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Updated Parameters of the Z^0 Lineshape and Lepton Forward-Backward Asymmetries from Combined Preliminary Data of the LEP Experiments

The LEP Electroweak Working Subgroup on Lineshape and Lepton Forward-Backward Asymmetries*

Internal Note

summarizing the combination of preliminary LEP data for the 27th International Conference on High Energy Physics, Glasgow, Scotland, 20-27 July 1994

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1 Introduction

We evaluate updated parameters of the Z^0 resonance derived from published data and new preliminary results. Most of these preliminary results are contributions to the 27th International Conference on High Energy Physics, Glasgow, Scotland, 20-27 July 1994. The emphasis of this note is a description of the combination of electroweak parameters on the Z^0 lineshape and lepton pair forward-backward asymmetries of the four LEP experiments, taking account of errors which are correlated among the experiments. Several aspects of such a combination have already been studied in [1, 2, 3].

2 Z^0 Lineshape and Lepton Forward-Backward Asymmetries

The results presented here are based on the data taken during the energy scans in 1990 and 1991 with centre-of-mass energies in a range $\sqrt{s} = M_Z \pm 3$ GeV, the high statistics data collected at the Z^0 peak in 1992, and a preliminary analysis of the high precision scan in 1993. During this scan more than 18 pb^{-1} were recorded by each experiment at two centre-of-mass energy points roughly 1.8 GeV above and below the Z^0 mass while the remainder was within 200 MeV of M_Z.

The total statistics and the systematic errors of the individual LEP collaborations are given in Tables 1 and 2. Details of the individual analyses can be found in [4, 5, 6, 7]. An important aspect of the lineshape analysis is a precise knowledge of the LEP centre-of-mass energies. The treatment of the LEP centre-of-mass energies by the four LEP experiments is based on [8, 9]. In order to determine Γ_Z we combine all of the recorded data, taking the measurement arising from the 1993 data alone to be uncorrelated with the measurements coming from previous years. However for the determination of M_Z we use the 1993 data alone. This follows the reccomandation of [9], and is due to the correlation with previous years having not yet been fully studied.

For the averaging of results the LEP experiments provide a standard set of 9 parameters describing the information contained in hadronic and leptonic cross-sections and leptonic forward-backward asymmetries [2]. The four sets of 9 parameters provided by the LEP experiments from fits to the data taken until the end of 1992 and preliminary data for 1993 are presented in Table 3. The full covariance matrix of the input parameters is given in Tables 4, 5 and 6. The combined parameter set and its correlation matrix are given in Tables 7 and 8.

If lepton universality is assumed the set of 9 parameters given above can be reduced to a set of 5 parameters. Due to mass corrections to $\Gamma_{\tau\tau}$ we expect, however, a small (0.2%) difference between the values for R_e and R_μ and the value for R_τ. We define $R_\ell \equiv \Gamma_{\mathrm{had}}/\Gamma_{\ell\ell}$, where $\Gamma_{\ell\ell}$ refers to the partial Z^0 width for the decay into a pair of massless charged leptons.

The data of each of the four LEP experiments and their combination are consistent with lepton universality. Based on this assumption Table 9 provides the five parameters M_Z, Γ_Z, σ^0_{h}, R_ℓ and $A^{0,\ell}_{\mathrm{FB}}$ for the individual LEP experiments. The four experiments now all use the above definition of $\Gamma_{\ell\ell}$ (for other choices c.f. [2]). The full covariance matrix for the combination of the 5-parameter results of the 4 experiments can be constructed from Tables 4, 10 and 11.

Tables 12 and 13 provide the five parameters M_{Z}, Γ_{Z}, σ_{h}^{0}, R_ℓ and $\mathrm{A_{FB}^{0,\ell}}$ and the corresponding correlation matrix for the combined result of the 4 LEP experiments. Figure 1 shows, for each leptonic species and for the combination assuming lepton universality, the resulting 68% probability contours in the R_ℓ-$\mathrm{A_{FB}^{0,\ell}}$ plane.

The parameter R_ℓ can be used to determine the strong coupling constant α_s. For $M_{\mathrm{Z}} = 91.1888$ GeV, $M_{\mathrm{t}} = 175$ GeV and $M_{\mathrm{H}} = 300$ GeV we obtain from $R_\ell = 20.795 \pm 0.040$ alone:

$$\alpha_s(M_{\mathrm{Z}}^2) = 0.126 \pm 0.006\,.$$

The error quoted accounts for experimental uncertainties only. A detailed discussion of theoretical uncertainties can be found in [10]. A more precise value of $\alpha_s(M_{\mathrm{Z}}^2)$ is given in the context of a global fit to all electroweak precision measurements in [11].

The parameters M_{Z}, Γ_{Z}, σ_{h}^{0}, R_ℓ and $\mathrm{A_{FB}^{0,\ell}}$ are convenient for fitting and averaging since they have minimal correlations amongst them. However a number of other parameters, which can be derived from the previous set, are of physical importance. Table 14 gives a number of commonly used parameters derived from the results of the 9-parameter (Tables 7 and 8) and the 5-parameter fit (Tables 12 and 13). Among these parameters we quote $\sin^2\theta_{eff}^{\mathrm{lept}}$ which is defined as:

$$\sin^2\theta_{eff}^{\mathrm{lept}} \equiv \frac{1}{4}(1 - g_{V_\ell}/g_{A_\ell})\,. \tag{1}$$

As an indication of the power of these new data we use them to derive the number of light neutrino species. Using the results of Tables 12 and 13 we find:

$$\Gamma_{\mathrm{inv}}/\Gamma_{\ell\ell} \quad = \quad 5.953 \pm 0.046\,.$$

Taking the Standard Model value for the ratio of the partial widths to neutrinos and charged leptons:

$$\Gamma_\nu/\Gamma_{\ell\ell} \quad = \quad 1.992 \pm 0.003\,,$$

where the central value is evaluated for $M_{\mathrm{Z}} = 91.1888$ GeV, $M_{\mathrm{t}} = 175$ GeV, $M_{\mathrm{H}} = 300$ GeV and $\alpha_s(M_{\mathrm{Z}}^2) = 0.123$ and the error quoted accounts for a variation of M_{t} in the range $100 < M_{\mathrm{t}}(\mathrm{GeV}) < 200$ and a variation of M_{H} in the range $60 < M_{\mathrm{H}}(\mathrm{GeV}) < 1000$, we find:

$$N_\nu \quad = \quad 2.988 \pm 0.023\,.$$

The parameters discussed above, however, do not describe the properties of the Z^0 completely, as they do not contain information about the interference of the Z^0 exchange with the photon exchange. In the case of charged leptons, assuming no modifications in the Born diagrams, the $\gamma\mathrm{Z}^0$ interference term could be expressed in terms of the leptonic effective vector and axial-vector couplings, g_{V_ℓ} and g_{A_ℓ}, which can be determined by the measurements discussed above. However, this is not the case for the inclusive hadronic final state, which is summed over all colours and open quark flavours. Thus, the hadronic $\gamma\mathrm{Z}^0$ interference term remains as an additional undetermined parameter unless it is fixed to its Standard Model expectation value.

Because of the asymmetric nature of the γZ^0 interference cross section as a function of the centre-of-mass energy when crossing the Z^0 pole, the hadronic γZ^0 interference term is strongly anti-correlated with M_Z. Therefore, its uncertainty substantially increases the total error on M_Z (by ≈ 8 MeV for a single LEP experiment) compared to the situation where it is taken from the Standard Model. The errors on the total and partial widths of the Z^0 increase only marginally.

Work is in progress to implement the γZ^0 interference term as an input parameter in the various fitting programs in order to include its effects in more detail in future analyses. For the results presented below, the hadronic γZ^0 interference term is fixed to its Standard Model value.

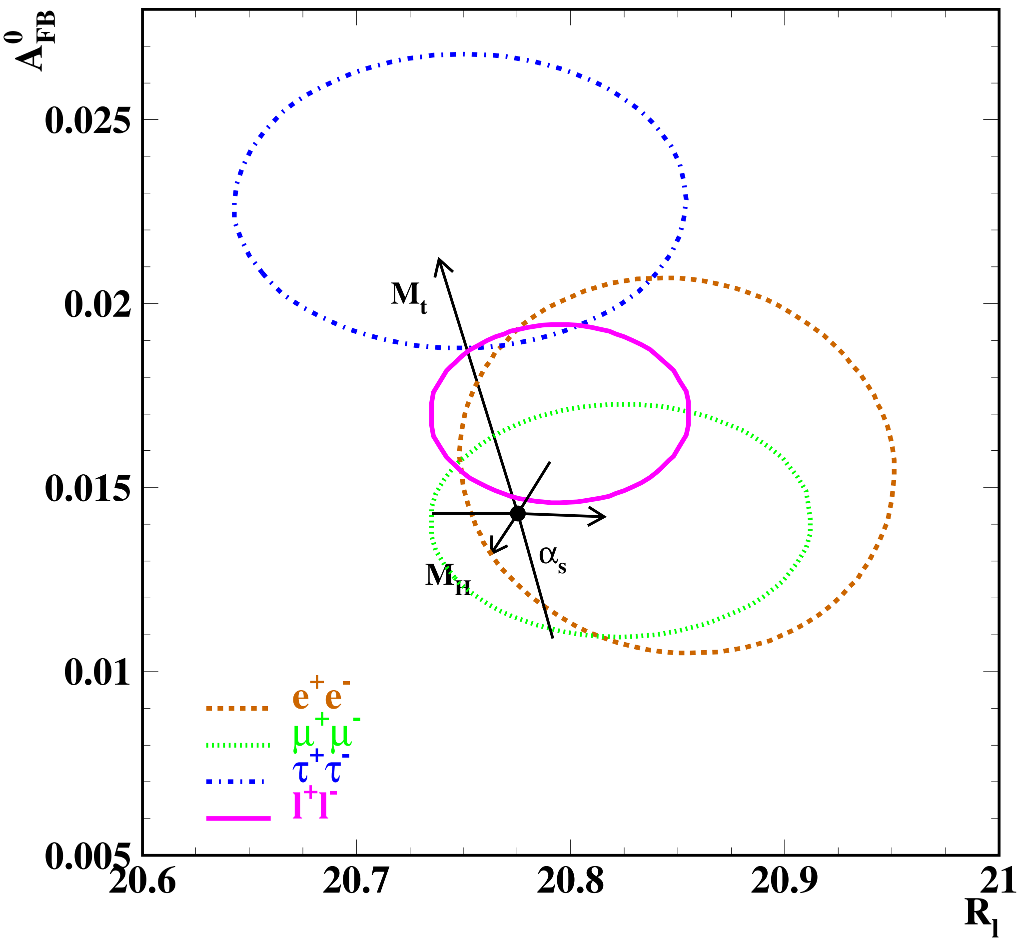

Figure 1: 68% probability contours in the R_ℓ-$A_{FB}^{0,\ell}$ plane. Also shown is the Standard Model prediction for $M_Z = 91.1888$ GeV, $M_t = 150$ GeV, $M_H = 300$ GeV, $\alpha_s = 0.123$. The lines with arrows correspond to the variation of the Standard Model prediction when M_t, M_H or α_s are varied in the intervals $50 < M_t(\mathrm{GeV}) < 250$, $60 < M_H(\mathrm{GeV}) < 1000$ and $\alpha_s(M_Z^2) = 0.123 \pm 0.006$, respectively. The arrows point in the direction of increasing values for M_t, M_H and α_s.

		ALEPH	DELPHI	L3	OPAL	LEP
$q\bar{q}$	'90-'91	451	356	423	454	1684
	'92	680	697	677	733	2787
	'93 prel.	653	677	658	653	2641
	total	1784	1730	1758	1840	7112
$\ell^+\ell^-$	'90-'91	55	37	40	58	190
	'92	82	69	58	88	297
	'93 prel.	79	71	62	81	293
	total	216	177	160	227	780

Table 1: The LEP statistics in units of 10^3 events used for the analysis of the Z^0 line shape and lepton forward-backward asymmetries.

	ALEPH		DELPHI		L3		OPAL	
	'92	'93 prel.	'92	'93 prel.	'92	'93 prel.	'92	'93 prel.
$\mathcal{L}^{\mathrm{exp.}\ (a)}$	0.15%	0.09%	0.38%	0.28%	0.5 %	0.16%	0.41%	0.07%
σ_{had}	0.14%	0.14%	0.13%	0.13%	0.15%	0.11 – 0.14%$^{(b)}$	0.20%	0.20%
σ_{e}	0.4 %	0.4%	0.59%	1.2%	0.3 %	0.25 – 0.76%$^{(b)}$	0.22%	0.35%
σ_{μ}	0.5 %	0.5%	0.37%	0.5%	0.5%	0.45 – 0.57%$^{(b)}$	0.19%	0.22%
σ_{τ}	0.3%	0.3%	0.63%	0.8%	0.7 %	0.54%	0.44%	0.51%
$A_{\mathrm{FB\ e}}$	0.0029	0.0029	0.003	0.003	0.002	0.005	0.002	0.002
$A_{\mathrm{FB}\ \mu}$	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.001
$A_{\mathrm{FB}\ \tau}$	0.0005	0.0005	0.0017	0.002	0.003	0.004	0.002	0.002

Table 2: The experimental systematic errors for the analysis of the Z^0 line shape and lepton forward-backward asymmetries. The errors quoted do not include the common uncertainty due to the LEP energy calibration. For the treatment of correlations between the errors for different years we refer to [4, 5, 6, 7].

$^{(a)}$Only the experimental error including the statistics of small angle Bhabha events is quoted. In addition, there is a theoretical error for the calculation of the small angle Bhabha cross section of 0.25% which is common to all experiments.

$^{(b)}$The indicated range for the 1993 selection expresses the variation of the systematic error as a function of energy.

	ALEPH	DELPHI	L3	OPAL
M_Z(GeV)	91.1915±0.0052	91.1870±0.0052	91.1900±0.0054	91.1862±0.0054
Γ_Z(GeV)	2.4959±0.0061	2.4951±0.0059	2.5040±0.0058	2.4945±0.0061
σ_h^0(nb)	41.59±0.13	41.26±0.17	41.44±0.15	41.47±0.16
R_e	20.67±0.13	20.96±0.16	20.94±0.13	20.90±0.13
R_μ	20.91±0.14	20.60±0.12	20.93±0.14	20.855±0.097
R_τ	20.69±0.12	20.64±0.16	20.70±0.17	20.91±0.13
$A_{FB}^{0,e}$	0.0212±0.0054	0.0207±0.0073	0.0109±0.0081	0.0060±0.0066
$A_{FB}^{0,\mu}$	0.0189±0.0039	0.0128±0.0037	0.0132±0.0048	0.0124±0.0035
$A_{FB}^{0,\tau}$	0.0253±0.0043	0.0209±0.0057	0.0299±0.0073	0.0193±0.0044
$\chi^2/(d.o.f.)$	172/178	154/132	111/131	10/6$^{(a)}$

Table 3: Line shape and asymmetry parameters from 9-parameter fits to the data of the four LEP experiments.

$^{(a)}$This parameter set has been obtained from a parameter transformation applied to the 15 parameters of the OPAL model independent fit, which treats the γZ^0 interference terms for leptons as free parameters. The extra parameters for the γZ^0 interference terms have not been fixed in the transformation. The $\chi^2/(d.o.f.)$ for the direct fit to the data is 87/125.

$\mathcal{V}$:	ALEPH	DELPHI	L3	OPAL
ALEPH	$\sigma_A A \sigma_A^T$	$(C_E + C_L)^T$	$(C_E + C_L)^T$	$(C_E + C_L)^T$
DELPHI	$C_E + C_L$	$\sigma_D D \sigma_D^T$	$(C_E + C_L)^T$	$(C_E + C_L)^T$
L3	$C_E + C_L$	$C_E + C_L$	$\sigma_L L \sigma_L^T$	$(C_E + C_L)^T$
OPAL	$C_E + C_L$	$C_E + C_L$	$C_E + C_L$	$\sigma_O O \sigma_O^T$

Table 4: The covariance matrix $\mathcal{V}$ used for the LEP average of parameters. The parameter errors of the individual experiments (σ_A, σ_D, σ_L, σ_O) and the correlation matrices of the individual experiments, A, D, L and O, (see Table 5 for the combination of 9 parameter results and Table 10 for the combination of 5 parameter results, respectively) include the effect of experiment specific and common uncertainties. The submatrix C_E (see Table 6 for the combination of 9 parameter results and Table 11 for the combination of 5 parameter results, respectively) accounts for common systematic uncertainties due to the LEP energy calibration. The submatrix C_L accounts for the common systematic error of the absolute normalization due to the theoretical uncertainty in the small angle Bhabha cross section. For simplicity we use for these preliminary results a submatrix C_L which is zero everywhere except for the diagonal element for σ_h^0, which is $C_L(\sigma_h^0, \sigma_h^0) = 0.0025 \cdot 0.0025 \cdot \sigma_h^{0\,2}$.

A:	M_{Z}	Γ_{Z}	σ_{h}^{0}	R_{e}	R_{μ}	R_{τ}	$A_{\mathrm{FB}}^{0,\mathrm{e}}$	$A_{\mathrm{FB}}^{0,\mu}$	$A_{\mathrm{FB}}^{0,\tau}$
M_{Z}	1.000	0.05	0.02	0.00	0.00	0.00	0.05	0.07	0.06
Γ_{Z}	0.05	1.00	−0.23	0.00	0.01	−0.01	0.00	0.00	0.00
σ_{h}^{0}	0.02	−0.23	1.00	0.11	0.10	0.12	0.00	0.00	0.00
R_{e}	0.00	0.00	0.11	1.00	0.06	0.07	0.01	0.00	0.00
R_{μ}	0.00	0.01	0.10	0.06	1.00	0.07	0.00	0.01	0.00
R_{τ}	0.00	−0.01	0.12	0.07	0.07	1.00	0.00	0.00	0.01
$A_{\mathrm{FB}}^{0,\mathrm{e}}$	0.05	0.00	0.00	0.01	0.00	0.00	1.00	0.03	0.04
$A_{\mathrm{FB}}^{0,\mu}$	0.07	0.00	0.00	0.00	0.01	0.00	0.03	1.00	0.03
$A_{\mathrm{FB}}^{0,\tau}$	0.06	0.00	0.00	0.00	0.00	0.01	0.04	0.03	1.00

D:	M_{Z}	Γ_{Z}	σ_{h}^{0}	R_{e}	R_{μ}	R_{τ}	$A_{\mathrm{FB}}^{0,\mathrm{e}}$	$A_{\mathrm{FB}}^{0,\mu}$	$A_{\mathrm{FB}}^{0,\tau}$
M_{Z}	1.000	0.05	0.01	0.01	0.00	0.00	0.02	0.03	0.02
Γ_{Z}	0.05	1.00	−0.18	0.00	0.00	0.00	0.00	−0.01	0.00
σ_{h}^{0}	0.01	−0.18	1.00	0.07	0.10	0.07	0.00	−0.01	−0.01
R_{e}	0.01	0.00	0.07	1.00	0.05	0.04	0.00	0.00	0.00
R_{μ}	0.00	0.00	0.10	0.05	1.00	0.05	0.00	0.02	0.00
R_{τ}	0.00	0.00	0.07	0.04	0.05	1.00	0.00	0.00	0.02
$A_{\mathrm{FB}}^{0,\mathrm{e}}$	0.02	0.00	0.00	0.00	0.00	0.00	1.00	0.02	0.01
$A_{\mathrm{FB}}^{0,\mu}$	0.03	−0.01	−0.01	0.00	0.02	0.00	0.02	1.00	0.02
$A_{\mathrm{FB}}^{0,\tau}$	0.02	0.00	−0.01	0.00	0.00	0.02	0.01	0.02	1.00

L:	M_{Z}	Γ_{Z}	σ_{h}^{0}	R_{e}	R_{μ}	R_{τ}	$A_{\mathrm{FB}}^{0,\mathrm{e}}$	$A_{\mathrm{FB}}^{0,\mu}$	$A_{\mathrm{FB}}^{0,\tau}$
M_{Z}	1.00	0.05	0.01	−0.11	0.00	0.00	0.11	0.03	0.02
Γ_{Z}	0.05	1.00	−0.23	0.00	0.00	0.01	−0.01	0.01	0.00
σ_{h}^{0}	0.01	−0.23	1.00	0.04	0.10	0.08	0.05	0.00	0.00
R_{e}	−0.11	0.00	0.04	1.00	0.06	0.05	−0.14	0.04	0.03
R_{μ}	0.00	0.00	0.10	0.06	1.00	0.05	0.00	0.01	0.00
R_{τ}	0.00	0.01	0.08	0.05	0.05	1.00	0.00	0.00	0.01
$A_{\mathrm{FB}}^{0,\mathrm{e}}$	0.11	−0.01	0.05	−0.14	0.00	0.00	1.00	−0.02	−0.02
$A_{\mathrm{FB}}^{0,\mu}$	0.03	0.01	0.00	0.04	0.01	0.00	−0.02	1.00	0.02
$A_{\mathrm{FB}}^{0,\tau}$	0.02	0.00	0.00	0.03	0.00	0.01	−0.02	0.02	1.00

O:	M_{Z}	Γ_{Z}	σ_{h}^{0}	R_{e}	R_{μ}	R_{τ}	$A_{\mathrm{FB}}^{0,\mathrm{e}}$	$A_{\mathrm{FB}}^{0,\mu}$	$A_{\mathrm{FB}}^{0,\tau}$
M_{Z}	1.00	0.15	0.04	−0.01	0.00	0.01	−0.01	0.06	0.05
Γ_{Z}	0.15	1.00	−0.26	0.00	0.01	0.01	0.02	0.01	0.00
σ_{h}^{0}	0.04	−0.26	1.00	0.19	0.31	0.23	0.04	0.00	0.00
R_{e}	−0.01	0.00	0.19	1.00	0.19	0.10	−0.13	0.04	0.03
R_{μ}	0.00	0.01	0.31	0.19	1.00	0.16	0.00	0.01	0.00
R_{τ}	0.01	0.01	0.23	0.10	0.16	1.00	0.00	0.00	0.01
$A_{\mathrm{FB}}^{0,\mathrm{e}}$	−0.01	0.02	0.04	−0.13	0.00	0.00	1.00	−0.02	−0.02
$A_{\mathrm{FB}}^{0,\mu}$	0.06	0.01	0.00	0.04	0.01	0.00	−0.02	1.00	0.02
$A_{\mathrm{FB}}^{0,\tau}$	0.05	0.00	0.00	0.03	0.00	0.01	−0.02	0.02	1.00

Table 5: The matrices A, D, L, and O used in the covariance matrix $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 9 parameter fits from the individual LEP experiments.

C_E:	σ_E	C								
		M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
M_Z(GeV)	4 MeV	1.	0.	0.	0.	0.	0.	0.	0.	0.
Γ_Z(GeV)	2.7 MeV	0.	1.	0.	0.	0.	0.	0.	0.	0.
σ_h^0(nb)	0	0.	0.	1.	0.	0.	0.	0.	0.	0.
R_e	0	0.	0.	0.	1.	0.	0.	0.	0.	0.
R_μ	0	0.	0.	0.	0.	1.	0.	0.	0.	0.
R_τ	0	0.	0.	0.	0.	0.	1.	0.	0.	0.
$A_{FB}^{0,e}$	0.0008	0.	0.	0.	0.	0.	0.	1.	−1.	−1.
$A_{FB}^{0,\mu}$	0.0008	0.	0.	0.	0.	0.	0.	−1.	1.	1.
$A_{FB}^{0,\tau}$	0.0008	0.	0.	0.	0.	0.	0.	−1.	1.	1.

Table 6: The matrix $C_E = \sigma_E C \sigma_E^T$ used in the covariance matrix, $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 9 parameter fits from the individual LEP experiments. Due to the t-channel contribution $A_{FB}^{0,e}$ has a slope with centre-of-mass energy which is of opposite sign to that of $A_{FB}^{0,\mu}$ and $A_{FB}^{0,\tau}$; hence the anticorrelation.

Parameter	Average Value
M_Z(GeV)	91.1888±0.0044
Γ_Z(GeV)	2.4974±0.0038
σ_h^0(nb)	41.49±0.12
R_e	20.850±0.067
R_μ	20.824±0.059
R_τ	20.749±0.070
$A_{FB}^{0,e}$	0.0156±0.0034
$A_{FB}^{0,\mu}$	0.0141±0.0021
$A_{FB}^{0,\tau}$	0.0228±0.0026

Table 7: Average line shape and asymmetry parameters from the data of the four LEP experiments given in Table 3, without the assumption of lepton universality. The $\chi^2/(d.o.f.)$ of the average is 26.8/27.

	M_Z	Γ_Z	σ_h^0	R_e	R_μ	R_τ	$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
M_Z	1.00	0.04	0.01	−0.02	−0.01	0.00	0.02	0.03	0.02
Γ_Z	0.04	1.00	−0.11	0.00	0.01	0.00	0.01	0.00	0.00
σ_h^0	0.01	−0.11	1.00	0.07	0.10	0.08	0.01	0.00	0.00
R_e	−0.02	0.00	0.07	1.00	0.09	0.06	−0.06	0.02	0.01
R_μ	−0.01	0.01	0.10	0.09	1.00	0.07	−0.01	0.01	0.00
R_τ	0.00	0.00	0.08	0.06	0.07	1.00	0.00	0.00	0.01
$A_{FB}^{0,e}$	0.02	0.01	0.01	−0.06	−0.01	0.00	1.00	−0.08	−0.06
$A_{FB}^{0,\mu}$	0.03	0.00	0.00	0.02	0.01	0.00	−0.08	1.00	0.12
$A_{FB}^{0,\tau}$	0.02	0.00	0.00	0.01	0.00	0.01	−0.06	0.12	1.00

Table 8: The correlation matrix for the set of parameters given in Table 7.

	ALEPH	DELPHI	L3	OPAL
M_Z(GeV)	91.1915±0.0052	91.1869±0.0052	91.1900±0.0054	91.1862±0.0054
Γ_Z(GeV)	2.4959±0.0061	2.4951±0.0059	2.5040±0.0058	2.4946±0.0061
σ_h^0(nb)	41.59±0.13	41.26±0.17	41.45±0.15	41.48±0.16
R_ℓ	20.730±0.078	20.69±0.086	20.859±0.088	20.864±0.076
$A_{FB}^{0,\ell}$	0.0216±0.0026	0.0160±0.0029	0.0168±0.0036	0.0137±0.0025
$\chi^2/(d.o.f.)$	175/182	160/136	117/135	13/10[a]

Table 9: Line shape and asymmetry parameters from 5-parameter fits to the data of the four LEP experiments, assuming lepton universality.
[a]This parameter set has been obtained from a parameter transformation to the 15 parameters of the OPAL model independent fit, which treats the γZ^0 interference terms for leptons as free parameters.

A:	M_Z	Γ_Z	σ_h^0	R_ℓ	$A_{FB}^{0,\ell}$
M_Z(GeV)	1.00	0.05	0.02	0.00	0.10
Γ_Z(GeV)	0.05	1.00	−0.23	−0.01	0.00
σ_h^0(nb)	0.02	−0.23	1.00	0.18	0.01
R_ℓ	0.00	−0.01	0.18	1.00	0.01
$A_{FB}^{0,\ell}$	0.10	0.00	0.01	0.01	1.00
D:	M_Z	Γ_Z	σ_h^0	R_ℓ	$A_{FB}^{0,\ell}$
M_Z(GeV)	1.00	0.05	0.01	0.00	0.04
Γ_Z(GeV)	0.05	1.00	−0.18	0.00	−0.01
σ_h^0(nb)	0.01	−0.18	1.00	0.13	−0.01
R_ℓ	0.00	0.00	0.13	1.00	0.01
$A_{FB}^{0,\ell}$	0.04	−0.01	−0.01	0.01	1.00
L:	M_Z	Γ_Z	σ_h^0	R_ℓ	$A_{FB}^{0,\ell}$
M_Z(GeV)	1.00	0.05	0.00	−0.06	0.08
Γ_Z(GeV)	0.05	1.00	−0.23	0.00	0.00
σ_h^0(nb)	0.00	−0.23	1.00	0.12	0.02
R_ℓ	−0.06	0.00	0.12	1.00	−0.01
$A_{FB}^{0,\ell}$	0.08	0.00	0.02	−0.01	1.00
O:	M_Z	Γ_Z	σ_h^0	R_ℓ	$A_{FB}^{0,\ell}$
M_Z(GeV)	1.00	0.15	0.04	0.00	0.06
Γ_Z(GeV)	0.15	1.00	−0.26	0.01	0.01
σ_h^0(nb)	0.04	−0.26	1.00	0.38	0.01
R_ℓ	0.00	0.01	0.38	1.00	0.01
$A_{FB}^{0,\ell}$	0.06	0.01	0.01	0.00	1.00

Table 10: The matrices A, D, L, and O used in the covariance matrix $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 5 parameter fits from the individual LEP experiments.

C_E:	σ_E	C				
		M_Z	Γ_Z	σ_h^0	R_ℓ	$A_{FB}^{0,\ell}$
M_Z(GeV)	4 MeV	1.	0.	0.	0.	0.
Γ_Z(GeV)	2.7 MeV	0.	1.	0.	0.	0.
σ_h^0(nb)	0	0.	0.	1.	0.	0.
R_ℓ	0	0.	0.	0.	1.	0.
$A_{FB}^{0,\ell}$	0.0008	0.	0.	0.	0.	1.

Table 11: The matrix $C_E = \sigma_E C \sigma_E^T$ used in the covariance matrix, $\mathcal{V}$ (see definition in Table 4) for the average of the parameters of the 5 parameter fits from the individual LEP experiments.

Parameter	Average Value
M_Z(GeV)	91.1888±0.0044
Γ_Z(GeV)	2.4974±0.0038
σ_h^0(nb)	41.49±0.12
R_ℓ	20.795±0.040
$A_{FB}^{0,\ell}$	0.0170±0.0016

Table 12: Average line shape and asymmetry parameters from the results of the four LEP experiments given in Table 3, assuming lepton universality. The $\chi^2/(d.o.f.)$ of the average is 18.4/15.

	M_Z	Γ_Z	σ_h^0	R_ℓ	$A_{FB}^{0,\ell}$
M_Z	1.00	0.04	0.01	−0.01	0.04
Γ_Z	0.04	1.00	−0.11	0.01	0.00
σ_h^0	0.01	−0.11	1.00	0.13	0.00
R_ℓ	−0.01	0.01	0.13	1.00	0.01
$A_{FB}^{0,\ell}$	0.04	0.00	0.00	0.01	1.00

Table 13: The correlation matrix for the set of parameters given in Table 12.

Without Lepton Universality:	
Γ_{ee}(MeV)	83.85±0.21
$\Gamma_{\mu\mu}$(MeV)	83.95±0.30
$\Gamma_{\tau\tau}$(MeV)	84.26±0.34
With Lepton Universality:	
Γ_l(MeV)	83.96±0.18
Γ_{had}(MeV)	1745.9±4.0
Γ_{inv}(MeV)	499.8±3.5
$g_{V_l}^2$	0.00144±0.00014
$g_{A_l}^2$	0.25118±0.00056
$\sin^2\theta_{eff}^{lept}$	0.23107±0.00090

Table 14: Average values of some derived parameters.

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