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A Combination of Preliminary LEP Electroweak Results for the 1995 Winter Conferences

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Abstract

This note presents a combination of published and preliminary electroweak results from the four LEP collaborations which were prepared for the 1995 winter conferences. Averages of electroweak LEP results from the measurement of hadronic and leptonic cross sections, the leptonic forward-backward asymmetries, the τ polarization asymmetries, the $b\overline{b}$ and $c\overline{c}$ partial widths and forward-backward asymmetries and the $q\overline{q}$ charge asymmetry are presented. The combined set of electroweak measurements is used to constrain the parameters of the Standard Model.

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1 Introduction

In a recent note [1] the four LEP experiments have presented parameters of the Z resonance derived from published and preliminary results based on data recorded until the end of 1993. Most of the preliminary results were contributions to the 27th International Conference on High Energy Physics, Glasgow, Scotland, 20-27 July 1994.

Since then several preliminary analyses have been completed and published. Also the calibration of the LEP energy scale for the high statistics scan of the Z resonance in 1993 has been finalized [2] resulting in improved uncertainties due to LEP energies on the Z mass, m_{Z}, and total width, Γ_{Z}.

Furthermore several new preliminary results from the 1994 running period have become available. In 1994 the LEP experiments approximately doubled their event statistics. Important progress has also been achieved for the theoretical error associated to the luminosity determination [3]. It is expected that an even better theoretical precision will finally be obtained. The interpretation of electroweak precision data requires a precise knowledge of the fine structure constant evaluated at the Z pole, $\alpha(m_{\mathrm{Z}}^2)$, for which several reevaluations have been performed recently [4–7]. Last but not least, the CDF and DØ collaborations have submitted their discovery papers for the top quark [8,9].

The data consist of the hadronic and leptonic cross sections, the leptonic forward-backward asymmetries, the τ polarization asymmetries, the $\mathrm{b\overline{b}}$ and $\mathrm{c\overline{c}}$ partial widths and forward-backward asymmetries and the $\mathrm{q\overline{q}}$ charge asymmetry. Many technical aspects of their combination have already been described in [1] and references therein.

This note is organised in the following manner. In section 2 the results on the Z lineshape and leptonic forward-backward asymmetries are presented, and section 3 contains the measurements of the τ polarization. Section 4 describes the parameters associated with heavy flavour analyses. In sections 5.1 and 5.2 several LEP electroweak measurements are combined to determine the effective neutral current coupling constants and to give a value of the effective electroweak mixing angle. The number of light neutrinos is determined in section 5.3. In section 5.4 the LEP data and also data from SLD [10], from neutrino interactions [11–13] and from measurements of the mass of the W boson [14–17] and the top quark [8,9] are used to constrain the parameters of the Standard Model.

2 Z Lineshape and Lepton Forward-Backward Asymmetries

The results presented here are based on the data taken during the energy scans in 1990 and 1991 with centre-of-mass energies, $\sqrt{s}$, in a range $|\sqrt{s} - m_{\mathrm{Z}}| < 3$ GeV, the high statistics data collected at the Z peak in 1992 and a preliminary analysis of the high precision scan in 1993. During this scan more than 18 pb^{-1} were recorded by each experiment at two centre-of-mass energy points roughly 1.8 GeV above and below the Z mass while about 15 pb^{-1} was within 200 MeV of m_{Z}. We also add several preliminary results based on data of the 1994 running period in which the LEP machine delivered approximately 65 pb^{-1} to each of the four LEP experiments at the Z peak.

The total statistics and the systematic errors of the individual LEP collaborations are given in Tables 1 and 2. Details of the individual analyses can be found in References 18–21. An important aspect of the lineshape analysis is a precise knowledge of the LEP centre-of-mass energies. The treatment of the LEP centre-of-mass energies by the four LEP experiments is based on [2]. For the 1994 data a preliminary LEP energy calibration is available [22]. In order to determine the Z mass, m_{Z}, and the total Z width, Γ_{Z}, all of the recorded data are combined, taking the energy uncertainty from

the 1993 data to be uncorrelated with the energy uncertainty coming from previous years, resulting in an overall LEP energy error on $m_{\rm Z}$ of $\approx$1.5 MeV and on $\Gamma_{\rm Z}$ of $\approx$1.7 MeV.

For the averaging of results the LEP experiments provide a standard set of 9 parameters describing the information contained in hadronic and leptonic cross sections and leptonic forward-backward asymmetries [1,23]. These parameters are convenient for fitting and averaging since they have minimal correlations amongst themselves. They are:

- The mass of the Z, $m_{\rm Z}$, and the total width, $\Gamma_{\rm Z}$, where the definition is based on the Breit Wigner denominator $(s - m_{\rm Z}^2 + is\Gamma_{\rm Z}/m_{\rm Z})$ [24].
- The hadronic pole cross section:
$$\sigma_{\rm h}^0 \equiv \frac{12\pi}{m_{\rm Z}^2}\frac{\Gamma_{\rm ee}\Gamma_{\rm had}}{\Gamma_{\rm Z}^2}\,.$$
Here $\Gamma_{\rm ee}$ and $\Gamma_{\rm had}$ are the partial widths of the Z for decays into electrons and hadrons.
- The ratios:
$$R_{\rm e} \equiv \Gamma_{\rm had}/\Gamma_{\rm ee} \qquad R_\mu \equiv \Gamma_{\rm had}/\Gamma_{\mu\mu} \qquad R_\tau \equiv \Gamma_{\rm had}/\Gamma_{\tau\tau}. \tag{1}$$
Here $\Gamma_{\mu\mu}$ and $\Gamma_{\tau\tau}$ are the partial widths of the Z for the decays $\mathrm{Z} \to \mu^+\mu^-$ and $\mathrm{Z} \to \tau^+\tau^-$. Even under the assumption of lepton universality a small (0.2%) difference between the values for $R_{\rm e}$ and R_μ, and the value for R_τ due to mass corrections to $\Gamma_{\tau\tau}$ is expected.
- The pole asymmetries, $A_{\rm FB}^{0,\rm e}$, $A_{\rm FB}^{0,\mu}$ and $A_{\rm FB}^{0,\tau}$ for the processes $\mathrm{e^+e^-} \to \mathrm{e^+e^-}$, $\mathrm{e^+e^-} \to \mu^+\mu^-$ and $\mathrm{e^+e^-} \to \tau^+\tau^-$. In terms of the effective vector and axial-vector neutral current couplings of fermions, $g_{V\rm f}$ and $g_{A\rm f}$, the pole asymmetries are expressed as:[1]
$$A_{\rm FB}^{0,\rm f} \equiv \frac{3}{4}\mathcal{A}_{\rm e}\mathcal{A}_{\rm f} \tag{2}$$
with:
$$\mathcal{A}_{\rm f} \equiv \frac{2g_{V\rm f}g_{A\rm f}}{g_{V\rm f}^2 + g_{A\rm f}^2}\,. \tag{3}$$

Note, that these parameters do not describe the Z production and decay completely, because they do not include the interference of the Z exchange with the photon exchange. For the results presented in this paper, the γZ interference terms are therefore fixed to their Standard Model values.[2]

The four sets of 9 parameters provided by the LEP experiments are presented in Table 3. The covariance matrix of these parameters is as described in our previous paper [23]. It is constructed from the covariance matrices of the individual LEP experiments and common systematic errors. These common errors arise from the theoretical uncertainty in the luminosity normalization affecting the hadronic pole cross section, $\Delta\sigma_{\rm h}^0/\sigma_{\rm h}^0 = 0.16\%$[3], from the uncertainty of the LEP centre-of-mass energy spread of 5 MeV, resulting in $\Delta\Gamma_{\rm Z} \approx 1.0$ MeV, and from the uncertainty in the LEP energy calibration, giving $\Delta m_{\rm Z} \approx 1.5$ MeV, $\Delta\Gamma_{\rm Z} \approx 1.7$ MeV, and $\Delta A_{\rm FB}^{0,\ell} \approx 0.0005$ for each lepton species ($\ell = \mathrm{e}, \mu, \tau$). Full correlation between $A_{\rm FB}^{0,\mu}$ and $A_{\rm FB}^{0,\tau}$ and full anti-correlation between $A_{\rm FB}^{0,\rm e}$ and $A_{\rm FB}^{0,\mu}$ or $A_{\rm FB}^{0,\tau}$ is used. This change of sign of the energy effect for $A_{\rm FB}^{0,\rm e}$ is an approximation of the effect of the t-channel interference

[1] Effects coming from photon exchange, as well as real and imaginary parts of the photon vacuum polarization are not included in the definition of $A_{\rm FB}^{0,\rm f}$, but are accounted for explicitly in the fitting formulae used by the experiments.

[2] If instead the γZ interference terms are entirely determined from LEP cross section data, the error on $m_{\rm Z}$ for each LEP experiment is increased to 13 MeV.

[3] The theoretical uncertainty depends on the acceptance of the luminosity monitor and the cuts applied to the data. The ALEPH, DELPHI, L3 and OPAL experiments quote 0.16%, 0.25%, 0.16% and 0.16%, respectively.

for a typical LEP experimental acceptance for the e^+e^- final state. The combined parameter set and its correlation matrix are given in Tables 4 and 5.

If lepton universality is assumed, the set of 9 parameters given above is reduced to a set of 5 parameters. R_ℓ is defined as $R_\ell \equiv \Gamma_{\rm had}/\Gamma_{\ell\ell}$, where $\Gamma_{\ell\ell}$ refers to the partial Z width for the decay into a pair of massless charged leptons.

The data of each of the four LEP experiments are consistent with lepton universality (the χ^2/d.o.f. are $6/4, 2/4, 5/4$ and $5/4$ for ALEPH, DELPHI, L3 and OPAL, respectively). Based on this assumption Table 6 provides the five parameters $m_{\rm Z}$, $\Gamma_{\rm Z}$, $\sigma^0_{\rm h}$, R_ℓ and $A^{0,\ell}_{\rm FB}$ for the individual LEP experiments. The four experiments all use the above definition of $\Gamma_{\ell\ell}$. Tables 7 and 8 provide the five parameters $m_{\rm Z}$, $\Gamma_{\rm Z}$, $\sigma^0_{\rm h}$, R_ℓ and $A^{0,\ell}_{\rm FB}$ and the corresponding correlation matrix for the combined result of the 4 LEP experiments. Figure 1 shows, for each lepton species and for the combination assuming lepton universality, the resulting 68% probability contours in the R_ℓ-$A^{0,\ell}_{\rm FB}$ plane.

For completeness, in Table 9 the partial decay widths of the Z boson are listed. Note, that the substantial improvement of $\Gamma_{\ell\ell}$ as compared to [1] is essentially due to a decrease of systematic errors on both, the LEP energy calibration and the theoretical calculation of the small angle Bhabha cross section.

		ALEPH	DELPHI	L3	OPAL	LEP
$q\overline{q}$	'90-'91	451	356	416	454	1677
	'92	680	697	678	733	2788
	'93 prel.	631	677	658	646	2612
	'94 prel.	[a]	[a]	1362	1524	2886
	total	1762	1730	3114	3357	9963
$\ell^+\ell^-$	'90-'91	55	37	40	58	190
	'92	82	69	58	88	297
	'93 prel.	78	71	62	82	293
	'94 prel.[b]	[a]	54	88	184	326
	total	215	231	248	412	1106

Table 1: The LEP statistics in units of 10^3 events used for the analysis of the Z line shape and lepton forward-backward asymmetries.
[a] No preliminary result quoted yet.
[b] DELPHI muon pairs, L3: electron and muon pairs.

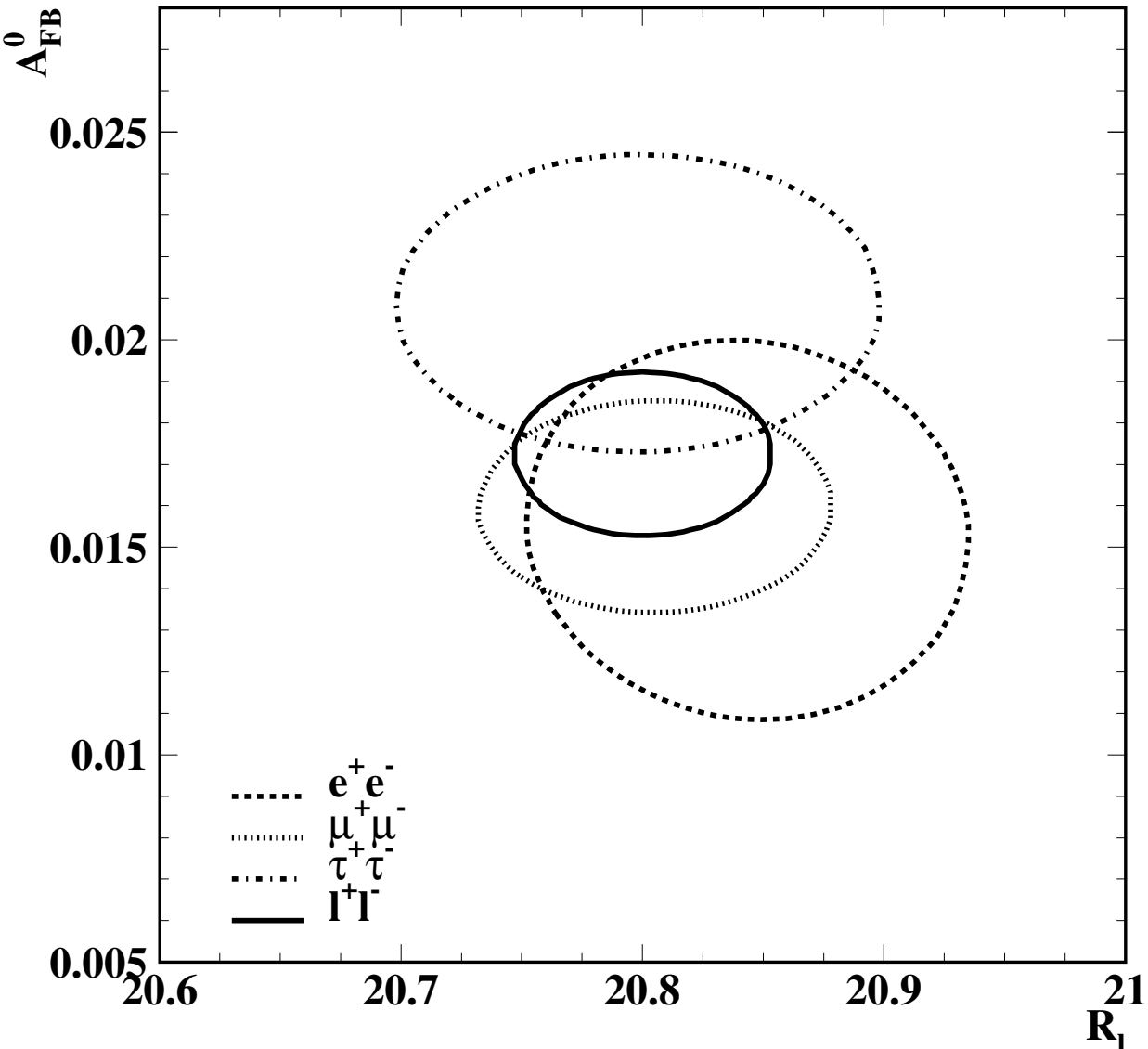

Figure 1: 68% probability contours in the R_ℓ-$A_{\rm FB}^{0,\ell}$ plane.

	ALEPH		DELPHI		L3		OPAL	
	'93	'94	'93 prel.	'94 prel.	'93 prel.	'94 prel.	'93 prel.	'94 prel.
$\mathcal{L}^{\text{exp.}}$ [b]	0.09%	[a]	0.28%	[a]	0.12%	0.15%	0.076%	0.079%
σ_{had}	0.08%	[a]	0.13%	[a]	0.11%	0.2%	0.15%	0.16%
σ_{e}	0.5 %	[a]	1.2%	[a]	0.30%	0.4%	0.23%	0.24%
σ_{μ}	0.25 %	[a]	0.3%	[a]	0.29%	[a]	0.16%	0.15%
σ_{τ}	0.34 %	[a]	0.8%	[a]	0.63%	[a]	0.43%	0.46%
$A_{\text{FB e}}$	0.003	[a]	0.003	[a]	0.004	0.004	0.0016	0.0016
$A_{\text{FB }\mu}$	0.001	[a]	0.001	0.0015	0.001	0.004	0.001	0.001
$A_{\text{FB }\tau}$	0.003	[a]	0.002	[a]	0.004	[a]	0.002	0.002

Table 2: The experimental systematic errors for the analysis of the Z line shape and lepton forward-backward asymmetries at the Z peak. The errors quoted do not include the common uncertainty due to the LEP energy calibration. The treatment of correlations between the errors for different years is described in References 18–21.
[a] No preliminary result quoted yet.
[b] In addition, there is a theoretical error for the calculation of the small angle Bhabha cross section of 0.16%, 0.25%, 0.16%, 0.16% for ALEPH, DELPHI, L3 and OPAL, respectively. 0.16% have been treated as common to all experiments.

	ALEPH	DELPHI	L3	OPAL
m_{Z}(GeV)	91.1917±0.0039	91.1862±0.0034	91.1934±0.0037	91.1852±0.0036
Γ_{Z}(GeV)	2.4932±0.0058[a]	2.4941±0.0055[a]	2.5040±0.0053[a]	2.4960±0.0052[a]
σ^0_{h}(nb)	41.62±0.10	41.27±0.17	41.41±0.11	41.47±0.10
R_e	20.63±0.13	20.86±0.16	20.91±0.12	20.90±0.10
R_μ	20.95±0.12	20.64±0.11	20.85±0.12	20.796±0.073
R_τ	20.68±0.12	20.64±0.16	20.71±0.17	21.00±0.11
$A^{0,\text{e}}_{\text{FB}}$	0.0218±0.0055	0.0221±0.0073	0.0125±0.0070	0.0081±0.0051
$A^{0,\mu}_{\text{FB}}$	0.0192±0.0039	0.0168±0.0030	0.0164±0.0041	0.0137±0.0027
$A^{0,\tau}_{\text{FB}}$	0.0217±0.0044	0.0210±0.0057	0.0305±0.0073	0.0183±0.0035
χ^2/d.o.f.	174/178	150/133	112/135	10/6[b]

Table 3: Line shape and asymmetry parameters from 9-parameter fits to the data of the four LEP experiments.
[a] Γ_{Z} has an additional error of 0.0010 GeV due to an uncertainty of 5 MeV in the LEP centre-of-mass energy spread which is common to all experiments.
[b] This parameter set has been obtained from a parameter transformation applied to the 15 parameters of the OPAL fit [21], which treats the γZ interference terms for leptons as additional free parameters. The extra parameters for the γZ interference terms have been fixed to their Standard Model values in the transformation. The χ^2/d.o.f. for the 15 parameter fit to the data is 87/132.

Parameter	Average Value
$m_{\rm Z}$(GeV)	91.1887±0.0022
$\Gamma_{\rm Z}$(GeV)	2.4971±0.0031 ± 0.0010
$\sigma^0_{\rm h}$(nb)	41.492±0.081
$R_{\rm e}$	20.843±0.060
R_μ	20.805±0.048
R_τ	20.798±0.066
$A^{0,\,{\rm e}}_{\rm FB}$	0.0154±0.0030
$A^{0,\,\mu}_{\rm FB}$	0.0160±0.0017
$A^{0,\,\tau}_{\rm FB}$	0.0209±0.0024

Table 4: Average line shape and asymmetry parameters from the data of the four LEP experiments given in Table 3, without the assumption of lepton universality. The χ^2/d.o.f. of the average is 34.7/27.

	$m_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	$R_{\rm e}$	R_μ	R_τ	$A^{0,\,{\rm e}}_{\rm FB}$	$A^{0,\,\mu}_{\rm FB}$	$A^{0,\,\tau}_{\rm FB}$
$m_{\rm Z}$	1.00	−0.07	0.01	0.03	−0.01	0.00	0.01	0.06	0.04
$\Gamma_{\rm Z}$	−0.07	1.00	−0.14	−0.01	0.00	0.00	0.00	0.00	0.00
$\sigma^0_{\rm h}$	0.01	−0.14	1.00	0.10	0.15	0.11	0.01	0.00	0.00
$R_{\rm e}$	0.03	−0.01	0.10	1.00	0.10	0.05	−0.06	0.01	0.01
R_μ	−0.01	0.00	0.15	0.10	1.00	0.07	0.00	0.01	0.00
R_τ	0.00	0.00	0.11	0.05	0.07	1.00	0.00	0.00	0.01
$A^{0,\,{\rm e}}_{\rm FB}$	0.01	0.00	0.01	−0.06	0.00	0.00	1.00	−0.03	−0.02
$A^{0,\,\mu}_{\rm FB}$	0.06	0.00	0.00	0.01	0.01	0.00	−0.03	1.00	0.06
$A^{0,\,\tau}_{\rm FB}$	0.04	0.00	0.00	0.01	0.00	0.01	−0.02	0.06	1.00

Table 5: The correlation matrix for the set of parameters given in Table 4.

	ALEPH	DELPHI	L3	OPAL
$m_{\rm Z}$(GeV)	91.1918±0.0039	91.1862±0.0034	91.1936±0.0037	91.1846±0.0035
$\Gamma_{\rm Z}$(GeV)	2.4929±0.0058$^{(a)}$	2.4942±0.0055$^{(a)}$	2.5040±0.0053$^{(a)}$	2.4959±0.0052$^{(a)}$
$\sigma^0_{\rm h}$(nb)	41.62±0.10	41.27±0.17	41.42±0.11	41.47±0.10
R_ℓ	20.746±0.073	20.677±0.083	20.836±0.080	20.851±0.059
$A^{0,\,\ell}_{\rm FB}$	0.0206±0.0026	0.0182±0.0025	0.0183±0.0032	0.0142±0.0020
χ^2/d.o.f.	180/182	152/137	117/139	15/10$^{(b)}$

Table 6: Line shape and asymmetry parameters from 5-parameter fits to the data of the four LEP experiments, assuming lepton universality.
$^{(a)}\Gamma_{\rm Z}$ has an additional error of 0.0010 GeV due to an uncertainty of 5 MeV in the LEP centre-of-mass energy spread which is common to all experiments.
$^{(b)}$This parameter set has been obtained from a parameter transformation to the 15 parameters of the OPAL fit, which treats the γZ interference terms for leptons as additional free parameters.

Parameter	Average Value
$m_{\rm Z}$(GeV)	91.1887±0.0022
$\Gamma_{\rm Z}$(GeV)	2.4971±0.0031 ± 0.0010
$\sigma^0_{\rm h}$(nb)	41.492±0.081
R_ℓ	20.800±0.035
$A^{0,\ell}_{\rm FB}$	0.0172±0.0013

Table 7: Average line shape and asymmetry parameters from the results of the four LEP experiments given in Table 6, assuming lepton universality. The χ^2/d.o.f. of the average is 22.0/15.

	$m_{\rm Z}$	$\Gamma_{\rm Z}$	$\sigma^0_{\rm h}$	R_ℓ	$A^{0,\ell}_{\rm FB}$
$m_{\rm Z}$	1.00	−0.07	0.01	0.01	0.06
$\Gamma_{\rm Z}$	−0.07	1.00	−0.14	0.00	0.00
$\sigma^0_{\rm h}$	0.01	−0.14	1.00	0.20	0.01
R_ℓ	0.01	0.00	0.20	1.00	0.00
$A^{0,\ell}_{\rm FB}$	0.06	0.00	0.01	0.00	1.00

Table 8: The correlation matrix for the set of parameters given in Table 7.

Without Lepton Universality:	
$\Gamma_{\rm ee}$(MeV)	83.86±0.17
$\Gamma_{\mu\mu}$(MeV)	84.01±0.24
$\Gamma_{\tau\tau}$(MeV)	84.04±0.31

With Lepton Universality:	
$\Gamma_{\ell\ell}$(MeV)	83.94±0.13
$\Gamma_{\rm had}$(MeV)	1746.0±3.1
$\Gamma_{\rm inv}$(MeV)	499.5±2.7

Table 9: Partial decay widths of the Z boson, derived from the results of the 9-parameter (Tables 4 and 5) and the 5-parameter fit (Tables 7 and 8).

3 The τ Polarisation

3.1 Introduction

The τ polarization $\mathcal{P}_\tau$ is determined by an indirect measurement of the longitudinal polarization of τ pairs produced in Z decays. It is defined as:

$$\mathcal{P}_\tau \equiv \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}\,, \tag{4}$$

where σ_R and σ_L are the τ-pair cross sections for the production of a right-handed and left-handed τ^-, respectively.

Neglecting small corrections for the effects of photon exchange and the γZ-interference and electromagnetic radiative corrections for initial and final state radiation, all of which are taken into account in the analyses, the angular distribution of $\mathcal{P}_\tau$ as a function of the angle θ between the e^- and the τ^-, for $\sqrt{s} = m_{\rm Z}$ is given by:

$$\mathcal{P}_\tau(\cos\theta) = -\frac{\mathcal{A}_\tau + \mathcal{A}_{\rm e}\frac{2\cos\theta}{1+\cos^2\theta}}{1 + \mathcal{A}_\tau\mathcal{A}_{\rm e}\frac{2\cos\theta}{1+\cos^2\theta}}\,, \tag{5}$$

with $\mathcal{A}_{\rm e}$ and $\mathcal{A}_\tau$ defined in equation (3). When averaged over all production angles $\mathcal{P}_\tau$ is a measurement of $\mathcal{A}_\tau$, while as a function of $\cos\theta$, $\mathcal{P}_\tau(\cos\theta)$ provides nearly independent determinations of both $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$, allowing thus a test of the universality of the couplings of the Z to e and τ.

Each experiment makes separate $\mathcal{P}_\tau$ measurements using the five τ decay modes $e\nu\overline{\nu}$, $\mu\nu\overline{\nu}$, $\pi\nu$, $\rho\nu$ and $a_1\nu$ [25–28]. The $\rho\nu$ and $\pi\nu$ are the most sensitive channels, contributing weights of about 40% each in the average. In addition, DELPHI have used an inclusive hadronic analysis. The combination is made on the results from each experiment already averaged over the τ decay modes measured by the experiment.

A discussion of the effects of possible common systematic errors between the experiments can be found in [1].

3.2 Results

Tables 10 and 11 show the most recent results for $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$ obtained by the four experiments [25–28] and their combination. No common systematics are included in these averages. For further discussion see [1]. The statistical correlation between the extracted values of $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$ is small ($\leq 4\%$).

The average values for $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$:

$$\mathcal{A}_\tau = 0.140 \pm 0.008\,, \tag{6}$$

$$\mathcal{A}_{\rm e} = 0.137 \pm 0.009\,, \tag{7}$$

are compatible, as is expected from lepton universality. Assuming lepton universality, the values for $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$ can be combined. This combination is performed neglecting any possible common systematic error between $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$ within a given experiment. Such errors are estimated to be small, but warrant further study. The combined result of $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$ gives:

$$\mathcal{A}_\ell = 0.139 \pm 0.006\,. \tag{8}$$

For the future, further study of the uncertainties from the effects of radiative corrections for the ρ decay-mode of the τ is desirable.

ALEPH	('90 - '92), final	$0.136 \pm 0.012 \pm 0.009$
DELPHI	('90 - '92), final	$0.148 \pm 0.017 \pm 0.014$
L3	('90 - '93), final	$0.150 \pm 0.013 \pm 0.009$
OPAL	('90 - '94), prel.	$0.134 \pm 0.010 \pm 0.009$
LEP Average		0.140 ± 0.008

Table 10: LEP results for $\mathcal{A}_\tau$. The χ^2/d.o.f. for the average is 0.8/3

ALEPH	('90 - '92), final	$0.129 \pm 0.016 \pm 0.005$
DELPHI	('90 - '92), final	$0.136 \pm 0.027 \pm 0.003$
L3	('90 - '93), final	$0.157 \pm 0.020 \pm 0.005$
OPAL	('90 - '94), prel.	$0.134 \pm 0.015 \pm 0.004$
LEP Average		0.137 ± 0.009

Table 11: LEP results for $\mathcal{A}_e$. The χ^2/d.o.f. for the average is 1.2/3

4 Electroweak Results with b and c Quarks

4.1 Introduction

The relevant quantities in the heavy quark sector are:

- The ratios of the b and c quark partial widths to the total hadronic partial width: $R_{\mathrm{b}} \equiv \Gamma_{\mathrm{b\bar{b}}}/\Gamma_{\mathrm{had}}$ and $R_{\mathrm{c}} \equiv \Gamma_{\mathrm{c\bar{c}}}/\Gamma_{\mathrm{had}}$.
- The forward-backward asymmetries, $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ and $A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$, which are combined at three average centre-of-mass energies: on-peak, above peak and below peak.
- The semileptonic branching ratios, BR($\mathrm{b} \to \ell$) and BR($\mathrm{b} \to \mathrm{c} \to \ell^+$), and the average $\mathrm{B}^0\overline{\mathrm{B}^0}$ mixing parameter, $\overline{\chi}$. These are often determined at the same time as the widths or asymmetries in multiparameter fits to lepton tag samples. They are included in the combination procedure to take into account their significant correlations with the other parameters.

There are therefore a total of 11 parameters to be determined in the combination procedure: the two partial widths, two asymmetries at each of three average centre-of-mass energies, two semileptonic branching ratios and the average mixing parameter.

4.2 Summary of measurements and averaging procedure

Since the Summer of 1994, several preliminary analyses have now been published, and some new preliminary results including more data have become available. However, no new tagging techniques or methods have been introduced. More details of the methods were given in the previous notes [1,29–31]. The measurements currently included in the combination are [32]:

- Lepton fits from all four LEP experiments [33–37].
- Event shape tag for R_{b} from ALEPH and L3 [38, 39].
- Lifetime (and lepton) double tag measurements for R_{b} from ALEPH, DELPHI and OPAL [34, 40, 41].
- Measurements of $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ based on lifetime tagged events with a hemisphere charge measurement from ALEPH and OPAL where the the mean b-hemisphere charge is derived from the charge distributions themselves [42, 43], and from DELPHI using lepton information in the opposite hemisphere [35].
- Analyses with D mesons to measure R_{c} from DELPHI and OPAL [44, 45].
- Analyses with a $\mathrm{D}^{*\pm}$ meson to measure $A_{\mathrm{FB}}^{\mathrm{c\bar{c}}}$ and $A_{\mathrm{FB}}^{\mathrm{b\bar{b}}}$ from ALEPH, DELPHI and OPAL [46–48].

These measurements are provided in the form of standard tables [29–32] by the four LEP collaborations. These tables include a detailed breakdown of the systematic error of each measurement and its dependence on other electroweak parameters. Where necessary, the experiments apply small corrections to their results in order to use agreed values and ranges for the input parameters used to calculate systematic errors. The measurements, corrected where necessary, are summarised in the

Appendix in Tables 19-28, where the statistical and systematic errors are quoted separately. The correlated systematic entries are from sources shared with one or more other results in the table and are derived from the full breakdown of common systematic uncertainties. The uncorrelated systematic entries are from the remaining sources.

The χ^2 minimization procedure used to derive the values of all the parameters listed in section 4.1 was unchanged [1, 31, 32]. The explicit dependences of each measurement on the other parameters was taken into account, for example the dependence of the value of $R_{\rm b}$ on the assumed value of $R_{\rm c}$ as described below. The full statistical and systematic covariance matrix for all the measurements was calculated. The correlation matrices relating several measurements made in the same analyses were used, in particular for the multiparameter lepton fits. Additional correlations arising from common sources of systematic uncertainty were estimated from a detailed breakdown of systematic errors [32]. This breakdown for the double-tag measurements of $R_{\rm b}$, plus the L3 event shape analysis which also measures this single parameter, is given for illustration in Table 12.

Since c-quark events form the main background in the $R_{\rm b}$ analyses, the value of $R_{\rm b}$ depends on the value of $R_{\rm c}$. If $R_{\rm b}$ and $R_{\rm c}$ are measured in the same analysis, this is reflected in the correlation matrix for the results. Otherwise, this dependence may be written as:

$$R_{\rm b} = R_{\rm b}^{\rm meas} + a(R_{\rm c})\frac{(R_{\rm c} - R_{\rm c}^{\rm used})}{R_{\rm c}}. \qquad (9)$$

In this expression, $R_{\rm b}^{\rm meas}$ is the result of the analysis which assumed a value of $R_{\rm c} = R_{\rm c}^{\rm used}$. The values of $R_{\rm c}^{\rm used}$ and the coefficients $a(x)$ are given in Table 19 where appropriate. The dependences of all other measurements on other electroweak parameters are treated in the same way.

The results for BR($\rm b \to \ell$) and BR($\rm b \to c \to \ell^+$) are now considered to be much more reliable than for summer 94 for the following reasons:

- A sign error in the quoted semileptonic decay model uncertainty has been corrected [36].
- The complete dependences of all the measurements on $\overline{\chi}$, BR($\rm b \to \ell$) and BR($\rm b \to c \to \ell^+$) are now taken into account in the same way as their dependence on the other electroweak parameters.

4.3 Corrections to asymmetries

The corrections to the forward-backward asymmetries are summarised here. The only change since the previous note is the inclusion of an improved calculation of the QCD correction [49].

Forward-backward asymmetries are translated to the following centre-of-mass energies:

$$\begin{aligned} \sqrt{s} &= 89.55 \text{ GeV } (\text{“}-2\text{”}), \\ \sqrt{s} &= 91.26 \text{ GeV } (\text{“pk”}), \\ \sqrt{s} &= 92.94 \text{ GeV } (\text{“}+2\text{”}). \end{aligned}$$

These were chosen so as to be close to the $\sqrt{s}$ values at which measurements were made so that the corrections are generally small. The predicted Standard Model $\sqrt{s}$ dependence from ZFITTER was used. After calculating the overall averages, the corrections described below were made to the average peak asymmetries to derive the pole asymmetries.

	ALEPH shape [38]	ALEPH lifetime [40]	DELPHI multiple [34]	L3 shape [39]	OPAL multiple [41]
Charm production	0.0	-0.85	-1.5	0.0	-0.89
D^0 lifetime	0.0	-0.28	-0.3	0.0	-0.22
D^+ lifetime	0.0	-0.36	-0.4	0.0	-0.28
D_s lifetime	0.0	-0.22	-0.3	0.0	-0.17
D decay multip.	0.0	-0.57	-0.5	0.0	-0.73
BR(D$\rightarrow$K^0)	0.0	0.0	$+0.7$	0.0	$+0.57$
$g \rightarrow b\overline{b}, c\overline{c}$	0.0	-0.33	-0.2	0.0	-0.48
Long-lived light hadrons	0.0	-0.24	-0.8	0.0	-0.49
BR($c \rightarrow \ell$)	$+0.6$	0.0	-0.3	0.0	-0.28
Semilept.mod. $c \rightarrow \ell$	-2.1	0.0	-0.2	0.0	-0.24
$\langle x_E(c) \rangle$	$+0.8$	-0.12	-0.4	$+1.8$	-0.72
Semilept model $b \rightarrow \ell$	-1.3	0.0	0.0	0.0	0.0
$\langle x_E(b) \rangle$	0.0	0.0	0.0	-3.1	0.0
Total corr. error	2.7	1.2	2.1	3.6	1.8

Table 12: Example of breakdown of the correlated systematic error for R_b from lifetime, multiple and shape double-tag measurements (in units of 10^{-3}). The sign is the sign of the correlation among the experiments.

In general, the measured asymmetries assume that the differential cross section has the form:

$$\frac{d\sigma}{d\cos\theta} \propto 1 + \cos^2\theta + \frac{8}{3} A_{FB} \cos\theta, \tag{10}$$

where θ is the angle between the direction of the incoming electron and outgoing quark. The event thrust axis is used as an estimate of the quark direction. Small corrections are then applied to relate the measured asymmetries to the pole asymmetries. Only the asymmetries measured at the peak ($\sqrt{s}$ = 91.26 GeV) have been used to determine the pole asymmetries. The corrections are summarised in Table 13 and explained below:

- Energy shift correction: The slope of the asymmetry as a function of $\sqrt{s}$ around m_Z depends essentially on the axial coupling and the charge of the initial and final state fermions and is thus independent of the pole asymmetry itself.

- QED corrections: Initial state radiation reduces the effective centre-of-mass energy. Thus a correction similar in nature to the energy shift must be applied.

- QCD corrections: The QCD corrections using the thrust axis have been calculated recently by Djouadi, Lampe and Zerwas [49] in first order QCD including mass corrections. They have been found to be $A_{FB}^{QCD} = A_{FB}^{no\,QCD}(1 + c\frac{\alpha_s}{\pi})$ with $c = -0.87$ for charm and $c = -0.79$ for bottom quarks. Assuming $\alpha_s(m_Z^2) = 0.12$ and a scale of $\mu^2 = (m_Z/2)^2$ the correction factor is $1 - 0.034 \pm 0.004$ ($1 - 0.037 \pm 0.004$) for b (c) quarks where the error is deduced from a variation of the renormalization scale between $(m_Z/4)^2$ and m_Z^2. This result is valid under the assumption of a full acceptance. However, it has not yet been studied carefully how far events with very hard gluon radiation contribute in the same way to the asymmetry measurement as two jet events. For that reason another 25% error is assigned to the size of the QCD correction.

  Although the measured asymmetries using a lepton or $D^\star$ meson tag need to be corrected in this way for the effects of QCD, the lifetime/hemisphere-charge measurements of the b asymmetry

take into account QCD effects as an inherent part of the analysis [35,42,43]. To form a consistent average, the QCD correction for the b asymmetry of +0.0033 is therefore subtracted from each of these hemisphere charge measurements before combining with the other measurements.

- γ exchange and γZ interference: These diagrams modify very slightly the asymmetry.

All corrections with the exception of the QCD corrections have been determined using ZFITTER [50].

Source	$\delta A_{\rm FB}^{\rm b\bar{b}}$	$\delta A_{\rm FB}^{\rm c\bar{c}}$
$\sqrt{s} = m_{\rm Z}$	–0.0013	–0.0034
QED corrections	+0.0041	+0.0104
QCD corrections	$+0.0033 \pm 0.0010$	$+0.0023 \pm 0.0007$
γ, γZ	–0.0003	–0.0008
Total	$+0.0058 \pm 0.0010$	$+0.0085 \pm 0.0007$

Table 13: Corrections to be applied to the quark asymmetries. The corrections are to be understood as $A_{\rm FB}^0 = A_{\rm FB}^{meas} + \sum_i (\delta A_{\rm FB})_i$

4.4 Results

Using the full averaging procedure gives the following combined results for the electroweak parameters:

$$\begin{aligned}
R_{\rm b} &= 0.2204 \pm 0.0020 \\
R_{\rm c} &= 0.1606 \pm 0.0095 \\
{\rm BR}({\rm b} \to \ell) &= (11.08 \pm 0.31)\% \\
{\rm BR}({\rm b} \to {\rm c} \to \ell^+) &= (8.22 \pm 0.46)\% \\
\overline{\chi} &= 0.1160 \pm 0.0060 \\
A_{\rm FB}^{\rm b\bar{b}}(-2) &= (4.5 \pm 1.4)\% \\
A_{\rm FB}^{\rm c\bar{c}}(-2) &= (-20.2 \pm 4.3)\% \\
A_{\rm FB}^{\rm b\bar{b}}({\rm pk}) &= (9.57 \pm 0.35)\% \\
A_{\rm FB}^{\rm c\bar{c}}({\rm pk}) &= (6.74 \pm 0.89)\% \\
A_{\rm FB}^{\rm b\bar{b}}(+2) &= (11.9 \pm 1.2)\% \\
A_{\rm FB}^{\rm c\bar{c}}(+2) &= (10.5 \pm 3.7)\%
\end{aligned}$$

With a χ^2/dof of $37.1/(60-11)$, and the correlation matrix given in table 14. The total corrections for the on peak asymmetries to give the pole asymmetries are $+0.0058 \pm 0.0010$ and $+0.0085 \pm 0.0007$ for b and c respectively. The main results to be used in electroweak fits are then:

$$\begin{aligned}
R_{\rm b} &= 0.2204 \pm 0.0020 \\
R_{\rm c} &= 0.1606 \pm 0.0095 \\
A_{\rm FB}^{0,\,{\rm b}} &= 0.1015 \pm 0.0036 \\
A_{\rm FB}^{0,\,{\rm c}} &= 0.0760 \pm 0.0089
\end{aligned}$$

with their correlations given in table 14. If $R_{\rm c}$ is fixed to 0.171, then the value of $R_{\rm b}$ is:

$$R_{\rm b} = 0.2196 \pm 0.0019 \ (R_{\rm c} = 0.171).$$

The dominant changes with respect to the results in [1] are due to the inclusion of new data [35,37,43].

	R_b	R_c	$Br(1)$	$Br(2)$	$\overline{\chi}$	$A_{FB}^{b\bar{b}}$ (−2)	$A_{FB}^{c\bar{c}}$ (−2)	$A_{FB}^{b\bar{b}}$ (pk)	$A_{FB}^{c\bar{c}}$ (pk)	$A_{FB}^{b\bar{b}}$ (+2)	$A_{FB}^{c\bar{c}}$ (+2)
R_b	1.000	−.356	−.329	.045	−.052	−.026	−.048	−.006	.110	−.022	.048
R_c	−.356	1.000	.266	.019	−.056	.007	.061	.064	−.134	.021	−.037
$Br(1)$	−.329	.266	1.000	−.119	.284	.001	.073	.090	.170	.008	.036
$Br(2)$	.045	.019	−.119	1.000	−.755	−.025	−.055	−.206	−.262	−.067	−.058
$\overline{\chi}$	−.052	−.056	.284	−.755	1.000	.042	.039	.300	.091	.101	.022
$A_{FB}^{b\bar{b}}(-2)$	−.026	.007	.001	−.025	.042	1.000	.047	−.003	−.024	−.047	−.069
$A_{FB}^{c\bar{c}}(-2)$	−.048	.061	.073	−.055	.039	.047	1.000	−.004	−.102	−.067	−.080
$A_{FB}^{b\bar{b}}$(pk)	−.006	.064	.090	−.206	.300	−.003	−.004	1.000	.234	.026	.078
$A_{FB}^{c\bar{c}}$(pk)	.110	−.134	.170	−.262	.091	−.024	−.102	.234	1.000	.051	.149
$A_{FB}^{b\bar{b}}(+2)$	−.022	.021	.008	−.067	.101	−.047	−.067	.026	.051	1.000	.196
$A_{FB}^{c\bar{c}}(+2)$	.048	−.037	.036	−.058	.022	−.069	−.080	.078	.149	.196	1.000

Table 14: Correlation matrix of the final result, where $Br(1)$ represents BR(b $\rightarrow \ell$) and $Br(2)$ represents BR(b $\rightarrow$ c $\rightarrow \ell^+$).

The pole forward-backward asymmetries are related to $\mathcal{A}_c$ and $\mathcal{A}_b$ by equation (2). Assuming lepton universality, the value of $\mathcal{A}_e = \mathcal{A}_\ell$ can be derived from the measured lepton forward-backward asymmetry:

$$A_{FB}^{0,\ell} = 0.0172 \pm 0.0013 \equiv \frac{3}{4}\mathcal{A}_\ell{}^2,$$

which gives:

$$\mathcal{A}_\ell = 0.1514 \pm 0.0057,$$

and from the tau polarization measurements, again assuming lepton universality:

$$\mathcal{A}_\ell = 0.139 \pm 0.006.$$

The average value of $\mathcal{A}_\ell$ is then:

$$\mathcal{A}_\ell = 0.1455 \pm 0.0041.$$

From the measured values of $A_{FB}^{0,b}$ and $A_{FB}^{0,c}$ this gives:

$$\begin{aligned} \mathcal{A}_b &= 0.930 \pm 0.042, \\ \mathcal{A}_c &= 0.696 \pm 0.084. \end{aligned}$$

in very good agreement with the Standard Model prediction $\mathcal{A}_b = 0.934$ and $\mathcal{A}_c = 0.666$ (m_Z = 91.1887 GeV, m_t = 180 GeV, m_H = 300 GeV).

5 Interpretation of Results

5.1 The Effective Z Couplings

The partial widths of the Z into leptons and the lepton forward-backward asymmetries (Section 2), the τ polarization and the τ polarization asymmetry (Section 3) can all be combined to determine the effective vector and axial vector couplings for e, μ and τ. The asymmetries (Equations 2 and 5) determine the ratio $g_{V\ell}/g_{A\ell}$ (Equation 3), while the sum of the squares of the couplings is derived from the leptonic partial widths:

$$\Gamma_{\ell\ell} = \frac{G_{\rm F} m_{\rm Z}^3}{6\pi\sqrt{2}}(g_{V\ell}^2 + g_{A\ell}^2)(1 + \delta_\ell^{QED}) \,, \tag{11}$$

where $\delta_\ell^{QED} = 3q_\ell^2\alpha(m_{\rm Z}^2)/(4\pi)$ accounts for final state photonic corrections.

The averaged results for the effective lepton couplings are given in Table 15. Figure 2 shows the 68% probability contours in the $g_{A\ell}$-$g_{V\ell}$ plane. The signs of $g_{A\ell}$ and $g_{V\ell}$ are based on the convention $g_{A{\rm e}} < 0$. With this convention the signs of the couplings of all charged leptons follow from LEP data alone. The measured ratios of the e, μ and τ couplings provide a test of lepton universality:

$$\begin{aligned} g_{V\mu}/g_{V{\rm e}} &= 1.01 \ \pm 0.15 \ , \ g_{A\mu}/g_{A{\rm e}} = 1.0008 \pm 0.0019, \\ g_{V\tau}/g_{V{\rm e}} &= 1.013 \pm 0.076, \ g_{A\tau}/g_{A{\rm e}} = 1.0022 \pm 0.0022. \end{aligned}$$

The neutrino coupling can be derived from the measured value of the invisible width of the Z, $\Gamma_{\rm inv}$, attributing it exclusively to the decay into three identical neutrino generations ($\Gamma_{\rm inv} = 3\Gamma_{\nu\nu}$) and assuming $g_{A\nu} = g_{V\nu} \equiv g_\nu$. The sign of g_ν is chosen to be in agreement with neutrino scattering data [51], resulting in

$$g_\nu = +0.5009 \pm 0.0014.$$

Without Lepton Universality:	
$g_{V{\rm e}}$	-0.0364 ± 0.0018
$g_{V\mu}$	-0.0367 ± 0.0044
$g_{V\tau}$	-0.0369 ± 0.0019
$g_{A{\rm e}}$	-0.50099 ± 0.00051
$g_{A\mu}$	-0.50141 ± 0.00079
$g_{A\tau}$	-0.50208 ± 0.00093

With Lepton Universality:	
$g_{V\ell}$	-0.0366 ± 0.0011
$g_{A\ell}$	-0.50123 ± 0.00042
g_ν	$+0.5009 \pm 0.0014$

Table 15: Results for the effective vector and axial vector couplings derived from the combined LEP data without and with the assumption of lepton universality.

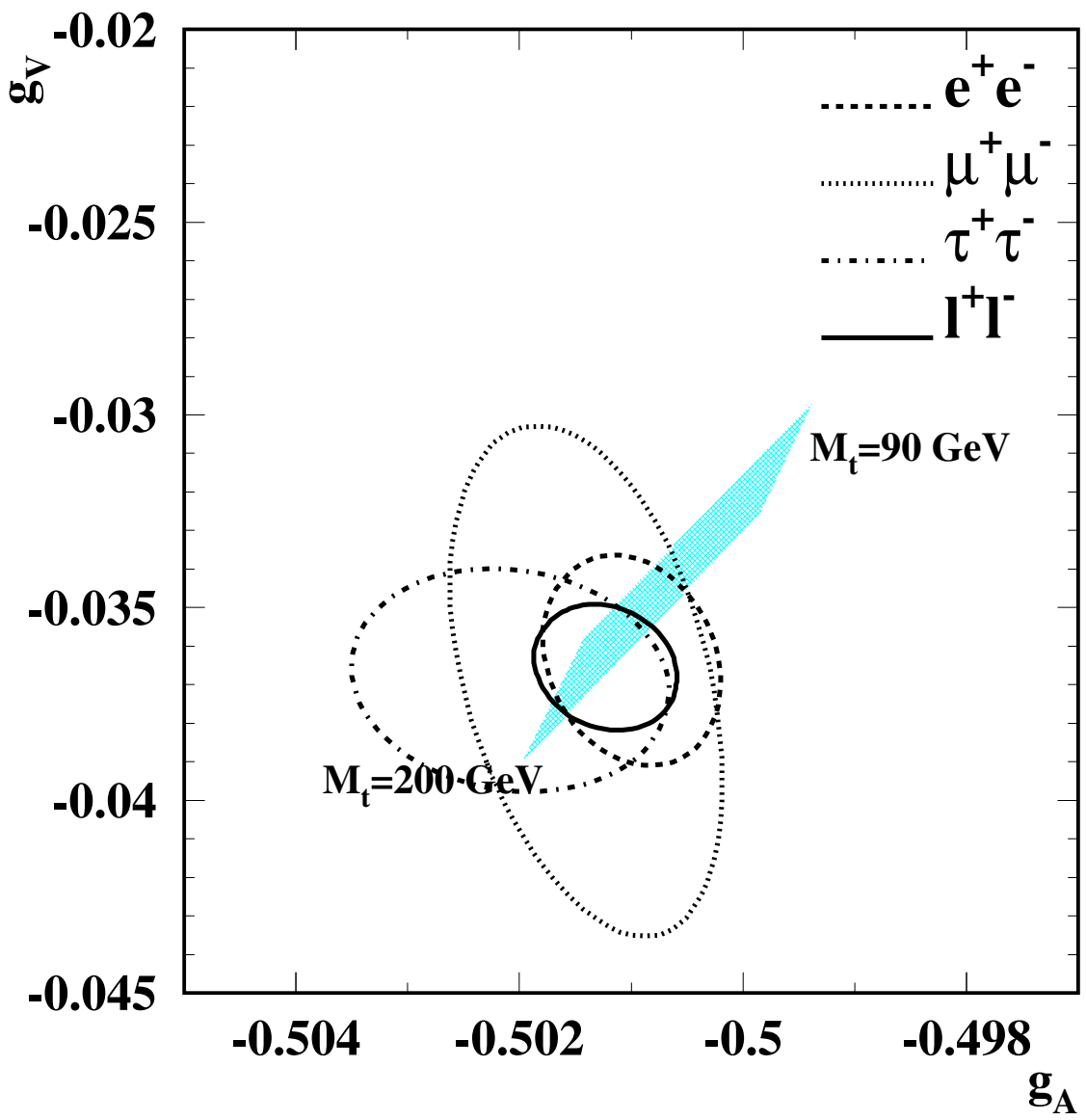

Figure 2: The 68% probability contours in the $g_{V\ell}$-$g_{A\ell}$ plane. The solid contour results from a fit assuming lepton universality. The shaded band represents the Standard Model prediction.

5.2 The Effective Electroweak Mixing Angle $\sin^2\theta^{\rm lept}_{\rm eff}$

The asymmetry measurements from LEP can be combined into a single observable, the effective electroweak mixing angle, $\sin^2\theta^{\rm lept}_{\rm eff}$, defined as:

$$\sin^2\theta^{\rm lept}_{\rm eff} \equiv \frac{1}{4}(1 - g_{V\ell}/g_{A\ell})\,, \tag{12}$$

without making any strong model-specific assumptions.

For a combined average of $\sin^2\theta^{\rm lept}_{\rm eff}$ from $A^{0,\ell}_{\rm FB}$, $\mathcal{A}_\tau$ and $\mathcal{A}_{\rm e}$ only the assumption of lepton universality, already inherent in the definition of $\sin^2\theta^{\rm lept}_{\rm eff}$, is needed. In practice no further assumption is involved if the quark forward-backward asymmetries, $A^{\rm b\bar{b}}_{\rm FB}$ and $A^{\rm c\bar{c}}_{\rm FB}$, are included in this average, as these asymmetries have a reduced sensitivity to corrections particular to the hadronic vertex. The results of these determinations of $\sin^2\theta^{\rm lept}_{\rm eff}$ and their combination are shown in Table 16. Also the comparison with the measurement of the left-right asymmetry, $A_{\rm LR}$, at SLD [10] is given. It should be noted that $A_{\rm LR}$ measures the same quantity as $\mathcal{A}_{\rm e}$ from τ polarization, with minimal model dependence. Both measurements have small systematic errors.

	$\sin^2\theta^{\rm lept}_{\rm eff}$	Average by Group of Observations	Cumulative Average	χ^2/d.o.f.
$A^{0,\ell}_{\rm FB}$	0.23096 ± 0.00073			
$\mathcal{A}_\tau$	0.2324 ± 0.0010			
$\mathcal{A}_{\rm e}$	0.2328 ± 0.0011	0.2318 ± 0.0005	0.2318 ± 0.0005	2.5/2
$A^{0,\rm b}_{\rm FB}$	0.23182 ± 0.00064			
$A^{0,\rm c}_{\rm FB}$	0.2310 ± 0.0021	0.2318 ± 0.0006	0.2318 ± 0.0004	2.6/4
$\langle Q_{\rm FB}\rangle^{(a)}$	0.2320 ± 0.0016	0.2320 ± 0.0016	0.2318 ± 0.0004	2.6/5
$A_{\rm LR}$ (SLD)	0.2294 ± 0.0010	0.2294 ± 0.0010	0.2315 ± 0.0004	7.6/6

Table 16: Comparison of several determinations of $\sin^2\theta^{\rm lept}_{\rm eff}$ from asymmetries. Averages are obtained as weighted averages assuming no correlations. The second column lists the $\sin^2\theta^{\rm lept}_{\rm eff}$ values derived from the quantities listed in the first column. The third column contains the averages of these numbers by groups of observations, where the groups are indicated by the horizontal lines. The last column shows the cumulative averages. The χ^2 per degree of freedom for the cumulative averages also is given. $^{(a)}$The LEP results for the hadronic charge asymmetry [52–55], $\langle Q_{\rm FB}\rangle$, and their method of combination are unchanged with respect to Reference 23.

5.3 Number of Neutrinos

An important aspect of our measurement concerns the information related to Z decays into invisible channels. Using the results of Tables 7 and 8 the ratio of the Z decay width into invisible particles and the leptonic decay width is determined:

$$\Gamma_{\rm inv}/\Gamma_{\ell\ell} \;=\; 5.951 \pm 0.032\,.$$

Dividing this by the Standard Model value for the ratio of the partial widths to neutrinos and charged leptons:

$$(\Gamma_{\nu\nu}/\Gamma_{\ell\ell})_{\rm SM} \quad = \quad 1.992 \pm 0.003\,,$$

where the central value is evaluated for $m_{\rm Z} = 91.1887$ GeV, $m_{\rm t} = 175$ GeV, $m_{\rm H} = 300$ GeV and the error quoted accounts for a variation of $m_{\rm t}$ in the range $100 < m_{\rm t}({\rm GeV}) < 200$ and a variation of $m_{\rm H}$ in the range $60 < m_{\rm H}({\rm GeV}) < 1000$, the number of light neutrinos is found to be:

$$N_\nu \quad = \quad 2.987 \pm 0.016.$$

5.4 Standard Model Constraints

The precise electroweak measurements performed at LEP can be used to check the validity of the Standard Model and, within its framework, to infer valuable information about its basic parameters. Their accuracy makes them sensitive to the top quark mass, $m_{\rm t}$, and to the mass of the Higgs boson, $m_{\rm H}$, through the loop corrections. The leading top quark dependence is quadratic and allows a determination of $m_{\rm t}$. The main dependence on $m_{\rm H}$ is logarithmic and therefore, with the present data accuracy, the constraints on $m_{\rm H}$ are still weak.

The various measurements are summarised in Table 17 and presented in Figure 3 and 4 together with their Standard Model prediction as a function of $m_{\rm t}$. The bands in the Standard Model predictions reflect the linear sum of the expected variations of each quantity due to a change of the strong coupling constant $\alpha_s(m_{\rm Z}^2) = 0.123 \pm 0.006$ [56] and $m_{\rm H}$ in the interval $60 \leq m_{\rm H}$ [GeV] ≤ 1000 for $m_{\rm Z} = 91.1887$ GeV.

Table 18 shows the constraints obtained on $m_{\rm t}$ and $\alpha_s(m_{\rm Z}^2)$ when fitting the measurements in Table 17 to the most up to date Standard Model calculations [58] as verified by the working group on 'Precision Calculations at LEP-I' [59]. The fits have been repeated for $m_{\rm H} = 60$, 300 and 1000 GeV and the difference in the fitted parameters is quoted as the second uncertainty. Presented in Table 18 are the results obtained using only LEP data (Table 17a), as well as those obtained by including the measurements of $m_{\rm W}$ from UA2 [14], CDF [15,16] and DØ [17][4], and the measurements of the neutrino neutral to charged current ratios from CDHS [11], CHARM [12] and CCFR [13] (Table 17b), as well those obtained by including the SLD result for the left-right asymmetry, $A_{\rm LR}$ [10] (Table 17c). The χ^2/d.o.f. for all these fits is acceptable.

The value of $m_{\rm t}$ resulting from these fits is in excellent agreement with the mass values $m_{\rm t} = 176 \pm 8$ (stat.) $\pm$ 10 (syst.) and $m_{\rm t} = 199^{+19}_{-21}$ (stat.) $\pm$ 22 (syst.) reported from the observation of the top quark at the TEVATRON by the CDF [8] and the DØ collaborations [9], respectively. For the sake of comparison with electroweak precision data the weighted average of both measurements, $m_{\rm t} = 180 \pm 12$ GeV, is used in the following.

Similarly, the value of $\alpha_s(m_{\rm Z}^2)$ resulting from the fits of Table 18 is in very good agreement with that obtained from event shape measurements at LEP ($\alpha_s(m_{\rm Z}^2) = 0.123 \pm 0.006$ [56]) and of similar precision. The strong coupling constant can also be determined from the parameter R_ℓ alone. For $m_{\rm Z} = 91.1887$ GeV, and imposing $m_{\rm t} = 180 \pm 12$ GeV as constraint, $\alpha_s = 0.128 \pm 0.005 \pm 0.002$ is obtained, where the second error accounts for the variation of the result when varying $m_{\rm H}$ in $60 \leq m_{\rm H}$ [GeV] ≤ 1000. A detailed discussion of theoretical uncertainties in the determination of α_s from R_ℓ can be found in Reference 60.

[4] See [57] for a combination of these $m_{\rm W}$ measurements.

	Measurement	Standard Model Fit	Pull
a) LEP			
line-shape and lepton asymmetries:			
m_Z [GeV]	91.1887 ± 0.0022	91.1885	0.0
Γ_Z [GeV]	2.4971 ± 0.0032	2.4979	−0.3
σ_h^0 [nb]	41.492 ± 0.081	41.441	0.6
R_ℓ	20.800 ± 0.035	20.783	0.5
$A_{FB}^{0,\ell}$	0.0172 ± 0.0013	0.0157	1.1
+ correlation matrix Table 8			
τ polarization:			
$\mathcal{A}_\tau$	0.140 ± 0.008	0.145	−0.6
$\mathcal{A}_e$	0.137 ± 0.009	0.145	−0.9
b and c quark results:			
R_b	0.2204 ± 0.0020	0.2157	2.4
R_c	0.1606 ± 0.0095	0.172	−1.2
$A_{FB}^{0,b}$	0.1015 ± 0.0036	0.1015	0.0
$A_{FB}^{0,c}$	0.0760 ± 0.0089	0.0724	0.4
+ correlation matrix Table 14			
$q\bar{q}$ charge asymmetry:			
$\sin^2\theta_{eff}^{lept}$ ($\langle Q_{FB} \rangle$)	0.2320 ± 0.0016	0.2318	0.1
b) $p\bar{p}$ and νN			
m_W [GeV] ($p\bar{p}$ [57])	80.26 ± 0.16	80.34	−0.6
$1 - m_W^2/m_Z^2$ (νN [11–13])	0.2253 ± 0.0047	0.2238	0.4
c) SLC			
$\sin^2\theta_{eff}^{lept}$ (A_{LR} [10])	0.2294 ± 0.0010	0.2318	−2.4

Table 17: Summary of measurements included in the combined analysis of Standard Model parameters. Section a) summarises LEP averages, section b) electroweak precision tests from $p\bar{p}$ colliders and νN-scattering, section c) gives the result for $\sin^2\theta_{eff}^{lept}$ from the measurement of the left-right polarization asymmetry at SLD. The Standard Model fit results in column 3 and the pulls (difference to measurement in units of the measurement error) in column 4 are derived from the fit including all data (Table 18, column 4) for a fixed value of $m_H = 300$ GeV.

Detailed studies of the theoretical uncertainties in the Standard Model predictions are carried out in the working group on 'Precision Calculations at LEP-I' [59]. The theoretical uncertainties are dominated by the uncertainty in the value of $\alpha(m_Z^2)$ due to the contribution of light quarks to the photon vacuum-polarization. Recently there have been several preprints on reevaluations of $\alpha(m_Z^2)$ [4–6]. The span of results ranges from $\alpha(m_Z^2) = 1/(129.08 \pm 0.10)$ [4] to $\alpha(m_Z^2) = 1/(128.896 \pm 0.090)$ [6]. For the results presented in this section, a value of $\alpha(m_Z^2) = 1/(128.896 \pm 0.090)$ [6] is used, where the error is propagated in the fits. This uncertainty causes an error of 0.00023 on the Standard Model prediction of $\sin^2\theta_{eff}^{lept}$ and an error of 4 GeV on m_t. The effect on the Standard Model prediction for $\Gamma_{\ell\ell}$ is negligible. Using instead $\alpha(m_Z^2) = 1/(129.08 \pm 0.10)$ [4] results in a shift of the

	LEP	LEP + $p\overline{p}$ and νN data	LEP + $p\overline{p}$ and νN data + A_{LR} from SLD
m_t (GeV)	$176 \pm 10\ ^{+17}_{-19}$	$174 \pm 9\ ^{+17}_{-19}$	$179 \pm 9\ ^{+17}_{-19}$
$\alpha_s(m_Z^2)$	$0.125 \pm 0.004\ \pm 0.002$	$0.126 \pm 0.004\ \pm 0.002$	$0.125 \pm 0.004\ \pm 0.002$
χ^2/d.o.f.	8.8/9	9.0/11	15/12
$\sin^2\theta_{eff}^{lept}$	$0.2320 \pm 0.0003\ ^{+0.0001}_{-0.0002}$	$0.2320 \pm 0.0003\ ^{+0.0001}_{-0.0002}$	$0.2318 \pm 0.0003\ ^{+0.0001}_{-0.0002}$
$1 - m_W^2/m_Z^2$	$0.2242 \pm 0.0012\ ^{+0.0003}_{-0.0002}$	$0.2245 \pm 0.0011\ ^{+0.0002}_{-0.0002}$	$0.2238 \pm 0.0011\ ^{+0.0003}_{-0.0002}$
m_W (GeV)	$80.32 \pm 0.06\ ^{+0.01}_{-0.01}$	$80.31 \pm 0.06\ ^{+0.01}_{-0.01}$	$80.34 \pm 0.06\ ^{+0.01}_{-0.01}$

Table 18: Results of fits to LEP and other electroweak precision data for m_t and $\alpha_s(m_Z^2)$. No external constraint on $\alpha_s(m_Z^2)$ has been imposed. The second column presents the results obtained using LEP data only (Table 17a). In the third column also the combined data from the $p\overline{p}$ collider and νN experiments (Table 17b) are included. The fourth column gives the result when the SLD measurement of the left-right asymmetry (Table 17c) is also added. The central values and the first errors quoted refer to m_H = 300 GeV. The second errors correspond to the variation of the central value when varying m_H in the interval $60 \leq m_H\ [\text{GeV}] \leq 1000$. The bottom part of the table lists derived results for $\sin^2\theta_{eff}^{lept}$, $1 - m_W^2/m_Z^2$ and m_W.

m_t values quoted in Table 18 by -9 GeV.[5] The $\alpha_s(m_Z^2)$ values quoted in Table 18 are stable against a variation of $\alpha(m_Z^2)$.

The measurement of R_b causes the largest χ^2 contribution of all LEP data in the Standard Model fits presented above (see also Table 17). There is a strong correlation (of -0.4) between the R_b and R_c measurements; the agreement between R_b and its Standard Model prediction improves from 2.4 to 2.1 standard deviations if the value of R_c is fixed to the Standard Model value $R_c = 0.171$. In this case one obtains $R_b = 0.2196 \pm 0.0019$.

Attributing the deviation of R_b to the b partial width, R_ℓ should also be affected since $\Gamma_{b\bar{b}}$ is a component of the total hadronic width [62]. In Figure 5 the measured value of R_b is plotted versus $\sin^2\theta_{eff}^{lept}$. If one assumes the Standard Model dependence on $\sin^2\theta_{eff}^{lept}$ for the light quark widths and taking $\alpha_s(m_Z^2) = 0.123 \pm 0.006$, R_ℓ imposes a constraint on the two variables. The one-sigma R_ℓ band is centred on the Standard Model prediction while the R_b band is slightly off-set. The overlap, however, depends on the value of $\alpha_s(m_Z^2)$. By lowering the assumed value of $\alpha_s(m_Z^2)$ to 0.112 the centres of the bands would intersect in a single point.

Figure 6 shows a comparison of the leptonic partial width from LEP, $\Gamma_{\ell\ell}$, and the the effective electroweak mixing angle from asymmetries measured at LEP and SLC, $\sin^2\theta_{eff}^{lept}$, with the Standard Model. Good agreement with the Standard Model prediction is observed. The star shows the prediction if among the electroweak radiative corrections only the photon vacuum polarization is included, showing evidence that LEP data are sensitive to genuinely electroweak corrections.

Figure 7 shows the χ^2 value for the Standard Model fits discussed in Table 18 column 4, as a function of m_t for the three values of m_H (60, 300 and 1000 GeV) considered. It can be seen that the minima of these curves occur at different values of χ^2. This suggests the possibility of extracting

[5] If the fits are repeated with the old value $\alpha(m_Z^2) = 1/(128.79 \pm 0.12)$ [61] which was used in [1] we observe a shift of ≈ 3 GeV in m_t.

constraints on the value of $m_{\rm H}$.

The main $m_{\rm H}$ dependence of the Standard Model predictions for the measurements listed in Table 17 is given by corrections proportional to $\log(m_{\rm H})$. The effects of $m_{\rm H}$ and $m_{\rm t}$, however, are correlated for most observables, which weakens the determination of $m_{\rm H}$ without a direct measurement of $m_{\rm t}$. Figure 8 shows the observed value of $\Delta\chi^2 \equiv \chi^2 - \chi^2_{min}$ as a function of $m_{\rm H}$, when the Tevatron value of $m_{\rm t}$ is used as an additional constraint in the fit. The observed $\Delta\chi^2$ curve exhibits a minimum for low values of $m_{\rm H}$. However, the entire range of $m_{\rm H}$ up to 1000 GeV is accommodated within an interval in $\Delta\chi^2$ of about four, approximately corresponding to a two-sided 95% probability range.

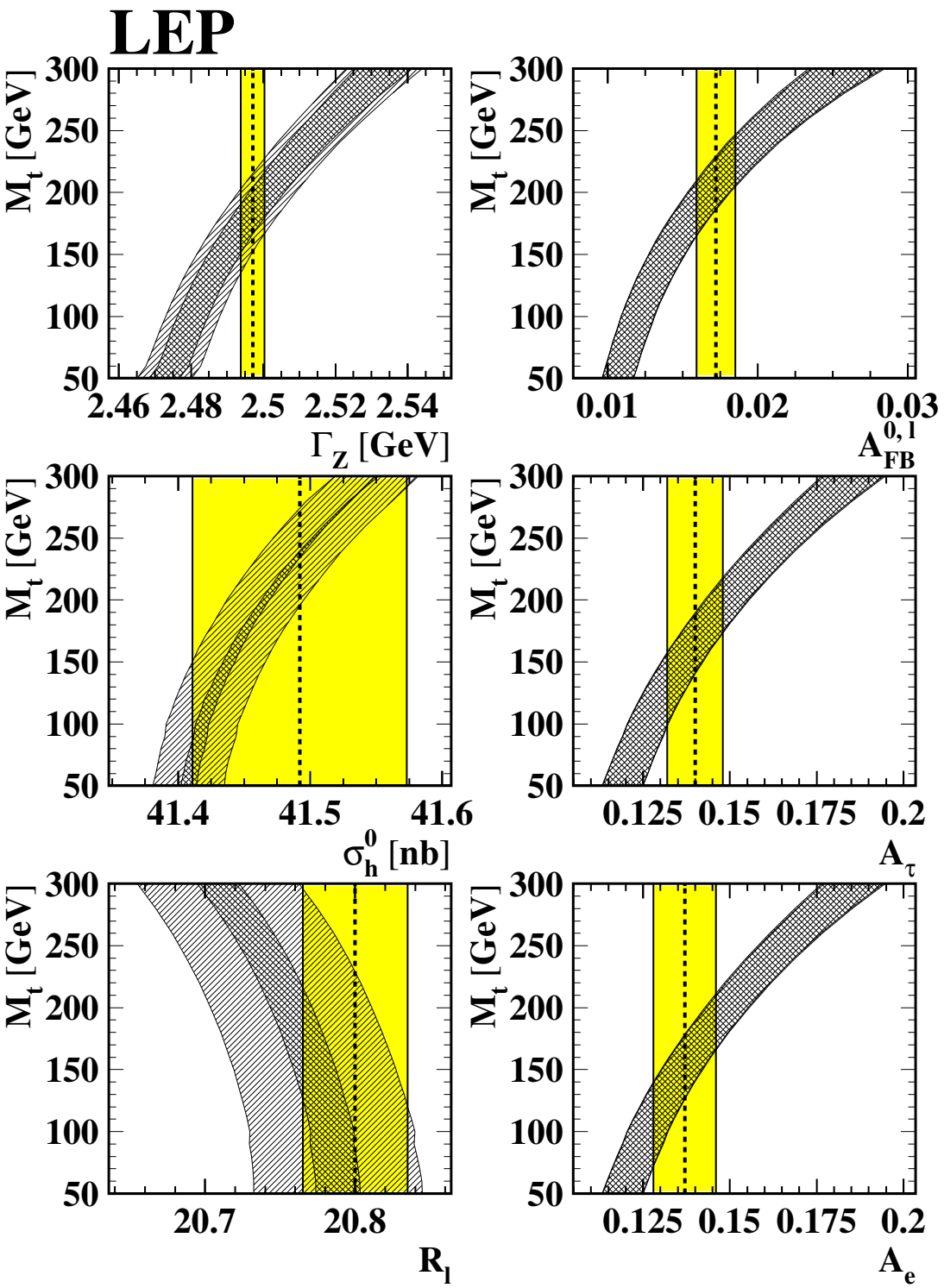

Figure 3: Comparison of LEP measurements with the Standard Model prediction as a function of m_t. The cross-hatched area shows the variation of the Standard Model prediction with m_H spanning the interval $60 < m_H\,(\mathrm{GeV}) < 1000$ and the singly-hatched area corresponds to a variation of $\alpha_s(m_Z^2)$ within the interval $\alpha_s(m_Z^2) = 0.123 \pm 0.006$. The total width of the band corresponds to the linear sum of both uncertainties. The experimental errors on the parameters are indicated as vertical bands.

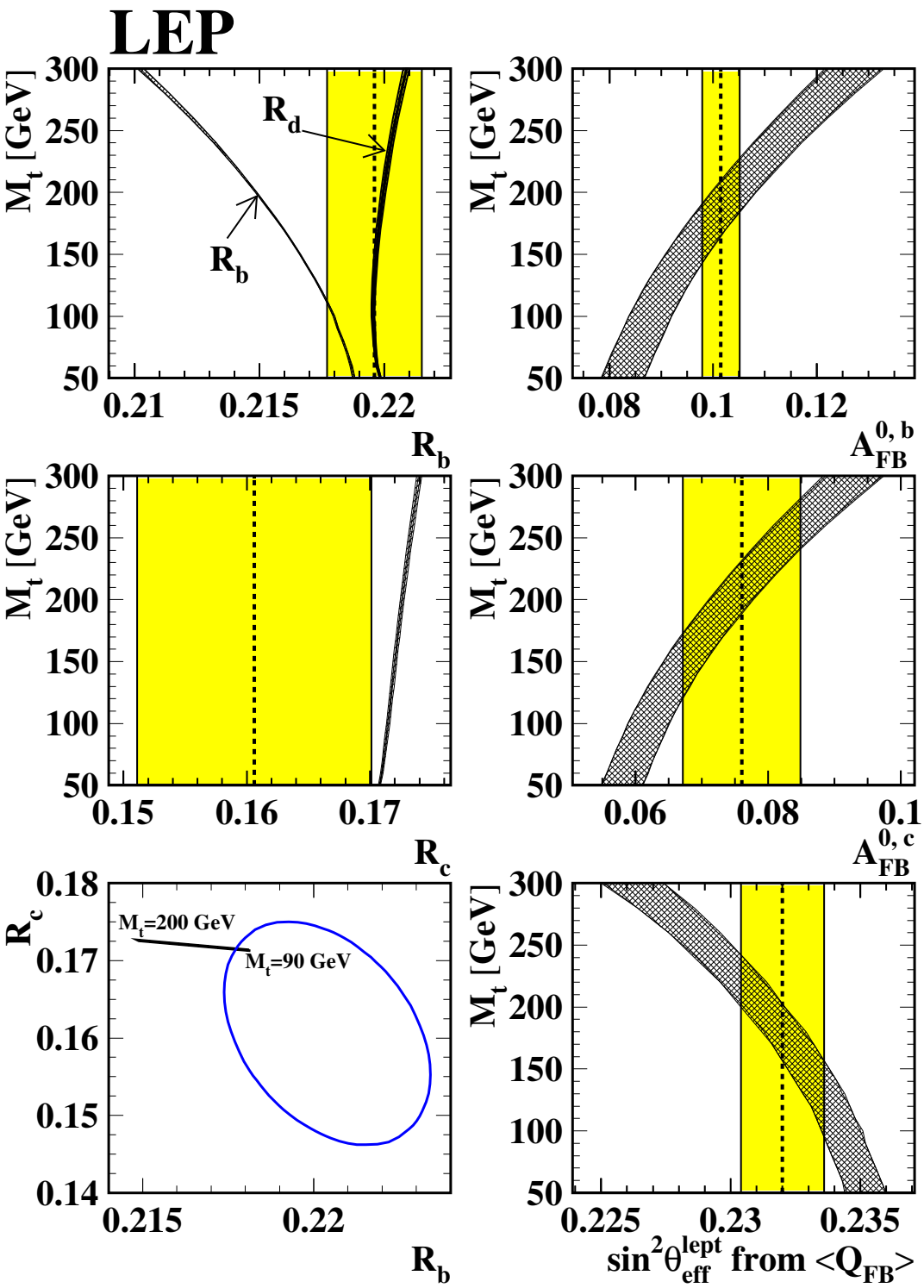

Figure 4: Comparison of LEP measurements with the Standard Model prediction as a function of m_{t} (*c.f.* figure 3). For ratios of hadronic partial widths this variation with m_{H} and $\alpha_s(m_{\mathrm{Z}}^2)$ nearly cancels. For the comparison of R_{b} with the Standard Model the value of R_{c} has been fixed to the Standard Model. Also shown is the 68% confidence level contour in the R_{b}-R_{c} plane together with the Standard Model prediction. To illustrate the impact of special vertex corrections to R_{b} the Standard Model prediction of R_{d} is also indicated.

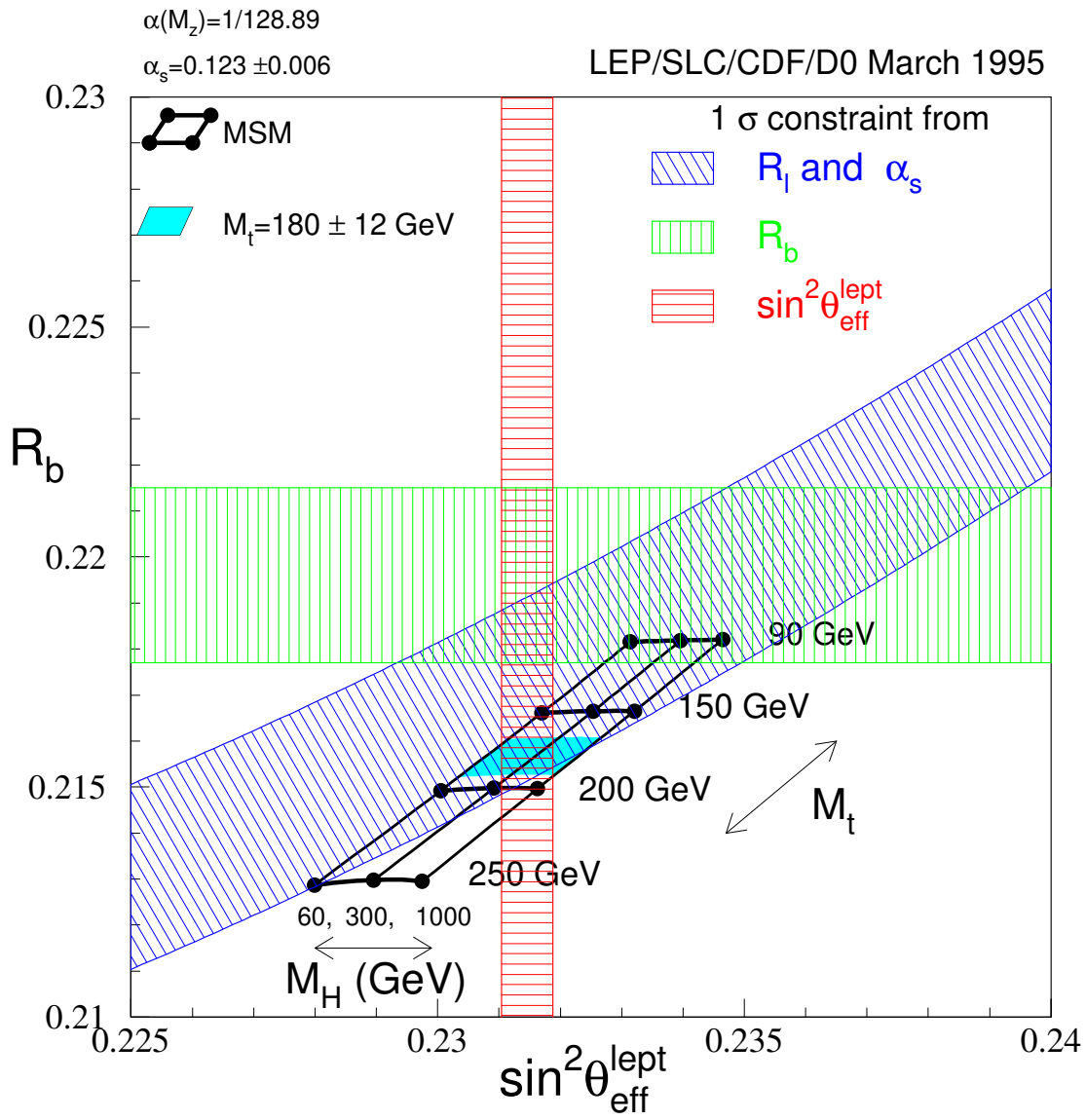

Figure 5: The LEP/SLD measurements of $\sin^2\theta^{\rm lept}_{\rm eff}$ and $R_{\rm b}$ and the Standard Model prediction. Also shown is the constraint resulting from the measurement of R_ℓ on these variables, assuming $\alpha_s(m_{\rm Z}^2) = 0.123 \pm 0.006$, as well as the Standard Model dependence of light quark partial widths on $\sin^2\theta^{\rm lept}_{\rm eff}$.

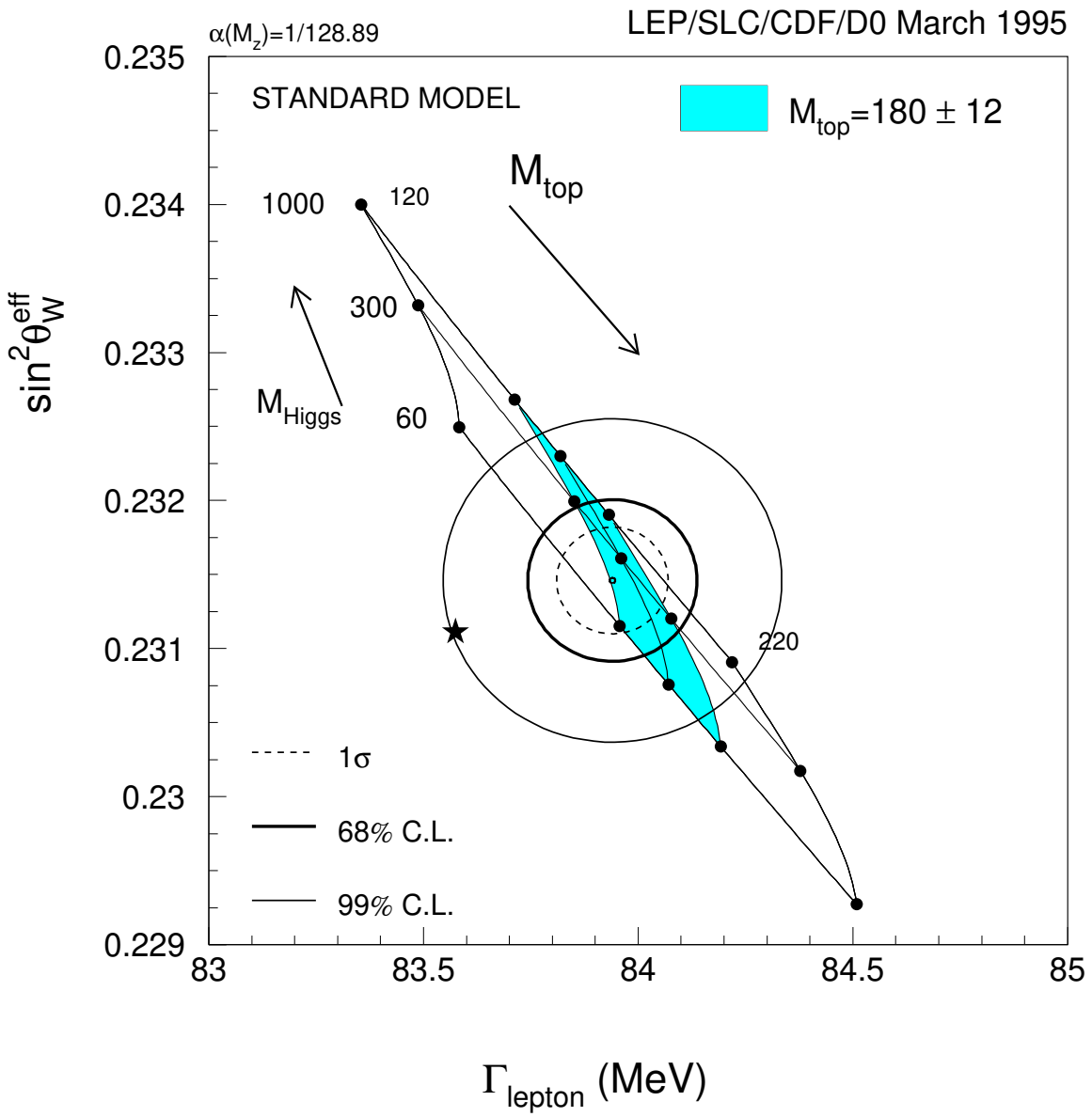

Figure 6: The LEP/SLD measurements of $\sin^2\theta^{\rm lept}_{\rm eff}$ and $\Gamma_{\ell\ell}$ and the Standard Model prediction. The star shows the predictions if among the electroweak radiative corrections only the photon vacuum polarization is included.

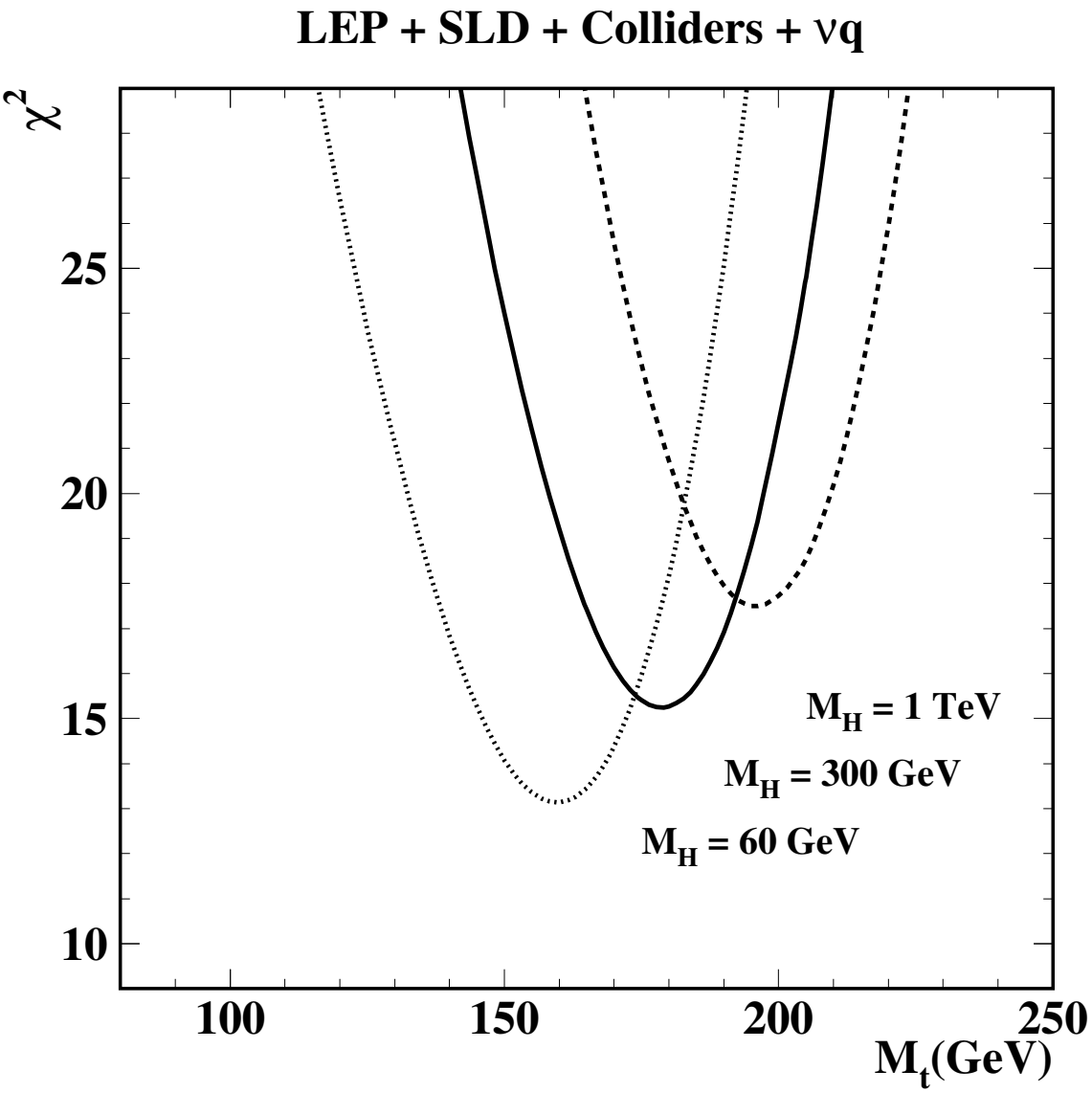

Figure 7: The χ^2 curves for the Standard Model fit in Table 18, column 4 to the electroweak precision measurements listed in Table 17 as a function of $m_{\rm t}$ for three different Higgs mass values spanning the interval $60 \leq m_{\rm H}\,[\mathrm{GeV}] \leq 1000$.

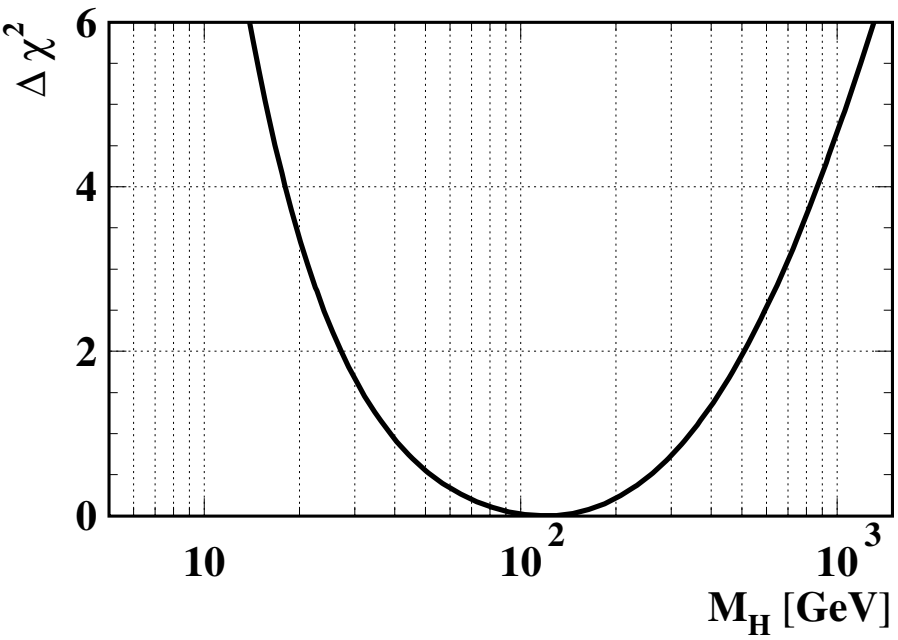

Figure 8: $\Delta\chi^2 = \chi^2 - \chi^2_{min}$ vs $m_{\rm H}$ for the data used in the last column of Table 18 and including the Tevatron value of $m_{\rm t}$.

Appendix

A The Measurements used in the Heavy Flavour Averages

Tagging	ALEPH 90-91 shape [38]	ALEPH 90-91 lepton [33]	ALEPH 92 lifetime [40]	DELPHI 90-92 multiple [34]	DELPHI 91-92 lepton [34]
R_b	0.2280	0.2162	0.2187	0.2217	0.2145
Statistics	0.0054	0.0062	0.0022	0.0020	0.0089
Uncorrelated	0.0036	0.0028	0.0022	0.0020	0.0063
Correlated	0.0032	0.0042	0.0012	0.0021	0.0023
Total Syst.	0.0048	0.0050	0.0025	0.0029	0.0067
$a(R_c)$	-0.004		-0.014	-0.020	
R_c^{used}	0.165		0.171	0.171	

Tagging	L3 91 shape [39]	L3 90-91† lepton [36]	OPAL 92-93 multiple [41]	OPAL 90-91 lepton [37]
R_b	0.2220	0.2187	0.2171	0.2250
Statistics	0.0030	0.0081	0.0021	0.0110
Uncorrelated	0.0053	0.0047	0.0011	0.0035
Correlated	0.0036	0.0034	0.0018	0.0057
Total Syst.	0.0064	0.0058	0.0021	0.0066
$a(R_c)$	-0.021	-0.023	-0.019	-0.013
R_c^{used}	0.171	0.171	0.171	0.171
$a(\mathrm{BR}(bc\ell))$		0.109		
$\mathrm{BR}(bc\ell)^{used}$		10.5		

Table 19: The measurements of R_b. Preliminary results are indicated by the symbol †. The correlated systematic entries are from sources shared with one or more other results in the table; the uncorrelated systematic entries are from the remaining sources. $a(x)$ denotes the dependence on the assumed x^{used}, which is also given. There is an additional +0.2 statistical and +0.2 systematic correlation coefficient between the first two ALEPH results [33,38].

Tagging	ALEPH 90-91 lepton [33]	DELPHI 91-92† D [44]	DELPHI 91-92 lepton [34]	OPAL 90-92 $D^{*\pm}$ [45]
R_c	0.1670	0.2090	0.1625	0.142
Statistics	0.0054	0.0190	0.0085	0.008
Uncorrelated	0.0149	0.0257	0.0177	0.014
Correlated	0.0114	0.0000	0.0110	0.000
Total Syst.	0.0188	0.0257	0.0209	0.014

Table 20: The measurements of R_c.

Tagging	ALEPH 90-93† lepton [33]	ALEPH 90-93† lepton [33]	ALEPH 90-93† lepton [33]	DELPHI 91-93† lepton [35]	L3 90-93† lepton [36]	OPAL 90-93† lepton [37]	OPAL 90-93† $D^{*\pm}$ [48]	OPAL 91-94† jet [43]
$\sqrt{s}$ (GeV)	88.36	89.42	90.21	89.43	89.56	89.54	89.54	89.54
$A_{\rm FB}^{\rm b\bar{b}}(-2)$	-3.1	3.3	9.3	6.3	7.0	3.3	24.0	6.3
Statistics	11.0	3.0	5.8	3.8	3.5	3.0	27.0	3.4
Uncorrelated	0.0	0.0	0.0	0.2	0.4	0.7	4.6	0.2
Correlated	0.1	0.1	0.3	0.2	0.1	0.2	0.9	0.1
Total Syst.	0.1	0.1	0.3	0.2	0.4	0.8	4.7	0.2
$a(R_{\rm b})$				-0.007	-1.642			-0.018
$R_{\rm b}^{\rm used}$				0.217	0.216			0.216
$a(R_{\rm c})$				0.001	0.913	-0.658		0.0001
$R_{\rm c}^{\rm used}$				0.171	0.169	0.171		0.173
$a(\overline{\chi})$				0.021				
$\overline{\chi}^{\rm used}$				0.121				
$a({\rm BR}({\rm b}\to\ell))$				-0.010	-0.928			
${\rm BR}({\rm b}\to\ell)^{\rm used}$				11.0	10.5			
$a({\rm BR}({\rm bc}\ell))$				0.002	-0.066			
${\rm BR}({\rm bc}\ell)^{\rm used}$				7.9	7.9			
$a(A_{\rm FB}^{\rm c\bar{c}}(-2))$					-0.267			-0.002
$A_{\rm FB}^{\rm c\bar{c}}(-2)^{\rm used}$					-2.9			-3.16

Table 21: The measurements of $A_{\rm FB}^{\rm b\bar{b}}(-2)$ (in units of 10^{-2}).

Tagging	ALEPH 90-93† lepton [33]	ALEPH 91-93 jet [42]	DELPHI 92-93† jet [35]	DELPHI 91-93† lepton [35]	DELPHI 91-92† $D^{*\pm}$ [47]	L3 90-93† lepton [36]	OPAL 91-94† jet [43]	OPAL 90-94† lepton [37]	OPAL 90-93† $D^{*\pm}$ [48]
$\sqrt{s}$ (GeV)	91.26	91.19	91.28	91.26	91.27	91.27	91.25	91.24	91.26
$A_{\rm FB}^{\rm b\bar{b}}({\rm pk})$	8.43	9.92	11.50	10.65	5.90	10.28	9.73	10.3	4.20
Statistics	0.68	0.85	1.30	1.10	6.20	1.00	0.67	0.9	7.90
Uncorrelated	0.08	0.25	0.57	0.33	2.09	0.36	0.08	0.2	2.45
Correlated	0.11	0.28	0.29	0.23	1.22	0.12	0.32	0.3	0.65
Total Syst.	0.14	0.38	0.64	0.40	2.42	0.38	0.39	0.4	2.53
$a(R_{\rm b})$		-0.031	-0.050	-0.029		-1.642	-0.010		
$R_{\rm b}^{\rm used}$		0.2188	0.217	0.217		0.216	0.216		
$a(R_{\rm c})$		+0.026	0.017	0.011		0.913	0.001	1.71	
$R_{\rm c}^{\rm used}$		0.171	0.171	0.171		0.169	0.173	0.171	
$a(\overline{\chi})$			0.031	0.034					
$\overline{\chi}^{\rm used}$			0.115	0.121					
$a({\rm BR}({\rm b}\to\ell))$			0.019	-0.039		-0.647			
${\rm BR}({\rm b}\to\ell)^{\rm used}$			11.0	11.0		10.5			
$a({\rm BR}({\rm bc}\ell))$			-0.009	0.005		-0.283			
${\rm BR}({\rm bc}\ell)^{\rm used}$			7.9	7.9		7.9			
$a(A_{\rm FB}^{\rm c\bar{c}}({\rm pk}))$						0.580	0.007		
$A_{\rm FB}^{\rm c\bar{c}}({\rm pk})^{\rm used}$						6.3	6.1		

Table 22: The measurements of $A_{\rm FB}^{\rm b\bar{b}}({\rm pk})$ (in units of 10^{-2}).

Tagging	ALEPH 90-93† lepton [33]	ALEPH 90-93† lepton [33]	ALEPH 90-93† lepton [33]	DELPHI 91-93† lepton [35]	L3 90-93† lepton [36]	OPAL 90-94† lepton [37]	OPAL 90-93† $D^{*\pm}$ [48]	OPAL 91-94† jet [43]
$\sqrt{s}$ (GeV)	92.07	93.02	93.93	93.02	92.93	92.94	92.94	92.94
$A_{FB}^{b\bar{b}}(+2)$	5.1	10.5	11.8	14.9	11.0	12.1	-41.0	17.3
Statistics	4.9	2.4	7.5	3.6	2.9	2.7	23.0	2.9
Uncorrelated	0.0	0.0	0.0	0.5	0.4	0.6	7.2	0.6
Correlated	0.2	0.3	0.3	0.4	0.1	0.4	2.5	0.4
Total Syst.	0.2	0.3	0.3	0.6	0.4	0.7	7.6	0.7
$a(R_b)$				-0.029	-1.642			-0.183
R_b^{used}				0.217	0.216			0.216
$a(R_c)$				-0.010	0.913	2.631		0.002
R_c^{used}				0.171	0.169	0.171		0.173
$a(\overline{\chi})$				0.040				
$\overline{\chi}^{used}$				0.121				
$a(BR(b \to \ell))$				-0.032	-0.630			
$BR(b \to \ell)^{used}$				11.0	10.5			
$a(BR(bc\ell))$				0.005	-0.303			
$BR(bc\ell)^{used}$				7.9	7.9			
$a(A_{FB}^{c\bar{c}}(+2))$					1.113			0.007
$A_{FB}^{c\bar{c}}(+2)^{used}$					12.1			12.0

Table 23: The measurements of $A_{FB}^{b\bar{b}}(+2)$ (in units of 10^{-2}).

Tagging	ALEPH 90-93† lepton [33]	ALEPH 89-91 $D^{*\pm}$ [46]	DELPHI 91-92† lepton [35]	DELPHI 91-92 $D^{*\pm}$ [47]	L3 90-91 lepton [36]	OPAL 90-94† lepton [37]	OPAL 90-93† $D^{*\pm}$ [48]
$\sqrt{s}$ (GeV)	91.26	91.25	91.27	91.27	91.24	91.24	91.26
$A_{FB}^{c\bar{c}}(pk)$	9.10	7.12	8.02	7.70	7.84	5.20	11.00
Statistics	2.00	2.11	2.20	2.90	3.70	1.00	3.70
Uncorrelated	1.54	0.69	1.25	1.14	2.40	0.81	0.97
Correlated	1.04	0.20	0.81	0.34	0.59	0.82	0.73
Total Syst.	1.86	0.72	1.49	1.19	2.47	1.15	1.22
$a(R_b)$			0.029		4.32		
R_b^{used}			0.217		0.216		
$a(R_c)$			-0.065		-6.76	-3.815	
R_c^{used}			0.171		0.169	0.171	
$a(BR(b \to \ell))$			0.049		3.501		
$BR(b \to \ell)^{used}$			11.0		10.5		
$a(BR(bc\ell))$			-0.040		-0.303		
$BR(bc\ell)^{used}$			7.9		7.9		
$a(A_{FB}^{b\bar{b}}(pk))$		-1.175			6.400		
$A_{FB}^{b\bar{b}}(pk)^{used}$		8.46			8.8		

Table 24: The measurements of $A_{FB}^{c\bar{c}}(pk)$ from $D^{\star}$ meson and lepton tag analyses (in units of 10^{-2}).

Tagging	OPAL 90-93† lepton [37]	OPAL 90-93† $D^{*\pm}$ [48]	OPAL 90-93† lepton [37]	OPAL 90-93† $D^{*\pm}$ [48]
$\sqrt{s}$ (GeV)	89.54	89.54	92.94	92.94
$A_{FB}^{c\bar{c}}$	-21.1	-7.0	9.0	32.0
Statistics	4.8	12.0	4.0	11.0
Uncorrelated	2.9	1.4	2.0	2.4
Correlated	0.4	0.5	0.2	1.6
Total Syst.	2.9	1.4	2.0	2.9
$a(R_c)$	11.97		-4.735	
R_c^{used}	0.171		0.171	

Table 25: The measurements of $A_{FB}^{c\bar{c}}(-2)$ and $A_{FB}^{c\bar{c}}(+2)$ (in units of 10^{-2}).

Tagging	ALEPH 90-93† lepton [33]	DELPHI 91-92 lepton [34]	L3 90-93† lepton [36]	OPAL 90-91† lepton [37]
BR(b $\to \ell$)(%)	11.20	11.21	11.44	10.50
Statistics	0.33	0.45	0.48	0.60
Uncorrelated	0.33	0.50	0.37	0.39
Correlated	0.26	0.48	0.22	0.54
Total Syst.	0.42	0.70	0.43	0.66
$a(R_c)$			0.006	0.224
R_c^{used}			0.171	0.171
a(BR(bcℓ))			0.015	
BR(bcℓ)^{used}			10.5	

Table 26: The measurements of BR(b $\to \ell$) from the lepton tag analyses.

Tagging	ALEPH 90-93† lepton [33]	DELPHI 91-92 lepton [34]	OPAL 90-91† lepton [37]
BR(b $\to$ c $\to \ell^+$) (%)	8.81	7.70	8.30
Statistics	0.25	0.49	0.40
Uncorrelated	0.40	0.95	0.57
Correlated	0.69	0.83	0.39
Total Syst.	0.80	1.26	0.69
$a(R_c)$			0.013
R_c^{used}			0.171

Table 27: The measurements of BR(b $\to$ c $\to \ell^+$) from the lepton tag analyses.

Tagging	ALEPH 90-93† lepton [33]	DELPHI 91-92 lepton [34]	L3 90-93† lepton [36]	OPAL 90-91† lepton [37]
$\overline{\chi}$	0.0993	0.1500	0.1253	0.1440
Statistics	0.0073	0.0200	0.0110	0.0220
Uncorrelated	0.0028	0.0107	0.0053	0.0055
Correlated	0.0051	0.0119	0.0025	0.0028
Total Syst.	0.0058	0.0160	0.0058	0.0062
$a(R_{\mathrm{b}})$			0.001	
$R_{\mathrm{b}}^{\mathrm{used}}$			0.216	
$a(R_{\mathrm{c}})$			0.001	-0.658
$R_{\mathrm{c}}^{\mathrm{used}}$			0.169	0.171
$a(\mathrm{BR}(\mathrm{b} \to \ell))$			0.046	
$\mathrm{BR}(\mathrm{b} \to \ell)^{\mathrm{used}}$			10.5	
$a(\mathrm{BR}(\mathrm{bc}\ell))$			-0.0342	
$\mathrm{BR}(\mathrm{bc}\ell)^{\mathrm{used}}$			7.9	

Table 28: The measurements of $\overline{\chi}$ from the lepton tag analyses.

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