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An Investigation of the Interference between Photon and Z-Boson Exchange

The S-Matrix Subgroup of the LEP Electroweak Working Group[1]

Internal Note

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Abstract

This note presents an investigation of the γZ interference contribution to total cross sections and forward-backward asymmetries. It is based on a combination of published and preliminary electroweak results from the four LEP collaborations which were prepared for the 1996 winter conferences.

[1]The present members of the subgroup are: A. Blondel, P. Clarke, M.W. Grünewald, M. Kienzle-Focacci, W. Lu, M. Martinez, M.-N. Minard, A. Olshevsky, Ch. Paus, M. Pepe-Altarelli, T.J. Smith, P. Renton, F. Teubert.

1 Introduction

In Section 2 of the note on LEP electroweak averages prepared for the 1996 winter conferences [1], an analysis of the Z lineshape and leptonic forward-backward asymmetries is presented. The formulae used in the fits presented in that Section are valid under quite general assumptions, namely that the scattering process arises from the s-channel exchange of two bosons, a massless photon with vector couplings to the electric charge and a massive Z boson with both vector and axial-vector couplings to fermions. For leptons these two parameters, together with $\alpha(m_{\mathrm{Z}}^2)$, fully determine both the Z exchange and the γZ interference terms. In contrast, a model dependence is introduced by the treatment of the γZ interference contribution to the hadronic cross section, because quark couplings to the Z are not individually measured for each flavour. This term is therefore taken from the Standard Model.

A more general approach is to parametrise the γZ interference term independent of the Z exchange amplitude. A theoretically well motivated framework for such an analysis based on S-Matrix theory can be found in [2]. Formulae for $\mathrm{e^+e^-} \rightarrow \mathrm{f\bar{f}}$ cross sections and forward-backward asymmetries are summarised in Appendix A.1.

Measurements at centre-of-mass energies far away from the Z pole are especially sensitive to the parameters describing the interference between photon and Z-boson exchange [3]. This analysis includes the results from the 1995 LEP run at centre-of-mass energies, $\sqrt{s}$, from 130 GeV to 140 GeV [4–7]. The total statistics of these high-energy measurements of the four LEP collaborations are given in Table 1.

For a large fraction of these high-energy events, initial-state radiation photons lower the effective centre-of-mass energy, $\sqrt{s'}$, of the annihilation process to values close to the Z mass, m_{Z} ('radiative return to the Z'). The radiative events are not particularly useful. By applying a cut on $\sqrt{s'}$ to exclude them, $\sqrt{s'} >> m_{\mathrm{Z}}$, the high-energy non-radiative events are isolated. These events are useful in constraining the γZ interference parameters.

2 General Framework and Results

The S-Matrix parameters r_{f}, j_{f} and g_{f}, which are real numbers, express the size of the Z exchange, γZ interference and photon exchange contributions, respectively. In our approach, the photon exchange contribution, determined by g_{f}, is fixed by QED, whereas r_{f} and j_{f} are treated as free parameters. This leaves four free parameters for each final state considered, two for cross sections ($r_{\mathrm{f}}^{\mathrm{tot}}$ describing partial widths, and $j_{\mathrm{f}}^{\mathrm{tot}}$) and two for forward-backward asymmetries ($r_{\mathrm{f}}^{\mathrm{fb}}$ describing pole asymmetries, and $j_{\mathrm{f}}^{\mathrm{fb}}$). In models with only vector and axial-vector couplings of the Z, these four S-Matrix parameters are not independent of each other [2]:[1]

$$
\begin{aligned}
r_{\mathrm{f}}^{\mathrm{tot}} &\propto \left[g_{A\mathrm{e}}^2 + g_{V\mathrm{e}}^2\right] \cdot \left[g_{A\mathrm{f}}^2 + g_{V\mathrm{f}}^2\right] && (1)\\
j_{\mathrm{f}}^{\mathrm{tot}} &\propto g_{V\mathrm{e}} g_{V\mathrm{f}} && (2)\\
r_{\mathrm{f}}^{\mathrm{fb}} &\propto g_{A\mathrm{e}} g_{V\mathrm{e}} \cdot g_{A\mathrm{f}} g_{V\mathrm{f}} && (3)\\
j_{\mathrm{f}}^{\mathrm{fb}} &\propto g_{A\mathrm{e}} g_{A\mathrm{f}} \, . && (4)
\end{aligned}
$$

For the results presented below, the parameters m_{Z}, Γ_{Z} as well as $r_{\mathrm{f}}^{\mathrm{tot}}$, $j_{\mathrm{f}}^{\mathrm{tot}}$, $r_{\mathrm{f}}^{\mathrm{fb}}$, $j_{\mathrm{f}}^{\mathrm{fb}}$ for leptons and $r_{\mathrm{had}}^{\mathrm{tot}}$, $j_{\mathrm{had}}^{\mathrm{tot}}$ for hadrons are treated as free and independent parameters. This allows to test the

[1] These relations are approximations. Exact expressions are listed in Appendix A.1.

above assumptions and to perform other tests. For example, it is now possible to discuss the energy dependence of total cross sections and forward-backward asymmetries independently.

The four sets of 16 parameters provided by the four LEP experiments are presented in Table 2. For the averaging procedure the parameter covariance matrices of the individual LEP experiments and systematic errors common to the four experiments are taken into account. Sources of common systematic errors are the theoretical uncertainty of the luminosity determination and the LEP beam energy calibration. The theoretical error of the luminosity determination of 0.16% introduces a correlation of the same size between all S-Matrix parameters r, j and g of different experiments. The common systematic errors and the correlations amongst the S-Matrix parameters arising from the LEP beam energy calibration are summarised in Appendix A.2. The combined parameter set and its correlation matrix are given in Tables 3 and 4.

The data of each of the four LEP experiments are consistent with lepton universality. If lepton universality is assumed, the set of 16 S-Matrix parameters is reduced to a set of 8 parameters. Table 5 shows the 8 parameters of the individual LEP experiments. Tables 6 and 7 provide the LEP averages and their correlation matrix. The Standard-Model predictions agree well with the LEP averages of the S-Matrix parameters.

3 The Hadronic γZ Interference Term

The largest correlation (-89%) arises between the mass of the Z boson and the hadronic γZ interference term, $j_{\mathrm{had}}^{\mathrm{tot}}$. This correlation leads to a sizeable increase in the error on the mass of the Z in fits where $j_{\mathrm{had}}^{\mathrm{tot}}$ is left free. Figure 1 shows the 68% probability contour in the (m_{Z}, $j_{\mathrm{had}}^{\mathrm{tot}}$)-plane. The error on the hadronic γZ interference term $j_{\mathrm{had}}^{\mathrm{tot}}$, and consequently the error on m_{Z}, are reduced by using total cross section measurements at centre-of-mass energies far away from the Z pole [2, 3]. The inclusion of the high-energy LEP data reduces the uncertainty on $j_{\mathrm{had}}^{\mathrm{tot}}$ by 25% with respect to our previous S-Matrix analysis [8]. The average value of $j_{\mathrm{had}}^{\mathrm{tot}}$, -0.22 ± 0.25, is now measured to be about 1.8 standard deviations below the prediction of the Standard Model. The reason is that two experiments measure large negative values for $j_{\mathrm{had}}^{\mathrm{tot}}$ (Tables 2 and 5).

Inclusion of low-energy data leads to a further improvement in the errors. The TOPAZ collaboration at TRISTAN (KEK) has performed a measurement of the hadronic total cross section at $\sqrt{s} = 57.77$ GeV, $\sigma_{\mathrm{tot}}^{0} = 143.6 \pm 1.5(stat.) \pm 4.5(syst.)$ pb [9]. If we include this measurement, the results for m_{Z} and $j_{\mathrm{had}}^{\mathrm{tot}}$ are:

$$m_{\mathrm{Z}} = 91.1937 \pm 0.0038 \text{ GeV} \quad (5)$$

$$j_{\mathrm{had}}^{\mathrm{tot}} = -0.06 \pm 0.18\,, \quad (6)$$

and the correlation between m_{Z} and $j_{\mathrm{had}}^{\mathrm{tot}}$ is reduced to -82%.

4 The Energy Dependence of the Leptonic Forward-Backward Asymmetries

The dependence of the leptonic forward-backward asymmetry on the centre-of-mass energy arises from the γZ interference, and is quantified by the slope parameter j_{ℓ}^{fb}. Among the parameters characterising the γZ interference this parameter is determined with the highest relative precision. The χ^2 values of the averages in Tables 3 and 6 have a large contribution from the energy dependence of the leptonic forward-backward asymmetries. In the following discussion of j_ℓ^{fb}, we neglect its small correlations with other S-Matrix parameters.

Figure 2 shows the forward-backward asymmetries for electrons, muons and taus as a function of the centre-of-mass energy. The measurements are extrapolated to full phase-space. In the case of electrons, t-channel and s/t-interference contributions are removed. Considering each energy point separately, the measurements for each lepton species are in good agreement with each other, and compatible with the prediction of the Standard Model. As a consequence, the values for $j_{\mathrm{e}}^{\mathrm{fb}}$, j_μ^{fb}, j_τ^{fb} are also in good agreement within each experiment (Table 2).

Assuming lepton universality, the results on the slope parameter j_ℓ^{fb} of the four LEP experiments are listed in Table 5. The weighted average of the four values for j_ℓ^{fb} agrees with the prediction of the Standard Model. However, the average has a χ^2/d.o.f. of 12.3/3, corresponding to a probability of about 0.63%. The reason for the high value of the χ^2 is shown in Figure 3, where the average leptonic forward-backward asymmetries are compared to the prediction of the Standard Model. One experiment measures a slope larger than the prediction, whereas the other three measure a slope slightly smaller than the prediction.

All experiments perform detailed checks on possible systematic effects in the measurements of the leptonic forward-backward asymmetries. No systematic bias is found so far, but further investigations are in progress. For the averaging of these preliminary data, this unlikely value for the χ^2 is assumed to be caused by a statistical fluctuation, and the leptonic forward-backward asymmetries are included in the analyses presented in [1]. Note, that the inclusion of the forward-backward asymmetries measured around the Z using the 1995 data has improved the agreement among the experiments compared to our previous S-Matrix analysis [8].

5 Summary

The S-Matrix framework allows a coherent treatment of data taken at the Z resonance and data taken in energy regions where the γZ interference is large. The results on the S-Matrix parameters provide a test of the assumed vector and axial-vector structure of the Z couplings. Fixing the hadronic γZ interference term $j_{\mathrm{had}}^{\mathrm{tot}}$ to its Standard-Model prediction, the central values and errors of the quantities discussed in Section 2 of [1] are obtained.

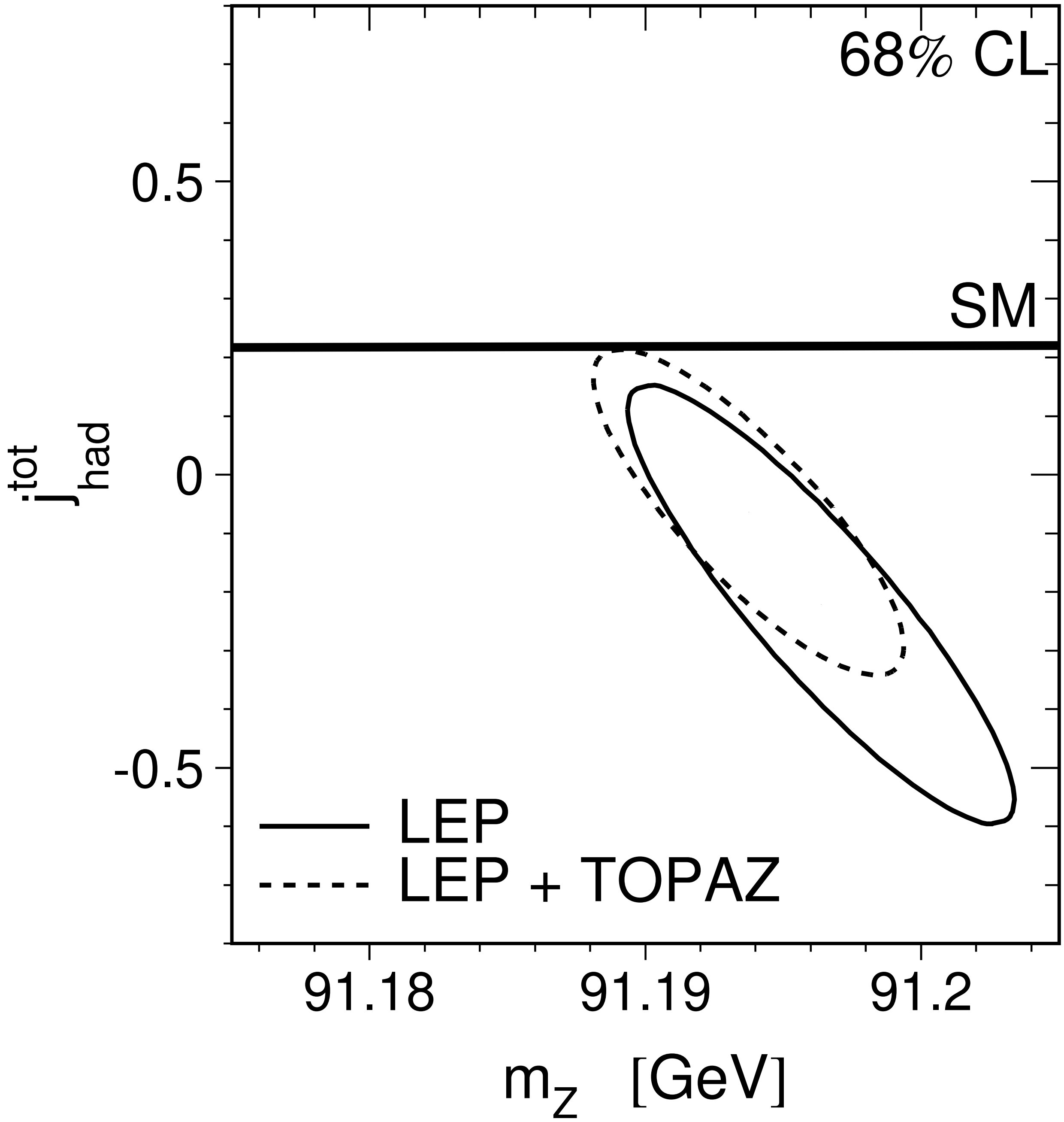

Figure 1: Contours of 68% confidence level between the mass of the Z boson, m_Z, and the hadronic γZ interference term for total cross sections, $j_{\mathrm{had}}^{\mathrm{tot}}$. Lepton universality is assumed. The solid contour corresponds to LEP data only, the dashed contour is obtained by including the TOPAZ measurement. The Standard-Model prediction for $j_{\mathrm{had}}^{\mathrm{tot}}$ is shown as the solid band.

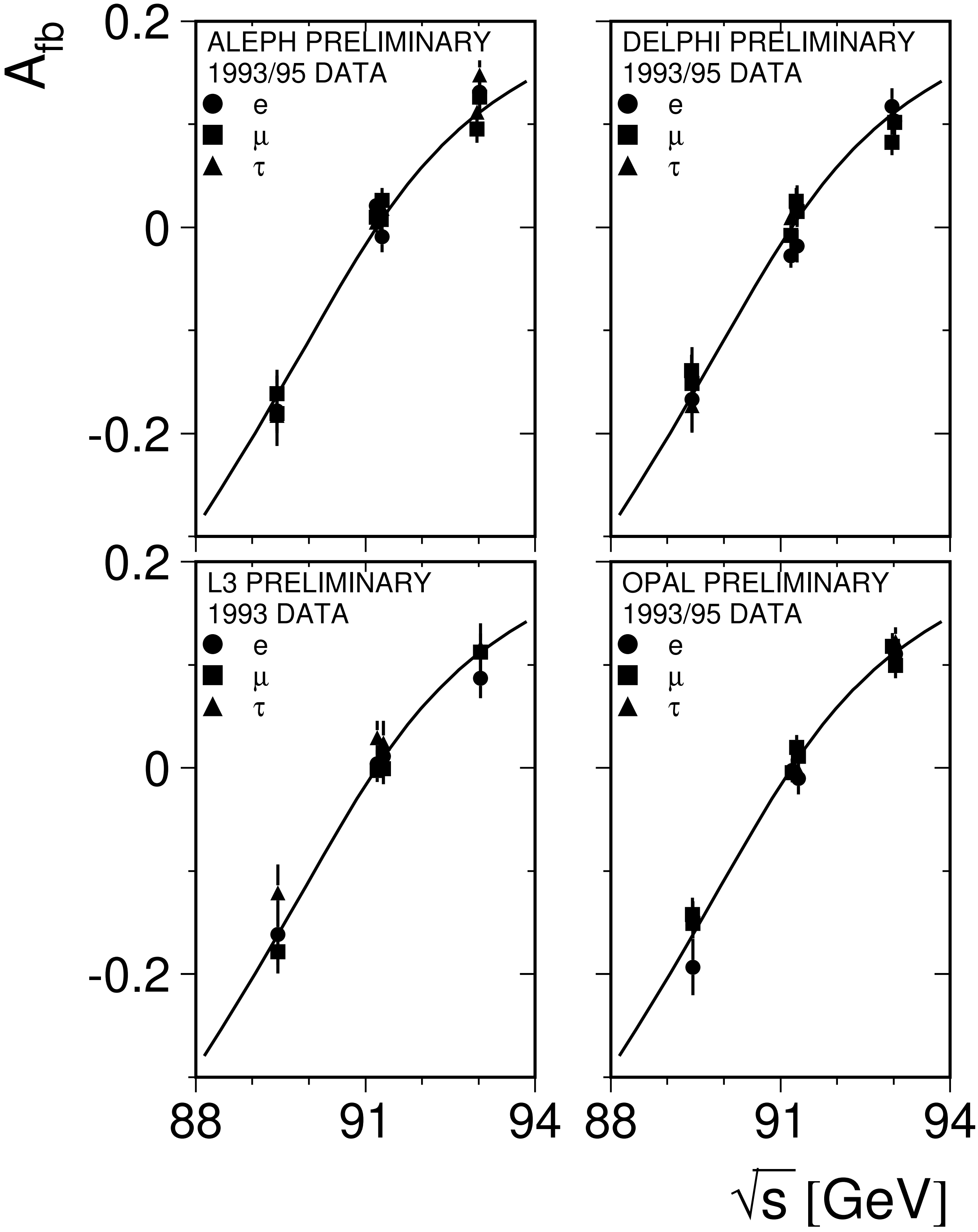

Figure 2: Leptonic forward-backward asymmetries as a function of the centre-of-mass energy $\sqrt{s}$ (Preliminary results from the analysis of the 1993 and, except for L3, 1995 energy-scan data). At each energy, the results are in good agreement between electrons, muons and taus. They are also compatible with the prediction of the Standard Model for $m_{\mathrm{Z}} = 91.19$ GeV, $m_{\mathrm{t}} = 180$ GeV, $m_{\mathrm{H}} = 300$ GeV, $\alpha_s(m_{\mathrm{Z}}^2) = 0.123$, and $1/\alpha(m_{\mathrm{Z}}^2) = 128.896$, shown as the solid line.

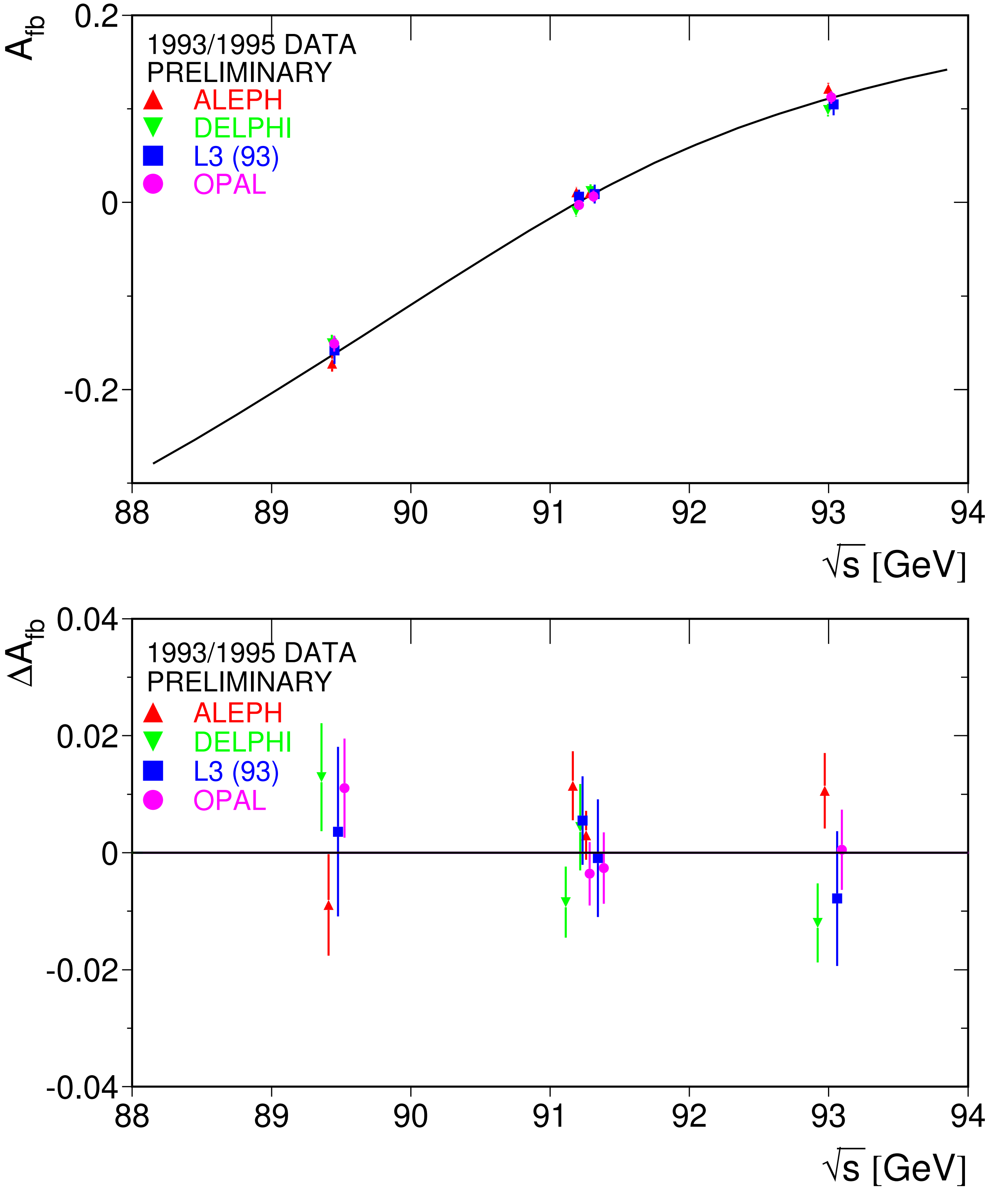

Figure 3: Leptonic forward-backward asymmetries as a function of the centre-of-mass energy $\sqrt{s}$, assuming lepton universality (Preliminary results from the analysis of the 1993 and, except for L3, 1995 energy-scan data). The line shows the prediction of the Standard Model for $m_Z = 91.19$ GeV, $m_t = 180$ GeV, $m_H = 300$ GeV, $\alpha_s(m_Z^2) = 0.123$, and $1/\alpha(m_Z^2) = 128.896$. The bottom plot shows the difference between the measurements and the Standard-Model prediction on an enlarged vertical scale. For better visibility the measurements are spread horizontally in the bottom plot.

		ALEPH	DELPHI	L3	OPAL	LEP
hadrons	total	1644	1514	1577	1488	6223
	s' cut	369	—	469	334	1172
leptons	total	[a]177	[a]156	374	[a]159	866
	s' cut	1242	[a]76	268	[a]76	1662

Table 1: The number of hadronic and charged-lepton events collected by the LEP experiments at centre-of-mass energies between 130 and 140 GeV. The s' cuts of the different experiments are: ALEPH: $\sqrt{s'/s} > 0.9$, DELPHI: $\sqrt{s'/s} > 0.85 - 0.90$, L3: $\sqrt{s'} > 110$ GeV, OPAL: $s'/s > 0.8$.
[a]Only muons and taus.

	ALEPH	DELPHI	L3	OPAL
m_Z (GeV)	91.2027±0.0077	91.1915±0.0098	91.1962±0.0077	91.1980±0.0080
Γ_Z (GeV)	2.4935±0.0058[a]	2.4896±0.0056[a]	2.5013±0.0055[a]	2.4932±0.0053[a]
$r_{\mathrm{had}}^{\mathrm{tot}}$	2.968±0.014	2.945±0.014	2.980±0.013	2.962±0.014
$r_{\mathrm{e}}^{\mathrm{tot}}$	0.1445±0.0010	0.1406±0.0012	0.1426±0.0011	0.1414±0.0012
r_{μ}^{tot}	0.14214±0.00089	0.14218±0.00089	0.14298±0.00099	0.14220±0.00075
r_{τ}^{tot}	0.14315±0.00090	0.1427±0.0013	0.1439±0.0013	0.14111±0.00097
$j_{\mathrm{had}}^{\mathrm{tot}}$	-0.40±0.40	-0.15±0.54	0.04±0.40	-0.45±0.41
$j_{\mathrm{e}}^{\mathrm{tot}}$	-0.011±0.063	-0.119±0.060	-0.001±0.058	-0.144±0.059
j_{μ}^{tot}	-0.044±0.042	0.034±0.043	-0.015±0.047	-0.037±0.040
j_{τ}^{tot}	-0.013±0.044	-0.011±0.052	0.064±0.054	-0.043±0.047
$r_{\mathrm{e}}^{\mathrm{fb}}$	0.00368±0.00087	0.0031±0.0010	0.0021±0.0015	0.0017±0.0011
r_{μ}^{fb}	0.00342±0.00053	0.00304±0.00055	0.00313±0.00075	0.00246±0.00054
r_{τ}^{fb}	0.00404±0.00076	0.00365±0.00095	0.0051±0.0013	0.00330±0.00067
$j_{\mathrm{e}}^{\mathrm{fb}}$	0.941±0.092	0.814±0.074	0.70±0.10	0.809±0.090
j_{μ}^{fb}	0.847±0.041	0.740±0.036	0.835±0.063	0.728±0.044
j_{τ}^{fb}	0.892±0.043	0.784±0.067	0.693±0.082	0.759±0.051
χ^2/d.o.f.	179/190	181/161	120/146	88/134

Table 2: S-Matrix parameters from 16-parameter fits to the data of the four LEP experiments, without the assumption of lepton universality.
[a]Γ_Z has an additional error of 0.0010 GeV due to an uncertainty of 5 MeV in the LEP center-of-mass energy spread which is common to all experiments.

Parameter	Average Value
m_Z (GeV)	91.1968±0.0048
Γ_Z (GeV)	2.4943±0.0031±0.0010
$r_{\mathrm{had}}^{\mathrm{tot}}$	2.9649±0.0083
$r_{\mathrm{e}}^{\mathrm{tot}}$	0.14256±0.00060
r_{μ}^{tot}	0.14241±0.00049
r_{τ}^{tot}	0.14262±0.00057
$j_{\mathrm{had}}^{\mathrm{tot}}$	-0.24±0.25
$j_{\mathrm{e}}^{\mathrm{tot}}$	-0.066±0.030
j_{μ}^{tot}	-0.013±0.022
j_{τ}^{tot}	-0.004±0.025
$r_{\mathrm{e}}^{\mathrm{fb}}$	0.00287±0.00053
r_{μ}^{fb}	0.00296±0.00029
r_{τ}^{fb}	0.00375±0.00042
$j_{\mathrm{e}}^{\mathrm{fb}}$	0.815±0.043
j_{μ}^{fb}	0.778±0.022
j_{τ}^{fb}	0.808±0.028

Table 3: Average S-Matrix parameters from the data of the four LEP experiments given in Table 2, without the assumption of lepton universality. The χ^2/d.o.f. of the average is 58/48.

	m_Z	Γ_Z	$r_{\mathrm{had}}^{\mathrm{tot}}$	$r_{\mathrm{e}}^{\mathrm{tot}}$	r_{μ}^{tot}	r_{τ}^{tot}	$j_{\mathrm{had}}^{\mathrm{tot}}$	$j_{\mathrm{e}}^{\mathrm{tot}}$	j_{μ}^{tot}	j_{τ}^{tot}	$r_{\mathrm{e}}^{\mathrm{fb}}$	r_{μ}^{fb}	r_{τ}^{fb}	$j_{\mathrm{e}}^{\mathrm{fb}}$	j_{μ}^{fb}	j_{τ}^{fb}
m_Z	1.00	-.28	-.21	-.14	-.17	-.15	-.89	-.37	-.47	-.42	.05	.14	.11	.00	-.02	-.01
Γ_Z	-.28	1.00	.77	.50	.64	.54	.28	.08	.15	.13	.00	-.01	-.01	.03	.06	.04
$r_{\mathrm{had}}^{\mathrm{tot}}$	-.21	.77	1.00	.63	.78	.67	.22	.05	.11	.10	.01	.00	.01	.04	.08	.06
$r_{\mathrm{e}}^{\mathrm{tot}}$	-.14	.50	.63	1.00	.52	.45	.14	.04	.07	.07	.03	-.01	.00	.05	.05	.04
r_{μ}^{tot}	-.17	.64	.78	.52	1.00	.55	.17	.04	.18	.09	.00	.02	.00	.03	.11	.06
r_{τ}^{tot}	-.15	.54	.67	.45	.55	1.00	.15	.03	.08	.17	.01	.01	.03	.03	.06	.09
$j_{\mathrm{had}}^{\mathrm{tot}}$	-.89	.28	.22	.14	.17	.15	1.00	.37	.47	.41	-.05	-.13	-.10	.00	.02	.01
$j_{\mathrm{e}}^{\mathrm{tot}}$	-.37	.08	.05	.04	.04	.03	.37	1.00	.20	.17	-.01	-.05	-.04	.03	.01	.00
j_{μ}^{tot}	-.47	.15	.11	.07	.18	.08	.47	.20	1.00	.21	-.03	-.02	-.05	-.01	.01	.00
j_{τ}^{tot}	-.42	.13	.10	.07	.09	.17	.41	.17	.21	1.00	-.02	-.06	-.03	.01	.02	.00
$r_{\mathrm{e}}^{\mathrm{fb}}$	.05	.00	.01	.03	.00	.01	-.05	-.01	-.03	-.02	1.00	.01	.01	.05	.00	.00
r_{μ}^{fb}	.14	-.01	.00	-.01	.02	.01	-.13	-.05	-.02	-.06	.01	1.00	.03	.00	.17	.01
r_{τ}^{fb}	.11	-.01	.01	.00	.00	.03	-.10	-.04	-.05	-.03	.01	.03	1.00	.01	.01	.13
$j_{\mathrm{e}}^{\mathrm{fb}}$	.00	.03	.04	.05	.03	.03	.00	.03	-.01	.01	.05	.00	.01	1.00	-.01	.00
j_{μ}^{fb}	-.02	.06	.08	.05	.11	.06	.02	.01	.01	.02	.00	.17	.01	-.01	1.00	.01
j_{τ}^{fb}	-.01	.04	.06	.04	.06	.09	.01	.00	.00	.00	.00	.01	.13	.00	.01	1.00

Table 4: The correlation matrix for the set of parameters given in Table 3.

	ALEPH	DELPHI	L3	OPAL
m_Z (GeV)	91.2023±0.0077	91.1907±0.0097	91.1963±0.0077	91.1950±0.0070
Γ_Z (GeV)	2.4936±0.0057[(a)]	2.4893±0.0056[(a)]	2.5011±0.0054[(a)]	2.4932±0.0053[(a)]
$r_{\rm had}^{\rm tot}$	2.968±0.014	2.944±0.014	2.979±0.013	2.961±0.014
$r_\ell^{\rm tot}$	0.14311±0.00075	0.14184±0.00080	0.14304±0.00080	0.14178±0.00070
$j_{\rm had}^{\rm tot}$	-0.38±0.40	-0.11±0.53	0.05±0.39	-0.30±0.39
$j_\ell^{\rm tot}$	-0.019±0.031	-0.013±0.034	0.014±0.033	-0.051±0.030
$r_\ell^{\rm fb}$	0.00366±0.00040	0.00316±0.00045	0.00333±0.00059	0.00260±0.00039
$j_\ell^{\rm fb}$	0.876±0.028	0.759±0.029	0.762±0.045	0.754±0.029
χ^2/d.o.f.	186/198	190/169	127/154	96/142

Table 5: S-Matrix parameters from 8-parameter fits to the data of the four LEP experiments, assuming lepton universality.
[(a)] Γ_Z has an additional error of 0.0010 GeV due to an uncertainty of 5 MeV in the LEP center-of-mass energy spread which is common to all experiments.

Parameter	Average Value	SM Prediction
m_Z (GeV)	91.1965±0.0048	—
Γ_Z (GeV)	2.4941±0.0031±0.0010	2.4973
$r_{\rm had}^{\rm tot}$	2.9644±0.0083	2.9681
$r_\ell^{\rm tot}$	0.14245±0.00044	0.14268
$j_{\rm had}^{\rm tot}$	-0.22±0.25	0.22
$j_\ell^{\rm tot}$	-0.020±0.017	0.004
$r_\ell^{\rm fb}$	0.00315±0.00022	0.00271
$j_\ell^{\rm fb}$	0.793±0.016	0.799

Table 6: Average S-Matrix parameters from the data of the four LEP experiments given in Table 2, assuming lepton universality. The χ^2/d.o.f. of the average is 65/56. The Standard-Model predictions are listed for $m_Z = 91.19$ GeV, $m_t = 180$ GeV, $m_H = 300$ GeV, $\alpha_s(m_Z^2) = 0.123$, and $1/\alpha(m_Z^2) = 128.896$.

	m_Z	Γ_Z	$r_{\rm had}^{\rm tot}$	$r_\ell^{\rm tot}$	$j_{\rm had}^{\rm tot}$	$j_\ell^{\rm tot}$	$r_\ell^{\rm fb}$	$j_\ell^{\rm fb}$
m_Z	1.00	-.30	-.22	-.20	-.89	-.62	.18	-.02
Γ_Z	-.30	1.00	.77	.69	.28	.19	-.01	.08
$r_{\rm had}^{\rm tot}$	-.22	.77	1.00	.86	.22	.14	.01	.11
$r_\ell^{\rm tot}$	-.20	.69	.86	1.00	.19	.17	.02	.13
$j_{\rm had}^{\rm tot}$	-.89	.28	.22	.19	1.00	.62	-.16	.02
$j_\ell^{\rm tot}$	-.62	.19	.14	.17	.62	1.00	-.09	.02
$r_\ell^{\rm fb}$	.18	-.01	.01	.02	-.16	-.09	1.00	.15
$j_\ell^{\rm fb}$	-.02	.08	.11	.13	.02	.02	.15	1.00

Table 7: The correlation matrix for the set of parameters given in Table 6.

A Details on the S-Matrix Analysis

A.1 The S-Matrix Approach

Within the S-Matrix approach, the lowest-order total cross section, σ^0_{tot}, and forward-backward asymmetry, A^0_{fb}, for $\mathrm{e^+e^-}$ annihilation into a fermion-antifermion pair are given as [2]:

$$
\begin{aligned}
\sigma^0_a(s) &= \frac{4}{3}\pi\alpha^2\left[\frac{g^a_{\mathrm{f}}}{s} + \frac{j^a_{\mathrm{f}}(s-\overline{m}^2_{\mathrm{Z}}) + r^a_{\mathrm{f}}\, s}{(s-\overline{m}^2_{\mathrm{Z}})^2 + \overline{m}^2_{\mathrm{Z}}\overline{\Gamma}^2_{\mathrm{Z}}}\right] \qquad \text{for a} = \mathrm{tot}, \mathrm{fb} && (7)\\
A^0_{\mathrm{fb}}(s) &= \frac{3}{4}\frac{\sigma^0_{\mathrm{fb}}(s)}{\sigma^0_{\mathrm{tot}}(s)}, && (8)
\end{aligned}
$$

where $\sqrt{s}$ is the centre-of-mass energy. The S–Matrix ansatz is defined using a Breit–Wigner denominator with s–independent width for the Z resonance, $s-\overline{m}^2_{\mathrm{Z}} + i\overline{m}_{\mathrm{Z}}\overline{\Gamma}_{\mathrm{Z}}$. The transformations of the resulting values for the mass and the total width of the Z boson to correspond to a Breit-Wigner denominator with s-dependent width, $s - m^2_{\mathrm{Z}} + is\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}}$, are:

$$
\begin{aligned}
m_{\mathrm{Z}} &\equiv \overline{m}_{\mathrm{Z}}\sqrt{1+\overline{\Gamma}^2_{\mathrm{Z}}/\overline{m}^2_{\mathrm{Z}}} \approx \overline{m}_{\mathrm{Z}} + 34.2\ \mathrm{MeV}\\
\Gamma_{\mathrm{Z}} &\equiv \overline{\Gamma}_{\mathrm{Z}}\sqrt{1+\overline{\Gamma}^2_{\mathrm{Z}}/\overline{m}^2_{\mathrm{Z}}} \approx \overline{\Gamma}_{\mathrm{Z}} + 0.9\ \mathrm{MeV}\,,
\end{aligned} \qquad (9)
$$

so that $\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}} = \overline{\Gamma}_{\mathrm{Z}}/\overline{m}_{\mathrm{Z}}$. Note, that we apply the transformations according to Equation 9 when listing numerical results for the mass and total width of the Z boson. QED radiative corrections are included by convolution with a radiator function [10].

The S-Matrix parameters r_{f}, j_{f} and g_{f}, which are real numbers, scale the Z exchange, γZ interference and photon exchange contributions, respectively. In models with only vector couplings of the photon to the electric charge, q_f, and only vector and axial-vector couplings of the Z, the S-Matrix parameters are [2]:[2]

$$
\begin{aligned}
r^{\mathrm{tot}}_{\mathrm{f}} &= \kappa^2\left[g^2_{A\mathrm{e}} + g^2_{V\mathrm{e}}\right]\left[g^2_{A\mathrm{f}} + g^2_{V\mathrm{f}}\right] - 2\kappa g_{V\mathrm{e}} g_{V\mathrm{f}} C_{Im}\\
j^{\mathrm{tot}}_{\mathrm{f}} &= 2\kappa g_{V\mathrm{e}} g_{V\mathrm{f}}\left(C_{Re} + C_{Im}\right)\\
g^{\mathrm{tot}}_{\mathrm{f}} &= q^2_{\mathrm{e}} q^2_f \left|F_A(m^2_{\mathrm{Z}})\right|^2\\
r^{\mathrm{fb}}_{\mathrm{f}} &= 4\kappa^2 g_{A\mathrm{e}} g_{V\mathrm{e}} g_{A\mathrm{f}} g_{V\mathrm{f}} - 2\kappa g_{A\mathrm{e}} g_{A\mathrm{f}} C_{Im}\\
j^{\mathrm{fb}}_{\mathrm{f}} &= 2\kappa g_{A\mathrm{e}} g_{A\mathrm{f}}\left(C_{Re} + C_{Im}\right)\\
g^{\mathrm{fb}}_{\mathrm{f}} &= 0\,,
\end{aligned} \qquad (10)
$$

where:

$$
\begin{aligned}
\kappa &= \frac{\mathrm{G_F m^2_Z}}{2\sqrt{2}\pi\alpha} \approx 1.50 && (11)\\
C_{Im} &= \frac{\Gamma_{\mathrm{Z}}}{m_{\mathrm{Z}}}\, q_{\mathrm{e}} q_f\ \mathrm{Im}\left\{\mathrm{F_A(m^2_Z)}\right\} && (12)\\
C_{Re} &= q_{\mathrm{e}} q_f\ \mathrm{Re}\left\{\mathrm{F_A(m^2_Z)}\right\} && (13)\\
F_A(m^2_{\mathrm{Z}}) &= \frac{\alpha(m^2_{\mathrm{Z}})}{\alpha}\,. && (14)
\end{aligned}
$$

[2]The relations for r and j listed here neglect the imaginary parts of the effective vector and axial-vector coupling constants of the Z boson to fermions.

In the last equation $\alpha(m_Z^2)$ denotes the complex running fine structure constant, and $\alpha \equiv \alpha(0)$. For the hadronic final state, the S-Matrix parameters r_{had}, j_{had} and g_{had} are summed over all colours and open quark flavours.

A.2 Correlated Errors due to the LEP Energy Calibration

The following procedure is used to determine the correlated errors on the S-Matrix parameters arising from the LEP energy calibration. The S-Matrix fit to the experimental measurements of cross sections and forward-backward asymmetries of a single LEP experiment determines the error matrix $\mathcal{E}$ of the set of S-Matrix parameters fitted. A second S-Matrix fit is performed, where the errors of all measurements except those arising from the LEP energy calibration are scaled by a factor of 0.5. This factor simulates the weight of the four LEP experiments in the average. Denoting the error matrix of the second fit with $\mathcal{E}'$, the error matrix $\mathcal{C}$ describing the correlated errors on the S-Matrix parameters arising from the LEP energy calibration is then obtained as:

$$\mathcal{C} = \frac{4}{3}\mathcal{E}' - \frac{1}{3}\mathcal{E}\,. \qquad (15)$$

Table 8 lists numerical values for the correlated errors on the S-Matrix parameters and for the corresponding correlation coefficients.

	Error	m_Z	Γ_Z	$r_{\mathrm{had}}^{\mathrm{tot}}$	r_e^{tot}	r_μ^{tot}	r_τ^{tot}	$j_{\mathrm{had}}^{\mathrm{tot}}$	j_e^{tot}	j_μ^{tot}	j_τ^{tot}	r_e^{fb}	r_μ^{fb}	r_τ^{fb}	j_e^{fb}	j_μ^{fb}	j_τ^{fb}
m_Z	3 MeV	1.0	-.5	-.5	-.3	-.5	-.5	-.9	-.9	-.9	-.9	.0	.0	.0	.0	.0	.0
Γ_Z	1.6 MeV	-.5	1.0	1.0	.7	1.0	1.0	.4	.4	.4	.4	.0	.0	.0	.0	.0	.0
$r_{\mathrm{had}}^{\mathrm{tot}}$	0.003	-.5	1.0	1.0	.7	1.0	1.0	.4	.4	.4	.4	.0	.0	.0	.0	.0	.0
r_e^{tot}	0.00018	-.3	.7	.7	1.0	.7	.7	.2	.2	.2	.2	.0	.0	.0	.0	.0	.0
r_μ^{tot}	0.00016	-.5	1.0	1.0	.7	1.0	1.0	.4	.4	.4	.4	.0	.0	.0	.0	.0	.0
r_τ^{tot}	0.00016	-.5	1.0	1.0	.7	1.0	1.0	.4	.4	.4	.4	.0	.0	.0	.0	.0	.0
$j_{\mathrm{had}}^{\mathrm{tot}}$	0.15	-.9	.4	.4	.2	.4	.4	1.0	1.0	1.0	1.0	.0	.0	.0	.0	.0	.0
j_e^{tot}	0.007	-.9	.4	.4	.2	.4	.4	1.0	1.0	1.0	1.0	.0	.0	.0	.0	.0	.0
j_μ^{tot}	0.007	-.9	.4	.4	.2	.4	.4	1.0	1.0	1.0	1.0	.0	.0	.0	.0	.0	.0
j_τ^{tot}	0.007	-.9	.4	.4	.2	.4	.4	1.0	1.0	1.0	1.0	.0	.0	.0	.0	.0	.0
r_e^{fb}	0.00013	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0
r_μ^{fb}	0.00006	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0
r_τ^{fb}	0.00006	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0
j_e^{fb}	0.001	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0
j_μ^{fb}	0.001	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0
j_τ^{fb}	0.001	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0

Table 8: The correlated errors on the S-Matrix parameters and the corresponding correlation coefficients arising from the LEP energy calibration which is common to all four LEP experiments.

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