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An Investigation of the Interference between Photon and Z-Boson Exchange

The S-Matrix Subgroup of the LEP Electroweak Working Group[1]

Internal Note

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Abstract

This note presents an investigation of the γZ interference contribution to total cross sections and forward-backward asymmetries. It is based on a combination of published and preliminary electroweak results from the four LEP collaborations which were prepared for the 1997 winter conferences.

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1 Introduction

In Section 2 of the note on LEP electroweak averages prepared for the 1997 winter conferences [1], an analysis of the Z lineshape and leptonic forward-backward asymmetries is presented. The formulae used in the fits presented in that section are valid under quite general assumptions, namely that the scattering process arises from the s-channel exchange of two bosons, a massless photon with vector couplings to the electric charge and a massive Z boson with both vector and axial-vector couplings to fermions. For leptons these two couplings, together with $\alpha(m_{\mathrm{Z}}^2)$, fully determine both the Z exchange and the γZ interference terms. In contrast, a model dependence is introduced by the treatment of the γZ interference contribution to the hadronic cross section, because quark couplings to the Z are not individually measured for each flavour. This term is therefore taken from the Standard Model.

A more general approach is to parametrise the γZ interference term independent of the Z exchange amplitude. Several model independent formalisms exist [2, 3]. For the analysis performed here the framework described in [3] based on the S-Matrix ansatz is used. Formulae for $\mathrm{e^+e^-} \to \mathrm{f\bar{f}}$ cross sections and forward-backward asymmetries are summarised in Appendix A.1.

Measurements at centre-of-mass energies far away from the Z pole are especially sensitive to the parameters describing the interference between photon and Z-boson exchange [4]. This analysis includes the results from the 1995 LEP run at centre-of-mass energies, $\sqrt{s}$, from 130 GeV to 140 GeV [5–7, 9] and the new results from the 1996 LEP run at $\sqrt{s}$ from 161 GeV to 172 GeV [8–10]. The total statistics of these high-energy measurements of the four LEP collaborations are given in Tables 1 and 2. The OPAL collaboration has not used their measurements between 161 GeV and 172 GeV for the fit procedure [11].

For a large fraction of these high-energy events, initial-state radiation photons lower the effective centre-of-mass energy, $\sqrt{s'}$, of the annihilation process to values close to the Z mass, m_{Z} ('radiative return to the Z'). These radiative events dilute the sensitivity of the cross section and forward-backward asymmetry measurements to the γZ interference parameters. By applying a cut on the effective centre-of-mass energy, $\sqrt{s'} \gg m_{\mathrm{Z}}$, the high-energy non-radiative events are isolated and used to constrain these parameters.

2 General Framework and Results

The S-Matrix parameters r_{f}, j_{f} and g_{f} are real numbers and express the size of the Z exchange, γZ interference and photon exchange contributions. In the approach presented here r_{f} and j_{f} are treated as free parameters while the photon exchange contribution, g_{f}, is fixed to its QED prediction. Each final state is thus described by four free parameters: two for cross sections, $r_{\mathrm{f}}^{\mathrm{tot}}$ and $j_{\mathrm{f}}^{\mathrm{tot}}$, and two for forward-backward asymmetries, $r_{\mathrm{f}}^{\mathrm{fb}}$ and $j_{\mathrm{f}}^{\mathrm{fb}}$. In models with only vector and axial-vector couplings of the Z-boson, these four S-Matrix parameters are not independent of each other and are approximatively [3]:

$$r_{\mathrm{f}}^{\mathrm{tot}} \propto \left[g_{A\mathrm{e}}^2 + g_{V\mathrm{e}}^2\right] \cdot \left[g_{A\mathrm{f}}^2 + g_{V\mathrm{f}}^2\right] , \tag{1}$$

$$j_{\mathrm{f}}^{\mathrm{tot}} \propto g_{V\mathrm{e}} g_{V\mathrm{f}} , \tag{2}$$

$$r_{\mathrm{f}}^{\mathrm{fb}} \propto g_{A\mathrm{e}} g_{V\mathrm{e}} \cdot g_{A\mathrm{f}} g_{V\mathrm{f}} , \tag{3}$$

$$j_{\mathrm{f}}^{\mathrm{fb}} \propto g_{A\mathrm{e}} g_{A\mathrm{f}} . \tag{4}$$

The exact expressions are listed in Appendix A.1.

The four sets of 16 parameters provided by the four LEP experiments are presented in Table 3. For the averaging procedure the parameter mean values, errors and correlations of the single experiments and in addition the correlations common to the different LEP experiments are used.

Sources of common systematic errors are the theoretical uncertainty of the luminosity determination and the LEP beam energy calibration. The common errors are taken into account only for the data taken in the vicinity of the Z pole. For the high energy measurements the statistical uncertainty is dominant. The theoretical error of the luminosity determination of 0.11% for the data taken in the vicinity of the Z pole introduces a correlation of the same size between all S-Matrix parameters r and j of the different experiments. The common systematic errors and the correlations amongst the S-Matrix parameters arising from the LEP beam energy calibration are summarised in Appendix A.2. The combined parameter set and its correlation matrix are given in Tables 4 and 5.

The data of each of the four LEP experiments are consistent with lepton universality. If lepton universality is assumed, the set of 16 S-Matrix parameters is reduced to a set of 8 parameters. In Table 6 the 8 parameters of the individual LEP experiments are shown. In Tables 7 and 8 the LEP averages and their correlation matrix are provided. The Standard-Model predictions agree well with the LEP averages of the S-Matrix parameters except for the $r_\ell^{\rm fb}$ parameter. It deviates by 2.1 standard deviations from the Standard-Model prediction.

3 The Hadronic γZ Interference Term

The largest correlation (-77%) arises between the mass of the Z boson, $m_{\rm Z}$, and the hadronic γZ interference term, $j_{\rm had}^{\rm tot}$. This correlation leads to a sizeable increase in the error on the Z-boson mass in fits where $j_{\rm had}^{\rm tot}$ is left free. In Figure 1 the 68% probability contour in the $m_{\rm Z}$–$j_{\rm had}^{\rm tot}$ plane is depicted. The error on the hadronic γZ interference term, and consequently the error on $m_{\rm Z}$, are reduced by using total cross section measurements at centre-of-mass energies far away from the Z pole [3,4]. With respect to the last publication [12] the error on the hadronic γZ interference term has further reduced by 32%. This is due to the inclusion of the data measured between 161 GeV and 172 GeV. The average value of $j_{\rm had}^{\rm tot}$, 0.15 ± 0.15, is in good agreement with the Standard-Model prediction.

The inclusion of data taken well below the Z peak leads to a small improvement in the errors. The TOPAZ collaboration at TRISTAN (KEK) has performed a measurement of the hadronic total cross section at $\sqrt{s} = 57.77$ GeV, $\sigma_{\rm tot}^0 = 143.6 \pm 1.5(stat.) \pm 4.5(syst.)$ pb [13]. Combining this measurement with the LEP results, $m_{\rm Z}$ and $j_{\rm had}^{\rm tot}$ are:

$$m_{\rm Z} = 91187.6 \pm 0.0031 \text{ GeV} \quad (5)$$

$$j_{\rm had}^{\rm tot} = 0.14 \pm 0.13\,, \quad (6)$$

and the correlation between $m_{\rm Z}$ and $j_{\rm had}^{\rm tot}$ is reduced to -72%.

4 Leptonic Forward-Backward Asymmetries

The leptonic forward-backward pole asymmetry described by the $r_\ell^{\rm fb}$ parameter shows the largest discrepancy with the Standard-Model prediction, 2.1 standard deviations, as given in Table 7. From Table 4 one concludes that the discrepancy arises from the tau channel only which is measured to be larger than the prediction by all experiments, see Table 3. This effect is also observed in the less

general $g_{A\ell} - g_{V\ell}$ ansatz which is described in [1]. There the forward-backward pole asymmetry of the tau channel is measured to be larger than the Standard-Model prediction.

The dependence of the leptonic forward-backward asymmetry on the centre-of-mass energy arises from the γZ interference, and is quantified by the slope parameter $j_\ell^{\rm fb}$. Among the parameters characterising the γZ interference this parameter is determined with the highest relative precision. The χ^2 values of the averages in Tables 4 and 7 have a large contribution from the energy dependence of the leptonic forward-backward asymmetries, $j_\ell^{\rm fb}$. In the following discussion of $j_\ell^{\rm fb}$, its small correlations with other S-Matrix parameters is neglected.

In Figure 2 the forward-backward asymmetries for electrons, muons and taus as a function of the centre-of-mass energy are depicted for the four LEP experiments. The measurements are extrapolated to full phase-space. In the case of electrons, t-channel and s/t-interference contributions are removed. The uncertainty of the leptonic forward-backward asymmetry measurements and also their averages are still dominated by statistics. Considering each energy point separately, the measurements for each lepton species are in agreement with each other, and compatible with the prediction of the Standard Model. The values for $j_{\rm e}^{\rm fb}$, $j_\mu^{\rm fb}$, $j_\tau^{\rm fb}$ are also in agreement within each experiment, see Table 3.

Assuming lepton universality, the results on the slope parameter $j_\ell^{\rm fb}$ of the four LEP experiments are listed in Table 6. The weighted average of the four values for $j_\ell^{\rm fb}$ agrees within 0.8 standard deviations with the prediction of the Standard Model. However, the average has a χ^2/DoF of 13.1/3, corresponding to a probability of 0.45%. The reason for this high χ^2 value is shown in Figure 3, where the average leptonic forward-backward asymmetries are compared to the prediction of the Standard Model. ALEPH measures a slope about 2.3 standard deviations larger than the prediction, whereas DELPHI and OPAL measure the slope about 1.9 and 2.4 standard deviations smaller than the prediction.

All experiments perform detailed checks on possible systematic effects in the measurements of the leptonic forward-backward asymmetries. No systematic bias is found so far, but further investigations are in progress. For the averaging of these preliminary data, this unlikely value for the χ^2 is assumed to be caused by a statistical fluctuation, and the leptonic forward-backward asymmetries are included in the analyses presented in [1].

5 Summary

The S-Matrix framework allows a coherent treatment of data taken at the Z resonance and data taken in energy regions where the γZ interference is large. The results on the S-Matrix parameters provide a test of the assumed vector and axial-vector structure of the Z couplings.

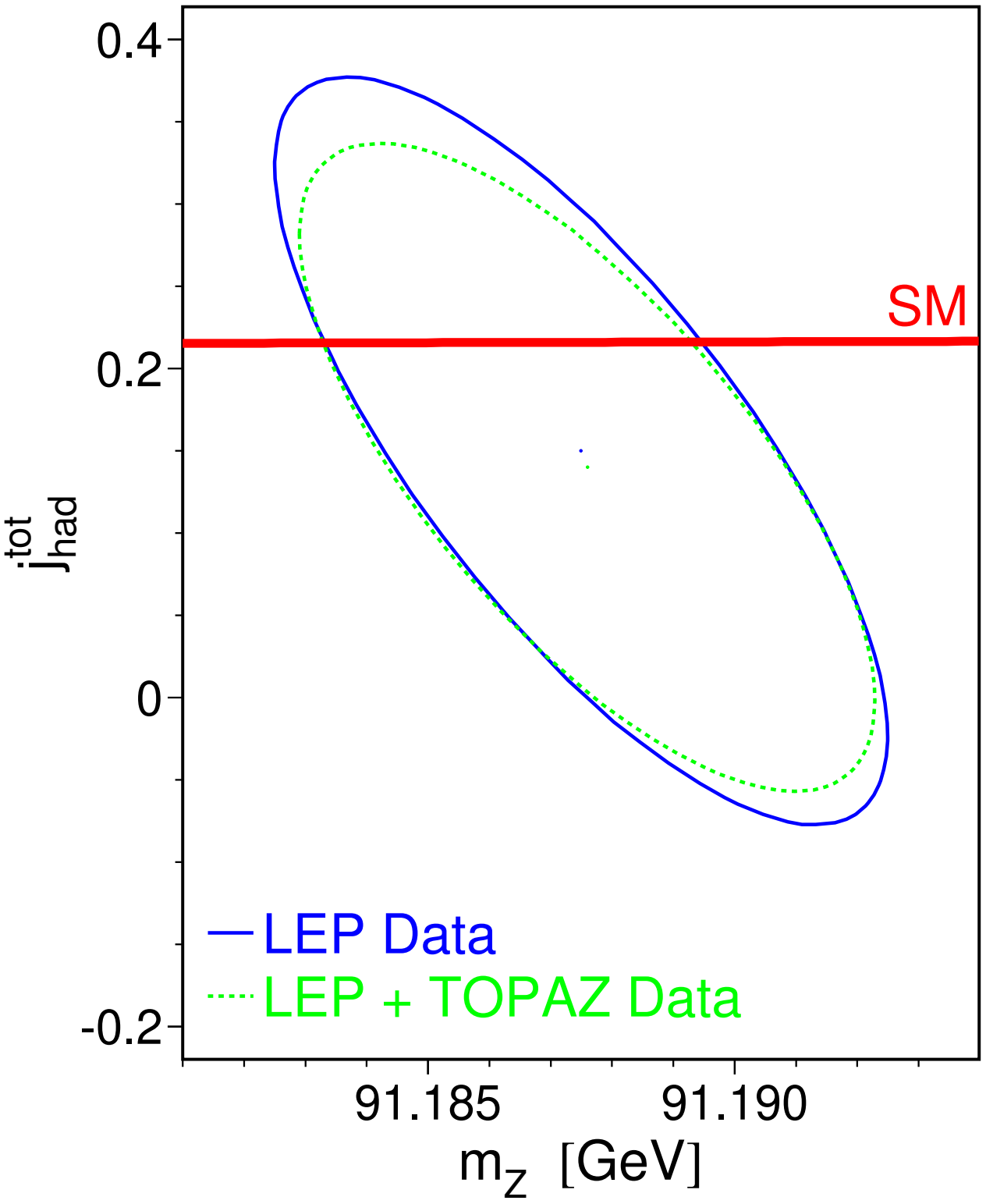

Figure 1: Contours of 68% confidence level between the mass of the Z boson, m_Z, and the hadronic γZ interference term for total cross sections, $j^{\rm tot}_{\rm had}$. Lepton universality is assumed. The solid contour corresponds to LEP data only, the dashed contour is obtained by including the TOPAZ measurement. The Standard-Model prediction for $j^{\rm tot}_{\rm had}$ is shown as the solid band.

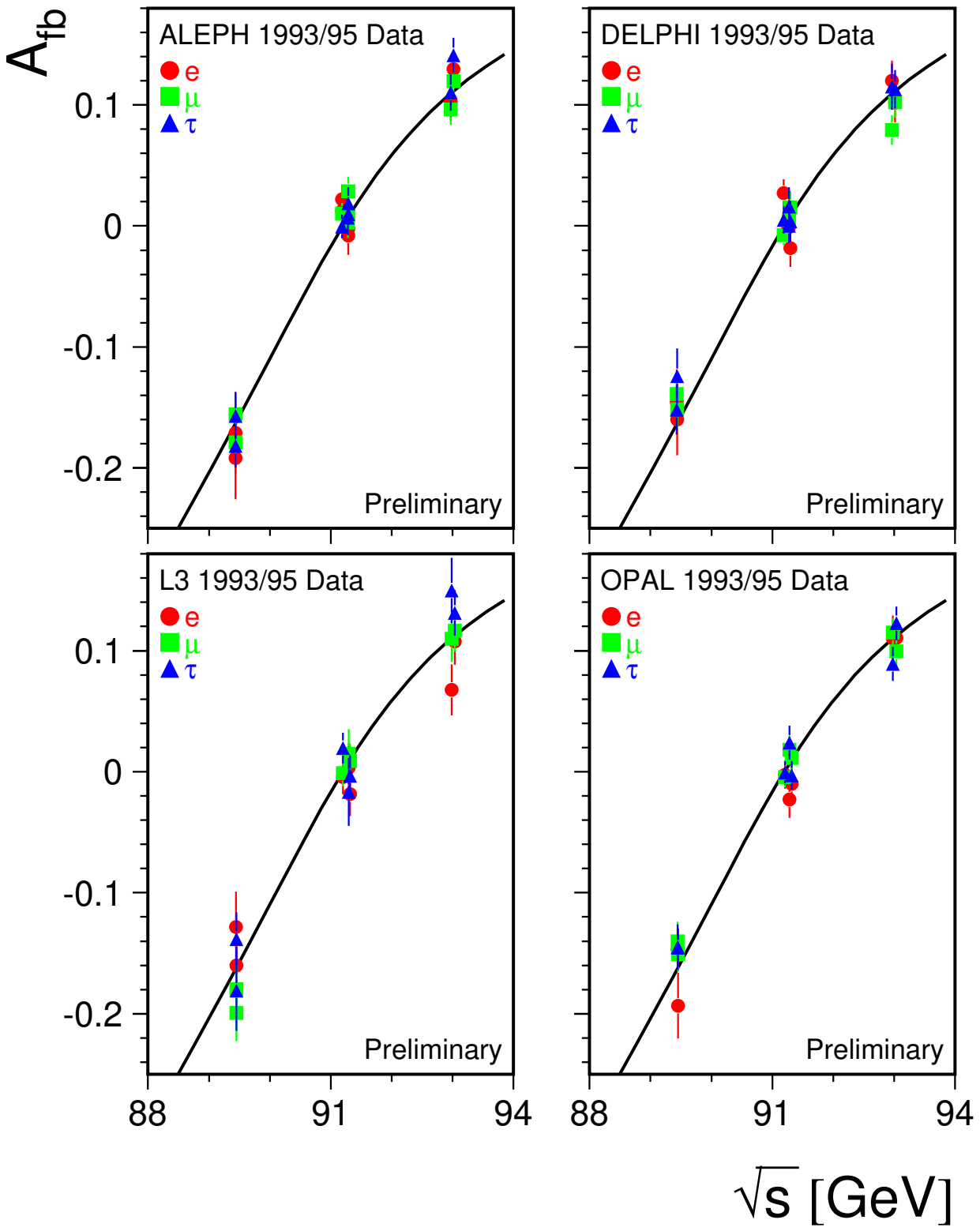

Figure 2: Leptonic forward-backward asymmetries as a function of the centre-of-mass energy $\sqrt{s}$ (Preliminary results from the analysis of the 1993 and 1995 energy-scan data). At each energy, the results are in good agreement between electrons, muons and taus. They are also compatible with the prediction of the Standard Model for $m_{\mathrm{Z}} = 91.1863$ GeV, $m_{\mathrm{t}} = 175$ GeV, $m_{\mathrm{H}} = 300$ GeV, $\alpha_s(m_{\mathrm{Z}}^2) = 0.123$, and $1/\alpha(m_{\mathrm{Z}}^2) = 128.896$, shown as the solid line.

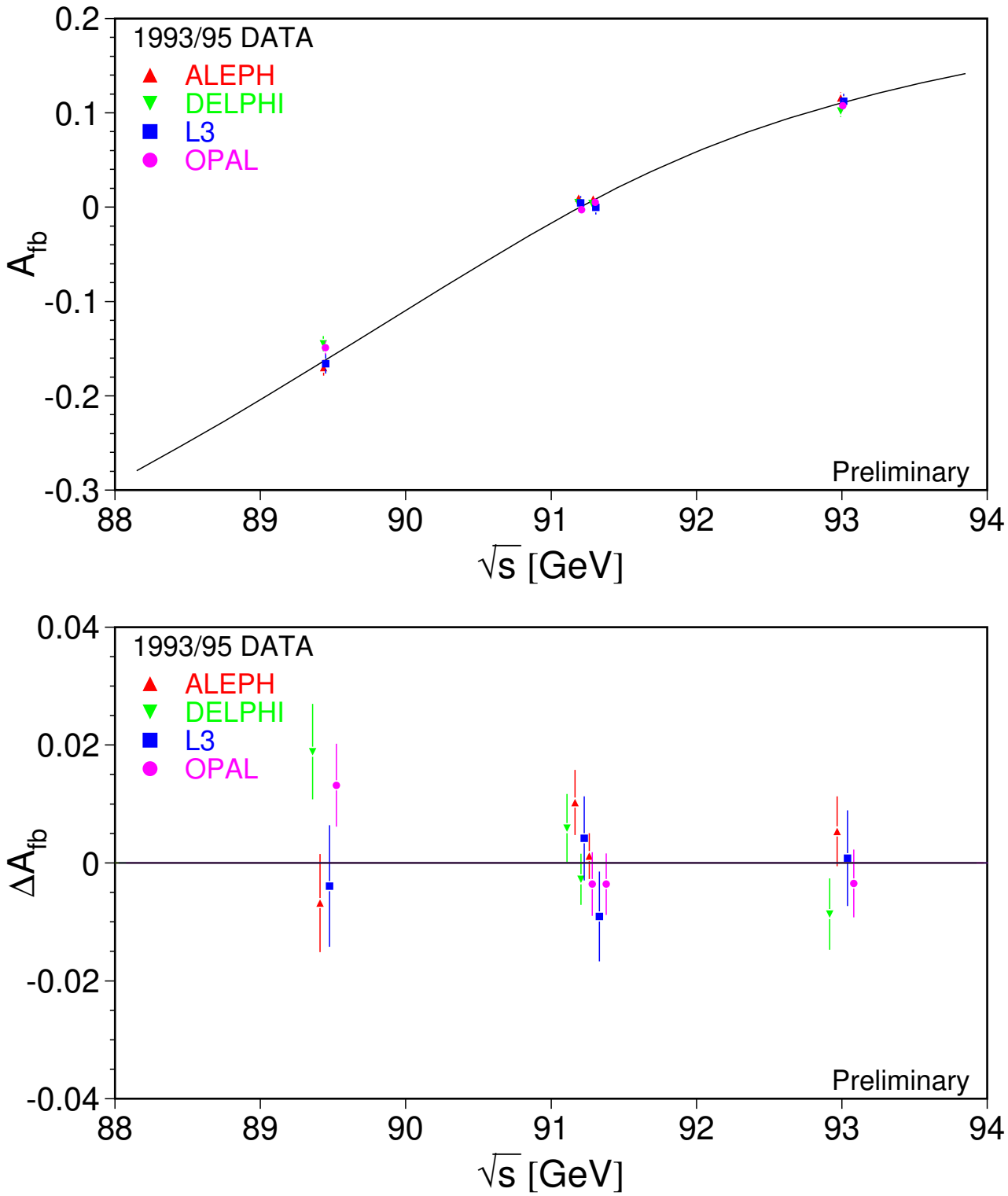

Figure 3: Leptonic forward-backward asymmetries as a function of the centre-of-mass energy $\sqrt{s}$, assuming lepton universality (Preliminary results from the analysis of the 1993 and 1995 energy-scan data). The line shows the prediction of the Standard Model for $m_{\mathrm{Z}} = 91.1863$ GeV, $m_{\mathrm{t}} = 175$ GeV, $m_{\mathrm{H}} = 300$ GeV, $\alpha_s(m_{\mathrm{Z}}^2) = 0.123$, and $1/\alpha(m_{\mathrm{Z}}^2) = 128.896$. The bottom plot shows the difference between the measurements and the Standard-Model prediction on an enlarged vertical scale. For better visibility the measurements are spread horizontally in the bottom plot.

		ALEPH	DELPHI	L3	OPAL	LEP
hadrons	total	1644	1595	1577	1488	6304
	s' cut	369	456	469	334	1628
taus & muons	total	177	165	128	159	629
	s' cut	77	75	60	76	288

Table 1: The number of hadronic and tau plus muon events collected by the LEP experiments at centre-of-mass energies between 130 and 140 GeV. The s' cuts of the different experiments are: ALEPH: $\sqrt{s'/s} > 0.9$, DELPHI: $\sqrt{s'/s} > 0.85 - 0.90$, L3: $\sqrt{s'} > 110$ GeV, OPAL: $s'/s > 0.8$.

		ALEPH	DELPHI	L3	LEP
hadrons	total	2879	2775	2781	8435
	s' cut	691	907	713	2311
taus & muons	total	347	236	251	834
	s' cut	180	103	121	404

Table 2: The number of hadronic and tau plus muon events collected by the LEP experiments at centre-of-mass energies between 161 and 172 GeV. The s' cuts of the different experiments are: ALEPH: $\sqrt{s'/s} > 0.9$, DELPHI: $\sqrt{s'/s} > 0.85 - 0.90$, L3: $\sqrt{s'} > 0.85$ GeV.

	ALEPH	DELPHI	L3	OPAL
m_Z (GeV)	91.1924±0.0058	91.1834±0.0056	91.1855±0.0056	91.1954±0.0079
Γ_Z (GeV)	2.4918±0.0047	2.4903±0.0042	2.5003±0.0043	2.4922±0.0054
$r_{\rm had}^{\rm tot}$	2.964±0.012	2.958±0.010	2.972±0.010	2.963±0.013
$r_{\rm e}^{\rm tot}$	0.14366±0.00087	0.1411±0.0010	0.14266±0.00088	0.1417±0.0011
$r_{\mu}^{\rm tot}$	0.14206±0.00076	0.14284±0.00070	0.14251±0.00083	0.14223±0.00075
$r_{\tau}^{\rm tot}$	0.14322±0.00077	0.1416±0.0010	0.1433±0.0011	0.14116±0.00096
$j_{\rm had}^{\rm tot}$	-0.12±0.30	0.38±0.28	0.34±0.28	-0.47±0.41
$j_{\rm e}^{\rm tot}$	-0.019±0.050	-0.031±0.045	-0.009±0.046	-0.148±0.061
$j_{\mu}^{\rm tot}$	0.001±0.030	0.060±0.031	0.037±0.036	-0.040±0.039
$j_{\tau}^{\rm tot}$	-0.006±0.032	0.063±0.044	0.047±0.039	-0.040±0.047
$r_{\rm e}^{\rm fb}$	0.00325±0.00081	0.00308±0.00096	0.0024±0.0013	0.0014±0.0010
$r_{\mu}^{\rm fb}$	0.00326±0.00047	0.00265±0.00051	0.00321±0.00067	0.00272±0.00051
$r_{\tau}^{\rm fb}$	0.00373±0.00042	0.00372±0.00076	0.00417±0.00096	0.00326±0.00067
$j_{\rm e}^{\rm fb}$	0.870±0.070	0.820±0.073	0.649±0.080	0.783±0.064
$j_{\mu}^{\rm fb}$	0.841±0.034	0.756±0.035	0.837±0.047	0.736±0.034
$j_{\tau}^{\rm fb}$	0.869±0.032	0.704±0.049	0.787±0.057	0.729±0.042
χ^2/DoF	205/230	213/195	156/183	96/143

Table 3: S-Matrix parameters from 16-parameter fits to the data of the four LEP experiments, without the assumption of lepton universality.

Parameter	Average Value
$m_{\rm Z}$ (GeV)	91.1876±0.0033
$\Gamma_{\rm Z}$ (GeV)	2.4936±0.0026
$r^{\rm tot}_{\rm had}$	2.9631±0.0067
$r^{\rm tot}_{\rm e}$	0.14244±0.00051
$r^{\rm tot}_{\mu}$	0.14243±0.00039
$r^{\rm tot}_{\tau}$	0.14246±0.00046
$j^{\rm tot}_{\rm had}$	0.15±0.15
$j^{\rm tot}_{\rm e}$	-0.033±0.025
$j^{\rm tot}_{\mu}$	0.021±0.017
$j^{\rm tot}_{\tau}$	0.017±0.019
$r^{\rm fb}_{\rm e}$	0.00277±0.00048
$r^{\rm fb}_{\mu}$	0.00291±0.00026
$r^{\rm fb}_{\tau}$	0.00332±0.00029
$j^{\rm fb}_{\rm e}$	0.787±0.035
$j^{\rm fb}_{\mu}$	0.788±0.018
$j^{\rm fb}_{\tau}$	0.797±0.020

Table 4: Average S-Matrix parameters from the data of the four LEP experiments given in Table 3, without the assumption of lepton universality. The χ^2/DoF of the average is 60/48.

	$m_{\rm Z}$	$\Gamma_{\rm Z}$	$r^{\rm tot}_{\rm had}$	$r^{\rm tot}_{\rm e}$	$r^{\rm tot}_{\mu}$	$r^{\rm tot}_{\tau}$	$j^{\rm tot}_{\rm had}$	$j^{\rm tot}_{\rm e}$	$j^{\rm tot}_{\mu}$	$j^{\rm tot}_{\tau}$	$r^{\rm fb}_{\rm e}$	$r^{\rm fb}_{\mu}$	$r^{\rm fb}_{\tau}$	$j^{\rm fb}_{\rm e}$	$j^{\rm fb}_{\mu}$	$j^{\rm fb}_{\tau}$
$m_{\rm Z}$	1.00	-.13	-.09	-.06	-.06	-.06	-.77	-.27	-.34	-.28	.03	.13	.07	.01	-.03	.04
$\Gamma_{\rm Z}$	-.13	1.00	.81	.52	.51	.43	.17	.01	.08	.07	.00	.00	.02	.02	.05	.03
$r^{\rm tot}_{\rm had}$	-.09	.81	1.00	.61	.65	.55	.16	.00	.07	.06	-.01	.01	.03	.03	.07	.05
$r^{\rm tot}_{\rm e}$	-.06	.52	.61	1.00	.42	.35	.10	.02	.06	.05	.03	.00	.02	.03	.05	.03
$r^{\rm tot}_{\mu}$	-.06	.51	.65	.42	1.00	.46	.11	-.01	.12	.05	.01	.02	.02	.03	.10	.05
$r^{\rm tot}_{\tau}$	-.06	.43	.55	.35	.46	1.00	.09	.01	.05	.11	-.01	.02	.07	.03	.04	.10
$j^{\rm tot}_{\rm had}$	-.77	.17	.16	.10	.11	.09	1.00	.27	.36	.30	-.04	-.12	-.07	.00	.03	-.03
$j^{\rm tot}_{\rm e}$	-.27	.01	.00	.02	-.01	.01	.27	1.00	.12	.09	.01	-.05	-.04	.03	.01	-.04
$j^{\rm tot}_{\mu}$	-.34	.08	.07	.06	.12	.05	.36	.12	1.00	.13	-.01	-.01	-.03	.01	.02	-.02
$j^{\rm tot}_{\tau}$	-.28	.07	.06	.05	.05	.11	.30	.09	.13	1.00	-.02	-.05	.02	.00	.02	.03
$r^{\rm fb}_{\rm e}$	.03	.00	-.01	.03	.01	-.01	-.04	.01	-.01	-.02	1.00	.00	.00	.01	.01	-.01
$r^{\rm fb}_{\mu}$	.13	.00	.01	.00	.02	.02	-.12	-.05	-.01	-.05	.00	1.00	.03	.02	.11	.03
$r^{\rm fb}_{\tau}$	.07	.02	.03	.02	.02	.07	-.07	-.04	-.03	.02	.00	.03	1.00	.00	.01	.45
$j^{\rm fb}_{\rm e}$	.01	.02	.03	.03	.03	.03	.00	.03	.01	.00	.01	.02	.00	1.00	.01	.00
$j^{\rm fb}_{\mu}$	-.03	.05	.07	.05	.10	.04	.03	.01	.02	.02	.01	.11	.01	.01	1.00	.01
$j^{\rm fb}_{\tau}$	.04	.03	.05	.03	.05	.10	-.03	-.04	-.02	.03	-.01	.03	.45	.00	.01	1.00

Table 5: The correlation matrix for the set of parameters given in Table 4.

	ALEPH	DELPHI	L3	OPAL
$m_{\rm Z}$ [GeV]	91.1920±0.0061	91.1831±0.0056	91.1854±0.0056	91.1925±0.0077
$\Gamma_{\rm Z}$ [GeV]	2.4917±0.0048	2.4899±0.0042	2.4999±0.0043	2.4922±0.0054
$r^{\rm tot}_{\rm had}$	2.965±0.012	2.957±0.010	2.971±0.010	2.962±0.013
$r^{\rm tot}_{\ell}$	0.14288±0.00063	0.14216±0.00061	0.14264±0.00066	0.14186±0.00069
$j^{\rm tot}_{\rm had}$	-0.10±0.32	0.39±0.28	0.34±0.28	-0.34±0.40
$j^{\rm tot}_{\ell}$	0.002±0.023	0.040±0.024	0.031±0.025	-0.052±0.031
$r^{\rm fb}_{\ell}$	0.00344±0.00034	0.00300±0.00039	0.00327±0.00050	0.00268±0.00038
$j^{\rm fb}_{\ell}$	0.856±0.025	0.749±0.027	0.788±0.033	0.740±0.025
χ^2/DoF	213/238	223/203	162/191	103/151

Table 6: S-Matrix parameters from 8-parameter fits to the data of the four LEP experiments, assuming lepton universality.

Parameter	Average Value	SM Prediction
$m_{\rm Z}$ [GeV]	91.1534±0.0033	—
$\Gamma_{\rm Z}$ [GeV]	2.4924±0.0026	2.4932
$r^{\rm tot}_{\rm had}$	2.9623±0.0067	2.9603
$r^{\rm tot}_{\ell}$	0.14239±0.00034	0.14253
$j^{\rm tot}_{\rm had}$	0.15±0.15	0.22
$j^{\rm tot}_{\ell}$	0.009±0.012	0.004
$r^{\rm fb}_{\ell}$	0.00304±0.00018	0.00266
$j^{\rm fb}_{\ell}$	0.789±0.013	0.799

Table 7: Average S-Matrix parameters from the data of the four LEP experiments given in Table 3, assuming lepton universality. The χ^2/DoF of the average is 65/56. The Standard-Model predictions are listed for $m_{\rm Z} = 91.1863$ GeV, $m_{\rm t} = 175.6$ GeV, $m_{\rm H} = 300$ GeV, $\alpha_s(m^2_{\rm Z}) = 0.118$, and $1/\alpha(m^2_{\rm Z}) = 128.896$.

	$m_{\rm Z}$	$\Gamma_{\rm Z}$	$r^{\rm tot}_{\rm had}$	$r^{\rm tot}_{\ell}$	$j^{\rm tot}_{\rm had}$	$j^{\rm tot}_{\ell}$	$r^{\rm fb}_{\ell}$	$j^{\rm fb}_{\ell}$
$m_{\rm Z}$	1.00	-.13	-.09	-.08	-.77	-.47	.13	.01
$\Gamma_{\rm Z}$	-.13	1.00	.80	.62	.17	.09	.02	.07
$r^{\rm tot}_{\rm had}$	-.09	.80	1.00	.78	.16	.07	.02	.09
$r^{\rm tot}_{\ell}$	-.08	.62	.78	1.00	.13	.11	.05	.12
$j^{\rm tot}_{\rm had}$	-.77	.17	.16	.13	1.00	.49	-.13	.00
$j^{\rm tot}_{\ell}$	-.47	.09	.07	.11	.49	1.00	-.05	.01
$r^{\rm fb}_{\ell}$	.13	.02	.02	.05	-.13	-.05	1.00	.27
$j^{\rm fb}_{\ell}$	.01	.07	.09	.12	.00	.01	.27	1.00

Table 8: The correlation matrix for the set of parameters given in Table 7.

A Details on the S-Matrix Analysis

A.1 The S-Matrix Approach

Within the S-Matrix approach, the lowest-order total cross section, $\sigma^0_{\rm tot}$, and forward-backward asymmetry, $A^0_{\rm fb}$, for $\rm e^+e^-$ annihilation into a fermion-antifermion pair are given as [3]:

$$\sigma^0_a(s) = \frac{4}{3}\pi\alpha^2\left[\frac{g^a_{\rm f}}{s} + \frac{j^a_{\rm f}(s-\overline{m}^2_{\rm Z}) + r^a_{\rm f}\,s}{(s-\overline{m}^2_{\rm Z})^2 + \overline{m}^2_{\rm Z}\overline{\Gamma}^2_{\rm Z}}\right] \qquad \text{for a} = \text{tot, fb} \tag{7}$$

$$A^0_{\rm fb}(s) = \frac{3}{4}\frac{\sigma^0_{\rm fb}(s)}{\sigma^0_{\rm tot}(s)}\,, \tag{8}$$

where $\sqrt{s}$ is the centre-of-mass energy. The S–Matrix ansatz is defined using a Breit–Wigner denominator with s–independent width for the Z resonance, $s - \overline{m}^2_{\rm Z} + i\overline{m}_{\rm Z}\overline{\Gamma}_{\rm Z}$. The transformations of the resulting values for the mass and the total width of the Z boson to correspond to a Breit-Wigner denominator with s-dependent width, $s - m^2_{\rm Z} + is\Gamma_{\rm Z}/m_{\rm Z}$, are:

$$\begin{aligned} m_{\rm Z} &\equiv \overline{m}_{\rm Z}\sqrt{1+\overline{\Gamma}^2_{\rm Z}/\overline{m}^2_{\rm Z}} &&\approx \overline{m}_{\rm Z} + 34.1\ \text{MeV} \\ \Gamma_{\rm Z} &\equiv \overline{\Gamma}_{\rm Z}\sqrt{1+\overline{\Gamma}^2_{\rm Z}/\overline{m}^2_{\rm Z}} &&\approx \overline{\Gamma}_{\rm Z} + \ 0.9\ \text{MeV}\,, \end{aligned} \tag{9}$$

so that $\Gamma_{\rm Z}/m_{\rm Z} = \overline{\Gamma}_{\rm Z}/\overline{m}_{\rm Z}$. Note, that the transformations according to Equation 9 are applied when listing numerical results for the mass and total width of the Z boson. QED radiative corrections are included by convolution with a radiator function [14].

The S-Matrix parameters $r_{\rm f}$, $j_{\rm f}$ and $g_{\rm f}$, which are real numbers, scale the Z exchange, γZ interference and photon exchange contributions, respectively. In models with only vector couplings of the photon to the electric charge, q_f, and only vector and axial-vector couplings of the Z, the S-Matrix parameters are [3]:[1]

$$\begin{aligned} r^{\rm tot}_{\rm f} &= \kappa^2\left[g^2_{A\rm e}+g^2_{V\rm e}\right]\left[g^2_{A\rm f}+g^2_{V\rm f}\right] - 2\kappa g_{V\rm e}g_{V\rm f}C_{Im} \\ j^{\rm tot}_{\rm f} &= 2\kappa g_{V\rm e}g_{V\rm f}\left(C_{Re}+C_{Im}\right) \\ g^{\rm tot}_{\rm f} &= q^2_{\rm e}q^2_f\left|F_A(m^2_{\rm Z})\right|^2 \\ r^{\rm fb}_{\rm f} &= 4\kappa^2 g_{A\rm e}g_{V\rm e}g_{A\rm f}g_{V\rm f} - 2\kappa g_{A\rm e}g_{A\rm f}C_{Im} \\ j^{\rm fb}_{\rm f} &= 2\kappa g_{A\rm e}g_{A\rm f}\left(C_{Re}+C_{Im}\right) \\ g^{\rm fb}_{\rm f} &= 0\,, \end{aligned} \tag{10}$$

where:

$$\kappa = \frac{\rm G_F m^2_Z}{2\sqrt{2}\pi\alpha} \approx 1.50 \tag{11}$$

$$C_{Im} = \frac{\Gamma_{\rm Z}}{m_{\rm Z}}\,q_{\rm e}q_f\ {\rm Im}\left\{\rm F_A(m^2_Z)\right\} \tag{12}$$

$$C_{Re} = q_{\rm e}q_f\ {\rm Re}\left\{\rm F_A(m^2_Z)\right\} \tag{13}$$

$$F_A(m^2_{\rm Z}) = \frac{\alpha(m^2_{\rm Z})}{\alpha}\,. \tag{14}$$

[1] The relations for r and j listed here neglect the imaginary parts of the effective vector and axial-vector coupling constants of the Z boson to fermions.

In the last equation $\alpha(m_Z^2)$ denotes the complex running fine structure constant, and $\alpha \equiv \alpha(0)$. For the hadronic final state, the S-Matrix parameters $r_{\rm had}$, $j_{\rm had}$ and $g_{\rm had}$ are summed over all colours and open quark flavours.

A.2 Correlated Errors due to the LEP Energy Calibration

The following procedure is used to determine the correlated errors on the S-Matrix parameters arising from the LEP energy calibration. The S-Matrix fit to the experimental measurements of cross sections and forward-backward asymmetries of a single LEP experiment determines the error matrix $\mathcal{E}$ of the set of S-Matrix parameters fitted. A second S-Matrix fit is performed, where the errors of all measurements except those arising from the LEP energy calibration are scaled by a factor of 0.5. This factor simulates the weight of the four LEP experiments in the average. Denoting the error matrix of the second fit with $\mathcal{E}'$, the error matrix $\mathcal{C}$ describing the correlated errors on the S-Matrix parameters arising from the LEP energy calibration is then obtained as:

$$\mathcal{C} = \frac{4}{3}\mathcal{E}' - \frac{1}{3}\mathcal{E}\,. \tag{15}$$

Table 9 lists numerical values for the correlated errors on the S-Matrix parameters and for the corresponding correlation coefficients.

	Error	m_Z	Γ_Z	$r_{\rm had}^{\rm tot}$	$r_e^{\rm tot}$	$r_\mu^{\rm tot}$	$r_\tau^{\rm tot}$	$j_{\rm had}^{\rm tot}$	$j_e^{\rm tot}$	$j_\mu^{\rm tot}$	$j_\tau^{\rm tot}$	$r_e^{\rm fb}$	$r_\mu^{\rm fb}$	$r_\tau^{\rm fb}$	$j_e^{\rm fb}$	$j_\mu^{\rm fb}$	$j_\tau^{\rm fb}$
m_Z	1.7 MeV	1.0	.0	.1	.2	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
Γ_Z	1.5 MeV	.0	1.0	1.0	.9	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
$r_{\rm had}^{\rm tot}$	0.0032	.1	1.0	1.0	.8	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
$r_e^{\rm tot}$	0.00019	.2	.9	.8	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
$r_\mu^{\rm tot}$	0.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
$r_\tau^{\rm tot}$	0.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
$j_{\rm had}^{\rm tot}$	0.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
$j_e^{\rm tot}$	0.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0
$j_\mu^{\rm tot}$	0.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0
$j_\tau^{\rm tot}$	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0
$r_e^{\rm fb}$	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0
$r_\mu^{\rm fb}$	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0
$r_\tau^{\rm fb}$	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0
$j_e^{\rm fb}$	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0
$j_\mu^{\rm fb}$	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0
$j_\tau^{\rm fb}$	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0

Table 9: The correlated errors on the S-Matrix parameters and the corresponding correlation coefficients arising from the LEP energy calibration which is common to all four LEP experiments.

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