EUROPEAN ORGANISATION FOR PARTICLE PHYSICS

LEPEWWG/LS/97-02
ALEPH 97-92 PHYSIC 97-82
DELPHI 97-153 PHYS 732
L3 Note 2164
OPAL Technical Note TN510
22 September 1997

An Investigation of the Interference between Photon and Z-Boson Exchange

The S-Matrix Subgroup of the LEP Electroweak Working Group[1]

Internal Note

Prepared from Contributions of the LEP Experiments to the 1997 Summer Conferences.

Abstract

This note presents an investigation of the γZ interference contribution to total cross sections and leptonic forward-backward asymmetries. It is based on a combination of published and preliminary electroweak results from the four LEP collaborations which were prepared for the 1997 summer conferences.

[1]The present members of the subgroup are: R. Clare, P. Clarke, D. Gele, M.W. Grünewald, M. Kobel, M.N. Minard, A. Olshevsky, Ch. Paus, M. Pepe-Altarelli, F. Teubert, P. Ward

1 Introduction

In Section 2 of the note on LEP electroweak averages prepared for the 1997 summer conferences [1], an analysis of the Z lineshape and leptonic forward-backward asymmetries is presented. The formulae used in the fits described in that section are valid under quite general assumptions, namely that the scattering process arises from the s-channel exchange of two bosons, a massless photon with vector couplings to the electric charge or a massive Z boson with both vector and axial-vector couplings to fermions. For leptons these two couplings, together with $\alpha(m_Z^2)$, fully determine both the Z exchange and the γZ interference terms. In contrast, a model dependence is introduced by the treatment of the γZ interference contribution to the hadronic cross section, because quark couplings to the Z are not individually measured for each flavour. This term is therefore taken from the Standard Model.

A more general approach is to parametrise the γZ interference term independently of the Z exchange amplitude. Several model independent formalisms exist [2, 3]. For the analysis performed here the framework described in [3] based on the S-Matrix ansatz is used. Formulae for $e^+e^- \to f\bar{f}$ cross sections and forward-backward asymmetries are summarised in Appendix A.1.

Measurements at centre-of-mass energies far away from the Z pole are especially sensitive to the parameters describing the interference between photon and Z-boson exchange [4]. This analysis includes the results from the 1995 LEP run at centre-of-mass energies, $\sqrt{s}$, from 130 GeV to 172 GeV [5–9]. The total statistics of these high-energy measurements of the four LEP collaborations are given in Tables 1 and 2.

For a large fraction of these high-energy events, initial-state radiation photons lower the effective centre-of-mass energy, $\sqrt{s'}$, of the annihilation process to values close to the Z mass, m_Z ('radiative return to the Z'). These radiative events dilute the sensitivity of the cross section and forward-backward asymmetry measurements to the γZ interference parameters. By applying a cut on the effective centre-of-mass energy, $\sqrt{s'} \gg m_Z$, the high-energy non-radiative events are isolated and used to constrain these parameters.

2 General Framework and Results

The S-Matrix parameters r_f, j_f and g_f are real numbers which express the size of the Z exchange, γZ interference and photon exchange contributions. In the approach presented here, r_f and j_f are treated as free parameters while the photon exchange contribution, g_f, is fixed to its QED prediction. Each final state is thus described by four free parameters: two for cross sections, r_f^{tot} and j_f^{tot}, and two for forward-backward asymmetries, r_f^{fb} and j_f^{fb}. In models with only vector and axial-vector couplings of the Z-boson, these four S-Matrix parameters are not independent of each other and are approximatively [3]:

$$r_f^{tot} \propto \left[g_{Ae}^2 + g_{Ve}^2\right] \cdot \left[g_{Af}^2 + g_{Vf}^2\right] , \quad (1)$$

$$j_f^{tot} \propto g_{Ve} g_{Vf} , \quad (2)$$

$$r_f^{fb} \propto g_{Ae} g_{Ve} \cdot g_{Af} g_{Vf} , \quad (3)$$

$$j_f^{fb} \propto g_{Ae} g_{Af} . \quad (4)$$

The exact expressions are listed in Appendix A.1.

The four sets of 16 parameters provided by the four LEP experiments are presented in Table 3. For the averaging procedure the parameter mean values, errors and correlations of the single experiments and in addition the correlations common to the different LEP experiments are used.

Sources of common systematic errors are the theoretical uncertainty on the luminosity determination and the LEP beam energy calibration. These common errors are taken care of in the averaging procedure. A common theoretical error on the luminosity determination of 0.11% is taken into account by introducing correlated uncertainties of the same size between all S-Matrix parameters r and j of the different experiments. This uncertainty is valid for the data taken in the vicinity of the Z pole. The uncertainty rises to 0.25% for the high energy measurements but is dominated by the much larger statistical errors. The common systematic errors and the correlations amongst the S-Matrix parameters arising from the LEP beam energy calibration are summarised in Appendix A.2. The combined parameter set and its correlation matrix are given in Tables 4 and 5.

The data of each of the four LEP experiments are consistent with lepton universality. If lepton universality is assumed, the set of 16 S-Matrix parameters is reduced to a set of 8 parameters. In Table 6 the 8 parameters of the individual LEP experiments are shown. In Tables 7 and 8 the LEP averages and their correlation matrix are provided. The Standard-Model predictions agree well with the LEP averages of the S-Matrix parameters.

3 The Hadronic γZ Interference Term

A large correlation (-75%) arises between the Z-boson mass, m_{Z}, and the hadronic γZ interference term, $j_{\mathrm{had}}^{\mathrm{tot}}$. This correlation leads to a sizeable increase in the error on the Z-boson mass in fits where $j_{\mathrm{had}}^{\mathrm{tot}}$ is left free. In Figure 1 the 68% probability contour in the m_{Z}–$j_{\mathrm{had}}^{\mathrm{tot}}$ plane is depicted for the complete data set and for the LEP1 data only as shown at the EPS conference in Brussels 1995 [10]. The error on the hadronic γZ interference term, and consequently the error on m_{Z}, are reduced by using total cross section measurements at centre-of-mass energies far away from the Z pole [3,4].

With respect to the last internal note [11] the error on the hadronic γZ interference term has slightly improved due to the inclusion of measurements between 161 GeV and 172 GeV by the OPAL collaboration [8]. For the Z-boson mass and the hadronic γZ interference term the following LEP average values are obtained:

$$
\begin{aligned}
m_{\mathrm{Z}} &= 91.1882 \pm 0.0031 \text{ GeV} && (5) \\
j_{\mathrm{had}}^{\mathrm{tot}} &= 0.14 \pm 0.14\,. && (6)
\end{aligned}
$$

The result of the Z-boson mass is consistent with the fit results presented in [1] ($m_{\mathrm{Z}} = 91.1867 \pm 0.0020$ GeV where the hadronic γZ interference term is fixed to its Standard Model prediction. The value of $j_{\mathrm{had}}^{\mathrm{tot}}$ is in good agreement with the Standard Model prediction of 0.22.

By now the LEP2 data constrain $j_{\mathrm{had}}^{\mathrm{tot}}$ well so that the inclusion of data taken well below the Z peak only leads to a small improvement in the errors. The TOPAZ collaboration at TRISTAN (KEK) has performed a measurement of the hadronic total cross section at $\sqrt{s} = 57.77$ GeV, $\sigma_{\mathrm{tot}}^{0} = 143.6 \pm 1.5(stat.) \pm 4.5(syst.)$ pb [12]. Combining this measurement with the LEP results gives:

$$
\begin{aligned}
m_{\mathrm{Z}} &= 91.1882 \pm 0.0029 \text{ GeV} && (7) \\
j_{\mathrm{had}}^{\mathrm{tot}} &= 0.14 \pm 0.12\,, && (8)
\end{aligned}
$$

and the correlation between m_{Z} and $j_{\mathrm{had}}^{\mathrm{tot}}$ is reduced to -71%.

4 Leptonic Forward-Backward Asymmetries

In Figure 2 the forward-backward asymmetries for electrons, muons and taus as a function of the centre-of-mass energy are depicted for the four LEP experiments. The measurements are extrapolated to full phase-space. In the case of electrons, t-channel and s/t-interference contributions are removed. The uncertainty of the leptonic forward-backward asymmetry measurements and also their averages are still dominated by statistics. Considering each energy point separately, the measurements for each lepton species are in agreement with each other, and compatible with the prediction of the Standard Model. The values for $r_\ell^{\rm fb}$ and $j_\ell^{\rm fb}$ are also in agreement within each experiment, see Table 3. In the following inconsistencies in these two parameters reported in the last internal note [11] are discussed in more detail.

The $r_\ell^{\rm fb}$ parameter disagreed in [11] with the Standard Model prediction by more than 2 standard deviations. This was caused by the tau pole forward-backward asymmetry which was consistently high for all experiments. Now, the disagreement with the Standard Model for this parameter is less than 1.4 standard deviations as shown in Table 7. This is due to the change of the ALEPH measurement from 0.00373 to 0.00288. This change has been observed after the reanalysis of the tau pair data before 1993 with the latest version of the analysis program. It is understood in terms of Bhabha contamination, which was originally estimated from Monte Carlo, but is now measured with data. From Table 4 it is concluded that the tau pole asymmetry still contributes most to this difference. The effect is also observed in the less general $g_{A\ell} - g_{V\ell}$ ansatz [1]. There, the forward-backward pole asymmetry of the tau channel is measured to be larger than the Standard-Model prediction.

The dependence of the leptonic forward-backward asymmetry on the centre-of-mass energy, expressed in the $j_\ell^{\rm fb}$ parameter, was the second problematic measurement [11]. Assuming lepton universality, the results on the slope parameter $j_\ell^{\rm fb}$ of the four LEP experiments are listed in Table 6. The weighted average of the four values for $j_\ell^{\rm fb}$ agrees within 1.5 standard deviations with the prediction of the Standard Model. However, the average has a χ^2/DoF of 9.9/3, corresponding to a probability of 1.9%. This probability has improved with respect to [11] where it was much below 1%. This improvement is due to the fact that the ALEPH and DELPHI measurements of $j_\ell^{\rm fb}$ have moved closer to the Standard Model prediction. The reason for the nevertheless relatively high χ^2 value is illustrated in Figure 3, where the average leptonic forward-backward asymmetries are compared to the prediction of the Standard Model. ALEPH measures a slope about 1.6 standard deviations larger than the prediction, whereas OPAL measures a slope about 2.6 standard deviations smaller than the prediction.

All experiments perform detailed checks on possible systematic effects in the measurements of the leptonic forward-backward asymmetries. No systematic bias is found so far, but further investigations are in progress. For the averaging of these preliminary data, this unlikely value for the χ^2 is assumed to be caused by a statistical fluctuation, and the leptonic forward-backward asymmetries are included in the analyses presented in [1].

5 Combined LEP Cross Sections

To allow searches for new phenomena using the LEP2 cross section measurements from all experiments, cross sections of hadronic final states and tau and muon–pair events from the four LEP experiments are combined. Combined cross section measurements are derived for events with high effective centre-of-mass energy. They are calculated for a unified cut value on the effective centre-of-mass energy of

$\sqrt{s'/s} > 0.85$, extrapolated to the full phase space and at the mean centre-of-mass energy of the four experiments. The measurements of each experiment are corrected to correspond to these conditions and then averaged.

Statistical and systematic errors of each single experiment, as well as uncertainties common to the LEP experiments due to the LEP energy calibration and the theoretical luminosity uncertainty, have been taken into account. The results of the average cross sections together with the correlation matrix are given in Tables 9 and 10.

Very recent cross section measurements from the 1997 data taking up to the 1997 summer conferences are also combined [13], however these cross section measurements are not included in the fits. The same procedure as described above has been applied. The results of the averaged preliminary cross sections as given in Table 11 are basically uncorrelated. All correlations with previous data are neglected. The errors on the tau-pair cross section are significantly larger than the ones for the muon-pair cross section since only L3 and OPAL submitted preliminary results for tau–pair production.

The interpretation of the cross section measurements have to be done with some care since there are theoretical uncertainties on the prediction. The effective centre-of-mass energy is not well defined when interference between initial state radiation, ISR, and final state radiation, FSR, occurs. Additional theoretical uncertainties are present for final state radiation in quark-pair production. All these effects are still small for a single LEP experiment but after combining the cross sections the uncertainty becomes relevant.

The OPAL collaboration has calculated these effects [8] and finds that their measurements should best be compared to theory predictions excluding ISR/FSR interference[1], applying corrections of +1.2% and about –1.6% to the measured cross-sections of hadrons and leptons, respectively. These estimates still suffer from sizeable errors which are due to missing calculations in theory and Monte Carlo programs. Since they depend on angular acceptance cuts and on the s' cut, the corrections are expected to be slightly different for the other LEP experiments. These corrections are therefore not applied to the combined cross-sections. Additional correlated errors of about 1.5%, anticorrelated between leptons and hadrons should therefore be added to these uncorrected averages.

6 Summary

The S-Matrix framework allows a coherent treatment of data taken at the Z resonance and data taken in energy regions where the γZ interference is large. The results on the S-Matrix parameters provide a test of the assumed vector and axial-vector structure of the Z couplings.

The results obtained in this framework are consistent with the results obtained in the less general approach using vector and axial-vector couplings to parametrise the Z-resonance. All parameters agree well with the Standard Model predictions. Also the combined high energy cross sections show good agreement with the Standard Model.

[1] ZFITTER flag INTF=0.

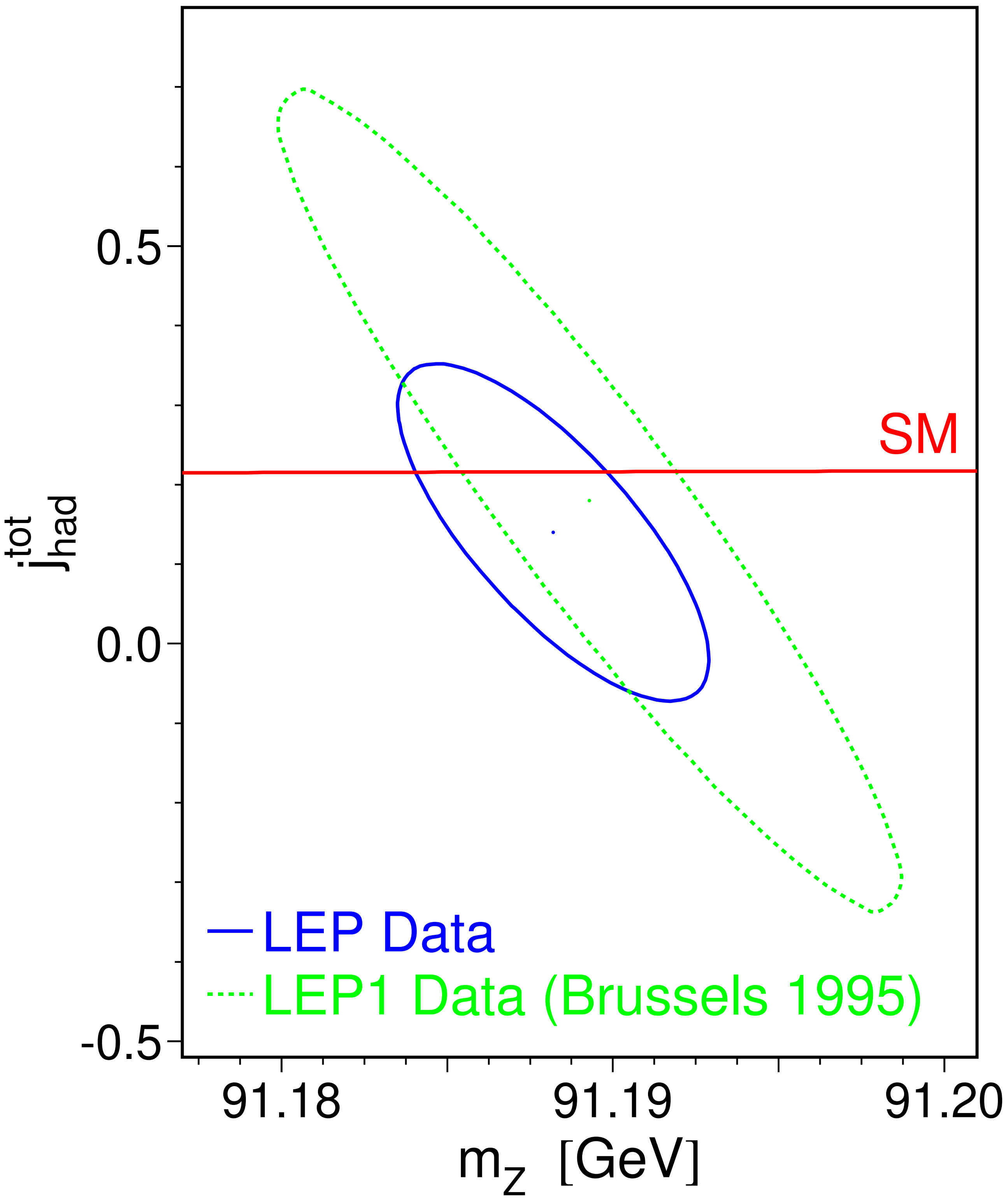

Figure 1: Contours of 68% confidence level between the mass of the Z boson, m_Z, and the hadronic γZ interference term for total cross sections, $j_{\rm had}^{\rm tot}$. Lepton universality is assumed. The solid contour corresponds to all LEP data while the dashed contour is obtained using only LEP1 results as presented at the EPS in Brussels 1995 [10]. The Standard-Model prediction for $j_{\rm had}^{\rm tot}$ is shown as the solid band.

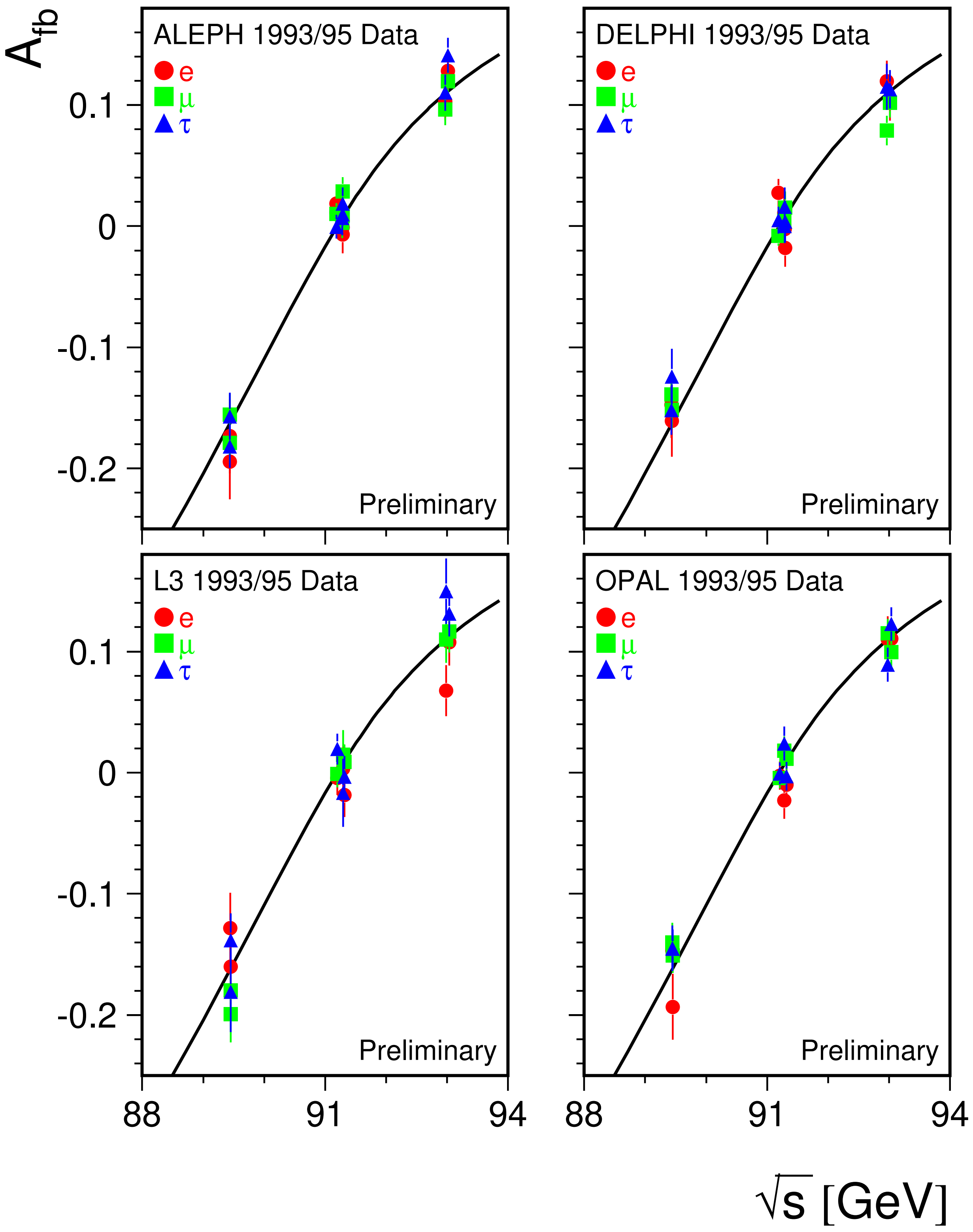

Figure 2: Leptonic forward-backward asymmetries as a function of the centre-of-mass energy $\sqrt{s}$ (Preliminary results from the analysis of the 1993 and 1995 energy-scan data). At each energy, the results are in good agreement between electrons, muons and taus. They are also compatible with the prediction of the Standard Model for $m_{\mathrm{Z}} = 91.1863$ GeV, $m_{\mathrm{t}} = 175$ GeV, $m_{\mathrm{H}} = 300$ GeV, $\alpha_s(m_{\mathrm{Z}}^2) = 0.123$, and $1/\alpha(m_{\mathrm{Z}}^2) = 128.896$, shown as the solid line.

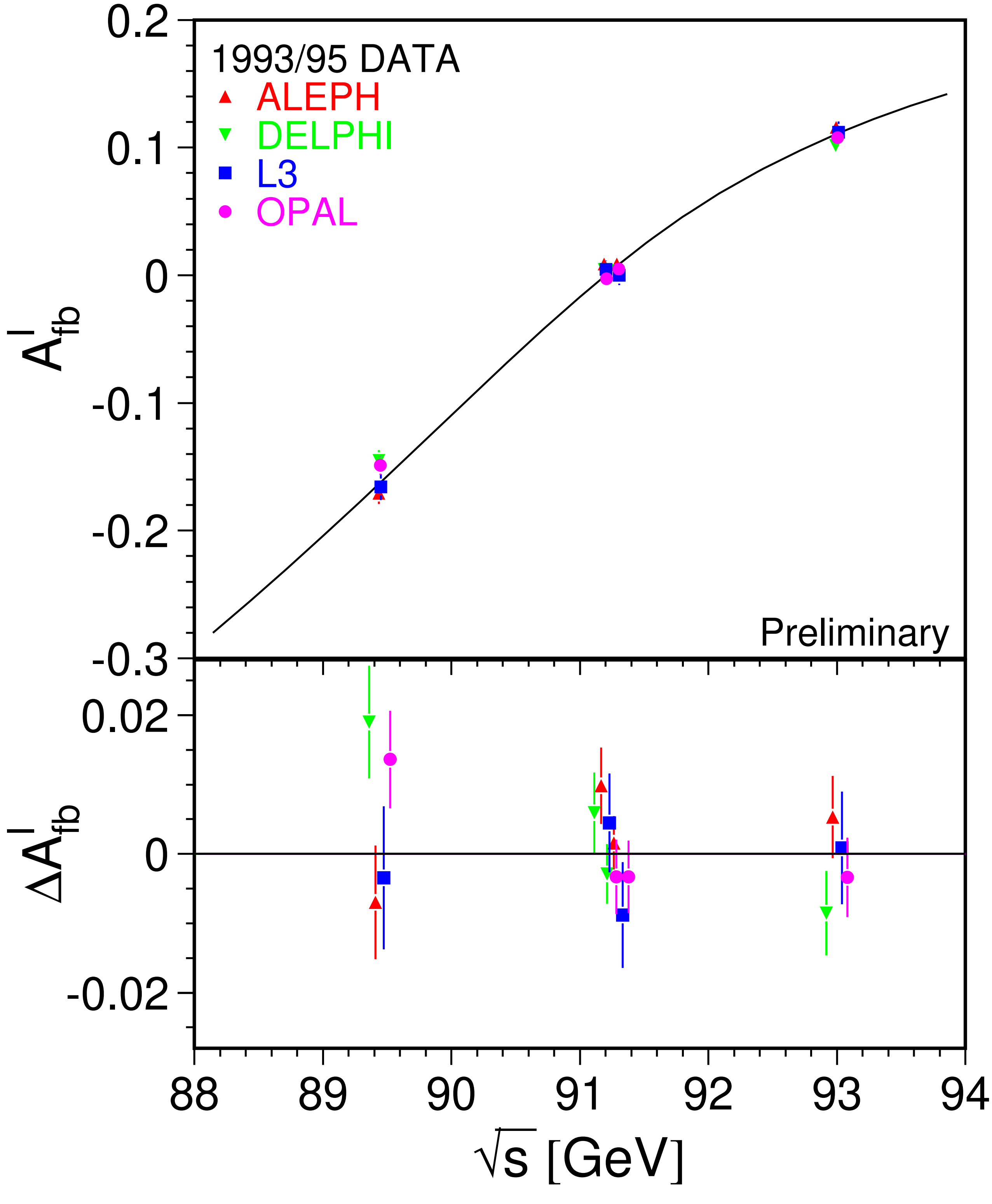

Figure 3: Leptonic forward-backward asymmetries as a function of the centre-of-mass energy $\sqrt{s}$, assuming lepton universality (Preliminary results from the analysis of the 1993 and 1995 energy-scan data). The line shows the prediction of the Standard Model for $m_Z = 91.1863$ GeV, $m_t = 175$ GeV, $m_H = 300$ GeV, $\alpha_s(m_Z^2) = 0.123$, and $1/\alpha(m_Z^2) = 128.896$. The bottom plot shows the difference between the measurements and the Standard Model prediction on an enlarged vertical scale. For better visibility the measurements are spread horizontally in the bottom plot.

		ALEPH	DELPHI	L3	OPAL	LEP
hadrons	total	1644	1583	1577	1505	6309
	s' cut	369	454	469	340	1632
taus & muons	total	177	158	128	167	630
	s' cut	77	73	60	80	290

Table 1: The number of hadronic and tau plus muon events collected by the LEP experiments at centre-of-mass energies between 130 and 140 GeV. The s' cuts of the different experiments are: ALEPH: $\sqrt{s'/s} > 0.9$, DELPHI: $\sqrt{s'/s} > 0.85$, L3: $\sqrt{s'}$ >110 GeV, OPAL: $s'/s > 0.8$. For the selection of the total data sample the experiments also apply mild s' cuts.

		ALEPH	DELPHI	L3	OPAL	LEP
hadrons	total	2795	2814	2728	2840	11177
	s' cut	727	770	710	709	2916
taus & muons	total	342	254	251	283	1130
	s' cut	177	117	121	145	560

Table 2: The number of hadronic and tau plus muon events collected by the LEP experiments at centre-of-mass energies between 161 and 172 GeV. The s' cuts of the different experiments are: ALEPH: $\sqrt{s'/s} > 0.9$, DELPHI: $\sqrt{s'/s} > 0.85$, L3: $\sqrt{s'/s} > 0.85$, OPAL: $s'/s > 0.8$. For the selection of the total data sample the experiments also apply mild s' cuts.

	ALEPH	DELPHI	L3[a]	OPAL[a]
m_Z (GeV)	91.1951±0.0056	91.1841±0.0056	91.1867±0.0056	91.1874±0.0055
Γ_Z (GeV)	2.4939±0.0044	2.4897±0.0041	2.5003±0.0043	2.4946±0.0044
$r_{\mathrm{had}}^{\mathrm{tot}}$	2.966±0.010	2.957±0.010	2.972±0.010	2.962±0.010
$r_{\mathrm{e}}^{\mathrm{tot}}$	0.14361±0.00076	0.1412±0.0010	0.14171±0.00088	0.1418±0.0011
r_{μ}^{tot}	0.14248±0.00062	0.14274±0.00069	0.14257±0.00083	0.14228±0.00066
r_{τ}^{tot}	0.14313±0.00067	0.14140±0.00097	0.1433±0.0011	0.14118±0.00088
$j_{\mathrm{had}}^{\mathrm{tot}}$	-0.18±0.27	0.36±0.28	0.30±0.28	0.03±0.27
$j_{\mathrm{e}}^{\mathrm{tot}}$	-0.007±0.041	-0.037±0.045	-0.011±0.045	-0.123±0.060
j_{μ}^{tot}	-0.018±0.030	0.052±0.030	0.028±0.036	-0.012±0.037
j_{τ}^{tot}	-0.012±0.032	0.017±0.037	0.042±0.039	-0.003±0.042
$r_{\mathrm{e}}^{\mathrm{fb}}$	0.00303±0.00072	0.00326±0.00096	0.0025±0.0013	0.0016±0.0010
r_{μ}^{fb}	0.00288±0.00048	0.00267±0.00053	0.00323±0.00067	0.00262±0.00050
r_{τ}^{fb}	0.00288±0.00055	0.00376±0.00075	0.00419±0.00094	0.00318±0.00066
$j_{\mathrm{e}}^{\mathrm{fb}}$	0.861±0.058	0.813±0.073	0.644±0.080	0.778±0.068
j_{μ}^{fb}	0.826±0.036	0.759±0.034	0.838±0.046	0.724±0.036
j_{τ}^{fb}	0.846±0.041	0.745±0.047	0.788±0.057	0.727±0.042
χ^2/DoF	180/189	233/195	156/183	109/155

Table 3: S-Matrix parameters from 16-parameter fits to the data of the four LEP experiments, without the assumption of lepton universality.

[a]For the averaging procedure the L3 values of m_Z and Γ_Z are shifted by +0.3 MeV and the OPAL value of m_Z by +0.5 MeV to account for the new energy calibration.

Parameter	Average Value
m_{Z} (GeV)	91.1884±0.0031
Γ_{Z} (GeV)	2.4945±0.0025
$r_{\mathrm{had}}^{\mathrm{tot}}$	2.9637±0.0063
$r_{\mathrm{e}}^{\mathrm{tot}}$	0.14229±0.00049
r_{μ}^{tot}	0.14253±0.00036
r_{τ}^{tot}	0.14247±0.00043
$j_{\mathrm{had}}^{\mathrm{tot}}$	0.13±0.14
$j_{\mathrm{e}}^{\mathrm{tot}}$	-0.028±0.023
j_{μ}^{tot}	0.013±0.016
j_{τ}^{tot}	0.010±0.018
$r_{\mathrm{e}}^{\mathrm{fb}}$	0.00270±0.00046
r_{μ}^{fb}	0.00279±0.00027
r_{τ}^{fb}	0.00329±0.00034
$j_{\mathrm{e}}^{\mathrm{fb}}$	0.786±0.034
j_{μ}^{fb}	0.780±0.019
j_{τ}^{fb}	0.778±0.023

Table 4: Average S-Matrix parameters from the data of the four LEP experiments given in Table 3, without the assumption of lepton universality. The χ^2/DoF of the average is 54/48.

	m_{Z}	Γ_{Z}	$r_{\mathrm{had}}^{\mathrm{tot}}$	$r_{\mathrm{e}}^{\mathrm{tot}}$	r_{μ}^{tot}	r_{τ}^{tot}	$j_{\mathrm{had}}^{\mathrm{tot}}$	$j_{\mathrm{e}}^{\mathrm{tot}}$	j_{μ}^{tot}	j_{τ}^{tot}	$r_{\mathrm{e}}^{\mathrm{fb}}$	r_{μ}^{fb}	r_{τ}^{fb}	$j_{\mathrm{e}}^{\mathrm{fb}}$	j_{μ}^{fb}	j_{τ}^{fb}
m_{Z}	1.00	-.10	-.07	-.03	-.06	-.05	-.75	-.25	-.30	-.28	.04	.11	.09	.01	-.02	-.01
Γ_{Z}	-.10	1.00	.80	.51	.51	.43	.16	.03	.07	.06	.00	.00	.00	.02	.05	.04
$r_{\mathrm{had}}^{\mathrm{tot}}$	-.07	.80	1.00	.59	.66	.55	.13	.01	.05	.05	.01	.01	.01	.03	.06	.05
$r_{\mathrm{e}}^{\mathrm{tot}}$	-.03	.51	.59	1.00	.42	.35	.09	.04	.04	.04	.03	.00	.00	.05	.04	.03
r_{μ}^{tot}	-.06	.51	.66	.42	1.00	.46	.10	.01	.14	.04	.01	.03	.01	.03	.10	.04
r_{τ}^{tot}	-.05	.43	.55	.35	.46	1.00	.09	.01	.04	.13	.01	.01	.03	.02	.04	.09
$j_{\mathrm{had}}^{\mathrm{tot}}$	-.75	.16	.13	.09	.10	.09	1.00	.27	.32	.30	-.04	-.11	-.09	.00	.03	.01
$j_{\mathrm{e}}^{\mathrm{tot}}$	-.25	.03	.01	.04	.01	.01	.27	1.00	.11	.10	.01	-.04	-.03	.02	.01	.00
j_{μ}^{tot}	-.30	.07	.05	.04	.14	.04	.32	.11	1.00	.12	-.01	.00	-.03	.00	.02	.00
j_{τ}^{tot}	-.28	.06	.05	.04	.04	.13	.30	.10	.12	1.00	-.01	-.04	.00	.00	.01	.01
$r_{\mathrm{e}}^{\mathrm{fb}}$	.04	.00	.01	.03	.01	.01	-.04	.01	-.01	-.01	1.00	.01	.01	.09	.00	.00
r_{μ}^{fb}	.11	.00	.01	.00	.03	.01	-.11	-.04	.00	-.04	.01	1.00	.03	.00	.17	.00
r_{τ}^{fb}	.09	.00	.01	.00	.01	.03	-.09	-.03	-.03	.00	.01	.03	1.00	.00	.00	.16
$j_{\mathrm{e}}^{\mathrm{fb}}$	.01	.02	.03	.05	.03	.02	.00	.02	.00	.00	.09	.00	.00	1.00	.00	.00
j_{μ}^{fb}	-.02	.05	.06	.04	.10	.04	.03	.01	.02	.01	.00	.17	.00	.00	1.00	.00
j_{τ}^{fb}	-.01	.04	.05	.03	.04	.09	.01	.00	.00	.01	.00	.00	.16	.00	.00	1.00

Table 5: The correlation matrix for the set of parameters given in Table 4.

	ALEPH	DELPHI	L3[a]	OPAL[a]
m_Z [GeV]	91.1951±0.0056	91.1837±0.0056	91.1854±0.0056	91.1861±0.0054
Γ_Z [GeV]	2.4939±0.0044	2.4896±0.0041	2.4999±0.0043	2.4945±0.0044
$r_{\rm had}^{\rm tot}$	2.966±0.010	2.956±0.010	2.971±0.010	2.962±0.010
$r_\ell^{\rm tot}$	0.14293±0.00055	0.14211±0.00061	0.14264±0.00066	0.14188±0.00060
$j_{\rm had}^{\rm tot}$	-0.18±0.27	0.38±0.28	0.34±0.28	0.08±0.27
$j_\ell^{\rm tot}$	-0.012±0.022	0.024±0.023	0.031±0.025	-0.013±0.027
$r_\ell^{\rm fb}$	0.00292±0.00033	0.00306±0.00040	0.00327±0.00050	0.00264±0.00037
$j_\ell^{\rm fb}$	0.840±0.025	0.761±0.026	0.788±0.033	0.733±0.025
χ^2/DoF	183/197	241/203	164/191	116/163

Table 6: S-Matrix parameters from 8-parameter fits to the data of the four LEP experiments, assuming lepton universality.
[a]For the averaging procedure the L3 values of m_Z and Γ_Z are shifted by +0.3 MeV and the OPAL value of m_Z by +0.5 MeV to account for the new energy calibration.

Parameter	Average Value	SM Prediction
m_Z [GeV]	91.1882±0.0031	—
Γ_Z [GeV]	2.4945±0.0024	2.4932
$r_{\rm had}^{\rm tot}$	2.9637±0.0062	2.9603
$r_\ell^{\rm tot}$	0.14245±0.00032	0.14253
$j_{\rm had}^{\rm tot}$	0.14±0.14	0.22
$j_\ell^{\rm tot}$	0.004±0.012	0.004
$r_\ell^{\rm fb}$	0.00292±0.00019	0.00266
$j_\ell^{\rm fb}$	0.780±0.013	0.799

Table 7: Average S-Matrix parameters from the data of the four LEP experiments given in Table 3, assuming lepton universality. The χ^2/DoF of the average is 59/56. The Standard-Model predictions are listed for $m_Z = 91.1882$ GeV, $m_t = 175.6$ GeV, $m_H = 300$ GeV, $\alpha_s(m_Z^2) = 0.118$, and $1/\alpha(m_Z^2) = 128.896$.

	m_Z	Γ_Z	$r_{\rm had}^{\rm tot}$	$r_\ell^{\rm tot}$	$j_{\rm had}^{\rm tot}$	$j_\ell^{\rm tot}$	$r_\ell^{\rm fb}$	$j_\ell^{\rm fb}$
m_Z	1.00	-.13	-.09	-.08	-.75	-.43	.14	-.02
Γ_Z	-.13	1.00	.80	.61	.16	.09	.00	.07
$r_{\rm had}^{\rm tot}$	-.09	.80	1.00	.77	.13	.06	.02	.09
$r_\ell^{\rm tot}$	-.08	.61	.77	1.00	.12	.12	.03	.12
$j_{\rm had}^{\rm tot}$	-.75	.16	.13	.12	1.00	.47	-.14	.03
$j_\ell^{\rm tot}$	-.43	.09	.06	.12	.47	1.00	-.05	.02
$r_\ell^{\rm fb}$	.14	.00	.02	.03	-.14	-.05	1.00	.15
$j_\ell^{\rm fb}$	-.02	.07	.09	.12	.03	.02	.15	1.00

Table 8: The correlation matrix for the set of parameters given in Table 7.

	$\sqrt{s}$ [GeV]	σ [pb]	$\sigma^{\rm SM}$ [pb]
σ_μ	130.26	9.4±1.0	8.4
σ_μ	136.24	8.7±1.0	7.3
σ_μ	161.32	5.05±0.41	4.62
σ_μ	172.10	3.92±0.35	3.96
σ_τ	130.26	10.4±1.3	8.4
σ_τ	136.24	7.1±1.1	7.3
σ_τ	161.32	5.82±0.55	4.62
σ_τ	172.10	3.87±0.44	3.96
$\sigma_{\rm h}$	130.26	77.6±3.1	82.6
$\sigma_{\rm h}$	136.24	65.1±2.8	66.5
$\sigma_{\rm h}$	161.32	36.5±1.1	35.3
$\sigma_{\rm h}$	172.10	29.8±1.0	28.9

Table 9: LEP combined cross sections for high effective s' selection with $\sqrt{s'/s} > 0.85$ at centre-of-mass energies well above the Z resonance. The quoted errors include statistical and systematic errors. The correlations between the measurements are given in Table 10. The cross sections are compared to Standard Model calculations excluding ISR/FSR interference. An additional fully correlated 1.5% systematic uncertainty to hadron and lepton-pair cross section have to be taken into account. The uncertainties are anticorrelated between lepton-pair and hadron cross sections.

	$\sqrt{s}$	σ_μ	σ_μ	σ_μ	σ_μ	σ_τ	σ_τ	σ_τ	σ_τ	$\sigma_{\rm h}$	$\sigma_{\rm h}$	$\sigma_{\rm h}$	$\sigma_{\rm h}$
$\sqrt{s}$	[GeV]	130	136	161	172	130	136	161	172	130	136	161	172
σ_μ	130	1.00	.04	.04	.04	.00	.00	.00	.00	.00	.00	.00	.00
σ_μ	136	.04	1.00	.04	.04	.00	.00	.00	.00	.00	.00	.00	.00
σ_μ	161	.04	.04	1.00	.06	.00	.00	.00	.00	.00	.00	.01	.01
σ_μ	172	.04	.04	.06	1.00	.00	.00	.00	.00	.00	.00	.01	.01
σ_τ	130	.00	.00	.00	.00	1.00	.03	.03	.03	.00	.00	.00	.00
σ_τ	136	.00	.00	.00	.00	.03	1.00	.03	.03	.00	.00	.00	.00
σ_τ	161	.00	.00	.00	.00	.03	.03	1.00	.04	.00	.00	.01	.01
σ_τ	172	.00	.00	.00	.00	.03	.03	.04	1.00	.00	.00	.01	.00
$\sigma_{\rm h}$	130	.00	.00	.00	.00	.00	.00	.00	.00	1.00	.15	.11	.10
$\sigma_{\rm h}$	136	.00	.00	.00	.00	.00	.00	.00	.00	.15	1.00	.10	.09
$\sigma_{\rm h}$	161	.00	.00	.01	.01	.00	.00	.01	.01	.11	.10	1.00	.13
$\sigma_{\rm h}$	172	.00	.00	.01	.01	.00	.00	.01	.00	.10	.09	.13	1.00

Table 10: The correlation matrix for the set of LEP combined cross sections given in Table 9.

	$\sqrt{s}$ [GeV]	σ [pb]	$\sigma^{\rm SM}$ [pb]
σ_μ	182.7	3.32±0.42	3.45
σ_τ	182.7	4.25±0.85	3.45
$\sigma_{\rm h}$	182.7	23.9±1.2	24.2

Table 11: LEP combined cross sections for high effective s' selection with $\sqrt{s'/s} > 0.85$ from the actual 1997 data taking period. The correlations between these measurements are negligible. For further explication see also the caption of Table 9.

A Details on the S-Matrix Analysis

A.1 The S-Matrix Approach

Within the S-Matrix approach, the lowest-order total cross section, σ^0_{tot}, and forward-backward asymmetry, A^0_{fb}, for $\mathrm{e^+e^-}$ annihilation into a fermion-antifermion pair are given as [3]:

$$\sigma^0_a(s) = \frac{4}{3}\pi\alpha^2\left[\frac{g^a_{\mathrm{f}}}{s} + \frac{j^a_{\mathrm{f}}(s-\overline{m}^2_{\mathrm{Z}}) + r^a_{\mathrm{f}}\, s}{(s-\overline{m}^2_{\mathrm{Z}})^2 + \overline{m}^2_{\mathrm{Z}}\overline{\Gamma}^2_{\mathrm{Z}}}\right] \qquad \text{for } a = \mathrm{tot}, \mathrm{fb} \tag{9}$$

$$A^0_{\mathrm{fb}}(s) = \frac{3}{4}\frac{\sigma^0_{\mathrm{fb}}(s)}{\sigma^0_{\mathrm{tot}}(s)}\,, \tag{10}$$

where $\sqrt{s}$ is the centre-of-mass energy. The S–Matrix ansatz is defined using a Breit–Wigner denominator with s–independent width for the Z resonance, $s-\overline{m}^2_{\mathrm{Z}} + i\overline{m}_{\mathrm{Z}}\overline{\Gamma}_{\mathrm{Z}}$. The transformations of the resulting values for the mass and the total width of the Z boson to correspond to a Breit-Wigner denominator with s-dependent width, $s - m^2_{\mathrm{Z}} + is\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}}$, are:

$$\begin{aligned} m_{\mathrm{Z}} &\equiv \overline{m}_{\mathrm{Z}}\sqrt{1+\overline{\Gamma}^2_{\mathrm{Z}}/\overline{m}^2_{\mathrm{Z}}} &&\approx \overline{m}_{\mathrm{Z}} + 34.1\ \mathrm{MeV} \\ \Gamma_{\mathrm{Z}} &\equiv \overline{\Gamma}_{\mathrm{Z}}\sqrt{1+\overline{\Gamma}^2_{\mathrm{Z}}/\overline{m}^2_{\mathrm{Z}}} &&\approx \overline{\Gamma}_{\mathrm{Z}} + \ \ 0.9\ \mathrm{MeV}\,, \end{aligned} \tag{11}$$

so that $\Gamma_{\mathrm{Z}}/m_{\mathrm{Z}} = \overline{\Gamma}_{\mathrm{Z}}/\overline{m}_{\mathrm{Z}}$. Note, that the transformations according to Equation 11 are applied when listing numerical results for the mass and total width of the Z boson. QED radiative corrections are included by convolution with a radiator function [14].

The S-Matrix parameters r_{f}, j_{f} and g_{f}, which are real numbers, scale the Z exchange, γZ interference and photon exchange contributions, respectively. In models with only vector couplings of the photon to the electric charge, q_f, and only vector and axial-vector couplings of the Z, the S-Matrix parameters are [3]:[2]

$$\begin{aligned} r^{\mathrm{tot}}_{\mathrm{f}} &= \kappa^2\left[g^2_{A\mathrm{e}} + g^2_{V\mathrm{e}}\right]\left[g^2_{A\mathrm{f}} + g^2_{V\mathrm{f}}\right] && - 2\kappa g_{V\mathrm{e}}g_{V\mathrm{f}}C_{Im} \\ j^{\mathrm{tot}}_{\mathrm{f}} &= 2\kappa g_{V\mathrm{e}}g_{V\mathrm{f}}\left(C_{Re}+C_{Im}\right) \\ g^{\mathrm{tot}}_{\mathrm{f}} &= q^2_{\mathrm{e}}q^2_f\left|F_A(m^2_{\mathrm{Z}})\right|^2 \\ r^{\mathrm{fb}}_{\mathrm{f}} &= 4\kappa^2 g_{A\mathrm{e}}g_{V\mathrm{e}}g_{A\mathrm{f}}g_{V\mathrm{f}} && - 2\kappa g_{A\mathrm{e}}g_{A\mathrm{f}}C_{Im} \\ j^{\mathrm{fb}}_{\mathrm{f}} &= 2\kappa g_{A\mathrm{e}}g_{A\mathrm{f}}\left(C_{Re}+C_{Im}\right) \\ g^{\mathrm{fb}}_{\mathrm{f}} &= 0\,, \end{aligned} \tag{12}$$

where:

$$\kappa = \frac{\mathrm{G_F}m^2_{\mathrm{Z}}}{2\sqrt{2}\pi\alpha} \approx 1.50 \tag{13}$$

$$C_{Im} = \frac{\Gamma_{\mathrm{Z}}}{m_{\mathrm{Z}}}\, q_{\mathrm{e}}q_f\ \mathrm{Im}\left\{F_A(m^2_{\mathrm{Z}})\right\} \tag{14}$$

$$C_{Re} = q_{\mathrm{e}}q_f\ \mathrm{Re}\left\{F_A(m^2_{\mathrm{Z}})\right\} \tag{15}$$

$$F_A(m^2_{\mathrm{Z}}) = \frac{\alpha(m^2_{\mathrm{Z}})}{\alpha}\,. \tag{16}$$

[2]The relations for r and j listed here neglect the imaginary parts of the effective vector and axial-vector coupling constants of the Z boson to fermions.

In the last equation $\alpha(m_Z^2)$ denotes the complex running fine structure constant, and $\alpha \equiv \alpha(0)$. For the hadronic final state, the S-Matrix parameters r_{had}, j_{had} and g_{had} are summed over all colours and open quark flavours.

A.2 Correlated Errors due to the LEP Energy Calibration

The following procedure is used to determine the correlated errors on the S-Matrix parameters arising from the LEP energy calibration. The S-Matrix fit to the experimental measurements of cross sections and forward-backward asymmetries of a single LEP experiment determines the error matrix $\mathcal{E}$ of the set of S-Matrix parameters fitted. A second S-Matrix fit is performed, where the errors of all measurements except those arising from the LEP energy calibration are scaled by a factor of 0.5. This factor simulates the weight of the four LEP experiments in the average. Denoting the error matrix of the second fit with $\mathcal{E}'$, the error matrix $\mathcal{C}$ describing the correlated errors on the S-Matrix parameters arising from the LEP energy calibration is then obtained as:

$$\mathcal{C} = \frac{4}{3}\mathcal{E}' - \frac{1}{3}\mathcal{E}\,. \tag{17}$$

For this years summer conferences the LEP centre-of-mass energy values and errors have been updated. The ALEPH and DELPHI experiments have used this updated energy error matrix while the L3 and OPAL experiments have used the old matrix applying a correction to the mean values of Z-boson mass and width. Therefore the actual correlation matrix has been determined by taking the average of the old and the new correlation matrices determined with the above described method.

In Table 12 the numerical values for the correlated errors on the S-Matrix parameters and for the corresponding correlation coefficients are listed.

	Error	m_Z	Γ_Z	r^{tot}_{had}	r^{tot}_{e}	r^{tot}_{μ}	r^{tot}_{τ}	j^{tot}_{had}	j^{tot}_{e}	j^{tot}_{μ}	j^{tot}_{τ}	r^{fb}_{e}	r^{fb}_{μ}	r^{fb}_{τ}	j^{fb}_{e}	j^{fb}_{μ}	j^{fb}_{τ}
m_Z	1.6 MeV	1.0	.0	.0	.2	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
Γ_Z	1.4 MeV	.0	1.0	1.0	.8	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
r^{tot}_{had}	0.0030	.0	1.0	1.0	.7	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
r^{tot}_{e}	0.00018	.2	.8	.7	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
r^{tot}_{μ}	0.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
r^{tot}_{τ}	0.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
j^{tot}_{had}	0.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0	.0
j^{tot}_{e}	0.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0	.0
j^{tot}_{μ}	0.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0	.0
j^{tot}_{τ}	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0	.0
r^{fb}_{e}	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0	.0
r^{fb}_{μ}	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0	.0
r^{fb}_{τ}	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0	.0
j^{fb}_{e}	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0	.0
j^{fb}_{μ}	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0	.0
j^{fb}_{τ}	0.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	.0	1.0

Table 12: The correlated errors on the S-Matrix parameters and the corresponding correlation coefficients arising from the LEP energy calibration which is common to all four LEP experiments.

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