EUROPEAN ORGANIZATION FOR PARTICLE PHYSICS

LEP EWWG / LS 99-01
June 25, 1999

Precise determination of Z–boson parameters from combined results of the LEP experiments

LEPEWWG: the LS and Afb sub-group

G. Duckeck, M. Grünewald, T. Kawamoto, R. Kellogg, G. Martinez,
J. Mnich, A. Olshevski, G. Quast, P. Renton, E. Tournefier

Abstract

The determination of the Z–boson mass and total decay width requires the averaging of results by individual experiments in the presence of common systematic errors. Uncertainties from the LEP energy calibration constitute the most important source of common uncertainty, followed by theoretical uncertainties in the calculation of the small-angle Bhabha cross section and the unavoidable corrections for QED radiative effects and t-channel diagrams and the s–t interference in the Bhabha final state. In this note we propose a method of combination for the final Z line shape and forward-backward asymmetries at LEP I and provide estimates of the systematic errors involved.

1 Introduction

Between the years 1989 and 1995, the e^+e^- collider LEP at CERN provided interactions at centre-of-mass energies ranging from 88 to 95 GeV, *i.e.*, around the mass of the Z boson. An important aspect of physics at LEP concerns the analysis of fermion-pair production in e^+e^- collisions [1]. The four LEP experiments ALEPH [2], DELPHI [3], L3 [4] and OPAL [5] analyse in particular hadron (quark-pair) production and the pair production of charged leptons, $\ell = e, \mu, \tau$.

At various centre-of-mass energies, total cross sections are measured for all processes, while forward-backward asymmetries are easily measured in lepton-pair production. These measurements of realistic observables allow the determination of various properties of the Z boson, such as its mass, total and partial decay widths, and coupling constants to fermions (pseudo-observables). The nine pseudo-observables considered here are: m_{Z}, Γ_{Z}, σ^{o}_{h}, R_ℓ and $\mathrm{A}^{0,\ell}_{\mathrm{FB}}$, $\ell = e, \mu, \tau$ as defined in [6]. For the extraction of these pseudo-observables, the experiments perform so-called model-independent fits to their measured realistic observables [7, 8, 9, 10].

To obtain the best possible precision the results of the four LEP experiments must be averaged. A combination on the basis of realistic observables constitutes an extremely complicated task, as it involves hundreds of measurements, each with specific cuts and experimental errors which are correlated among different energies and years and also among the LEP experiments. Therefore, the combination of the four LEP experiments is performed on the basis of the four sets of the pseudo-observables.

This note describes the associated combination procedure adopted by the LEP electroweak working group. The study is based on preliminary and confidential results by the four LEP experiments; they are considered close enough to the final results to draw conclusions on the precision of the averaging procedure. Some additional fits to the experimental data were provided which had not been available for previous combination attempts. A large effort is devoted to the treatment of systematic errors and their correlation among the LEP experiments, such that the combination procedure yields an optimal estimator for the averages.

This note is organised as follows: In Sec. 2 we discuss the sources of correlated systematic errors between experiments and how they affect the pseudo-observables. In Sec. 3, studies of variant methods for combining the results are presented. In the end, the extracted pseudo-observables will be be used in the framework of a specific Model, the minimal Standard Model (mSM), and the implications on the Z mass will be discussed in Sec. 4. Conclusions and our recommended combination procedure are given in Sec. 5.

2 Common errors

Common errors among all LEP collaborations arise from the calibration of the beam energy, from the theoretical error on the calculation of the small-angle Bhabha cross section used as the normalisation reaction, from theoretical uncertainties in the t-channel and s–t interference contribution to the differential large-angle Bhabha cross section, from theoretical uncertainties in the calculations of QED radiative effects, and finally from small uncertainties in the parametrisation of the electro-weak cross section near the Z resonance in terms of the set of pseudo-observables the four collaborations have agreed upon. These common errors must be quantified in order to combine the results of the experiments.

2.1 Common energy errors

Errors from the energy calibration of the LEP beams affect all experiments coherently. The error correlations between different data taking periods and experiments are fortunately well–known, but complex, and are described by the covariance matrices given by the Working Group on LEP Energy Calibration [11, 12, 13]. For the purpose of combining the experimental results at the parameter level, these energy errors need to be transformed into errors on the extracted pseudo-observables. The first step in the proposed transformation procedure is to scale the energy errors by factors of $1 \pm \epsilon$, while maintaining the experimental errors fixed. Typical values of ϵ are between 5 and 20 %. Performing the standard fits using these scaled errors generates two pseudo–observable covariance matrices, $V_{\pm}$, from which the covariance matrix due to energy errors, V_E, can be separated from the other errors, V_{exp}, using the relation $(V_{\pm}) = (1 \pm \epsilon)^2 (V_E) + (V_{\mathrm{exp}})$. As already demonstrated [14] for a data set restricted to the 1993–1995 hadronic cross sections, the common energy matrix does not depend strongly on which collaboration supplies the data used for the extraction. This conclusion also holds for the full data set, as will be discussed in section 3.

Historically [15] the common energy error matrix has been derived by scaling the experimental errors using an effective value of $\epsilon =^{+0.0}_{-0.5}$, motivated by the fact that this corresponds to shifting the experimental error scale from that of a single experiment to the effective value of four similar experiments combined. We now believe [16], however, that the appreciable weight shifts which occurred among the base measurements of each experiment under such strong error scaling and the resulting shifts in central values led to an underestimate of the common energy uncertainties.

To quantify this issue a test was made [14] in which each experiment fit their hadronic cross section measurements to determine an independent m_Z for 1993–94 and 1995. In combining these results, the error matrix allowed the measurement of each year and each experiment to take its appropriate weight, and this value of m_Z agreed well with that obtained from a direct fit at cross section level to the data of the four collaborations. We now extend this technique to include an independent determination of m_Z from the pre–1993 data as well, leading to an "eleven-parameter fit".

The only practical difference between evaluating the common energy errors around the full or the halved experimental errors is m_Z, where the difference in the extracted error amounts to only 0.1 MeV. On the other hand, the contribution of the experimental errors to the total error on the fitted parameters also changes when reducing the experimental errors by a factor of two; this change is not a simple factor of two, because the weighting of individual data points is no longer optimal for a single experiment. As a consequence, the experimental errors become slightly different from the ones given by each collaboration for their own data set.

Given this complication and the smallness of the effect, we now choose to evaluate the common energy error matrix for our central results about the normal experimental errors, using the smallest practical values of ϵ.

We also find it useful to control possible systematics by observing any shifts in the central values of the fit parameters which occur when the independent experimental errors are halved, as will be discussed in section 3.4.

Tab. 1, 2 and 3 show the errors on the pseudo-observables extracted from the individual experimental data sets, using the average LEP energy error matrix, as specified in Ref. [13]. Using the average matrix for the purpose of combination has the advantage of naturally taking into account the small parts of the energy-related errors that are correlated or anti-correlated among the experiments. There is remarkably good agreement for the nine-parameter results, except for the correlation coefficient between R_{e} and m_{Z}. The difference was traced back to the treatment of the t-channel and s–t interference correction, which depends on the value of m_{Z} through the s-t-interference contribution, as will be discussed in more detail below. This is the origin of additional energy-related contributions to the correlation between m_{Z} and R_{e} (or $\mathrm{A}_{\mathrm{FB}}^{0,\mathrm{e}}$) in L3 and OPAL, as can be seen from the relevant entries in Tab. 1.

The agreement of the energy-related errors for the eleven parameter results is also good among the experiments. Noticeable are the differences in the covariance matrix elements between the Z mass values and σ_{h}^{o}. They were traced back to a slightly different treatment of the role of m_{Z} when transforming the hadronic pole cross section into the partial widths Γ_{ee} and Γ_{had}, which are used in the fit programs to parameterise the differential cross sections around the Z resonance. σ_{h}^{o} is given by

$$\sigma_{\mathrm{h}}^{o} = \frac{12\pi}{m_{\mathrm{Z}}^{2}} \frac{\Gamma_{\mathrm{ee}}\Gamma_{\mathrm{had}}}{\Gamma_{\mathrm{Z}}^{2}} .$$

In this relation, ALEPH uses the m_{Z} value relevant for the given period, L3 uses the average of the three masses, OPAL uses the most precise, *i.e.* the 95 value, and DELPHI uses a fixed m_{Z}. Although not nice, it was verified that these differences do not lead to effects larger than 3 % of the combined error on the central value of any of the 11 parameters.

2.2 Common t-channel errors

Two different approaches are adopted by the collaborations in treating the t-channel and s–t interference contributions in the electron pair channel. ALEPH and DELPHI subtract these t-channel contributions from the data and the remaining s-channel component is extrapolated to the full acceptance. These t-channel corrected data are fitted in the same way as the other lepton pairs. L3 and OPAL instead fit the observed electron cross section within cuts to the sum of the predicted s- and t-channel and s–t interference contributions. In both cases the s-channel cross-section and asymmetry are calculated using ZFITTER. The t and s–t contributions are calculated in the Standard Model using the program

		m_Z	Γ_Z	σ_h^o	R_e
ALEPH	m_Z	0.0017			
	Γ_Z	−0.0005	0.0014		
	σ_h^o	−0.0027	−0.0028	0.011	
	R_e	−0.0029	−0.0012	0.0083	0.013
DELPHI	m_Z	0.0017			
	Γ_Z	−0.0005	0.0012		
	σ_h^o	−0.0026	−0.0021	0.010	
L3	m_Z	0.0016			
	Γ_Z	−0.0005	0.0013		
	σ_h^o	−0.0015	−0.0028	0.0101	
	R_e	0.0018	−0.0019	0.0074	0.0129
OPAL	m_Z	0.0017			
	Γ_Z	−0.0005	0.0012		
	σ_h^o	−0.0011	−0.0028	0.0108	
	R_e	0.0019	−0.0019	0.0083	0.0121
average	m_Z	0.0017			
	Γ_Z	−0.0005	0.0013		
	σ_h^o	−0.0020	−0.0026	0.011	
	R_e	0.0003	−0.0018	0.0080	0.013

Table 1: *Common energy errors, from individual data sets and average (9 parameter fits). Values are given as the signed square root of the covariance matrix elements; elements above the diagonal have been omitted for simplicity.*

ALIBABA [17], where the value of m_Z must be specified. L3 and OPAL parametrise the t and s–t contributions as a function of m_Z. This allows the t, s–t correction to follow their own determination of m_Z in the fits, which results in a correlation between between m_Z and R_e or $A_{FB}^{0,e}$. In the t, s–t subtraction of ALEPH and DELPHI a fixed value of m_Z was chosen a priori (from the latest combination of preliminary LEP results) before performing the fits. The extra correlation introduced by explicitely taking into account in the fits the m_Z dependence of the t-channel is about +12 % for the correlation m_Z–R_e and −11 % for m_Z–$A_{FB}^{0,e}$.

The theoretical uncertainty on the t-channel correction is discussed in detail in [18]. The size of the uncertainty is typically 1.1 pb for the forward cross-section and 0.3 pb for the backward cross-section and depends slightly on the acceptance cuts [19]. All collaborations incorporate the theory uncertainty as an additional error on the electron pair cross-section and asymmetry. In order to evaluate the common error due to the t, s–t theory error, each collaboration performed two fits, with and without the theory error, and the quadratic differences of the covariance matrix elements for R_e and $A_{FB}^{0,e}$ are taken as an estimate of the common error; these are summarised in table 4.

Since the error estimates from the individual experiments are all very similar, it was

		$A_{FB}^{0,e}$	$A_{FB}^{0,\mu}$	$A_{FB}^{0,\tau}$
ALEPH	$A_{FB}^{0,e}$	0.0003		
	$A_{FB}^{0,\mu}$	−0.0002	0.0003	
	$A_{FB}^{0,\tau}$	−0.0002	0.0003	0.0003
DELPHI	$A_{FB}^{0,e}$	0.0003		
	$A_{FB}^{0,\mu}$	−0.0003	0.0003	
	$A_{FB}^{0,\tau}$	−0.0003	0.0003	0.0003
L3	$A_{FB}^{0,e}$	0.0005		
	$A_{FB}^{0,\mu}$	−0.0003	0.0003	
	$A_{FB}^{0,\tau}$	−0.0003	0.0002	0.0002
OPAL	$A_{FB}^{0,e}$	0.0004		
	$A_{FB}^{0,\mu}$	−0.0003	0.0003	
	$A_{FB}^{0,\tau}$	−0.0003	0.0003	0.0003
average	$A_{FB}^{0,e}$	0.0004		
	$A_{FB}^{0,\mu}$	−0.0003	0.0003	
	$A_{FB}^{0,\tau}$	−0.0003	0.0003	0.0003

Table 2: *Common energy errors for forward-backward asymmetries. The anti-correlation between electron and muon or tau asymmetries arises from the different energy dependence of the electron asymmetry due to the t-channel contribution.*

agreed to use the average as the common error.

2.3 Common luminosity errors

The four collaborations use similar techniques to measure the luminosity of their data samples by counting the number of small–angle Bhabha scattered electrons. The experimental details of the four measurements differ sufficiently that no correlation is believed to exist in the experimental component of the luminosity errors. All four collaborations, however, use BHLUMI 4.04, the best available theoretical model of small–angle Bhabha scattering [20] to calculate the acceptance of their luminosity counters for the bulk of their data. Therefore significant correlations exist in the errors assigned to the scale of the measured cross sections due the uncertainty in this common theoretical calculation.

This uncertainty is estimated to be 0.061 % [20] without applying a correction for the production of light fermion pairs, which are not calculated in BHLUMI, and enter only as a contribution to the estimated error. A recent calculation of the contribution of light pairs [21] has allowed OPAL to explicitly correct for light pairs and reduce its theoretical luminosity uncertainty to 0.054 %. This 0.054 % error is taken to be correlated with the errors of the other three experiments, who between themselves share a mutual correlated error of 0.06 %

These errors almost exclusively affect the hadronic pole cross section, and contribute

		Γ_Z	σ_h^o	R_e	$m_Z^{\leq 92}$	m_Z^{93-94}	m_Z^{95}
ALEPH	$m_Z^{\leq 92}$	−0.0011	−0.0021	−0.0048	0.0057		
	m_Z^{93-94}	−0.0008	−0.0030	−0.0037	0.00083	0.0028	
	m_Z^{95}	−0.0001	−0.0022	−0.0022	0.00045	0.0013	0.0015
DELPHI	$m_Z^{\leq 92}$	−0.0011	0.0015		0.0052		
	m_Z^{93-94}	−0.0006	0.0018		0.0007	0.0031	
	m_Z^{95}	−0.0002	0.0021		0.0005	0.0013	0.0015
L3	$m_Z^{\leq 92}$	−0.0013	0.0046	0.0038	0.0049		
	m_Z^{93-94}	−0.0007	−0.0019	0.0035	0.0016	0.0029	
	m_Z^{95}	−0.0001	−0.0017	0.0013	0.0004	0.0014	0.0015
OPAL	$m_Z^{\leq 92}$	−0.0011	0.0025	0.0036	0.0058		
	m_Z^{93-94}	−0.0010	0.0034	0.0034	0.0006	0.0030	
	m_Z^{95}	0.0000	−0.0014	0.0013	−0.0003	0.0012	0.0015
average	$m_Z^{\leq 92}$	−0.0012	0.0016	0.0009	0.0054		
	m_Z^{93-94}	−0.0008	0.0001	0.0011	0.0009	0.0030	
	m_Z^{95}	−0.0001	−0.0008	0.0001	0.0003	0.0013	0.0015

Table 3: *Common energy errors for multiple-m_Z fit, from individual data sets and average. The matrix elements for Γ_Z, σ_h^o and R_e and for the asymmetries are very similar to the nine-parameter case and can be taken from Tab. 1 and 2.*

about half its total error after combination. The common luminosity error also makes a small contribution to the covariance matrix element between Γ_Z and σ_h^o (-0.0017^2 nb GeV) and a corresponding error of ± 0.4 MeV on Γ_Z. This was neglected in the common error tables given above and was checked to have no noticeable effect on the combined result.

2.4 Common theory errors

An additional class of common theoretical errors arises from the approximations and special choices made in the fitting codes. These comprise contributions from QED radiative corrections, including initial-state pair radiation, and the parameterisation of the differential cross section around the Z resonance in terms of pseudo-observables defined precisely at the peak and for pure Z exchange only. In order to estimate the uncertainties from the parametrisation of the electroweak cross sections near the Z resonance the two most advanced calculational tools, ZFITTER [22], and TOPAZ0 [23] can be compared. In addition, there are "parametric uncertainties" from the uncertainty in mSM parameters which are needed to fix certain parts of the calculations, like imaginary parts of the couplings or the size of the γ–Z interference term in the hadron line shape.

2.4.1 QED errors

QED effects can be estimated by comparing two different schemes:

		R_{e}	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$
ALEPH	R_{e}	0.025	
	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	−0.0056	0.0013
DELPHI	R_{e}	0.020	
	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	−0.0050	0.0013
L3	R_{e}	0.020	
	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	−0.0047	0.0012
OPAL	R_{e}	0.025	
	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	−0.0057	0.0015
average	R_{e}	0.022	
	$A_{\mathrm{FB}}^{0,\,\mathrm{e}}$	−0.0053	0.0013

Table 4: *Estimates of the t-channel errors of individual experiments and average, for ALIBABA.*

1. KF: O(... $\alpha^2 L^2$,$\alpha^2 L$,$\alpha^2 L^0$) calculations [24] including exponentiation of Kuraev-Fadin [25]. Recently O($\alpha^3 L^3$) [26] terms were added to the calculations in ZFITTER and TOPAZ0. [1)]

2. YFS: the 2$^{\mathrm{nd}}$ order inclusive exponentiation scheme of reference [28], based on the YFS approach [29]. Third order terms are known and have a small effect only [30].

Differences between these schemes amount to at most 0.1 MeV on m_{Z} and Γ_{Z} and 0.01 % on σ_{h}^{o}.

Comparing the implementation of initial state pair radiation in TOPAZ0 or ZFITTER with the approach taken in reference [31] leads to the dominant QED-related differences. They amount to 0.3 MeV in m_{Z} and to 0.5 MeV in Γ_{Z}. Differences in σ_{h}^{o} are smaller than the error estimate of 0.02 % in the above reference, which we therefore take as the error.

In summary, we use common QED theoretical errors on m_{Z}, Γ_{Z} and σ_{h}^{o} of 0.3 MeV, 0.5 MeV and 0.008 nb, respectively. Note that these numbers are from an ALEPH study, and that similar estimates are needed from the other experiments.

2.4.2 Parametrisation of line shape and asymmetries

To address the uncertainties that arise from the parametrisation of the electroweak cross sections near the Z resonance, ZFITTER and TOPAZ0 are compared. Conservatively, the full difference between the two codes is taken as the systematic uncertainty. In a recent, very detailed comparison [6], cross sections from Standard Model calculations and from a fairly model-independent parametrisation were compared. In principle, each of the two could be used in an iterative fit procedure and a direct comparison of the results would provide an error estimate on the pseudo-observables. In practice, however, TOPAZ0 is rather slow and therefore not well suited to be used in a fit. Therefore, a different approach

[1)] Third-order terms for the KF scheme had already been calculated earlier [27].

	TOPAZ0 4.4 – ZFITTER 5.20	TOPAZ0 4.4 – ZFITTER 6.10
m_{Z} [GeV]	0.0001	−0.0002
Γ_{Z} [GeV]	0.0001	0.0001
σ_{h}^{o} [nb]	0.006	0.001
R_{e}	0.004	0.002
R_{μ}	0.004	0.003
R_{τ}	0.004	0.005
$\mathrm{A}_{\mathrm{FB}}^{0,\mathrm{e}}$	0.0001	−0.0001
$\mathrm{A}_{\mathrm{FB}}^{0,\mu}$	0.0001	0.0000
$\mathrm{A}_{\mathrm{FB}}^{0,\tau}$	0.0001	−0.0002

Table 5: *Differences between TOPAZ0 and ZFITTER, reflecting theoretical uncertainties from the fitting code, evaluated for the ALEPH cuts around the ALEPH best-fit point for the pseudo-observables.*

was taken: cross sections were calculated with TOPAZ0 and then fitted with ZFITTER. Errors were assigned to the calculated cross sections to reflect the integrated luminosity taken at each energy by a typical LEP experiment. The pseudo-observables were chosen to be the LEP averages; this choice is important because the agreement of the two codes depends on how well the relations between the pseudo-observables represent those of the Standard Model.

The dominant difference between the two codes results from the implementation of the cross section parametrisation in terms of pseudo-observables, where the energy points away from the peak of the resonance are of particular importance. Variations of the electroweak calculations have only a negligible effect. The differences evaluated by the ALEPH collaboration are given in Tab. 5 and may serve as an estimate of the uncertainties involved. It is important to note that such differences depend on the particular set of pseudo-observables at which they are determined [6], and will finally be required from all experiments. Tab. 5 will then evolve to a common error matrix.

2.4.3 Parametric errors

The LEP experiments have agreed to use a standard set of mSM parameters to fix the so-called "Standard Model remnants" in the fitting codes, which is $m_{\mathrm{Z}} = 91.187$ GeV, $M_{\mathrm{t}} = 175$ GeV, $M_{\mathrm{H}} = 150$ GeV, $\alpha_s(m_{\mathrm{Z}}) = 0.119$ and $\alpha^{(5)}(MZ) = 1/128.878$ [2)]. Varying these parameters within their present experimental errors leads to observable effects only on the Z mass, which is affected through the γ–Z interference term. The dominant dependence is on M_{H}, followed by $\alpha^{(5)}$. Considering a Higgs mass range between 10 GeV and 1000 GeV and varying $\alpha^{(5)}$ by ±0.090 results in a "parametric" error of ±0.3 MeV on m_{Z}.

[2)] $\alpha^{(5)}(m_{\mathrm{Z}})$ is the electromagnetic coupling constant at the scale of the Z mass for five quark flavours.

3 Combination of results

3.1 Independence of choice of the energy error matrix

The estimated energy errors for the nine and eleven parameter fits, as shown in Tab. 1, 2 and 3 differ slightly depending on which experimental data set is used to derive them. Combinations may be attempted based on each of them or on the average. The central values and errors of each of the averaged parameters agree well within 5 % of the error on that average. It is therefore most appropriate to take the average of the error estimates over the experiments as the common energy errors.

3.2 Correlations between m_Z and R_e, $A_{FB}^{0,e}$

An additional difficulty arises from the dependence of the t-channel on the Z mass via the s-t interference, which was mentioned above and is treated differently by ALEPH or DELPHI and L3 or OPAL. This also leads to differences in the m_Z–R_e and m_Z–$A_{FB}^{0,e}$ matrix elements of the common energy errors.

It was shown that these correlation coefficients properly take into account the changes in R_e and $A_{FB}^{0,e}$ when m_Z takes its average value in the combination of the four experiments. However, there is also an effect of R_e on the Z mass when is is replaced by R_ℓ in the case of assumed lepton universality, *i.e.* in the five-parameter fit. For individual experiments, this effect is of the order of 0.5 MeV; the shift is driven by the (statistical) difference between R_e and R_ℓ. Similarly, changing R_e and $A_{FB}^{0,e}$ from the values of a single experiment to the LEP average introduces a m_Z shift in the presence of these particular correlation coefficients.

In a study, the m_Z–R_e and m_Z–$A_{FB}^{0,e}$ correlations were artificially modified. Adding these correlations to the ALEPH and DELPHI correlation matrices as well as adjusting the common energy errors and performing the averaging results in a shift of m_Z of $+0.4$ MeV. An additional shift in m_Z of -0.2 MeV occurs when condensing the nine parameters into five under the assumption of lepton universality. If these correlations are removed from both the L3 and OPAL matrices and from the common energy errors, then a shift of almost identical size in the same direction is observed, whereas the shift downwards in the five-parameter average disappears.

This peculiar behaviour is understood: The effect is dominated by the particular results obtained by ALEPH and OPAL. The R_e and m_Z values of OPAL and ALEPH are on opposite sides of the average. Thus, the two essentially cancel when the same kind of correlation is assumed. On the other hand, with the unmodified error matrices, ALEPH in contrary to OPAL does not have this correlation coefficient and therefore does not compensate the m_Z shift. Due to the asymmetrical treatment of the correlation coefficients the expected reduction of statistical fluctuations in the average does not occur.

It is therefore clear that the present situation represents the worst case and introduces a bias in m_Z. A symmetrical treatment of the m_Z–R_e and m_Z–$A_{FB}^{0,e}$ correlations is desirable. This can be achieved by either adding them “by hand” to the ALEPH and DELPHI results, or by removing them from the L3 and OPAL matrices.

3.3 Comparison of nine and eleven parameter fits

In 1993 and 1995, the two years when LEP performed precision scans to measure the Z line shape, the experimental errors are very comparable, but the LEP energy was appreciably better understood in 1995 than in 1993. In determining the optimum value of m_{Z}, therefore, the four experiments combined should give more weight to the 1995 data than they each do in their independent determinations. This can be achieved by an eleven-parameter fits, where three values of the Z mass are introduced, $m_{\mathrm{Z}}^{\leq 92}$, m_{Z}^{93-94} and m_{Z}^{95}. Now, each of these mass values has its specific energy error reflecting the largely different systematic errors on the absolute energy scale of LEP. In the combination, the relative importance of energy related and independent experimental errors on the mass values is properly taken into account. When the three values of m_{Z} are condensed into a single one, a better measurement of the Z mass is obtained than from the nine-parameter fits. The difference of -0.2 MeV w.r.t. the m_{Z} value from the nine-parameter corresponds to only 10 % of the combined error. All other parameters are identical to their values from the nine-parameter fit within less than 5 % of the combined error.

3.4 Shifts for halved experimental errors

When the experiments are averaged at the parameter level, information on the individual contribution of particular data points is lost. For the combined data set of all four experiments, the relative importance of independent experimental errors and of common systematic errors changes, e.g. for data points which are dominated by experimental errors for a single experiment, but which become systematics dominated for the combined data set of the four experiments.

The example of m_{Z} given in the previous subsection provides a good illustration of such effects. It was shown already in Ref. [14] that averaging the shifts in m_{Z} which each experiment observed when halving its experimental errors to simulate the generic "combined" experiment also reproduced the results of the full fit to the combined hadronic cross section measurements.

While m_{Z} is now properly treated by the eleven-parameter fits, other pseudo-observables may suffer from similar changes due to weight shifts. We propose to use the changes in central values when halving the independent experimental errors in each experiment as a monitoring tool. The average of these changes over the four LEP experiments serves to control the differences between an average at the parameter level compared to a full cross section average. Of course, this assumes that all experiments enter into the average with the same weight of 1/4. The shifts observed in a preliminary set of results are summarised in Tab. 6. The average shifts in σ_{h}^{o}, R_{e}, $\mathrm{A}_{\mathrm{FB}}^{0,\mu}$ and $\mathrm{A}_{\mathrm{FB}}^{0,\tau}$ amount to about 10 % of the combined errors, in all other cases they are smaller.

The shift in m_{Z} observed in the eleven-parameter fit agrees with the expected shift of -0.1 MeV seen in Tab. 6.

	A	D	L	O	average	% of error
m_Z	-0.0007	$+0.0008$	-0.0004	-0.0001	-0.0001	-5
Γ_Z	-0.0002	$+0.0002$	-0.0003	$+0.0001$	-0.0001	-4
σ_h^o	$+0.007$	-0.005	$+0.009$	$+0.003$	$+0.004$	$+11$
R_e	$+0.009$	0.000	$+0.002$	$+0.005$	$+0.004$	$+8$
R_μ	0.000	-0.001	0.000	$+0.001$	0.000	0
R_τ	0.000	$+0.004$	0.000	$+0.003$	$+0.002$	$+4$
$A_{FB}^{0,e}$	-0.0001	$+0.0001$	0.000	-0.0001	0.0000	-1
$A_{FB}^{0,\mu}$	$+0.0003$	$+0.0002$	$+0.0001$	$+0.0001$	$+0.0002$	$+14$
$A_{FB}^{0,\tau}$	$+0.0002$	$+0.0002$	$+0.0001$	$+0.0001$	$+0.0002$	$+12$

Table 6: *Shifts in central values of the fitted pseudo-observables seen when halving the independent experimental errors, for individual experiments and average.*

4 m_Z as a parameter of the Standard Model

When the set of pseudo-observables is used in the frame-work of the Standard Model, the role of m_Z changes from an independent parameter to that of a Lagrangian parameter of the theory, which now imposes additional constraints. Therefore m_Z is expected to change when mSM constraints are imposed in fits with all parameters except M_H and α_s constrained within their present experimental errors.

As a cross check of the validity of the combined set of pseudo-observables as an input to Standard Model fits each collaboration provided results from direct Standard Model fits to their cross section and asymmetry data with M_H and α_s as free parameters. These results can then be compared with those obtained by using the set of pseudo-observables as fit input instead. Most noticeably, a shift in m_Z is observed. One source of this was already mentioned: the interference term depends on M_H, which is allowed to move freely in the first fit, but is fixed to 150 GeV if the pseudo-observables serve as the input. Correcting for this (0.4 MeVper change in $\log_{10}(M_H/\mathrm{GeV})$ of two units), some shifts remain, which originate from the loss of information when the experimental data are condensed into only nine parameters. The observed differences in m_Z are shown in Tab. 7. After correction for the different values of the Higgs mass the net average difference in m_Z between the two fit procedures is only 0.3 MeV or 15 % of the combined error.

5 Conclusion

This section presents the proposed combination procedure for the line shape and asymmetry measurements at LEP 1 and summarises the input required from the experiments to perform this task.

The following represents our proposal:

- The combination should be based on the nine-parameter fits, where each collaboration

	mSM fits		
	PO	RO	comment
ALEPH	+0.2	−0.5 (−0.4)	low M_{H}
DELPHI	0.0	+0.8 (+1.1	very low M_{H}
L3	−0.3	−0.2 (−0.1)	low M_{H}
OPAL	−0.3	0.0 (+0.1)	high M_{H}
av.	$\simeq -0.1$	0.0 (+0.2)	(): MH=150 GeV

Table 7: *Shifts in m_{Z} w.r.t. to the nine-parameter fits; the second column is for the mSM fit to the pseudo-observables ("PO"), which averages to −0.1* MeV*, and the third column is from the fits using directly the cross sections and asymmetries (realistic observables, "RO") as input to the mSM fit; the number in brackets is corrected back to $M_{\mathrm{H}} = 150$* GeV*.*

should use the common LEP energy error matrix instead of its special sub-matrix from the 28×28 energy matrix, as documented in Ref. [13].

- Eleven-parameter fits with separate Z masses for the periods prior to 1993, for 1993–94 and for 1995 are needed to treat properly the largely different errors on the absolute energy scale. In addition, the average over the four experiments of the separate Z mass values provides a valuable check of the energy calibration of LEP. The effect on the Z mass is −0.2 MeV; the average mass from the eleven-parameter fit could therefore replace the value from the nine-parameter fit.
- The averages of the individual estimates for the common uncertainties will be used in general (see Tab. 1, 2, 3 and 4). For the luminosity, the common error between OPAL and any other collaboration should be taken to be the 10 % smaller OPAL error. It is necessary that these error determinations are made officially available by each collaboration.
- Theoretical errors arising from QED radiative effects, from the fitting codes and from uncertainties in the mSM remnants should not be contained in the results given by the experiments; these will be added to the combined results. Note that the relevant studies still need to be completed.
- Special nine-parameter fits are needed from each experiment, in which all independent experimental errors (*i.e.* those not common among the LEP experiments) are reduced by a factor of two to simulate the case of a generic LEP experiment, where common errors become relatively more important than the individual errors. The average over the experiments of the shifts observed w.r.t. the standard nine-parameter fit serves to estimate the difference between the proposed combination and a statistically optimal average.
- Standard Model fits using as input directly the cross section and asymmetry measurements in general lead to different values for the Z mass w.r.t. the nine-parameter fits. It is important to quantify this effect for the common LEP value of m_{Z}. This check will justify the use of the nine pseudo-observables instead of the full cross section and asymmetry results for the determination of the Lagrangian parameters of the mSM.

In addition, these fits provide a valuable check on the internal consistency of the fit programs. In these fits, the top mass should be constrained to $M_{\mathrm{t}} = 174 \pm 5$ GeV, and the electromagnetic coupling constant at the scale of the Z mass, for five quark flavours, to $1/\alpha^{(5)}(MZ) = 128.878 \pm 0.090$; m_{Z}, M_{H} and α_s are the free parameters.

- Among the many details in the experimental fit procedures which are treated slightly differently one aspect is of particular importance. The dependence of the s–t interference correction in the Bhabha channel on m_{Z} causes a significant shift of 0.4 MeV if the present fit results are used without modification (see section 3.2). It is important to symmetrise the treatment of the m_{Z}–R_{e} and m_{Z}–$\mathrm{A_{FB}^{0,e}}$ correlations to obtain an unbiased average. Since these correlations have a physical origin, we request them to be specified also by ALEPH and DELPHI, which at present merely include the m_{Z}-dependence of the t-channel in the diagonal errors of R_{e} and $\mathrm{A_{FB}^{0,e}}$.

The precision of the central values achieved by the proposed combination is estimated to be within 5 % of the combined error for all quantities. It approximates a statistically optimal average within 10 % or better.

ACKNOWLEDGEMENTS

We wish to thank the LEP collaborations for providing the preliminary results of special fits to their data allowing us to perform these tests.

References

[1] *Z Physics at LEP1*, CERN 89-08, ed. G. Altarelli *et al.*, Vol. 1–3 (1989) and references therein;
Reports of the working group on precision calculations for the Z resonance, eds. D. Bardin, W. Hollik and G. Passarino, CERN Yellow Report 95-03, Geneva, 31 March 1995, and references therein.

[2] 1996. ALEPH Coll. (D. Decamp *et al.*), Nucl. Inst. Meth. **A294** (1990) 121;
(D. Buskulic *et al.*), *Performance of the ALEPH Detector at LEP*, Nucl. Inst. Meth. **A360** (1995) 481-506;
Construction and Performance of the new ALEPH Vertex Detector, Proceedings of the 5th International Conference on Advanced Technology and Particle Physics, Como, Italy, 7-11 Oct. 1996;
The ALEPH Handbook 95, Vol. 1 and 2, available from the ALEPH Secretariat, CERN.

[3] 1996. DELPHI Coll. (P. Aarnio *et al.*), Nucl. Inst. Meth. **A303** (1991) 233;
(P. Abreu *et al.*), *Performance of the DELPHI Detector* Nucl. Inst. Meth. **A378** (1996) 57-100.

[4] 1996. L3 Coll. (B. Adeva *et al.*), Nucl. Inst. Meth. **A289** (1990) 35;
(M. Acciarri) *et al.*, Nucl. Inst. Meth. **A351** (1994) 300;

(M. Chemarin) *et al.*, Nucl. Inst. Meth. **A349** (1994) 345;
(A. Adam) *et al.*, Nucl. Inst. Meth. **A383** (1996) 342;
(I.C. Brock) *et al.*, Nucl. Inst. Meth. **A381** (1996) 236.

[5] 1994. OPAL Coll. (K. Ahmet *et al.*), Nucl. Inst. Meth. **A305** (1991) 275;
(P. Allport *et al.*), Nucl. Inst. Meth. **A324** (1993) 34;
(P. Allport *et al.*), Nucl. Inst. Meth. **A346** (1994) 476;
(B.E. Anderson *et al.*), IEEE Trans. Nucl. Sci. **41** (1994) 845.

[6] D. Bardin, G. Passarino, *Upgrading of Precision Calculations for Electroweak Observables*, hep-ph/9803425 ;
D. Bardin, G. Passarino and M. Grünewald, *Precision Calculation Project Report*, hep-ph/9902452 .

[7] ALEPH Collaboration, (D. Decamp *et al.*), Z. Phys. **C48** (1990) 365;
ALEPH Collaboration, Z. Phys. **C53** (1992) 1;
ALEPH Collaboration, (D. Buskulic *et al.*), Z. Phys. **C60** (1993) 71;
ALEPH Collaboration, Z. Phys. **C62** (1994) 539;
ALEPH Collaboration, *LEP I results on Z resonance parameters and lepton forward-backward asymmetries*, ALEPH 98-068 CONF 98-038, contributed paper to ICHEP 98 Vancouver **ICHEP'98 #284**.

[8] DELPHI Collaboration, P. Aarnio *et al.*, Nucl. Phys. **B367** (1991) 511;
DELPHI Collaboration, (P. Abreu *et al.*), Nucl. Phys. **B417** (1994) 3;
DELPHI Collaboration, Nucl. Phys. **B418** (1994) 403;
DELPHI Collaboration, DELPHI Note 95-62 PHYS 497, contributed paper to EPS-HEP-95 Brussels, **eps0404**;
DELPHI Collaboration, DELPHI Note 97-130 CONF 109, contributed paper to EPS-HEP-97, Jerusalem, **EPS-463**.

[9] L3 Collaboration, (B. Adeva *et al.*), Z. Phys. **C51** (1991) 179;
L3 Collaboration, (O. Adriani *et al.*), Phys. Rep. **236** (1993) 1;
L3 Collaboration, (M. Acciarri *et al.*), Z. Phys. **C62** (1994) 551;
L3 Collaboration, *Preliminary L3 Results on Electroweak Parameters using 1990-96 Data*, L3 Note 2065, March 1997, available via http://l3www.cern.ch/note/note-2065.ps.gz.

[10] OPAL Collaboration, (G. Alexander *et al.*), Z. Phys. **C52** (1991) 175;
OPAL Collaboration, (P.D. Acton *et al.*), Z. Phys. **C58** (1993) 219;
OPAL Collaboration, (R. Akers *et al.*), Z. Phys. **C61** (1994) 19;
OPAL Collaboration, *Precision Measurements of the Z^0 Lineshape and Lepton Asymmetry*, OPAL Physics Note PN358, July 1998;
OPAL Collaboration, *Precision Luminosity for OPAL Z^0 Lineshape Measurements with a Silicon-Tungsten Luminometer*, OPAL Physics Note PN364, July 1998.

[11] Working group on LEP energy, L. Arnaudon *et al.*, *The Energy Calibration of LEP in 1992*, CERN SL/93-21 (DI), April 1993.

[12] Working group on LEP energy, R. Assmann *et al.*, *The energy calibration of LEP in the 1993 scan*, Z. Phys. **C66** (1995) 567-582.

[13] Working group on LEP energy, R. Billen *et al.*, *Calibration of Centre-of-Mass Energies at LEP1 for Precise Measurements of Z properties*, Eur. Phys. J. **C6** (1999) 187.

[14] G. Duckeck, R. Kellogg, A. Olshevski, C. Paus, G. Quast, P. Renton and D. Strom (LEPEWWG: the m_Z and Γ_Z task force), *Precise Determination of Z–Boson Mass and Width at LEP*, LEPEWWG/LS 98-01.

[15] Abbaneo D. *et al.*, *A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model*, CERN-PPE/97-154.

[16] Abbaneo D. *et al.*, *A Combination of Preliminary Electroweak Measurements and Constraints on the Standard Model*, CERN-EP/99-15.

[17] W.Beenakker, F.A.Berends and S.C. van der Marck, Nucl. Phys. **B349** (1991) 323.

[18] W.Beenakker and G.Passarino, Phys. Letters **B425** (1998) 199-207.

[19] W.Beenakker and G.Passarino, private communications.

[20] B.F.L Ward *et al.*, *New Results on the Theoretical Precision of the LEP/SLC Luminosity*, UTHEP-96-501 (1998), to be published in Phys. Lett.

[21] G. Montagna *et al.*, *Light Pair Corrections to Small-Angle Bhabha Scattering in Realistic Set-up at LEP*, FNT/T-99/06, to be submitted to Phys Lett. B.

[22] D. Bardin *et al.*, Z. Phys. **C44** (1989) 493;
Comp. Phys. Comm. **59** (1990) 303;
Nucl. Phys. **B351** (1991) 1;
Phys. Lett. **B255** (1991) 290 and CERN-TH 6443/92 (May1992);
DESY Report 99-070.

[23] G. Montagna, O. Nicrosini, G. Passarino, F. Piccinni and R. Pittau, Nucl. Phys. **B401** (1993) 3; Comp. Phys. Comm. **76** (1993) 328; Comp. Phys. Comm. **93** (1996) 120;
G. Montagna, O. Nicrosini, G. Passarino and F. Piccinini, hep-ph/9804211.

[24] F.A. Berends, W.L. van Neerven and G.J.H. Burgers, Nucl. Phys. **B297** (1988) 429 and **B304** (1988) 921 (E).

[25] E.A. Kuraev and V.S.Fadin, Sov. J. Nucl. Phys. **41** (1985) 466.

[26] G. Montagna, O. Nicrosini and F. Piccinini, Phys. Lett. **B406** (1997) 243.

[27] M. Skrzypek, Acta Phys. Polonica, **B23** (1992) 135.

[28] S. Jadach, M. Skrzypek and B.F.L. Ward, Phys. Lett. **B257** (1991) 173.

[29] D.R. Yennie, S.C. Frautschi and H. Suura, Ann. Phys. (NY) **13** (1961) 379.

[30] S.Jadach, B.Pietrzyk and M. Skrzypek, *On the precision of calculations of Initial State Radiation in the LEP Z line-shape fits*, LAPP-EXP-99.01, to be published in Phys. Lett. B.

[31] S. Jadach, M. Skrzypek and M. Martinez, Phys. Lett. **B280** (1992) 129.