FB}}^{0,\,\mathrm{b}} &= 0.0967 \pm 0.0038 \\
\mathrm{A}_{\mathrm{FB}}^{0,\,\mathrm{c}} &= 0.0760 \pm 0.0091
\end{aligned}$$

with their correlation matrix

1.0	-.401	-.047	.082
-.401	1.0	.083	-.107
-.047	.083	1.0	.084
.082	-.107	.084	1.0

The value of R_{b} with R_{c} fixed to its expected Standard Model value is

$$R_{\mathrm{b}} = 0.2192 \pm 0.0018 \; (R_{\mathrm{c}} = 0.171)$$

In the following appendices, the relevant results from the papers and notes by the four LEP experiments are tabulated. The errors are quoted in the order: uncorrelated, experiment internal (may be correlated with error on other measurements by the same experiments) and finally fully correlated (can lead to a correlation between results by different experiments) Finally, any dependences on other electroweak parameters are given.

The results in these tables have in some cases been corrected **by the experiments** to use agreed default values and ranges for input parameters. The experiments were also responsible for combining where necessary the results of their different analyses and calculating their internal correlation matrices.

A ALEPH tables

	R_b
results	0.2187
statistics	0.00220
Monte Carlo stat	0.00135
track resolution	0.00145
internal	0.00091
$\langle x_E(c)\rangle$	-0.00012
$f(D^+)$	-0.00073
$f(D^0+D^+)$	-0.00041
$f(\Lambda_c)$	-0.00019
D^0 lifetime	-0.00028
D^+ lifetime	-0.00036
D_s lifetime	-0.00022
B decay multip.	-0.00029
D decay multip.	-0.00057
$g \to b\overline{b}$	-0.00027
$g \to c\overline{c}$	-0.00019
Long-lived light hadrons	-0.00024
Total systematic	0.0026
Assume $R_c = 0.171$	- 0.01378 dR_c/R_c

Table 2: Error breakdown for ALEPH lifetime double-tag R_b [10]

	R_b
results	0.228
statistics	0.0054
internal	0.0025
Monte Carlo Stat	0.0023
elec effic	-0.0001
elec misid	+0.0010
muon effic	-0.0002
punch	-0.0017
track resoln	0.
Non-hadronic backgr	-0.0003
acceptance corr	-0.0003
Detector effects	-0.0007
$\langle x_E(\mathrm{b})\rangle$	0.
$\langle x_E(\mathrm{c})\rangle$	0.
b frag function	0.
c frag function	+0.0008
$\mathrm{Br}(\mathrm{b} \to \ell)$	0.
$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+)$	0.
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	0.
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	0.
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	0.
$\mathrm{Br}(\mathrm{c} \to \ell)$	+0.0006
$\mathrm{Br}(\mathrm{c} \to \mathrm{D}^\star \to \mathrm{K}\pi\pi_\mathrm{s})$	0.
Semilept model $\mathrm{b} \to \ell$	-0.0013
Semilept model $\mathrm{c} \to \ell$	-0.0021
$\mathrm{b} \to \mathrm{D}$ model.	0.
$f(\mathrm{D}^0)$	0.
$f(\mathrm{D}^+)$	0.
$f(\mathrm{D}^0 + \mathrm{D}^+)$	0.
$f(\mathrm{D_s})$	0.
$f(\Lambda_\mathrm{c})$	0.
D^0 lifetime	0.
D^+ lifetime	0.
$\mathrm{D_s}$ lifetime	0.
Λ_c lifetime	0.
B decay multip.	0.
D decay multip.	0.
$\mathrm{g} \to \mathrm{b}\overline{\mathrm{b}}$	0.
$\mathrm{g} \to \mathrm{c}\overline{\mathrm{c}}$	0.
Long-lived light hadrons	0.
Light quark frag	0.
Total systematic	0.0040
Assume $R_\mathrm{c} = 0.165 \pm 0.020$	-0.0005

Table 3: Error breakdown for ALEPH neural-net R_b [14]; note that this is 20% statistically correlated with the ALEPH R_b from the multiparameter lepton fit.

	$A^{\rm b}_{FB}$(pk)
results	0.0992
roots	91.187
statistics	0.0085
Monte Carlo stat	0.0022
elec id efficiency	0
elec misid	0
elec conversions	0
muon id efficiency	0
muon id background	0
track resoln	0.0017
internal	0.0012
$\langle x_E(\mathrm{b})\rangle$	0
$\langle x_E(\mathrm{c})\rangle$	0
b frag function	0
c frag function	0
$\mathrm{Br}(\mathrm{b} \rightarrow \ell)$	0
$\mathrm{Br}(\mathrm{b} \rightarrow \mathrm{c} \rightarrow \ell^{+})$	0
$\mathrm{Br}(\mathrm{b} \rightarrow \overline{\mathrm{c}} \rightarrow \ell^{-})$	0
$\mathrm{Br}(\mathrm{b} \rightarrow \tau \rightarrow \ell)$	0
$\mathrm{Br}(\mathrm{b} \rightarrow \mathrm{J}/\psi \rightarrow \ell^{+}\ell^{-})$	0
$\mathrm{Br}(\mathrm{c} \rightarrow \ell)$	0
$\mathrm{Br}(\mathrm{c} \rightarrow \mathrm{D}^{\star} \rightarrow \mathrm{K}\pi\pi_{\mathrm{s}})$	0
Semilept model $\mathrm{b} \rightarrow \ell$	0
Semilept model $\mathrm{c} \rightarrow \ell$	0
$\mathrm{b} \rightarrow \mathrm{D}$ model.	0
$f(\mathrm{D}^0)$	0
$f(\mathrm{D}^+)$	0
$f(\mathrm{D}^0 + \mathrm{D}^+)$	0
$f(\mathrm{D_s})$	0
$f(\Lambda_{\mathrm{c}})$	0
D^0 lifetime	0
D^+ lifetime	0
$\mathrm{D_s}$ lifetime	0
Λ_{c} lifetime	0
B decay multip.	0
D decay multip.	0
$\mathrm{g} \rightarrow \mathrm{b\overline{b}}$	0
$\mathrm{g} \rightarrow \mathrm{c\overline{c}}$	0
Long-lived light h	0
Light quark frag	0.0023
Total systematic	0.0042
Assume $R_{\mathrm{b}} = 0.2188$	-0.0307 $dR_{\mathrm{b}}/R_{\mathrm{b}}$
Assume $R_{\mathrm{c}} = 0.171$	+0.0257 $dR_{\mathrm{c}}/R_{\mathrm{c}}$

Table 4: Error breakdown for ALEPH jet charge $A^{\rm b}_{FB}$(pk) [16]

	$R_{\rm b}$	$R_{\rm c}$	$\overline{\chi}$	Br(b $\to \ell$)	Br(bcℓ)	$A^{\rm b}_{FB}$(pk)	$A^{\rm c}_{FB}$(pk)
roots						91.26	91.26
results	0.2162	0.1670	0.0993	0.1120	0.0881	8.43	9.1
statistics	0.0062	0.0054	0.0073	0.0033	0.0025	0.68	2.0
Monte Carlo stat	0.	0.	0.0027	0.	0.	0.03	0.
elec effic	0.0002	-0.0047	0.	-0.0018	-0.0015	0.	0.16
muon effic	-0.0002	-0.0040	0.	-0.0015	-0.0018	0.	0.07
elec conversions	-0.0006	-0.0062	0.	0.0013	-0.0006	0.	0.31
elec misid	+0.0002	-0.0024	-0.0002	-0.0008	-0.0001	0.	0.11
pi/K decays	-0.0016	-0.0057	-0.0005	0.0004	-0.0015	0.01	-0.37
punchthrough	-0.0028	-0.0103	-0.0006	0.0016	-0.0028	0.03	+0.42
lepton effic	0.	0.	0.0001	0.	0.	0.	0.
kaon production	0.	0.	-0.0002	0.	0.	+0.01	0.
backgr chrg corr	0.	0.	0.0002	0.	0.	0.	0.
btocbartol model	0.	0.	0.0002	0.	0.	0.	0.
background asy	0.	0.	0.	0.	0.	-0.02	1.38
track resoln	0.	0.	0.	0.	0.	0.03	0.
internal	0.	0.	0.	0.	0.	0.06	0.
$\langle x_E(\rm b)\rangle$	0.	0.	0.	0.	0.	0.	0.
$\langle x_E(\rm c)\rangle$	0.	0.	0.	0.	0.	+0.07	0.
b frag function	0.	0.	0.	0.	0.	-0.01	0.
c frag function	0.	0.	0.	0.	0.	0.	0.
Br(b $\to \ell$)	0.	0.	0.0030	0.	0.	-0.04	0.
Br(b $\to$ c $\to \ell^+$)	0.	0.	-0.0047	0.	0.	-0.01	0.
Br(c $\to \ell$)	0.0010	-0.0093	0.	-0.0005	-0.0002	0.05	0.04
Br(b $\to \tau \to \ell$)	-0.0003	-0.0006	0.	-0.0003	-0.0009	0.	0.11
Br(b $\to \overline{\rm c} \to \ell^-$)	-0.0006	-0.0050	0.0002	-0.0003	-0.0020	0.	0.47
Br(b $\to$ J/$\psi \to \ell^+\ell^-$)	+0.0003	+0.0006	0.	-0.0002	-0.0006	0.	-0.02
BR(b $\to$ u)	-0.0022	-0.0006	0.0016	+0.0004	+0.0041	-0.03	-0.15
Br(c $\to$ D$^\star \to$ K$\pi\pi_{\rm s}$)	0.	0.	0.	0.	0.	0.	0.
Semilept model c $\to \ell$	-0.0008	-0.0010	0.0045	-0.0003	+0.0002	-0.05	-0.17
b $\to$ D model.	-0.0005	-0.0012	0.0004	+0.0006	-0.0004	-0.01	-0.08
Semilept model b $\to \ell$	-0.0027	+0.0041	-0.0017	-0.0025	+0.0051	-0.03	-0.90
$f(\rm D^0)$	0.	0.	0.	0.	0.	0.	0.
$f(\rm D^+)$	0.	0.	0.	0.	0.	0.	0.
$f(\rm D^0+D^+)$	0.	0.	0.	0.	0.	0.	0.
$f(\rm D_s)$	0.	0.	0.	0.	0.	0.	0.
$f(\Lambda_{\rm c})$	0.	0.	0.	0.	0.	0.	0.
D^0 lifetime	0.	0.	0.	0.	0.	0.	0.
D$^+$ lifetime	0.	0.	0.	0.	0.	0.	0.
D$_{\rm s}$ lifetime	0.	0.	0.	0.	0.	0.	0.
$\Lambda_{\rm c}$ lifetime	0.	0.	0.	0.	0.	0.	0.
B decay multip.	0.	0.	0.	0.	0.	0.	0.
D decay multip.	0.	0.	0.	0.	0.	0.	0.
g $\to$ b$\overline{\rm b}$	0.	0.	-0.0001	0.	0.	+0.01	0.
g $\to$ c$\overline{\rm c}$	0.	0.	-0.0001	0.	0.	+0.01	0.
Long-lived light h	0.	0.	0.	0.	0.	0.	0.
Light quark frag	0.	0.	0.	0.	0.	0.	0.
Total systematic	0.0050	0.0188	0.0081	0.0042	0.0080	0.14	1.9

Table 5: Results of the published ALEPH multiparameter lepton fit and the ALEPH measurement of $A^{\rm b\overline{b}}_{\rm FB}$ and $\overline{\chi}$ using high p_T leptons. These two analyses are combined here by ALEPH for the purpose of averaging with other LEP results [7]

	R_b	R_c	χ	$b \to \ell$	$b \to c \to \ell$	A_b	A_c
R_b	1.00	-0.48	0.00	-0.94	-0.38	0.05	0.05
R_c	-0.48	1.00	0.00	0.47	-0.31	0.00	-0.07
χ	0.00	0.00	1.00	0.27	-0.23	0.25	0.00
$b \to \ell$	-0.94	0.47	0.27	1.00	0.25	-0.04	-0.04
$b \to c \to \ell$	-0.38	-0.31	-0.23	0.25	1.00	-0.01	-0.01
A_b	0.05	0.00	0.25	-0.04	-0.01	1.00	0.01
A_c	0.05	-0.07	0.00	-0.04	-0.01	0.01	1.00

Table 6: ALEPH statistical correlation matrix for lepton measurements.

	R_b	R_c	χ	$b \to \ell$	$b \to c \to \ell$	A_b	A_c
R_b	1.00	0.30	0.00	-0.20	-0.24	0.27	0.12
R_c	0.30	1.00	-0.02	-0.05	0.16	-0.02	-0.29
χ	0.00	-0.02	1.00	0.28	-0.38	-0.12	-0.02
$b \to \ell$	-0.20	-0.05	0.28	1.00	0.02	-0.02	0.23
$b \to c \to \ell$	-0.24	0.16	-0.38	0.02	1.00	-0.24	-0.45
A_b	0.27	-0.02	-0.12	-0.02	-0.24	1.00	0.19
A_c	0.12	-0.29	-0.02	0.23	-0.45	0.19	1.00

Table 7: ALEPH systematic correlation matrix for lepton measurements.

	A^c_{FB}(pk)
results	7.12
$\sqrt{s}$	91.25
statistics	2.11
D* back rate	0.20
D* back asy	0.65
Rate c-D* / b-D*	0.10
W to D*	0.10
effective chi	0.20
Total systematic	0.72
Assume $A^b_{FB}(\mathrm{pk}) = 8.46 \pm 0.72\%$	-0.10

Table 8: Error table for ALEPH D* measurement of A^c_{FB}(pk) [21]

B DELPHI tables

results	0.2214
statistics	0.0020
Hemisph correl	0.0012
track Resoln	0.0015
internal	0.0006
$\langle x_E(\mathrm{b})\rangle$	0
$\langle x_E(\mathrm{c})\rangle$	-0.0004
b frag function	0
c frag function	0
$\mathrm{Br}(\mathrm{b} \to \ell)$	0
$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+)$	0
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	0
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	0
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	0
$\mathrm{Br}(\mathrm{c} \to \ell)$	0.0003
$\mathrm{Br}(\mathrm{c} \to \mathrm{D}^\star \to \mathrm{K}\pi\pi_\mathrm{s})$	0
Semilept model $\mathrm{b} \to \ell$	0.0003
Semilept model $\mathrm{c} \to \ell$	-0.0002
$\mathrm{b} \to \mathrm{D}$ model.	0
$f(\mathrm{D}^0)$	-0.0001
$f(\mathrm{D}^+)$	-0.0011
$f(\mathrm{D}^0 + \mathrm{D}^+)$	-0.0007
$f(\mathrm{D_s})$	-0.0005
$f(\Lambda_\mathrm{c})$	0.
D^0 lifetime	-0.0002
D^+ lifetime	-0.0003
$\mathrm{D_s}$ lifetime	-0.0002
Λ_c lifetime	0.
B decay multip.	0.
D decay multip.	-0.0005
$\mathrm{g} \to \mathrm{b}\overline{\mathrm{b}}$	-0.0002
$\mathrm{g} \to \mathrm{c}\overline{\mathrm{c}}$	-0.0001
Long-lived light h	-0.0007
Light quark frag	0
$\mathrm{Br}(D \to K^0)$	+0.0007
Total systematic	0.0028
Assume $R_\mathrm{c} = 0.171 \pm 0.014$	-0.0015

Table 9: Error breakdown for DELPHI R_b [11]

	R_c
results	0.209
statistics	0.019
Monte Carlo stat	0.0054
track resolution	0.0100
b hadron lifetime	0.0013
$\langle x_E(\mathrm{b})\rangle$	-0.0065
Light quark frag	0.0021
Statistical error from Υ_{4S} data	0.0127
Systematics from Υ_{4S} data	0.0180
Total systematic	0.026

Table 10: Breakdown of errors for DELPHI's R_c measured using D mesons [19]

results	R_b 0.2145	$\mathrm{Br}(b \to \ell)$ 11.21	$\mathrm{Br}(bc\ell)$ 7.70	R_c 0.1625	$\overline{\chi}$ 0.150
statistics	0.0089	0.45	0.49	0.0085	0.020
Monte Carlo stat	0.	0.	0.	0.	0.
muon backgr	-0.0054	$+0.30$	-0.91	-0.0160	$+0.010$
muon effic ec	-0.0004	-0.03	-0.04	-0.0014	0.
muon effic bar	$+0.0014$	-0.32	-0.25	-0.0038	0.
elec backgr	-0.0029	$+0.24$	$+0.05$	-0.0031	-0.003
$BR(b \to u)$	0	0	-0.03	$+0.0002$	-0.004
$\langle x_E(b) \rangle$	0.	0.	0.	0.	0.
$\langle x_E(c) \rangle$	$+0.0015$	-0.17	$+0.28$	-0.0056	-0.003
b frag function	-0.0008	$+0.11$	-0.03	$+0.0008$	-0.002
c frag function	0.	0.	0.	0.	0.
$\mathrm{Br}(b \to \ell)$	0.	0.	0.	0.	0.
$\mathrm{Br}(b \to c \to \ell^+)$	0.	0.	0.	0.	0.
$\mathrm{Br}(b \to \overline{c} \to \ell^-)$	$+0.0003$	$+0.01$	-0.33	$+0.0013$	$+0.003$
$\mathrm{Br}(b \to \tau \to \ell)$	-0.0002	-0.03	-0.13	-0.0002	$+0.001$
$\mathrm{Br}(b \to J/\psi \to \ell^+\ell^-)$	$+0.0002$	-0.06	$+0.02$	0	-0.002
$\mathrm{Br}(c \to \ell)$	$+0.0014$	-0.06	-0.04	-0.0098	-0.002
$\mathrm{Br}(c \to D^\star \to K\pi\pi_s)$	0.	0.	0.	0.	0.
Semilept model $b \to \ell$	$+0.0001$	$+0.40$	-0.69	-0.0004	$+0.008$
Semilept model $c \to \ell$	$+0.0007$	-0.16	-0.03	$+0.0049$	-0.006
$b \to D$ model.	0.	0.	0.	0.	0.
$f(D^0)$	0.	0.	0.	0.	0.
$f(D^+)$	0.	0.	0.	0.	0.
$f(D^0 + D^+)$	0.	0.	0.	0.	0.
$f(D_s)$	0.	0.	0.	0.	0.
$f(\Lambda_c)$	0.	0.	0.	0.	0.
D^0 lifetime	0.	0.	0.	0.	0.
D^+ lifetime	0.	0.	0.	0.	0.
D_s lifetime	0.	0.	0.	0.	0.
Λ_c lifetime	0.	0.	0.	0.	0.
B decay multip.	0.	0.	0.	0.	0.
D decay multip.	0.	0.	0.	0.	0.
$g \to b\overline{b}$	0.	0.	0.	0.	0.
$g \to c\overline{c}$	0.	0.	0.	0.	0.
Long-lived light h	0.	0.	0.	0.	0.
Light quark frag	0.	0.	0.	0.	0.
Total systematic	0.0067	0.70	1.26	0.021	0.016

Table 11: DELPHI lepton fit results [12]

	R_b	$\mathrm{Br}(b \to \ell)$	$\mathrm{Br}(bc\ell)$	R_c	$\overline{\chi}$
R_b	1.00	-0.95	-0.38	-0.42	-0.09
$\mathrm{Br}(b \to \ell)$	-0.95	1.00	0.29	0.37	0.11
$\mathrm{Br}(bc\ell)$	-0.38	0.29	1.00	-0.40	-0.17
R_c	-0.42	0.37	-0.40	1.00	0.17
$\overline{\chi}$	-0.09	0.11	-0.17	0.17	1.00

Table 12: DELPHI lepton fit statistical correlation matrix

	$A_{FB}^{\mathrm{b}}(\mathrm{pk})$	$A_{FB}^{\mathrm{c}}(\mathrm{pk})$	$A_{FB}^{\mathrm{b}}(-2)$	$A_{FB}^{\mathrm{b}}(+2)$
$\sqrt{s}$	91.255	91.269	89.43	93.017
results	0.1065	0.0802	0.0629	0.1489
statistics	0.011	0.022	0.038	0.036
muon id	0.0021	0.0051	0.0013	0.0040
elec id	0.0014	0.0025	0.0	0.0
track resoln	0.0013	0.0009	0.0006	0.0020
backgr asy	-0.001	+0.0102	-0.0001	0.0001
internal	0.0013	0.0045	0.0006	0.0020
$\langle x_E(\mathrm{b})\rangle$	-0.0001	-0.0005	+0.0002	-0.0001
$\langle x_E(\mathrm{c})\rangle$	+0.0011	+0.0001	-0.0001	+0.0014
b frag function	0.	0.	0.	0.
c frag function	0.	0.	0.	0.
$\mathrm{Br}(\mathrm{b}\to\ell)$	-0.0012	0.0015	-0.0003	-0.0010
$\mathrm{Br}(\mathrm{b}\to\mathrm{c}\to\ell^+)$	0.0003	-0.0025	0.0001	0.0003
$\mathrm{Br}(\mathrm{b}\to\overline{\mathrm{c}}\to\ell^-)$	0.0007	0.0038	0.0001	0.0017
$\mathrm{Br}(\mathrm{b}\to\tau\to\ell)$	0.	0.	0.	0.
$\mathrm{Br}(\mathrm{b}\to\mathrm{J}/\psi\to\ell^+\ell^-)$	0.	0.	0.	0.
$\mathrm{Br}(\mathrm{c}\to\ell)$	0.0004	-0.0037	0.0003	0.0007
$\mathrm{Br}(\mathrm{c}\to\mathrm{D}^\star\to\mathrm{K}\pi\pi_\mathrm{s})$	0.	0.	0.	0.
Semilept model $\mathrm{b}\to\ell$	0.0004	-0.0060	0.0015	0.0015
Semilept model $\mathrm{c}\to\ell$	0.0018	-0.0013	0.0004	0.0022
$\mathrm{b}\to\mathrm{D}$ model.	0.	0.	0.	0.
$f(\mathrm{D}^0)$	0.	0.	0.	0.
$f(\mathrm{D}^+)$	0.	0.	0.	0.
$f(\mathrm{D}^0+\mathrm{D}^+)$	0.	0.	0.	0.
$f(\mathrm{D_s})$	0.	0.	0.	0.
$f(\Lambda_\mathrm{c})$	0.	0.	0.	0.
D^0 lifetime	0.	0.	0.	0.
D^+ lifetime	0.	0.	0.	0.
$\mathrm{D_s}$ lifetime	0.	0.	0.	0.
Λ_c lifetime	0.	0.	0.	0.
B decay multip.	0.	0.	0.	0.
D decay multip.	0.	0.	0.	0.
$\mathrm{g}\to\mathrm{b}\overline{\mathrm{b}}$	0.	0.	0.	0.
$\mathrm{g}\to\mathrm{c}\overline{\mathrm{c}}$	0.	0.	0.	0.
Long-lived light h	0.	0.	0.	0.
Light quark frag	0.	0.	0.	0.
Total systematic	0.0042	0.015	0.0023	0.0061
Assume $R_\mathrm{b}=0.217\pm0.003$	-0.0004	+0.0004	-0.0001	-0.0004
Assume $R_\mathrm{c}=0.171\pm0.014$	+0.0009	-0.0053	+0.0001	-0.0008
Assume $\overline{\chi}=0.121\pm0.0165$	0.0046	0.	+0.0028	+0.0065

Table 13: DELPHI measurements of asymmetries using a lepton tag [18]

	A^{b}_{FB}(pk)	A^{c}_{FB}(pk)	$A^{b}_{FB}(-2)$	$A^{b}_{FB}(+2)$
A^{b}_{FB}(pk)	1.000	0.23	0.	0.
A^{c}_{FB}(pk)	0.23	1.000	0.	0.
$A^{b}_{FB}(-2)$	0.	0.	1.000	0.
$A^{b}_{FB}(+2)$	0.	0.	0.	1.000

Table 14: DELPHI lepton tag asymmetries statistical correlation matrix

	A^{b}_{FB}(pk)	A^{c}_{FB}(pk)
$\sqrt{s}$	91.27	91.27
results	.046	.081
statistics	.059	.029
Monte Carlo stat	-.009	.005
internal	.015	.006
detector effects	.002	.004
rate of pi_s	.009	-.006
D* b sample	.005	-.002
D* back rate	.002	.003
b hadron lifetime	-.004	.002
effective chi	.012	-.003
thrust resoln	.001	.001
$\langle x_E(\mathrm{b})\rangle$	-.001	.001
$\langle x_E(\mathrm{c})\rangle$	.002	.001
Total systematic	.024	.012

Table 15: Asymmetry measurements with a D* tag from DELPHI [23]

	A^{b}_{FB}(pk)	A^{c}_{FB}(pk)
A^{b}_{FB}(pk)	1.0	-.38
A^{c}_{FB}(pk)	-.38	1.0

Table 16: Statistical correlation matrix for asymmetry measurements with a D* tag from DELPHI

	A^{b}_{FB}(pk)	A^{c}_{FB}(pk)
A^{b}_{FB}(pk)	1.00	-.22
A^{c}_{FB}(pk)	-.22	1.0

Table 17: Systematic correlation matrix for asymmetry measurements with a D* tag from DELPHI

	A_{FB}^{b}(pk)
results	0.115
$\sqrt{s}$	91.28
statistics	0.018
Monte Carlo stat	0
lepton id	-0.0015
track resoln	0.
internal	0.0057
$\langle x_E(\mathrm{b})\rangle$	-0.0001
$\langle x_E(\mathrm{c})\rangle$	+0.0023
b frag function	0
c frag function	0
$\mathrm{Br}(\mathrm{b} \to \ell)$	0.0006
$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+)$	-0.0006
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	0.0001
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	0
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	0
$\mathrm{Br}(\mathrm{c} \to \ell)$	0.0002
$\mathrm{Br}(\mathrm{c} \to \mathrm{D}^\star \to \mathrm{K}\pi\pi_\mathrm{s})$	0
Semilept model $\mathrm{b} \to \ell$	-0.0003
Semilept model $\mathrm{c} \to \ell$	0.0004
$\mathrm{b} \to \mathrm{D}$ model.	0
$f(\mathrm{D}^0)$	0
$f(\mathrm{D}^+)$	0
$f(\mathrm{D}^0 + \mathrm{D}^+)$	0
$f(\mathrm{D_s})$	0
$f(\Lambda_\mathrm{c})$	0
D^0 lifetime	0
D^+ lifetime	0
$\mathrm{D_s}$ lifetime	0
Λ_c lifetime	0
B decay multip.	0
D decay multip.	0
$\mathrm{g} \to \mathrm{b\overline{b}}$	0
$\mathrm{g} \to \mathrm{c\overline{c}}$	0
Long-lived light h	0
Light quark frag	0.0018
Total systematic	0.0067
Assume R_b = 0.217	-0.0483 $dR_\mathrm{b}/R_\mathrm{b}$
Assume R_c = 0.171	+0.0173 $dR_\mathrm{c}/R_\mathrm{c}$
Assume $\overline{\chi}$ = 0.115	+0.0311 $d\overline{\chi}/\overline{\chi}$

Table 18: DELPHI A_{FB}^{b}(pk) from jet charge [18]

C L3 tables

	R_b
results	0.222
statistics	0.003
internal	0.0021
$\langle x_E(\mathrm{b})\rangle$	-0.0031
$\langle x_E(\mathrm{c})\rangle$	+0.0018
b frag function	0.
c frag function	0.
$\mathrm{Br(b \to \ell)}$	-0.0010
$\mathrm{Br(b \to c \to \ell^+)}$	0.
$\mathrm{Br(b \to \overline{c} \to \ell^-)}$	0.
$\mathrm{Br(b} \to \tau \to \ell)$	0.
$\mathrm{Br(b \to J/}\psi \to \ell^+\ell^-)$	0.
$\mathrm{Br(c \to \ell)}$	0.
$\mathrm{Br(c \to D^\star \to K}\pi\pi_s)$	0.
Semilept model $\mathrm{b} \to \ell$	0.
Semilept model $\mathrm{c} \to \ell$	0.
$\mathrm{b \to D}$ model.	0.
$f(\mathrm{D}^0)$	0.
$f(\mathrm{D}^+)$	0.
$f(\mathrm{D}^0 + \mathrm{D}^+)$	0.
$f(\mathrm{D_s})$	0.
$f(\Lambda_c)$	0.
D^0 lifetime	0.
D^+ lifetime	0.
$\mathrm{D_s}$ lifetime	0.
Λ_c lifetime	0.
B decay multip.	0.
D decay multip.	0.
$\mathrm{g \to b\overline{b}}$	0.
$\mathrm{g \to c\overline{c}}$	0.
Long-lived light hadrons	0.
Light quark frag.	+0.0049
Total systematic	0.007
Assume $R_c = 0.171 \pm 0.009$	-0.0011

Table 19: Error breakdown for L3 neural net R_b [15]

	R_b	$Br(b \to \ell)$
results	0.2187	0.1144
statistics	0.0081	0.0048
internal	0.0047	0.0028
$\langle x_E(b) \rangle$	0.0029	-0.0024
$\langle x_E(c) \rangle$	0.0004	-0.0002
$Br(b \to c \to \ell^+)$	0.0052	0.0007
$Br(c \to \ell)$	-0.0012	0.0003
Semilept model $b \to \ell$	-0.0009	0.0021
Semilept model $c \to \ell$	0.0007	0.0003
Total systematic	0.0080	0.0043
Assume $R_c 0.171 \pm 0.014$	-0.0019	0.0005

Table 20: Error breakdown for L3 R_b and $Br(b \to \ell)$ measurement using leptons [8]

	R_b	$Br(b \to \ell)$
R_b	1.	-0.98
$Br(b \to \ell)$	-0.98	1.

Table 21: Statistical correlation matrix for L3 R_b and $Br(b \to \ell)$

	R_b	$Br(b \to \ell)$
R_b	1.	-0.32
$Br(b \to \ell)$	-0.32	1.

Table 22: Systematic correlation matrix for L3 R_b and $Br(b \to \ell)$

	$A^b_{FB}(-2)$	$A^b_{FB}(pk)$	$A^b_{FB}(+2)$	$A^c_{FB}(pk)$	$\overline{\chi}$
$A^b_{FB}(-2)$	1.00	0.00	0.00	0.0	0.06
$A^b_{FB}(pk)$	0.00	1.00	0.00	0.0	0.30
$A^b_{FB}(+2)$	0.00	0.00	1.00	0.0	0.10
$A^c_{FB}(pk)$	0.0	0.0	0.0	1.00	0.00
$\overline{\chi}$	0.06	0.30	0.10	0.00	1.00

Table 23: Statistical correlation matrix for lepton fit for asymmetries from L3

	$A^b_{FB}(-2)$	$A^b_{FB}(pk)$	$A^b_{FB}(+2)$	$A^c_{FB}(pk)$	$\overline{\chi}$
$A^b_{FB}(-2)$	1.00	1.00	1.00	0.00	0.14
$A^b_{FB}(pk)$	1.00	1.00	1.00	0.00	0.27
$A^b_{FB}(+2)$	1.00	1.00	1.00	0.00	0.27
$A^c_{FB}(pk)$	0.00	0.00	0.00	1.00	0.00
$\overline{\chi}$	0.14	0.27	0.27	0.00	1.00

Table 24: Systematic correlation matrix for lepton fit for asymmetries from L3

	$A_{FB}^{\mathrm{b}}(-2)$	$A_{FB}^{\mathrm{b}}(\mathrm{pk})$	$A_{FB}^{\mathrm{b}}(+2)$	$A_{FB}^{\mathrm{c}}(\mathrm{pk})$	$\overline{\chi}$
Assume R_{b}	0.216 ± 0.005	0.216 ± 0.005	0.216 ± 0.005	0.216 ± 0.005	0.216 ± 0.005
Assume R_{c}	0.169 ± 0.005	0.169 ± 0.005	0.169 ± 0.005	0.169 ± 0.005	0.169 ± 0.005
Assume $A_{FB}^{\mathrm{c}}(-2)$	-2.90 ± 1.50	0.	0.	0.	0.
Assume $A_{FB}^{\mathrm{c}}(\mathrm{pk})$	0.	6.30 ± 1.50	0.	0.	0.
Assume $A_{FB}^{\mathrm{c}}(+2)$	0.	0.	12.10 ± 1.50	0.	0.
Assume $A_{FB}^{\mathrm{b}}(\mathrm{pk})$	0.	0.	0.	8.8 ± 1.1	0.

Table 25: Standard Model inputs to lepton fit for asymmetries from L3

	$A_{FB}^{b}(-2)$	$A_{FB}^{b}(\mathrm{pk})$	$A_{FB}^{b}(+2)$	$A_{FB}^{c}(\mathrm{pk})$	$\overline{\chi}$
$\sqrt{s}$	89.56	91.27	92.93	91.24	0.
results	7.02	10.28	11.02	7.84	0.1253
statistics	3.50	1.00	2.90	3.70	0.0110
internal	+0.3500	+0.3600	+0.3610	+2.4000	+0.005160
$\langle x_E(\mathrm{b})\rangle$	+0.0293	+0.0227	+0.0227	−0.4444	−0.001000
$\langle x_E(\mathrm{c})\rangle$	−0.0120	−0.0120	−0.0120	−0.2000	+0.000030
b frag function	0.	0.	0.	0.	0.
c frag function	0.	0.	0.	0.	0.
$\mathrm{Br}(\mathrm{b} \to \ell)$	−0.0442	−0.0308	−0.0300	+0.1667	+0.002200
$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^{+})$	−0.0100	−0.0430	−0.0460	−0.5000	−0.005190
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^{-})$	−0.0110	−0.0085	−0.0080	0.	+0.000430
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	0.	0.	0.	0.	0.
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^{+}\ell^{-})$	0.	0.	0.	0.	0.
$\mathrm{Br}(\mathrm{c} \to \ell)$	+0.0467	+0.0467	+0.0471	−0.3333	+0.000033
$\mathrm{Br}(\mathrm{c} \to \mathrm{D}^{\star} \to \mathrm{K}\pi\pi_{\mathrm{s}})$	0.	0.	0.	0.	0.
Semilept model $\mathrm{b} \to \ell$	+0.0990	+0.1010	+0.1010	0.	+0.000300
Semilept model $\mathrm{c} \to \ell$	−0.0550	−0.0430	−0.0420	0.	+0.001860
$\mathrm{b} \to \mathrm{D}$ model.	+0.0410	+0.0470	+0.0470	0.	+0.001530
$f(\mathrm{D}^{0})$	0.	0.	0.	0.	0.
$f(\mathrm{D}^{+})$	0.	0.	0.	0.	0.
$f(\mathrm{D}^{0}+\mathrm{D}^{+})$	0.	0.	0.	0.	0.
$f(\mathrm{D}_{\mathrm{s}})$	0.	0.	0.	0.	0.
$f(\Lambda_{\mathrm{c}})$	0.	0.	0.	0.	0.
D^{0} lifetime	0.	0.	0.	0.	0.
D^{+} lifetime	0.	0.	0.	0.	0.
D_{s} lifetime	0.	0.	0.	0.	0.
Λ_{c} lifetime	0.	0.	0.	0.	0.
B decay multip.	0.	0.	0.	0.	0.
D decay multip.	0.	0.	0.	0.	0.
$\mathrm{g} \to \mathrm{b}\overline{\mathrm{b}}$	0.	0.	0.	0.	0.
$\mathrm{g} \to \mathrm{c}\overline{\mathrm{c}}$	0.	0.	0.	0.	0.
Long-lived light h	0.	0.	0.	0.	0.
Light quark frag	0.	0.	0.	0.	0.
Total systematic	0.38	0.39	0.39	2.5	0.0081
R_{b}	−0.0380	−0.0380	−0.0380	+0.1000	+0.000020
R_{c}	+0.0270	+0.0270	+0.0270	−0.2000	+0.000020
$A_{FB}^{c}(-2)$	+0.1380	0.	0.	0.	0.
$A_{FB}^{c}(\mathrm{pk})$	0.	+0.1380	0.	0.	0.
$A_{FB}^{c}(+2)$	0.	0.	+0.1380	0.	0.
$A_{FB}^{b}(\mathrm{pk})$	0.	0.	0.	0.8	0.

Table 26: Results of the lepton fits from L3 [8]

D OPAL tables

	R_b
Result	0.2171
Statistics	0.00207
Monte Carlo stat	0.00049
Elec id efficiency	0.00012
Elec misid	0.00027
Elec conversions	0.00027
Muon id efficiency	0.00005
Muon id background	0.00041
Track resolution	0.00071
Efficiency correlation	0.00069
Event selection	0.00025
$\langle x_E(\mathrm{b})\rangle$	0.0
$\langle x_E(\mathrm{c})\rangle$	−0.00072
b frag function	0.0
c frag function	0.0
$\mathrm{Br}(\mathrm{b} \to \ell)$	0.0
$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+)$	0.0
$\mathrm{Br}(\mathrm{b} \to \overline{\mathrm{c}} \to \ell^-)$	0.0
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	0.0
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	0.0
$\mathrm{Br}(\mathrm{c} \to \ell)$	−0.00028
$\mathrm{Br}(\mathrm{c} \to \mathrm{D}^\star \to \mathrm{K}\pi\pi_s)$	0.0
Semilept model $\mathrm{b} \to \ell$	0.0
Semilept model $\mathrm{c} \to \ell$	−0.00024
$\mathrm{b} \to \mathrm{D}$ model.	0.0
$f(\mathrm{D}^0)$	+0.00002
$f(\mathrm{D}^+)$	−0.00065
$f(\mathrm{D}^0+\mathrm{D}^+)$	−0.00036
$f(\mathrm{D_s})$	−0.00049
$f(\Lambda_c)$	0.
D^0 lifetime	−0.00022
D^+ lifetime	−0.00028
$\mathrm{D_s}$ lifetime	−0.00017
Λ_c lifetime	−0.00005
B decay multip.	0.0
D decay multip.	−0.00073
$\mathrm{Br}(D \to K^0)$	+0.00057
$\mathrm{g} \to \mathrm{b}\overline{\mathrm{b}}$	0.
$\mathrm{g} \to \mathrm{c}\overline{\mathrm{c}}$	−0.00048
Long-lived light h	−0.00049
Light quark frag	0.0
Total systematic	0.00210
R_c	−0.01867

Table 27: Error breakdown for OPAL double-tag R_b. Entries marked "0.0" are negligible. [13]

	R_b	$Br(b \to \ell)$	$Br(b \to c \to \ell^+)$	$\overline{\chi}$
$\sqrt{s}$	–	–	–	–
Results	.2252	10.53	8.253	.1436
Statistics	.011	.60	.40	.022
Monte Carlo statistics	.003	.22	.14	.003
Λ_b polarisation	0.0	–.01	.08	0.0
$Br(b \to c \to \ell^+)$ syst.	–.0014	.072	–.49	.0042
Muon ID efficiency	.001	.24	.13	.001
Muon polar angle	.001	–.02	–.01	0.0
Elec ID efficiency	–.001	.18	.14	.001
e bremsstrahlung	0.0	0.0	0.0	0.0
Muon ID background	.002	–.09	.06	–.001
Elec misid	.001	–.02	.03	0.0
Elec conversions	.001	–.02	–.09	0.0
e decay background	0.0	.01	–.08	.001
Elec source dep	0.0	0.0	0.0	0.0
Track resolution	0.0	0.0	0.0	0.0
Track charge error	0.0	0.0	0.0	0.0
$\langle x_E \rangle_b$	0.0	0.0	0.0	0.0
$\langle x_E \rangle_c$	–.002	.04	.05	.001
b frag function	–.001	–.16	–.07	.001
c frag function	0.0	0.0	0.0	0.0
$Br(b \to \ell)$	0.0	0.0	0.0	0.0
$Br(b \to c \to \ell^+)$	0.0	0.0	0.0	0.0
$Br(b \to \overline{c} \to \ell^-)$	.001	–.06	–.07	0.0
$Br(b \to \tau \to \ell)$	.001	–.07	–.04	0.0
$Br(b \to J/\psi \to \ell^+\ell^-)$	0.0	0.0	0.0	0.0
$Br(c \to \ell)$	–.00045	–.0045	.0182	.00091
$Br(c \to D^\star \to K\pi\pi_s)$	0.0	0.0	0.0	0.0
Semilept model $b \to \ell$	–.004	–.50	.20	–.002
Semilept model $c \to \ell$	–.001	–.03	–.31	–.001
$b \to D$ model.	0.0	0.0	0.0	0.0
$f(D^0)$	0.0	0.0	0.0	0.0
$f(D^+)$	0.0	0.0	0.0	0.0
$f(D^0 + D^+)$	0.0	0.0	0.0	0.0
$f(D_s)$	0.0	0.0	0.0	0.0
$f(\Lambda_c)$	0.0	0.0	0.0	0.0
D^0 lifetime	0.0	0.0	0.0	0.0
D^+ lifetime	0.0	0.0	0.0	0.0
D_s lifetime	0.0	0.0	0.0	0.0
Λ_c lifetime	0.0	0.0	0.0	0.0
B decay multip.	0.0	0.0	0.0	0.0
D decay multip.	0.0	0.0	0.0	0.0
$g \to b\overline{b}$	0.0	0.0	0.0	0.0
$g \to c\overline{c}$	0.0	0.0	0.0	0.0
Long-lived light h	0.0	0.0	0.0	0.0
Light quark frag	0.0	0.0	0.0	0.0
Total systematic	.0066	0.66	0.69	0.0062
R_c	–0.014	+0.23	+0.32	0.014

Table 28: Results of the multiparameter lepton fit – part I [9]

	$A^{b}_{FB}(-2)$	$A^{c}_{FB}(-2)$	$A^{b}_{FB}(\mathrm{pk})$	$A^{c}_{FB}(\mathrm{pk})$	$A^{b}_{FB}(+2)$	$A^{c}_{FB}(+2)$
$\sqrt{s}$	89.54	89.54	91.26	91.26	92.94	92.94
Results	3.661	-17.81	8.724	4.300	11.08	7.601
Statistics	3.000	4.500	1.000	1.300	2.600	3.800
Monte Carlo stat	0.0	0.0	0.0	0.0	0.0	0.0
Λ_b polarisation	−.020	−.050	.010	−.050	.020	−.050
Br($b \to c \to \ell^+$) syst.	.080	.4500	.040	.1900	.020	.1800
Muon id efficiency	0.0	0.0	0.0	0.0	0.0	0.0
Muon polar angle	0.0	0.0	0.0	0.0	0.0	0.0
Elec id efficiency	0.0	0.0	0.0	0.0	0.0	0.0
e bremsstrahlung	0.0	0.0	0.0	0.0	0.0	0.0
Muon id background	−.090	−.5600	.1300	.2600	.1800	.3700
Elec misid	−.080	−.8400	.080	.2300	.2300	.1400
Elec conversions	−.3600	-1.260	.1300	.3700	.2600	.8500
e decay background	0.0	0.0	0.0	0.0	0.0	0.0
Elec source dep	.010	−.2600	.040	−.050	.040	−.060
Track resolution	−.010	−.1200	.020	.070	.010	.1200
Track charge error	.010	.070	.030	.010	.040	.030
$\langle x_E \rangle_b$	−.010	−.3000	−.030	−.090	−.070	−.1400
$\langle x_E \rangle_c$	−.2100	.2900	.1600	−.1400	.2700	−.1800
b frag function	0.0	0.0	0.0	0.0	0.0	0.0
c frag function	0.0	0.0	0.0	0.0	0.0	0.0
Br($b \to \ell$)	0.0	0.0	0.0	0.0	0.0	0.0
Br($b \to c \to \ell^+$)	0.0	0.0	0.0	0.0	0.0	0.0
Br($b \to \bar{c} \to \ell^-$)	−.050	−.1700	.070	.3600	.1100	.5200
Br($b \to \tau \to \ell$)	0.0	−.070	.020	.1500	.010	.2100
Br($b \to J/\psi \to \ell^+\ell^-$)	.040	.2200	−.010	.060	−.030	.030
Br($c \to \ell$)	−.0273	.596	.0909	−.150	.146	−.227
Br($c \to D^\star \to K\pi\pi_s$)	0.0	0.0	0.0	0.0	0.0	0.0
Semilept model $b \to \ell$	−.050	.2000	−.020	−.5000	.080	−.6400
Semilept model $c \to \ell$	−.090	.2700	−.1200	−.020	.2200	.020
$b \to D$ model.	0.0	0.0	0.0	0.0	0.0	0.0
$f(D^0)$	0.0	0.0	0.0	0.0	0.0	0.0
$f(D^+)$	0.0	0.0	0.0	0.0	0.0	0.0
$f(D^0 + D^+)$	0.0	0.0	0.0	0.0	0.0	0.0
$f(D_s)$	0.0	0.0	0.0	0.0	0.0	0.0
$f(\Lambda_c)$	0.0	0.0	0.0	0.0	0.0	0.0
D^0 lifetime	0.0	0.0	0.0	0.0	0.0	0.0
D^+ lifetime	0.0	0.0	0.0	0.0	0.0	0.0
D_s lifetime	0.0	0.0	0.0	0.0	0.0	0.0
Λ_c lifetime	0.0	0.0	0.0	0.0	0.0	0.0
B decay multip.	0.0	0.0	0.0	0.0	0.0	0.0
D decay multip.	0.0	0.0	0.0	0.0	0.0	0.0
$g \to b\bar{b}$	0.0	0.0	0.0	0.0	0.0	0.0
$g \to c\bar{c}$	0.0	0.0	0.0	0.0	0.0	0.0
Long-lived light h	0.0	0.0	0.0	0.0	0.0	0.0
Light quark frag	0.0	0.0	0.0	0.0	0.0	0.0
Total systematic	0.46	1.9	0.32	0.87	0.57	1.3
R_c	−0.6	+11.2	+1.2	−2.6	+2.1	−4.1

Table 29: Results of the multiparameter lepton fit – part II [9]

	$A_{FB}^{b\bar{b}}$(pk)
$\sqrt{s}$	91.28
Result	0.095
Statistics	0.011
Monte Carlo stat	0.0022
Elec id efficiency	0
Elec misid	0
Elec conversions	0
Muon ID efficiency	0
Muon ID background	0
Track resolution	0.0011
Internal	0.0020
$\langle x_E(\mathrm{b})\rangle$	0
$\langle x_E(\mathrm{c})\rangle$	0
b frag function	0
c frag function	0
$\mathrm{Br}(\mathrm{b} \to \ell)$	0
$\mathrm{Br}(\mathrm{b} \to \mathrm{c} \to \ell^+)$	0
$\mathrm{Br}(\mathrm{b} \to \bar{\mathrm{c}} \to \ell^-)$	0
$\mathrm{Br}(\mathrm{b} \to \tau \to \ell)$	0
$\mathrm{Br}(\mathrm{b} \to \mathrm{J}/\psi \to \ell^+\ell^-)$	0
$\mathrm{Br}(\mathrm{c} \to \ell)$	0
$\mathrm{Br}(\mathrm{c} \to \mathrm{D}^\star \to \mathrm{K}\pi\pi_\mathrm{s})$	0
Semilept model $\mathrm{b} \to \ell$	0
Semilept model $\mathrm{c} \to \ell$	0
$\mathrm{b} \to \mathrm{D}$ model.	0
$f(\mathrm{D}^0)$	0
$f(\mathrm{D}^+)$	0
$f(\mathrm{D}^0 + \mathrm{D}^+)$	0
$f(\mathrm{D_s})$	0
$f(\Lambda_\mathrm{c})$	0
D^0 lifetime	0
D^+ lifetime	0
$\mathrm{D_s}$ lifetime	0
Λ_c lifetime	0
B decay multip.	0.0004
D decay multip.	0
$\mathrm{g} \to \mathrm{b}\bar{\mathrm{b}}$	0
$\mathrm{g} \to \mathrm{c}\bar{\mathrm{c}}$	0
Long-lived light h	0
Light quark frag	0.0028
Total systematic	0.0044
R_b	−0.09768
R_c	+0.01214

Table 30: Asymmetry measurement using jet charge and a vertex tag [17]

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{c\bar{c}}(-2)$	$A_{FB}^{b\bar{b}}(pk)$	$A_{FB}^{c\bar{c}}(pk)$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(+2)$
$\sqrt{s}$	89.54	89.54	91.26	91.26	92.94	92.94
Results	-.2400	-.070	.042	.1100	-.4100	.3200
Statistics	.2700	.1200	.079	.037	.2300	.1100
$D^\star$ back acceptance	.015	.0010	.0050	.0040	.021	.013
$D^\star$ backgr rate	.027	.011	.0070	.0040	.023	.012
$D^\star$ backgr asymmetry	-.016	.0020	-.016	-.0010	-.016	-.0010
Detector effects	.0060	.0020	.0060	.0020	.0060	.0020
$x(D^\star)$ scale	.0070	.0010	.0060	.0020	.028	.010
Rate c to $D^\star$	-.0040	-.0010	-.0020	-.0010	-.016	-.0050
Rate b to $D^\star$	.0080	.0010	.0040	.0020	.031	.010
Shape b to D^*	.013	.0070	.013	.0070	.013	.0070
W to D^*	-.013	0.0	.0030	0.0	-.022	0.0
Effective mixing χ	-.021	0.0	.0040	0.0	-.035	0.0
Thrust resoln	.0010	.0010	.0010	.0010	.0010	.0010
$\langle x_E \rangle_b$	.0020	-.0020	.0030	-.0070	.0090	-.012
$\langle x_E \rangle_c$	-.0090	.0040	-.0040	.0020	-.023	.010
b frag function	0.0	0.0	0.0	0.0	0.0	0.0
c frag function	0.0	0.0	0.0	0.0	0.0	0.0
$Br(b \to \ell)$	0.0	0.0	0.0	0.0	0.0	0.0
$Br(b \to c \to \ell^+)$	0.0	0.0	0.0	0.0	0.0	0.0
$Br(b \to \bar{c} \to \ell^-)$	0.0	0.0	0.0	0.0	0.0	0.0
$Br(b \to \tau \to \ell)$	0.0	0.0	0.0	0.0	0.0	0.0
$Br(b \to J/\psi \to \ell^+\ell^-)$	0.0	0.0	0.0	0.0	0.0	0.0
$Br(c \to \ell)$	0.0	0.0	0.0	0.0	0.0	0.0
$Br(c \to D^\star \to K\pi\pi_s)$	0.0	0.0	0.0	0.0	0.0	0.0
Semilept model $b \to \ell$	0.0	0.0	0.0	0.0	0.0	0.0
Semilept model $c \to \ell$	0.0	0.0	0.0	0.0	0.0	0.0
$b \to D$ model.	0.0	0.0	0.0	0.0	0.0	0.0
$f(D^0)$	0.0	0.0	0.0	0.0	0.0	0.0
$f(D^+)$	0.0	0.0	0.0	0.0	0.0	0.0
$f(D^0 + D^+)$	0.0	0.0	0.0	0.0	0.0	0.0
$f(D_s)$	0.0	0.0	0.0	0.0	0.0	0.0
$f(\Lambda_c)$	0.0	0.0	0.0	0.0	0.0	0.0
D^0 lifetime	0.0	0.0	0.0	0.0	0.0	0.0
D^+ lifetime	0.0	0.0	0.0	0.0	0.0	0.0
D_s lifetime	0.0	0.0	0.0	0.0	0.0	0.0
Λ_c lifetime	0.0	0.0	0.0	0.0	0.0	0.0
B decay multip.	0.0	0.0	0.0	0.0	0.0	0.0
D decay multip.	0.0	0.0	0.0	0.0	0.0	0.0
$g \to b\bar{b}$	0.0	0.0	0.0	0.0	0.0	0.0
$g \to c\bar{c}$	0.0	0.0	0.0	0.0	0.0	0.0
Long-lived light h	0.0	0.0	0.0	0.0	0.0	0.0
Light quark frag	0.0	0.0	0.0	0.0	0.0	0.0
Total systematic	.04733	.01425	.02534	.01221	.07584	.02893

Table 31: Asymmetry measurements with a $D^\star$ tag [22]

	R_c
Result	0.141
Statistics	0.008
Internal	0.0019
Track resolution	0.0010
$D^\star$ backgr. rate	0.0029
Effective mixing χ	0.0005
b hadron lifetime	0.0025
c frag function	0.0080
$Br(c \to D^\star \to K\pi\pi_s)$	−0.0110
Total systematic	0.0014

Table 32: Breakdown of errors for OPAL R_c measured using a $D^\star$ tag [20]

	R_b	R_c	$Br(\ell)$	$Br(c\ell)$	$A_{FB}^{b\bar{b}}$ (−2)	$A_{FB}^{c\bar{c}}$ (−2)	$A_{FB}^{b\bar{b}}$ (pk)	$A_{FB}^{c\bar{c}}$ (pk)	$A_{FB}^{b\bar{b}}$ (+2)	$A_{FB}^{c\bar{c}}$ (+2)
R_b	1.00	−0.93	0.07	−0.12	0.00	−0.11	−0.12	0.09	−0.07	0.05
R_c	−0.93	1.00	−0.15	0.14	0.00	0.10	0.12	−0.08	0.07	−0.04
$Br(\ell)$	0.07	−0.15	1.00	−0.14	−0.01	−0.01	−0.08	0.01	−0.04	0.00
$Br(c\ell)$	−0.12	0.14	−0.14	1.00	0.08	0.01	0.53	−0.01	0.27	−0.01
$A_{FB}^{b\bar{b}}(-2)$	0.00	0.00	−0.01	0.08	1.00	0.28	0.00	0.00	0.00	0.00
$A_{FB}^{c\bar{c}}(-2)$	−0.11	0.10	−0.01	0.01	0.28	1.00	0.00	0.00	0.00	0.00
$A_{FB}^{b\bar{b}}(pk)$	−0.12	0.12	−0.08	0.53	0.00	0.00	1.00	0.23	0.00	0.00
$A_{FB}^{c\bar{c}}(pk)$	0.09	−0.08	0.01	−0.01	0.00	0.00	0.23	1.00	0.00	0.00
$A_{FB}^{b\bar{b}}(+2)$	−0.07	0.07	−0.04	0.27	0.00	0.00	0.00	0.00	1.00	0.27
$A_{FB}^{c\bar{c}}(+2)$	0.05	−0.04	0.00	−0.01	0.00	0.00	0.00	0.00	0.27	1.00

Table 33: Statistical correlation matrix for the OPAL multiparameter fit to lepton data. $Br(\ell)$ denotes $Br(b \to \ell)$, $Br(c\ell)$ denotes $Br(b \to c \to \ell^+)$.

	$A_{FB}^{b\bar{b}}(-2)$	$A_{FB}^{c\bar{c}}(-2)$	$A_{FB}^{b\bar{b}}(pk)$	$A_{FB}^{c\bar{c}}(pk)$	$A_{FB}^{b\bar{b}}(+2)$	$A_{FB}^{c\bar{c}}(+2)$
$A_{FB}^{b\bar{b}}(-2)$	1.000	-.7430	0.0	0.0	0.0	0.0
$A_{FB}^{c\bar{c}}(-2)$	-.7430	1.000	0.0	0.0	0.0	0.0
$A_{FB}^{b\bar{b}}(pk)$	0.0	0.0	1.000	-.7430	0.0	0.0
$A_{FB}^{c\bar{c}}(pk)$	0.0	0.0	-.7430	1.000	0.0	0.0
$A_{FB}^{b\bar{b}}(+2)$	0.0	0.0	0.0	0.0	1.000	-.7520
$A_{FB}^{c\bar{c}}(+2)$	0.0	0.0	0.0	0.0	-.7520	1.000

Table 34: OPAL $D^\star$ asymmetry statistical correlation matrix

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